Problem Set 1 Solution

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Question 1. Analytical solution for the cake-eating problem with no income flows

The first order condition gives

$$u'(c_t) = \beta R V'(a_{t+1}) \tag{0.1}$$
$$\tag{0.2}$$

And using the envelope condition, we get

$$V'(a_t) = u'(c_t) \tag{0.3}$$

Therefore, combining the two equations we get

$$u'(c_t) = \beta R u'(c_{t+1}) \tag{0.4}$$
$$c_t^{-\gamma} = \beta R c_{t+1}^{-\gamma} \tag{0.5}$$
$$c_t = (\beta R)^{-1/\gamma} c_{t+1} \tag{0.6}$$
$$c_t = (\beta R)^{(t-1)/\gamma} c_1 \tag{0.7}$$

Using life time budget constraint, we get

$$\sum_{k=1}^{T} (\beta R)^{(k-1)/\gamma} c_1 = a_1 \tag{0.8}$$
$$c_1 = \frac{a_1}{\sum_{k=1}^{T} (\beta R)^{(k-1)/\gamma}} \tag{0.9}$$
$$c_t = (\beta R)^{(t-1)/\gamma} \frac{a_1}{\sum_{k=1}^{T} (\beta R)^{(k-1)/\gamma}}. \tag{0.10}$$

The solution can be simplified by using

$$\sum_{k=1}^{T} (\beta R)^{(k-1)/\gamma} = \frac{1-(\beta R)^{T/\gamma}}{1-(\beta R)^{1/\gamma}}. \tag{0.11}$$

Question 2. Understanding numerical errors

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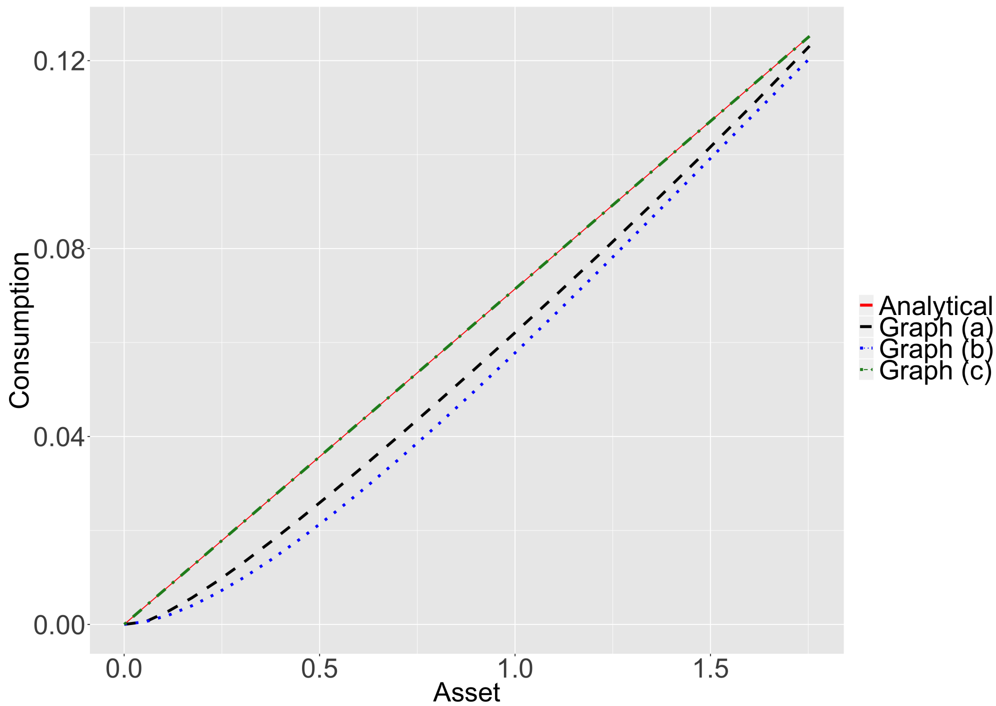
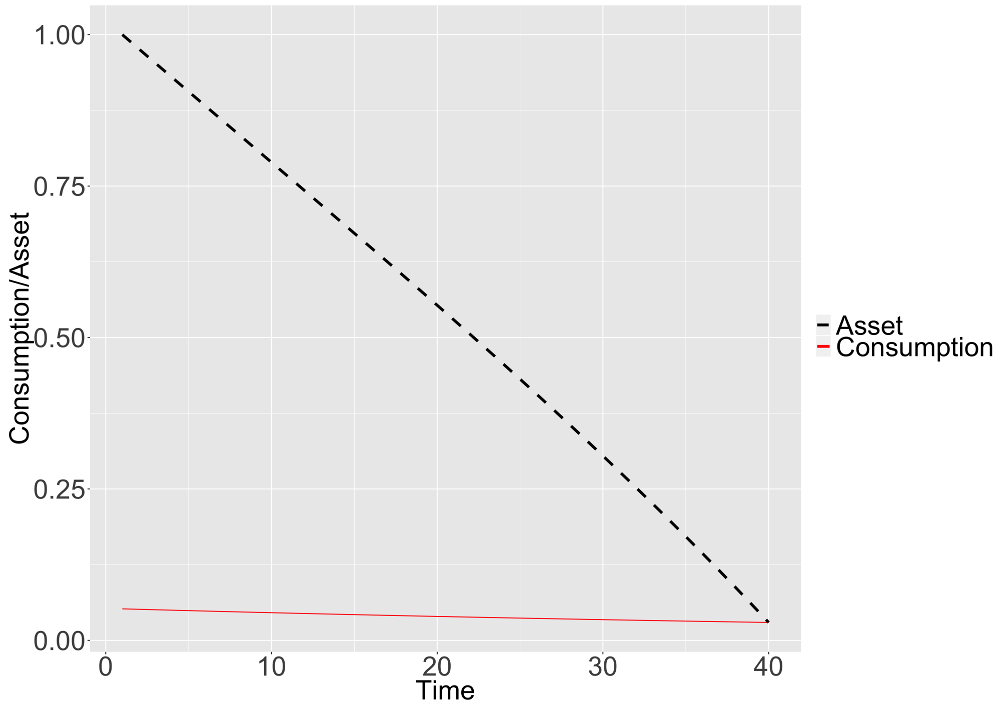


Due to numerical errors, graph (a) and (b) are different from the true optimal consumption plan. However, graph (c) is almost identical with true optimal consumption plan. Using linear transformation helps reducing numerical error.

Question 3. Simulating consumption and asset path using the solution


1.00
0.75
0.50
0.25
0.00
Consumption/Asset
0
10
20
30
40
Time
Asset
Consumption