Let S be a scheme, $p : A \to S$ a ring scheme over S. Write $m : A \times_S A \to A$ for the multiplication morphism of A, $1_S : S \to A$ for the identity section of the multiplication, and $p_1 : A \times_S A \to A$ for the first projection. For any ring R, we shall write $R^\times :\stackrel{\mathrm{def}}{=} \{r \in R | r \text{ is invertible}\}$. In this note, we prove that if $p : A \to S$ is flat and of finite presentation, then the functor $\mathrm{Hom}_S(-, A)^\times$ is representable by an open subscheme of A.

It is clearly that the following lemma holds:

Lemma 0.1. *The functor*

$$\begin{aligned} \mathrm{Hom}_S(-, A)^\times : \mathsf{Sch}_S &\to \mathsf{Grp} \\ T &\mapsto \mathrm{Hom}_S(T, A)^\times \end{aligned}$$

is representable by $A^\times :\stackrel{def}{=} (A \times_S A) \times_{m,A,1_S} A$.

Write $i : A^\times \to A \times_S A$ for the first projection. Then, since $\mathrm{Hom}_S(T, A)^\times$ is a subset of $\mathrm{Hom}_S(T, A)$, the following assertions hold:

Lemma 0.2. *The morphism $p_1 \circ i : A^\times \to A$ is a monomorphism of schemes.*

For any S-scheme X and any S-scheme T, we shall write $X(T) :\stackrel{\mathrm{def}}{=} \mathrm{Hom}_S(T, X)$ for the set of T-valued points. The following lemma follows from elementary scheme theory:

Lemma 0.3. *Let*

$$\begin{tikzcd} T_0 \arrow[r, "t_0"] \arrow[d, "i"'] & X \arrow[d] \\ T \arrow[r] & S \end{tikzcd}$$

be a commutative diagram of schemes such that i is a square nilpotent closed immersion, i.e., the ideal sheaf $\mathcal{I}$ of i satisfies $\mathcal{I}^2 = 0$. Then

Proposition 0.4. *Assume that $p : A \to S$ is flat and of finite presentation. Then the morphism $p_1 \circ i : A^\times \to A$ is an open immersion.*

Proof. By Lemma 0.2, to prove Proposition 0.4, it suffices to prove that $p_1 \circ i$ is smooth. Let $a : T \to A$ be a morphism of S-schemes, $j : T_0 \to T$ a square nilpotent closed immersion, and $a_0 : T_0 \to A^\times$ a morphism of A-schemes:

$$\begin{tikzcd} T_0 \arrow[r, "a_0"] \arrow[d, "j"'] & A^\times \arrow[d, "p_1 \circ i"] \\ T \arrow[r, "a"] & A. \end{tikzcd}$$