Ex03

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- The unit step sequence input $x[n]$ and the impulse response $h[n]$ of a discrete-time LT1 system are given by

$$x[n] = u[n], \quad h[n] = \alpha^n u[n], \quad 0 < \alpha < 1 \tag{0-1}$$

Hint:

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \dfrac{1-\alpha^N}{1-\alpha}, & (\alpha \neq 1) \\ N, & (\alpha = 1) \end{cases} \tag{0-2}$$

1. Compute the convolution sum, or the system output $y[n] = x[n] * h[n]$.

Answer:

$$\begin{aligned} y[n] &= x[n] * h[n] && \text{(0-3)} \\ &= \sum_{m=-\infty}^{\infty} x[m]h[n-m] && \text{(0-4)} \\ &= \sum_{m=-\infty}^{\infty} u[m]\alpha^{n-m}u[n-m] && \text{(0-5)} \\ &= \sum_{m=0}^{\infty} \alpha^{n-m}u[n-m] && \text{(0-6)} \\ & && \text{(0-7)} \end{aligned}$$

$n < m$ の時、$y[n] = 0$

$m < n$ の時、

$$
\begin{aligned}
y[n] &= \sum_{m=0}^{n} \alpha^{n-m} && (0\text{-}8)\\
&= \alpha^n \sum_{m=0}^{n} \alpha^{-m} && (0\text{-}9)\\
&= \alpha^n \left(\alpha^{-n} + \sum_{m=0}^{n-1} \alpha^{-m} \right) && (0\text{-}10)\\
&= 1 + \alpha^n \sum_{m=0}^{n-1} \alpha^{-m} && (0\text{-}11)\\
&= 1 + \alpha^n \frac{1-\alpha^{-n}}{1-\alpha^{-1}} && (0\text{-}12)\\
&= \frac{\alpha^n - 1}{\alpha - 1} && (0\text{-}13)
\end{aligned}
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y[n] = \frac{\alpha^n - 1}{\alpha - 1} u[n] \tag{0-14}
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