Higher symmetries and anomalies in $\mathfrak{so}$ QCD and $\mathcal{N}$=1 duality

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We study higher symmetries and anomalies of 4d $\mathfrak{so}(2n_c)$ gauge theory with $N_f = 2n_f$ flavors. We find that they depend on the parity of n_c and n_f, on the global form of the gauge group, and the discrete theta angle. The contribution from the fermions plays a central role in our analysis. Furthermore, our conclusion applies to $\mathcal{N}$=1 supersymmetric cases as well, and we see that higher symmetries and anomalies match across the duality $\mathfrak{so}(2n_c) \leftrightarrow \mathfrak{so}(2n_f - n_c + 4)$ of Intriligator and Seiberg.

Contents

1 Introduction and summary

Our understanding of the concept of symmetries in quantum field theories has been greatly improved in the last several years. We now have the concept of p-form symmetries acting on p-dimensional objects [GKSW14]. This concept gives a unifying point of view to both ordinary symmetries acting on point operators for $p = 0$ and center symmetries of gauge theories acting on Wilson line operators for $p = 1$. In addition, the 't Hooft magnetic flux [tH79] can now be thought of as a background gauge field for the 1-form center symmetry. It is also realized more recently that 0-form symmetries and 1-form symmetries can not only coexist in a direct product but also mix in a more intricate manner. They can have mixed anomalies between them. They can also combine to form a symmetry structure called 2-groups [CDI18, BCH18].

In this paper we study these issues in the case of 4d $\mathfrak{so}(N_c)$ gauge theories with N_f flavors of fermion fields in vector representation. Let us quickly recall the 0-form and 1-form symmetries these theories have.

As for the 1-form symmetry, we first need to recall that such theories come in three versions, Spin, SO_+ and SO_-, distinguished by the global form of the gauge group (Spin vs. SO) and by the choice of a discrete theta angle (SO_+ vs. SO_-) [AST13][1]. They also differ by the nontrivial line operator they possess: the Spin theory has the Wilson line W in the spinor representation, the SO_+ theory has the 't Hooft line H which is mutually non-local with respect to W, and the SO_- theory has the dyonic line $D = WH$. Furthermore, these line operators are charged under corresponding $\mathbb{Z}_2$ 1-form symmetries, which we can all electric, magnetic and dyonic 1-form symmetries.

As for the 0-form symmetry, we focus our attention on the $\mathrm{SU}(N_f)$ symmetry acting on N_f flavors of matter fields in the vector representation. There can be and definitely are other discrete symmetries, but we will not consider them in this paper for brevity.

Our main question is then how the $\mathbb{Z}_2$ 1-form symmetry and the $\mathrm{SU}(N_f)$ 0-form symmetry are related.[2] We concentrate on the case when N_c and N_f are both even, $N_c = 2n_c$ and $N_f = 2n_f$.

[1]This is when the theories are considered on spin manifolds. On more general manifolds a further distinction needs to be made [ARS19]. For simplicity we only consider spin manifolds in this paper.

[2]A partial answer was given in [HL20], but the contribution from fermions was not taken into account in that reference.

2 Charges of line operators in $\mathfrak{so}$ QCD

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