Continuous Variables :Analyzing and Interpreting Interaction Models

Julie Irwin

SCP 2009

San Diego

Agenda

- Examine how interaction models work in regression and why scaling matters,
- Examine how to graph and interpret interactions,
- Show how to test for particular effects within the interaction (without resorting to piecemeal tests)

Simple Additive Models

$Y = B_0 + B_1x + B_2z + e$

Example:

Y = happiness with a purchase

x = the extent to which the purchase is material or experiential (continuous)

z = whether the purchase was positive or negative (discrete)

Nicolao, Leonardo, Julie R. Irwin and Joseph Goodman. “Happiness for Sale: Do Experiential or Material Purchases Lead to Greater Retrospective Happiness?” *Journal of Consumer Research* (forthcoming).

Simple additive models are… simple

- No weird scaling issues.
- Easy to interpret.

- Also, boring.

Interaction Models

$$Y = B_0 + B'_1 x + B'_2 z + B_3 x{*}z + e$$

- Now we can ask whether the effect of purchase type on happiness depends on valence of the purchase.
- (much more interesting)

But what about interpreting and graphing the results?

- One easy (but wrong) choice is to perform a median split. Do not do this.*

Interpreting interactions with continuous variables (and coded discrete variables) actually is quite straight-forward, once you understand how the models work.

(see Irwin, Julie R. and Gary H. McClelland (2003), "Negative Consequences of Dichotomizing Continuous Predictor Variables," *Journal of Marketing Research, 40, 366-371 for all the gory details.)*

Misleading Heuristics

- The key to understanding interaction models is to unlearn misleading heuristics that apply to simple additive models but that do not apply when there is an interaction term.

(Irwin, Julie R. and Gary H. McClelland (2001), “Misleading Heuristics for Moderated Multiple Regression Models,” *Journal of Marketing Research*, *38*, 100-109.)

$Y = B_0 + B_1x + B_2z + e$

- Suppose x' = x-8 and we substitute x' for x in the above model.
  - Does B_2 change?
  - Do any of the significance tests change?

$Happiness = B_0 + B_1purchasetype + B_2valence + e$

- Does the effect of valence change when we subtract 8 from the purchase type measure?
- (seems like a really weird question to even ask…)

$$Y = B_0 + B'_1x + B'_2z + B_3x*z + e$$

If we substitute x' into this equation, does anything change?

$$Y = B_0 + B'_1p + B'_2v + B_3p*v + e$$

If we substitute purchase type - 7 into this equation (instead of purchase type), does anything change? Would the t-test for valence change?

The Scaling Heuristic

- “scaling changes do not affect significance tests, slopes, etc. of that variable or of any other variable in the model.”
- Yes, this heuristic is accurate for simple models without interactions, but not for models with interactions.

```
data temp;
input happiness purchasetype valence;

cards;
9 8 1
9 7 1
5 3 1
7 6 1
3 1 1
6 1 1
5 3 1
7 2 1
7 4 1
8 5 1
1 5 0
3 9 0
4 4 0
4 7 0
2 3 0
1 9 0
3 9 0
3 1 0
2 2 0
1 9 0
```

	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1.252747253	0.87261979	1.44	0.1693
valence	**4.556043956**	0.71499959	**6.37**	**<.0001**
purchasetype	0.197802198	0.12569024	1.57	0.1340

Positive relationship for valence:
Happier with more positive purchases. (good, the data make sense)
No effect for purchase type.

With the interaction in the model, original scaling

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Intercept	2.729257642	0.82128836	3.32	0.0043
valence	1.352223840	1.11651370	1.21	0.2434
purchasetype	-0.056768559	0.12553759	-0.45	0.6572
valence*purchasetype	**0.686398189**	**0.20613778**	**3.33**	**0.0042**

!

There is an interaction (yay) but it appears our manipulation check has now "failed."

Now valence isn't significant! What is going on?

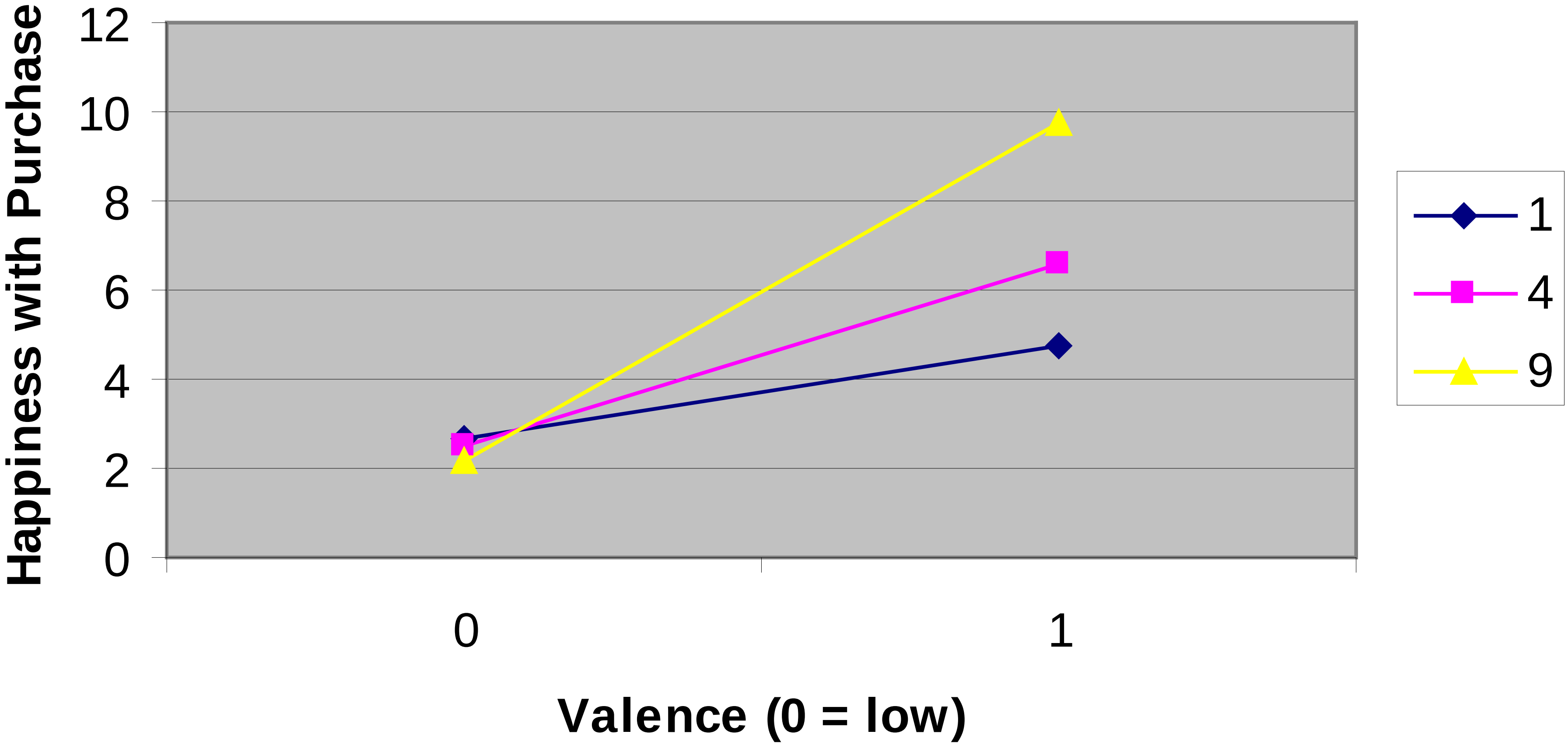
First let's graph our interaction…

Graphing the Interaction

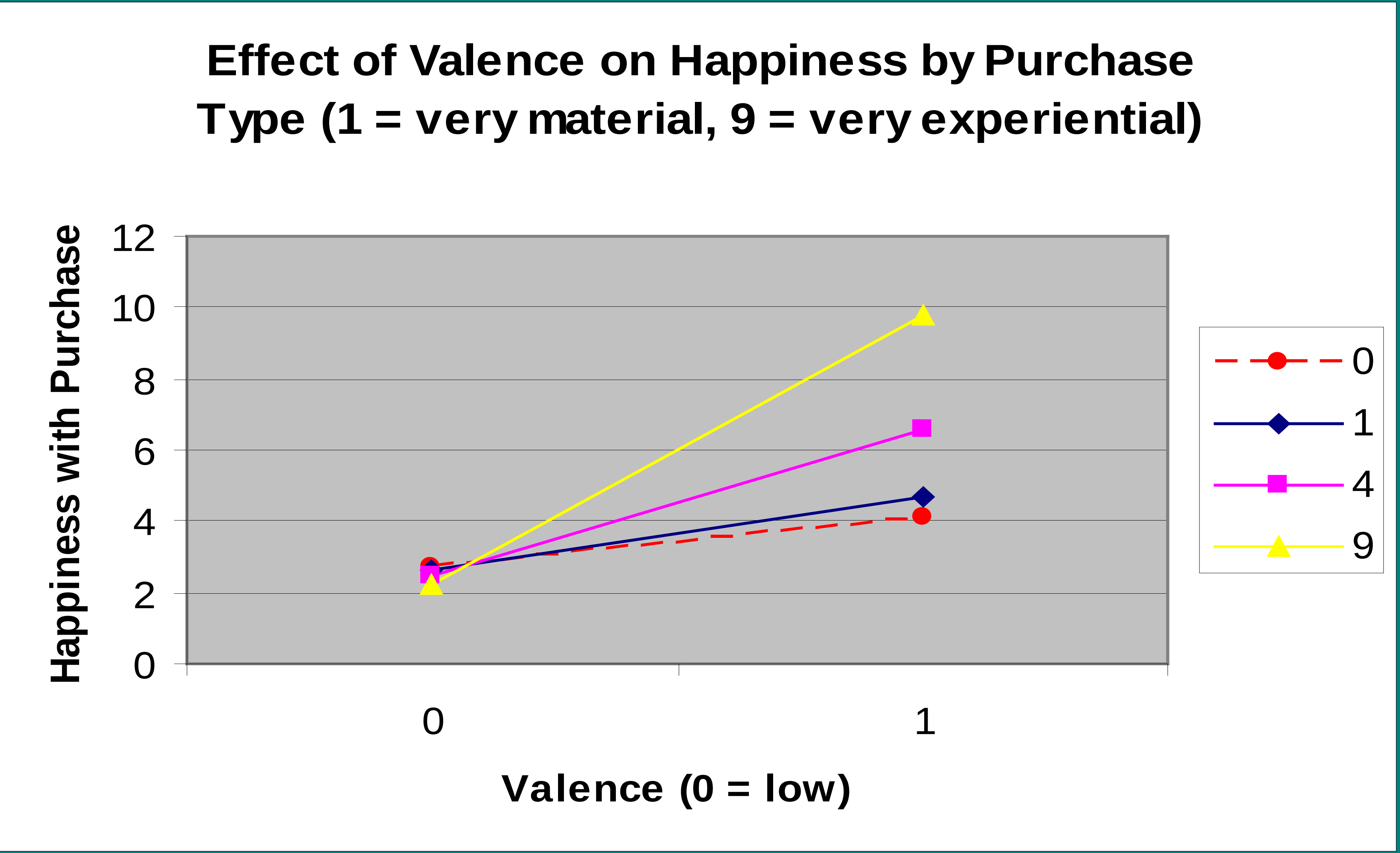
	Parameter Estimate (slope)
Intercept	2.73
valence	1.35
purchasetype	-0.06
valence*purchasetype	0.69

		Valence	
Purchase Type		0	1
	1	2.67	4.71
	4	2.49	6.6
	9	2.19	9.75

Effect of Valence on Happiness by Purchase Type (1 = very material, 9 = very experiential)



So, why did the valence t-test "change" when we added the interaction term?



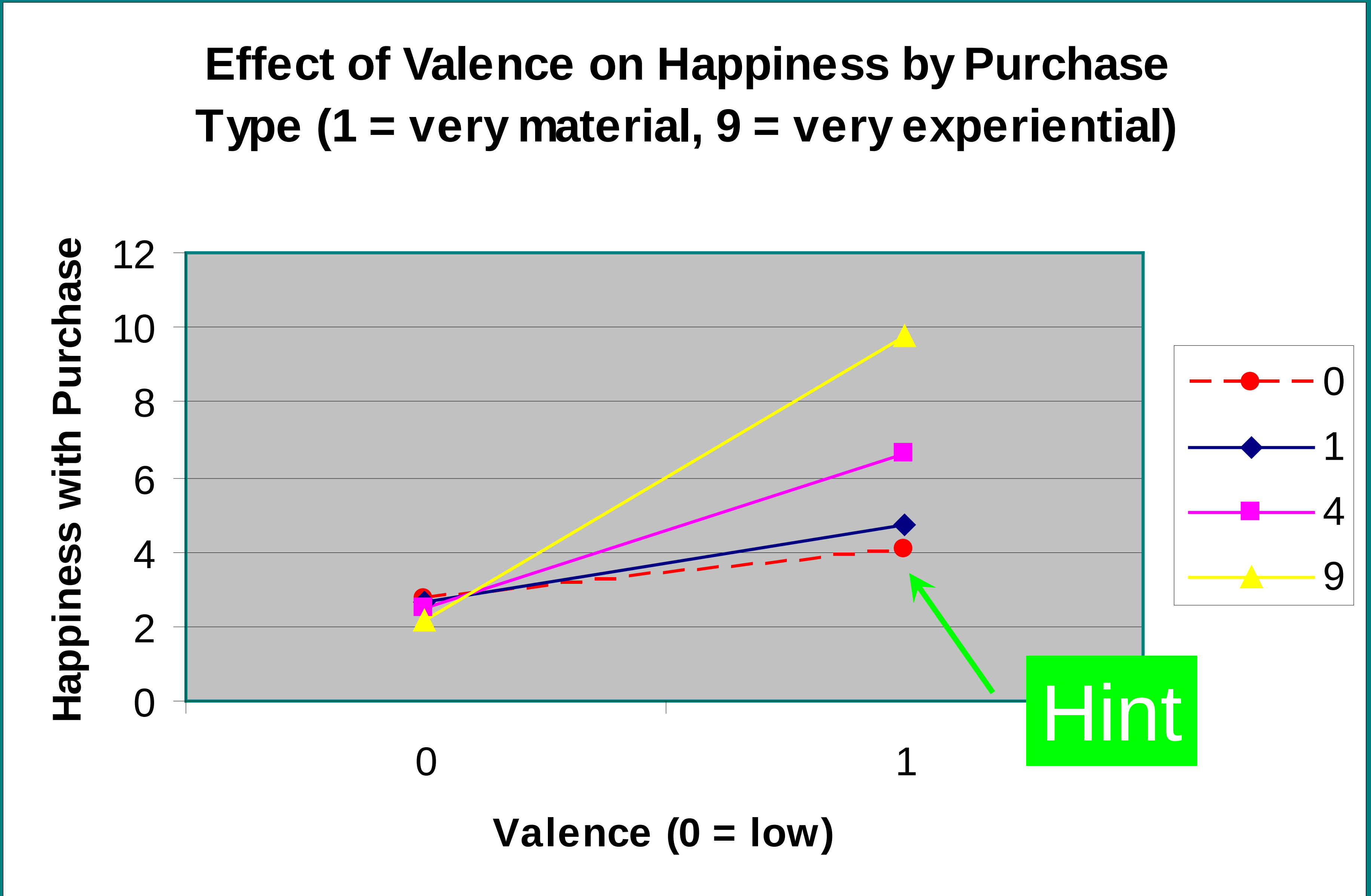
Now for some algebra…

$$Y = B_0 + B'_1 p + B'_2 v + B_3 p{*}v + e$$

$$Y = B_0 + B'_2 p + (B_1 + B'_3 p)v + e$$

The B_1 test is the test of x when z is zero.

Now we know what we were testing and why it wasn't significant…



Performing more Useful Tests: The Spotlight Method

- Query: What is the effect of valence at the average of material/experiential?
- Average = 4.9, so just move the 0 point like this:

- Purchasetype1 = Purchasetype – 4.9

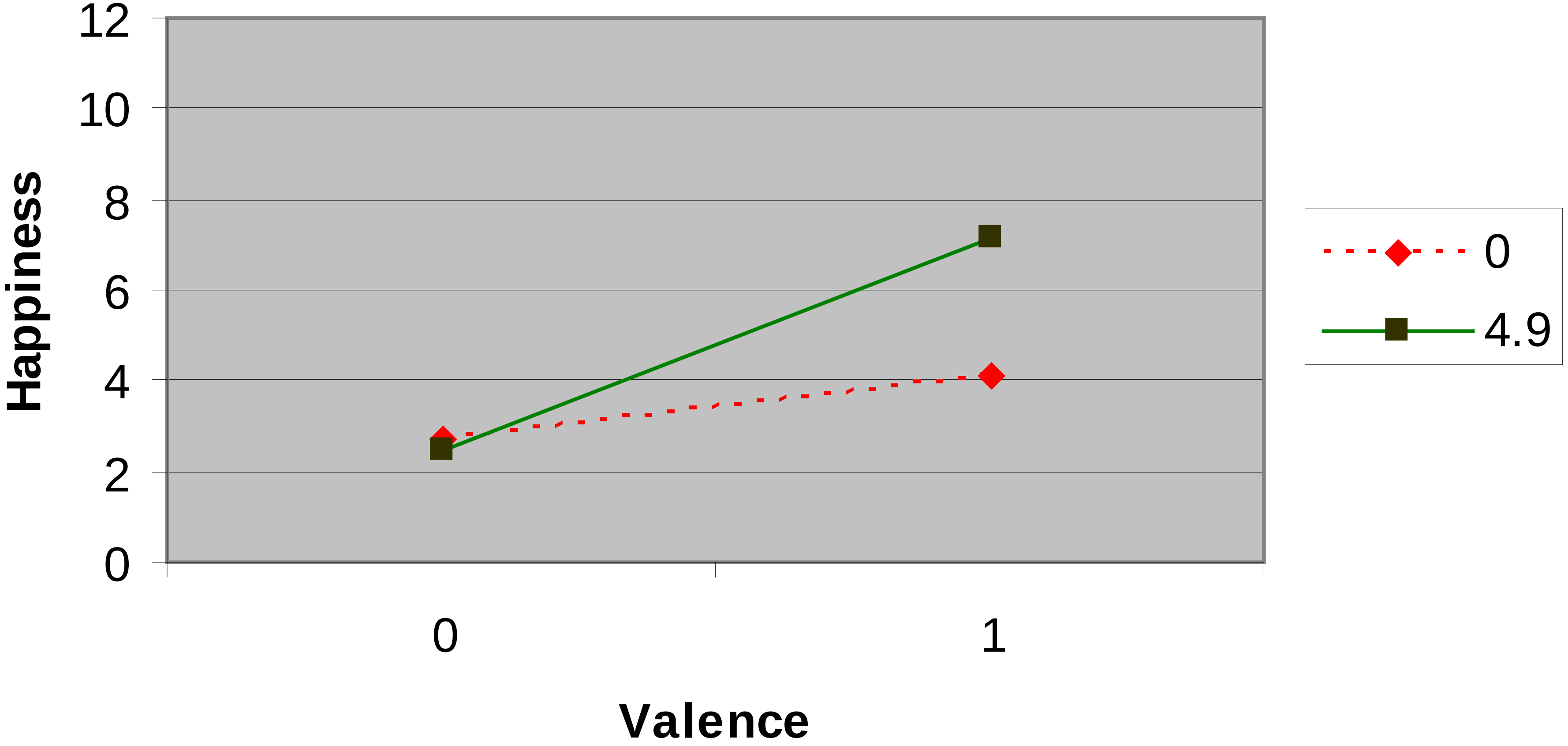
- Our first “spotlight” test.

Results Using the average of Purchasetype

	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	2.451091703	0.39638879	6.18	<.0001
valence	**4.715574964**	**0.56845106**	**8.30**	**<.0001**
purchasetype1	-0.056768559	0.12553759	-0.45	0.6572
valence*purchasetype	0.686398189	0.20613778	3.33	0.0042

So, at the average of purchasetype, valence is significant.

Purchase Type Spotlight at 4.9 (with test at purchasetype = 0 as a comparison)



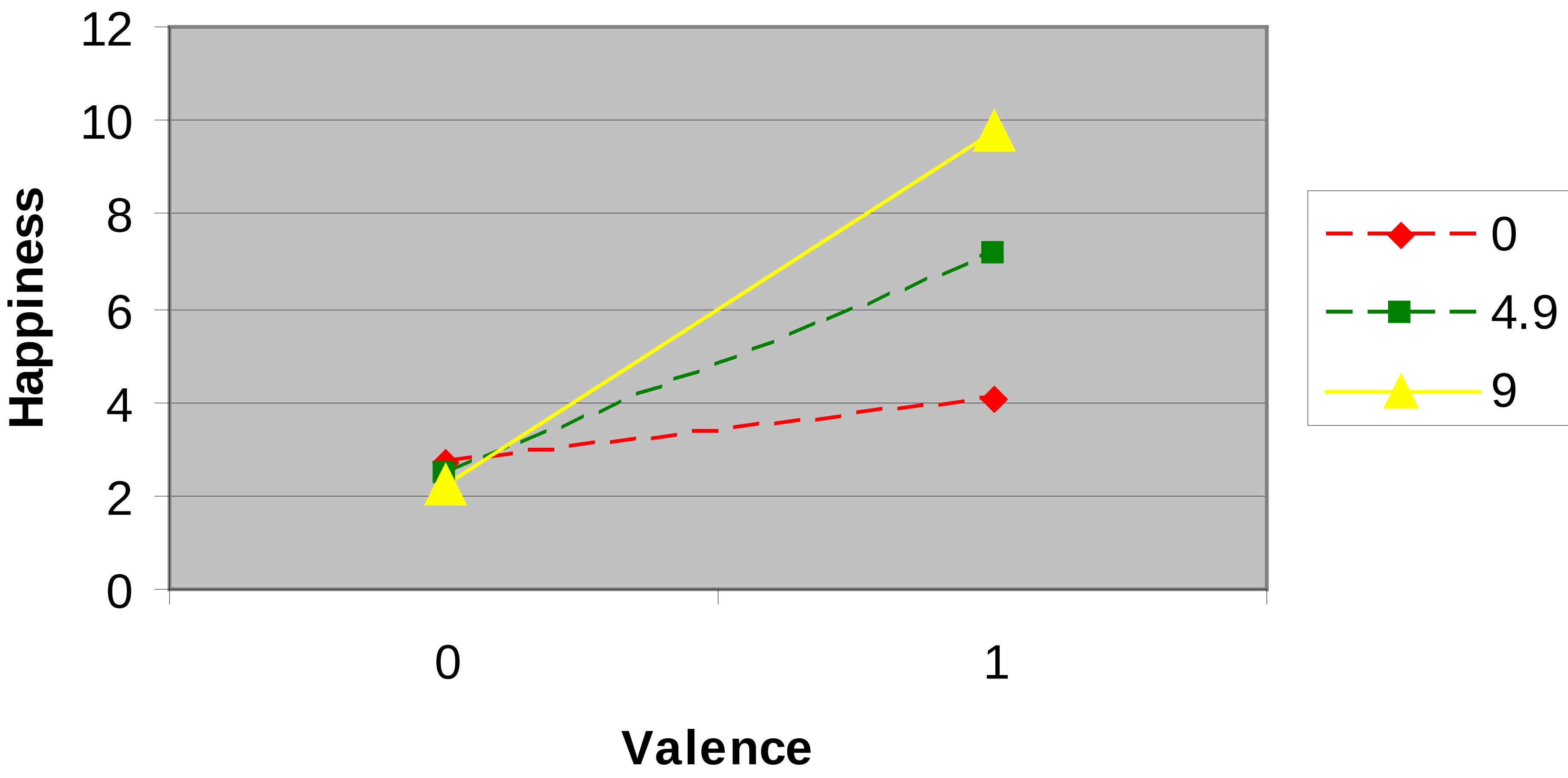
Let's do one more…

- What about at purchasetype = 9 (very experiential)?
- Purchasetype2 = purchasetype – 9
- Our second spotlight.

Results when Purchasetype = 9

<table>
<tr><th></th><th>Parameter Estimate</th><th>Standard Error</th><th>t Value</th><th>Pr > |t|</th></tr>
<tr><td>Intercept</td><td>2.22</td><td>0.55</td><td>4.01</td><td>0.0010</td></tr>
<tr><td>valence</td><td>7.53</td><td>1.06</td><td>7.12</td><td><.0001</td></tr>
<tr><td>purchasetype2</td><td>-0.06</td><td>0.13</td><td>-0.45</td><td>0.6572</td></tr>
<tr><td>valence*purchasetype</td><td>0.69</td><td>0.21</td><td>3.33</td><td>0.0042</td></tr>
</table>

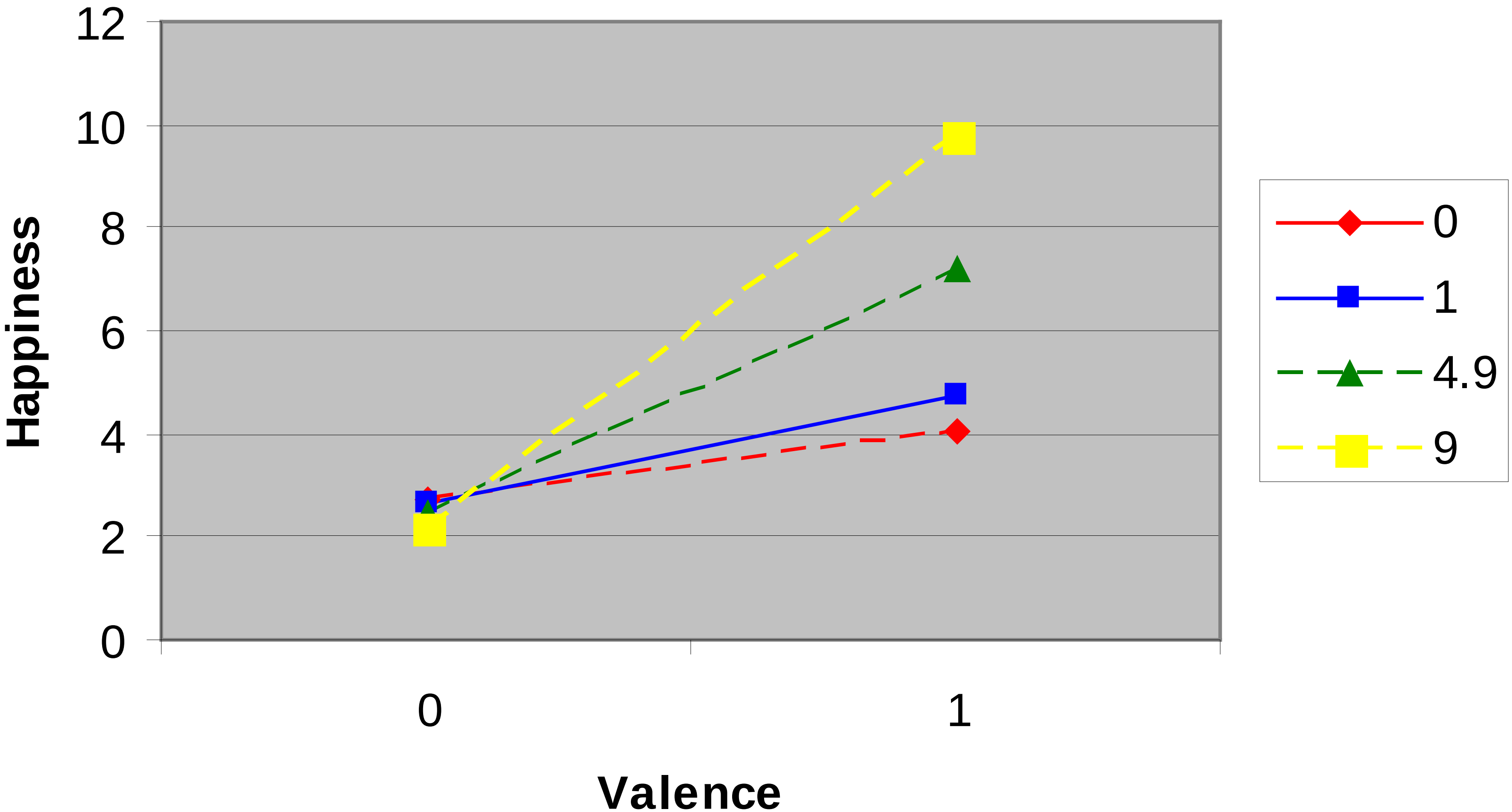
Spotlights at 4.9 and 9
(with test at 0 for comparison)



Spotlight at 1

	Parameter Estimate	Error	t Value	Pr > |t|
Intercept	1.88	1.22	1.54	0.1420
valence	**11.65**	**2.20**	**5.29**	**<.0001**
purchasetype3	-0.06	0.13	-0.45	0.6572
valence*purchasetype	0.69	0.21	3.33	0.0042

All Three Spotlights



How do you pick a spotlight?

- Most people will be interested in effects of one variable at the average of the other variable(s).
  - These effects approximate “main effects”
- The easy interpretability of the spotlight at the average is why people often mean-center their independent variables, as we did with the first spotlight.
- Unless you have a really good reason not to, instinctively mean-center all of the IV’s in a moderated multiple regression.

Comparing the mean centered model with the main effect

Effect of Valence at Average Level of Purchase Type

	Parameter Estimate	t Value	Pr > |t|
valence	4.72	8.3	<.0001

Main Effect of Valence from Model without Interaction

valence	4.56	6.37	<.0001

How to pick spotlights, pt. 2

- To interpret interactions, you will also want to look at effects above and below the mean.
  - You could look 1 std above and 1 std below (as long as these are within the data range).
  - You could also look at the endpoints of the scale (1 and 9 in our case).
  - You could also look anywhere else in the data with theoretical significance.
  - (Just do not look beyond the range of your data.)

Some points about the spotlight method

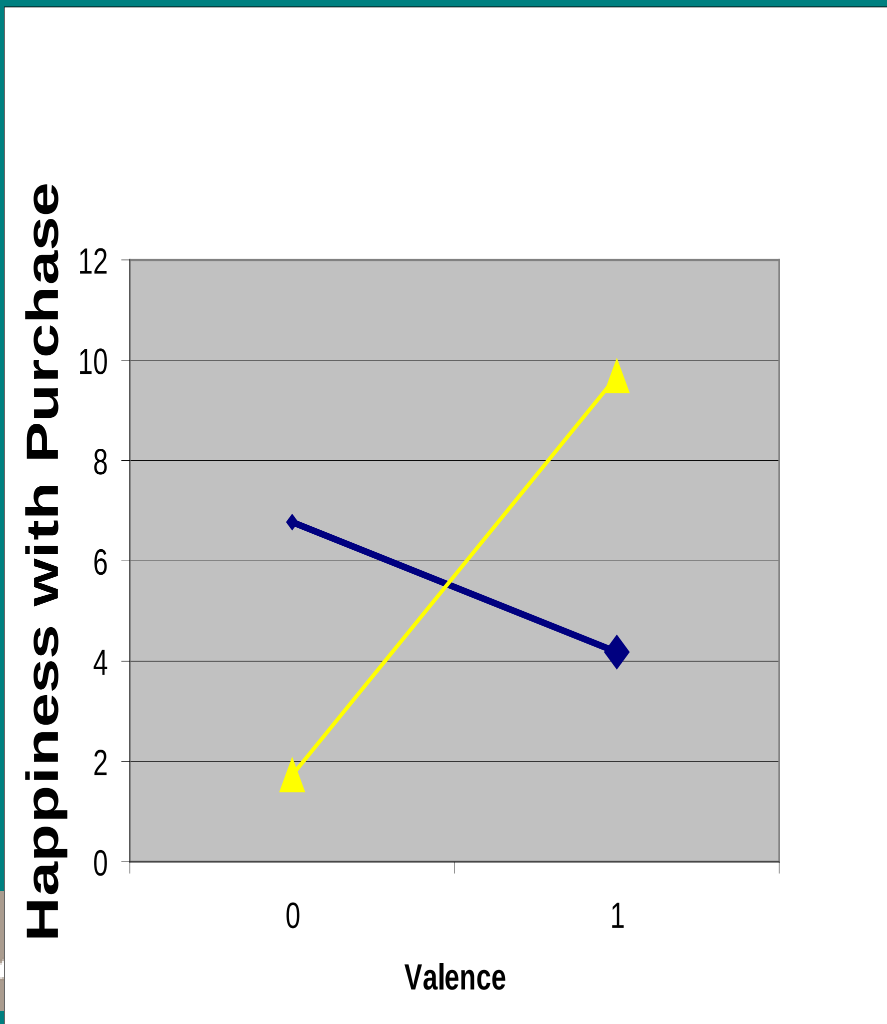
- Spotlights are a super-powerful way to see what is happening without using subgroups of data or splitting anything up.
  - Using subgroups, splitting things = bad (weak, unstable, potentially very misleading, bad)
- It isn't "multiple t-tests." It is used to understand the already-significant interaction.
  - So, do not report a gazillion spotlights as if they were all showing something different.
  - Also do not fuss at your reviewees for not using a special kind of t-test for their spotlights.

Expanding the Method

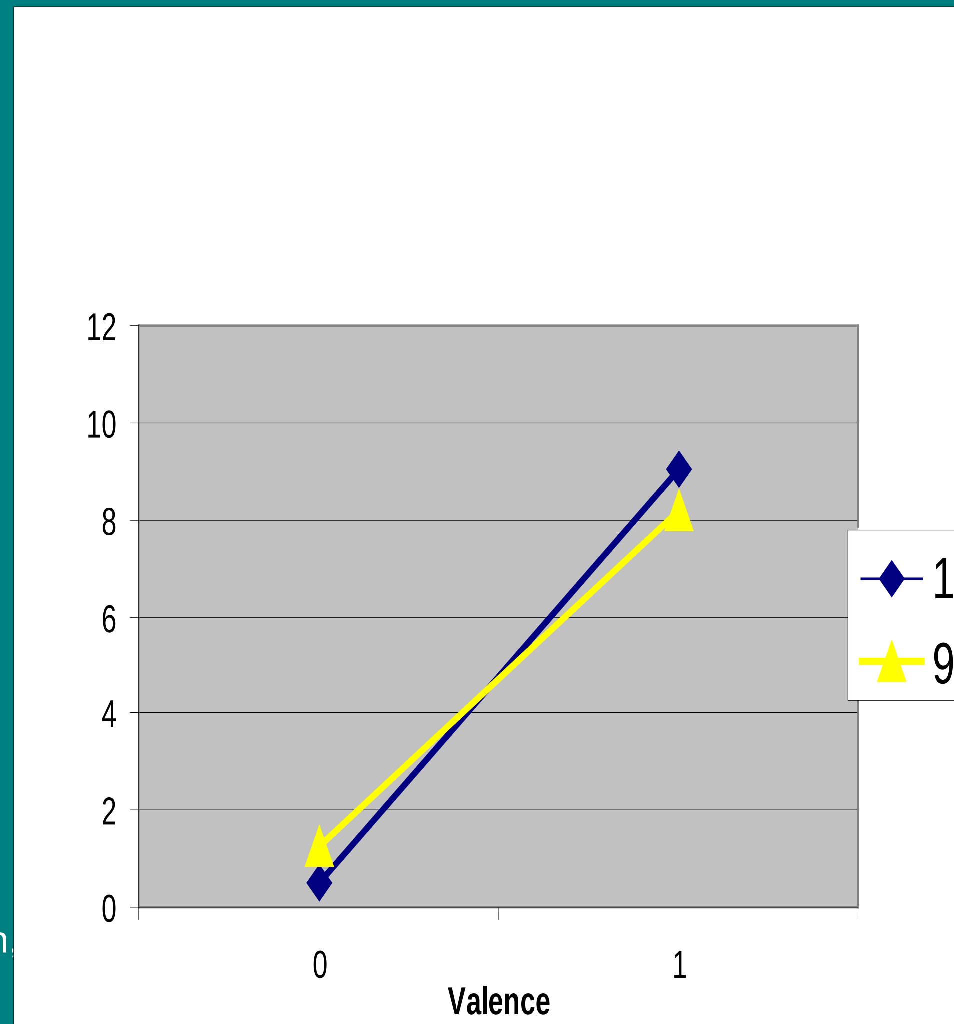
- For logit models, everything works the same (just probe at the relevant spotlights). Logit models are as affected by scaling of the IV's as are OLS models, so everything said here applies.
- For three-way interactions, probe at two spotlights at a time.
  - It is really helpful to graph the 3-way interaction first!
  - You will usually want to do 4 models (2 spotlights each for the two moderating variables)
  - Now you have the test for all of the relevant lines, and the two two-way interactions of interest.

Effect of Valence on Happiness by Purchase Type

Materialism = 1



Materialism = 9



purchasetype=1, materialism =1

	Parameter Estimate	t Value	Pr > |t|
Intercept	6.77	3.95	0.0004
p1	-0.63	-2.48	0.0188
valence	**-2.62**	**-1.45**	**0.1558**
p1*valence	**1.32**	**4.56**	**<.0001**
m1	-0.77	-1.98	0.0560
p1*m1	0.09	1.65	0.1093
valence*m1	1.39	3.41	0.0018
p1*valence*m1	-0.19	-3.01	0.0050

Purchasetype =9, materialism= 1

	Parameter Estimate	t Value	Pr > |t|
Intercept	1.77	3.49	0.0014
p9	-0.63	-2.48	0.0188
valence	**7.92**	**8.52**	**<.0001**
p9*valence	**1.32**	**4.56**	**<.0001**
m1	-0.02	-0.16	0.8738
p9*m1	0.09	1.65	0.1093
valence*m1	-0.15	-0.81	0.4262
p9*valence*m9	-0.19	-3.01	0.0050

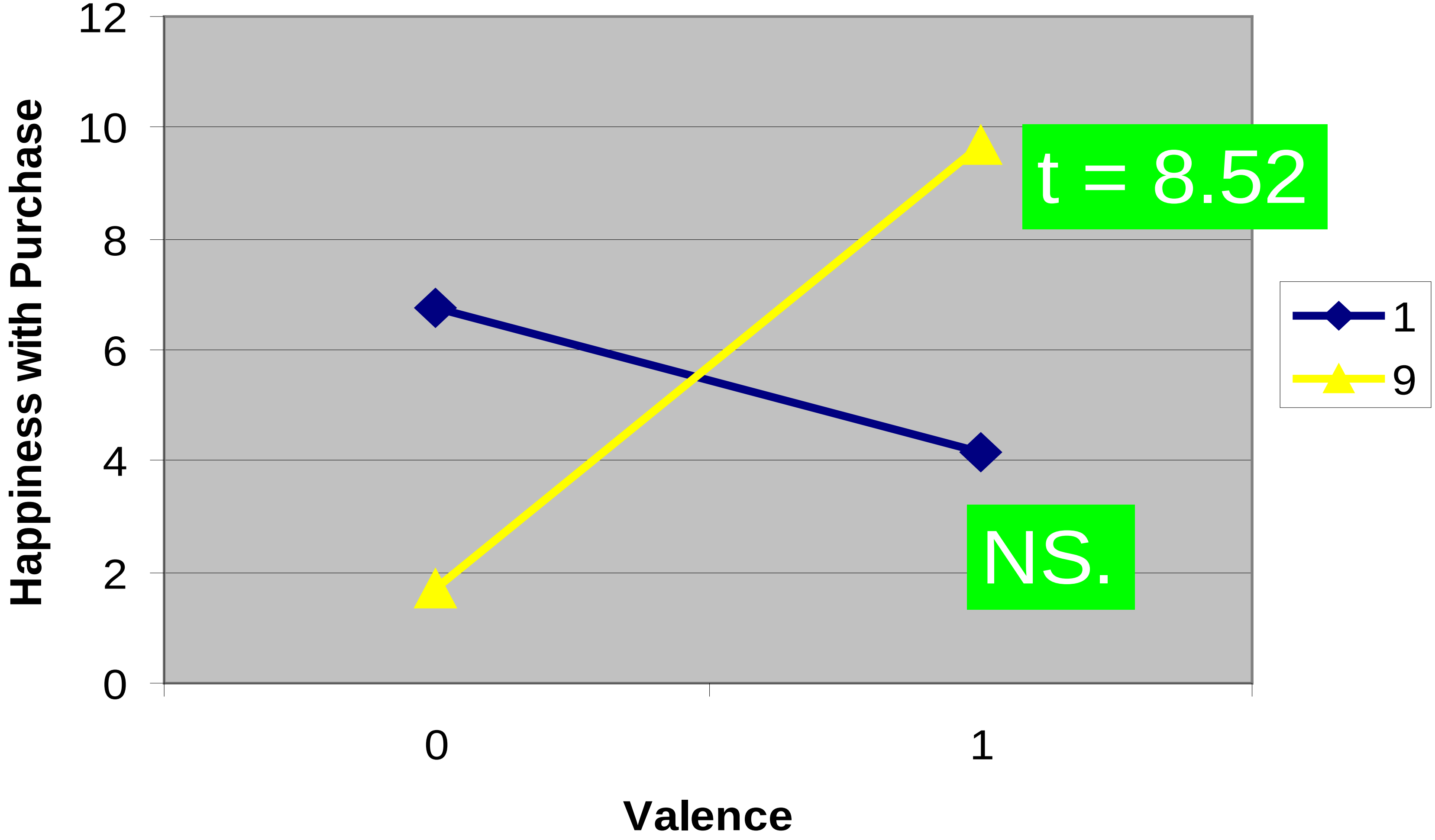
Purchase Type = 1, Materialism = 9

	Parameter Estimate	t Value	Pr > |t|
Intercept	0.59	0.30	0.7663
p1	0.13	0.45	0.6532
valence	**8.51**	**4.13**	**0.0002**
p1*valence	**-0.22**	**-0.69**	**0.4954**
m9	-0.77	-1.98	0.0560
p1*m9	0.09	1.65	0.1093
valence*m9	1.39	3.41	0.0018
p1*valence*m9	-0.19	-3.01	0.0050

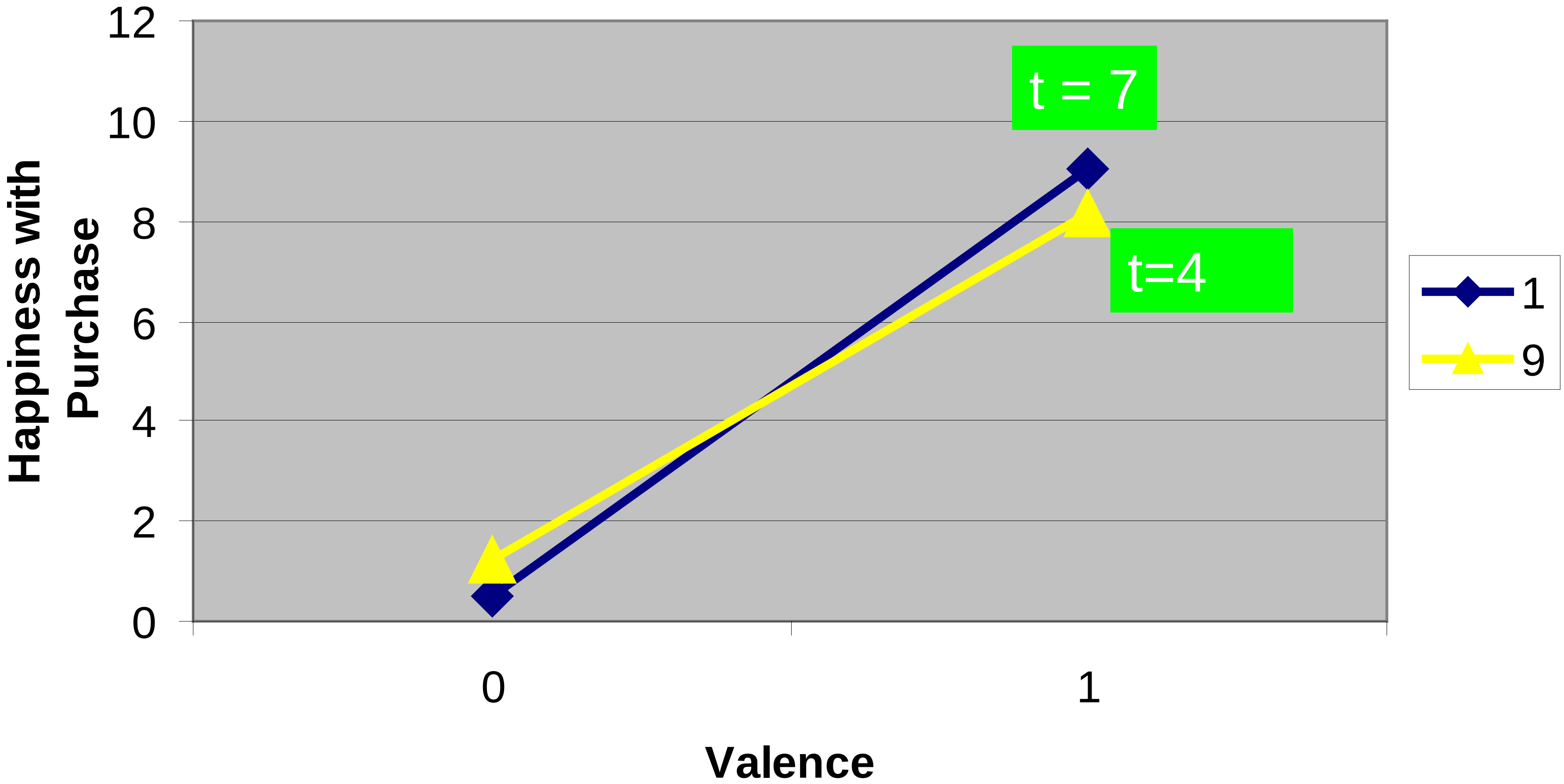
Purchasetype =9, materialism =9

	Parameter Estimate	t Value	Pr > |t|
Intercept	1.63	3.02	0.01
p9	0.13	0.45	0.65
valence	**6.72**	**7.11**	**<.01**
p9*valence	-0.22	-0.69	0.50
m9	-0.02	-0.16	0.87
p9*m9	0.095	1.65	0.11
valence*m9	-0.15	-0.81	0.43
p9*valence*m9	-0.20	-3.01	0.01

Effect of Valence on Happiness by Purchase Type, Less Materialistic Consumers (materialism = 1)
Two-way interaction significant



Effect of Valence on Happiness by Purchase Type, More Materialistic Consumers (materialism = 9)
No two-way interaction.



A important note about dummy codes in interaction models*

- Unless you are using them as a classification variable (i.e., they are not being used as actual numbers) dummy codes are **only** useful if you are planning to spotlight one group in an interaction.
- Otherwise, all of your purported “main effects” are actually the effects of a variable at one level of the other (i.e., when the other is 0). That is, you are not testing what you think you are testing.
- Gary always says “there’s a reason they call them dummy codes.”

- * pass this on to your quant/strategy friends.

Summary

- It is easy to figure out interactions with continuous and discrete variables.
- It is not necessary (in fact it is harmful) to split data or use subgroups to figure out the effects when the spotlight method allows you to
  - Make use of the full power of your data
  - Understand exactly what your interactions mean.
- (mean center, unless you are shining the spotlight somewhere in order to interpret the interaction)