

# Problem Set 5 Solution

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## Question 1.

(a)

```
> coef(oout) # estimate
[1] 0.3080911 5.1336746
> stdEr(oout) # standard error
[1] 0.009924786 0.127096105
```

(b)

```
> coef(oout) # estimate
[1] 0.3044062 5.0667203
> stdEr(oout) # standard error
[1] 0.01604319 0.07813442
```

(c)

```
> coef(oout) # estimate
[1] 0.3089259 5.2111210
> stdEr(oout) # standard error
[1] 0.01601892 0.07811093
```

## Question 2.

By taking the derivative w.r.t.  $\theta$ ,

$$0 = \sum_n \sum_s \frac{1}{\sum_{s'=1}^S \pi_{s'} \Pi_{t=1}^T \mathcal{L}_{ns't}(\theta)} \left( \sum_{t'} \pi_{s'} \Pi_{t \neq t'} \mathcal{L}_{ns't}(\theta) \right) \frac{\partial \mathcal{L}_{ns't}}{\partial \theta} \quad (0.1)$$

Note that

$$\frac{\partial \log \mathcal{L}_{ns't}}{\partial \theta} = \frac{1}{\mathcal{L}_{ns't}} \frac{\partial \mathcal{L}_{ns't}}{\partial \theta} \quad (0.2)$$

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Plugging equation 0.2 into 0.3, we obtain

$$0 = \sum_n \sum_s \underbrace{\frac{\pi_s \prod_{t=1}^T \mathcal{L}_{nst}(\theta)}{\sum_{s'=1}^S \pi_{s'} \prod_{t=1}^T \mathcal{L}_{ns't}(\theta)}}_{=q_{ns} \text{prob of ind } n \text{ of being type } s} \left( \frac{\sum_{t'} \partial \ln \mathcal{L}_{nst'}}{\partial \theta} \right) \quad (0.3)$$

**Question 3.**

```
> coef(oout) # estimate
[1] 0.09489552 4.03155725
> stdEr(oout) # standard error
[1] 0.001865697 0.014965900
```