

Ex02

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2019 年 4 月 16 日

- The input step signal $x(t)$ and the impulse response $h(t)$ of a continuous time LT1 system are given by

$$x(t) = u(t), \quad h(t) = e^{-\alpha t}u(t), \quad \alpha > 0 \quad (0-1)$$

1. Compute the output $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$

Answer:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \quad (0-2)$$

$$= \int_{-\infty}^{\infty} u(\tau)e^{-\alpha(t-\tau)}u(t - \tau)d\tau \quad (0-3)$$

$$= \int_{-\infty}^{\infty} e^{-\alpha(t-\tau)}u(\tau)u(t - \tau)d\tau \quad (0-4)$$

$t < 0$ の時、 $u(\tau)u(t - \tau) = 0$ なので、 $y(t) = 0$

$0 \leq t$ の時、

$$u(\tau)u(t - \tau) = \begin{cases} 1 & (0 \leq \tau \leq t) \\ 0 & otherwise \end{cases}$$

よって、

$$y(t) = \int_0^t e^{-\alpha(t-\tau)}d\tau \quad (0-5)$$

$$= \frac{1}{\alpha}[1 - e^{-\alpha t}] \quad (0-6)$$

したがって、

$$y(t) = \frac{1}{\alpha}[1 - e^{-\alpha t}]u(t) \quad (0-7)$$