Yuka Murata
USC ID: 6434262018
March 29, 2019

CSCI 420 Computer Graphics
Programming Assignment 2
Simulating a Roller Coaster

Problem Statement

Derive the steps that lead to the physically realistic equation of updating the u:

$$u_{new} = u_{old} + \frac{\Delta t\sqrt{2g\Delta h}}{\| \frac{dp}{du} \|}$$

see here.

Proof

In a free fall, the following two equations can be derived in physics:

$$v = gt \qquad (i)$$
$$\Delta h = \frac{1}{2}gt^2 \qquad (ii)$$

From the equation (i) and (ii), the following equation can be derived by removing t:

$$v = \sqrt{2g\Delta h}$$

Hence, the traveling distance by free fall is

$$dist = v\Delta t$$
$$= \Delta t\sqrt{2g\Delta h}$$

When parameter *u* is increased by 1, the traveling distance is the size of tangent vector at *u*. This fact leads to the following relation where *t(u)* is a tangent vector at *u*:

$$1: \| t(u) \| = \Delta u : \Delta t\sqrt{2g\Delta h}$$
$$\Delta u = \frac{\Delta t\sqrt{2g\Delta h}}{\| t(u) \|}$$
$$= \frac{\Delta t\sqrt{2g\Delta h}}{\| \frac{dp}{du} \|} \qquad (iii)$$

From the equation (iii), the new u value is derived as:

$$u_{new} = u_{old} + \Delta u$$
$$u_{new} = u_{old} + \frac{\Delta t\sqrt{2g\Delta h}}{\| \frac{dp}{du} \|}$$