# Ex09

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- Given a causal discrete-time LT1 system $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n]$, where $x[n]$ and $y[n]$ are the input and output of the system, respectively.

1. Determine the transfer function $H(z)$
2. Find its impulse response $h[n]$
3. Fint its step response $s[n]$

Answer:

1. Taking the z-transform of the system equation, we have

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z) \tag{0-1}$$

   Thus,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})} \tag{0-2}$$

2. Using partial-fraction expansion, we have

$$\frac{H(z)}{z} = \frac{z}{(z-\frac{1}{2})(z-\frac{1}{4})} = \frac{c_1}{z-\frac{1}{2}} + \frac{c_2}{z-\frac{1}{4}} \tag{0-3}$$

   where, $c_1 = 2$,$c_2 = -1$

   Thus,

$$H(z) = \frac{2z}{z-\frac{1}{2}} - \frac{z}{z-\frac{1}{4}} = \frac{2}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}} \tag{0-4}$$

   Taking the inverse z-transform [*1] of $H(z)$, we get

$$h(n) = 2\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n] \tag{0-5}$$

3. From the table in slide P19, we have the z-transform pair for step signal $x[n] = u[n] \overset{zT}{\Longleftrightarrow} X(z) = \frac{z}{z-1}$ Thus, the steo response

$$Y(z) = H(z)X(z) = \frac{2z^2}{(z-\frac{1}{2})(z-1)} - \frac{z^2}{(z-\frac{1}{4})(z-1)} \tag{0-6}$$

[*1] $z[a^n u[n]] = \frac{1}{1-az^{-1}}$, $(|z| > |a|)$

Using partial-fraction expansion, we have

$$\frac{Y(z)}{z} = \frac{2z}{(z-\frac{1}{2})(z-1)} - \frac{z}{(z-\frac{1}{4})(z-1)} \tag{0-7}$$

$$= \frac{c_1}{z-\frac{1}{2}} + \frac{c_2}{z-1} + \frac{c_3}{z-\frac{1}{4}} \tag{0-8}$$

where, $c_1 = -2, c_2 = \dfrac{8}{3}, c_3 = \dfrac{1}{3}$
Thus,

$$Y(z) = \frac{-2z}{z-\frac{1}{2}} + \frac{8z}{(3(z-1)} + \frac{z}{3(z-\frac{1}{4})} \tag{0-9}$$

Taking the inverse z-transform of $Y(z)$, we obtain the step response $s[n]$

$$s[n] = y[n] = -2\left(\frac{1}{2}\right)^n u[n] + \frac{8}{3}u[n] + \frac{1}{3}\left(\frac{1}{4}\right)^n u[n] \tag{0-10}$$