homework-2

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library(bis557)
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Q1

$$y = \beta_0 + \beta_1 x \quad \Rightarrow \quad Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}_{n \times 1}$$

based on OLS, $\beta = (X^T X$

$$X^T = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_n \end{bmatrix}_{2 \times n} \qquad X^T X$$

$$(X^T X)^{-1} = \begin{bmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{bmatrix}_{2 \times 2} \cdot \frac{}{n}$$

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = (X^T X)^{-1} X^T Y = \frac{1}{n \sum x_i^2}$$

$$= \frac{1}{n \sum x_i^2 - (\sum x_i)^2} \begin{bmatrix} (\sum x_i^2) \\ (-\sum x_i \end{bmatrix}$$

$$\therefore \beta_0 = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i}{}$$

$$n\sum x_i^2 - (\sum x_i)^2$$

$$\boxed{\beta_1 = \frac{-\sum x_i \sum y_i + \cdots}{n\sum x_i^2 - \cdots}}$$

Q1

Q5 $L(\beta) = \frac{1}{2n} \|Y - X\beta\|_2^2 + \lambda \|\beta\|_1$

$\hat{\beta}^{lasso} = \min L(\beta)$

$L(\beta) = \frac{1}{2n} (Y - X\beta)^T (Y - X\beta) + \lambda \|\beta\|$

$= \frac{1}{2n} (Y^T Y - 2Y^T X \beta^2 + \beta X^T X \beta) + \lambda \|\beta\|$

$$0 = \frac{\partial L(\beta)}{\partial \beta} = \begin{cases} \frac{1}{2n}(-2Y^T X + 2X^T X \hat{\beta}) + \lambda & \text{if } \beta \geq 0 \\ \frac{1}{2n}(-2Y^T X + 2X^T X \hat{\beta}) - \lambda & \text{if } \beta \leq 0 \end{cases}$$

$\therefore$ for $\beta \geq 0$, $\beta = (X^T X)^{-1} (2n\lambda - 2Y^T X) \geq 0$

$\because |X_j^T Y| \leq n\lambda$, $\beta \geq 0$

$\therefore 2n\lambda - 2Y^T X$ must be zero

$\therefore \beta = 0$.

for $\beta \leq 0$, $\beta = (X^T X)^{-1} (Y^T X + \lambda) \leq 0$.

$\because |X_j^T Y| \leq n\lambda$ $\therefore$ $Y^T X + \lambda \geq 0$.

$\therefore 0 \geq \beta = (X^T X)^{-1} (Y^T X + \lambda) \geq 0$.

$\therefore \beta = 0$

Based on above discussion, $\forall\, |X_j^T Y| \leq n\lambda$,

β^{lasso} must be zero.

Q5