Quadratic Growth Curve Example

In these growth curve examples, I do not allow the quadratic term to vary over time. The reason for this is that the model is not identified (non-positive degrees of freedom) if there are two random slopes (plus the intercept) the case when there are only three time points. If one tries to include too many random effects, HLM gives an explicit warning message about degrees of freedom. In SPSS, the output for the significance test of the second random slope is suppressed. The time variable is also centered in this example. Researchers may or may not want to center the time codes. The advantage is that centering the time variable reduces collinearity between the linear and quadratic time variables. The disadvantage is that the intercept is interpreted as the value of the dependent variable at the middle time point rather than at baseline.

SPSS

```
MIXED depress WITH ctime ctimesq
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV HISTORY
  /FIXED = ctime ctimesq | SSTYPE(3)
  /RANDOM = INTERCEPT  ctime  | SUBJECT(rid) COVTYPE(UN).
```

Mixed Model Analysis

Information Criteria[a]

-2 Restricted Log Likelihood	4900.299
Akaike's Information Criterion (AIC)	4908.299
Hurvich and Tsai's Criterion (AICC)	4908.356
Bozdogan's Criterion (CAIC)	4930.497
Schwarz's Bayesian Criterion (BIC)	4926.497

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: depress Summed CESD score.

Fixed Effects

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Intercept	11.73319	.6030842	388.447	19.455	.000	10.5474703	12.9189058
ctime	-1.91123	.2889570	233	-6.614	.000	-2.4805289	-1.3419241
ctimesq	-.9514957	.4757587	233	-2.000	.047	-1.8888344	-.0141570

a. Dependent Variable: depress Summed CESD score.

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Residual		35.31003	3.2714115	10.794	.000	29.4466464	42.3409347
Intercept + ctime [subject = rid]	UN (1,1)	49.79824	5.8074856	8.575	.000	39.6229307	62.5865997
	UN (2,1)	-2.88350	2.2800132	-1.265	.206	-7.3522466	1.5852411
	UN (2,2)	1.8830838	2.4397232	.772	.440	.1486117	23.8608708

a. Dependent Variable: depress Summed CESD score.

HLM

Summary of the model specified

Level-1 Model

$DEPRESS_{ij} = \beta_{0j} + \beta_{1j}*(CTIME_{ij}) + \beta_{2j}*(CTIMESQ_{ij}) + r_{ij}$

Level-2 Model

$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10} + u_{1j}$
$\beta_{2j} = \gamma_{20}$

Mixed Model

$DEPRESS_{ij} = \gamma_{00}$
$+ \gamma_{10}*CTIME_{ij}$
$+ \gamma_{20}*CTIMESQ_{ij} + u_{0j} + u_{1j}*CTIME_{ij} + r_{ij}$

Final Results - Iteration 4

Iterations stopped due to small change in likelihood function

$\sigma^2 = 35.23966$

τ

INTRCPT1,β_0	49.82527	-2.88216
CTIME,β_1	-2.88216	1.95763

τ (as correlations)

INTRCPT1,β_0	1.000	-0.292
CTIME,β_1	-0.292	1.000

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.809
CTIME,β_1	0.100

The value of the log-likelihood function at iteration 4 = -2.449231E+003

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	11.733188	0.634164	18.502	233	<0.001
For CTIME slope, β_1					
INTRCPT2, γ_{10}	-1.911226	0.288339	-6.628	233	<0.001
For CTIMESQ slope, β_2					
INTRCPT2, γ_{20}	-0.951496	0.474741	-2.004	233	0.046

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	7.05870	49.82527	233	1221.24362	<0.001
CTIME slope, u_1	1.39915	1.95763	233	258.36673	0.122
level-1, r	5.93630	35.23966			

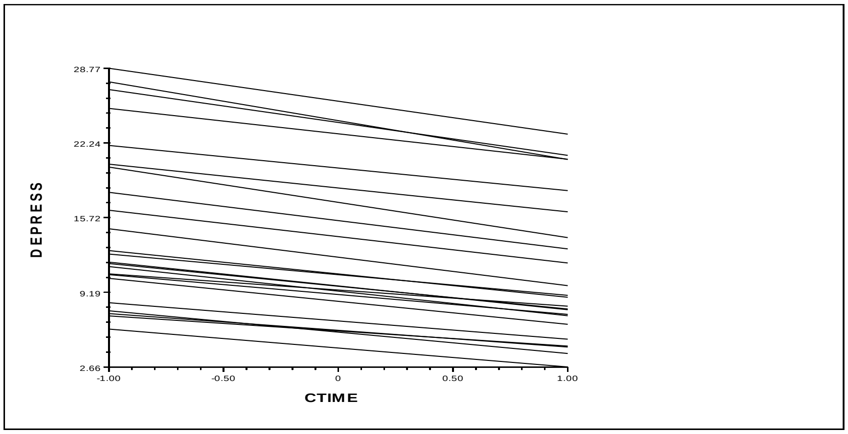
Statistics for current covariance components model

Deviance = 4898.461714
Number of estimated parameters = 4

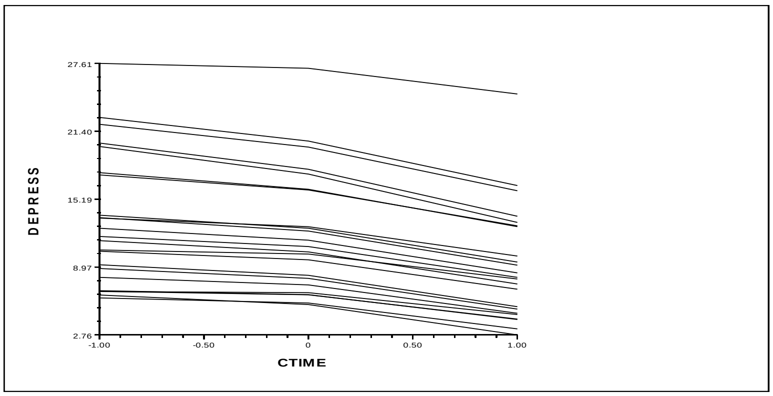
A growth curve model was tested to investigate whether there was a nonlinear change in depression over time. Both linear and quadratic components were included in the model. The time variable was centered at the mid-point of the study to reduce collinearity between the linear and quadratic components. Because of the limited number of parameters that could be estimated with three time points, the linear slope was allowed to vary across individuals, but the quadratic slope was not allowed to vary.[1] Robust standard errors were used to account for non-normality (Liang & Zeger, 1986). At the midpoint of the study, the mean depression score was 11.73. There was a significant linear decline in depression over the three waves, γ_{10} = -1.91, SE = .29, p < .001, suggesting a decrease in depression scores of nearly two points every six months. The quadratic effect was also significant, γ_{20} = -.95, SE = .47, p < 05, and, based on the examination of Figure 1, suggested an accelerated decline in depression over time. The linear slope did not vary significant across individuals, τ^2_{11} = 1.96, χ^2 = 258.37, p = .122, suggesting a similar rate of linear decline among participants.

[1] Because there tends to be low reliability and there are limits on the number of random effects that can be estimated I typically do not recommend testing quadratic models when there are only three time points. I do here just for didactic purposes..

Individual linear slopes



Individual quadratic curves



Average quadratic curve (Figure 1).