Combinatorial choices

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Outline

Predicting choice behavior

Decision rule

Homo economicus

Rational and narrowly self-interested economic actor who is optimizing her outcome

Behavioral assumptions

- The decision maker solves an optimization problem.
- The analyst needs to define
  - the decision variables,
  - the objective function,
  - the constraints.

Continuous case: classical microeconomics

Optimization problem

$$\max_{q} \widetilde{U}(q;\theta)$$

subject to

$$p^T q \leqslant I,\ \ q \geqslant 0.$$

Demand function

- Solution of the optimization problem.
- KKT optimality conditions:

$$q^* = f(I, p; \theta)$$

Discrete choices

How does it work for discrete choices?

Utility maximization

Optimization problem

$$\max_{q,w} \widetilde{U}(q, w; \theta)$$

subject to

$$\begin{aligned} & p^T q + c^T w \leqslant I \\ & \textstyle\sum_j w_j = 1 \\ & w_j \in \{0, 1\}, \forall j. \end{aligned}$$

where $c^T = (c_1, \ldots, c_i, \ldots, c_J)$ contains the cost of each alternative.

Derivation of the demand functions

- Mixed integer optimization problem
- No optimality condition
- Impossible to derive demand functions directly

Derivation of the demand functions

Step 1: condition on the choice of the discrete good

- Fix the discrete good(s), that is select a feasible w.
- Derive the conditional demand functions from KKT.

Step 2: enumerate all alternatives

- Enumerate all alternatives.
- Compute the conditional indirect utility function U_i.
- Select the alternative with the highest U_i.

Enumerate all alternatives ??????

Starbucks has 383 billion unique latte combinations. [Merritt, 2023]

Activity-based models

- Activity participation
- Activity type
- Activity location
- Activity timing
- Activity duration
- Activity scheduling
- Activity frequency
- Travel mode choice
- Route choice
- Departure time choice
- Trip chaining / Tour formation
- Vehicle usage
- Parking choice
- Joint activity participation
- Ride-sharing / Carpooling decision
- Household resource allocation
- Teleworking decision
- Trip cancellation or rescheduling
- Use of on-demand mobility services
- ... and many more

Outline

Activities

Why do people travel?

- Most of the time, not for the sake of it.
- Activities.
- Spread in space and time.

Activity-based models: literature

Econometric models

- Discrete choice models.
- Curse of dimensionality.
- Decomposition: sequence of choices
  - Activity pattern
  - Primary tour: time of day
  - Primary tour: destination and mode
  - Secondary tour: time of day
  - Secondary tour: destination and mode
  - e.g. [Bowman and Ben-Akiva, 2001]

Rule-based models

- If the selected activity is at location L,
- and the travel time from current location C to L exceeds $T_{\max}$,
- then reject the activity–location combination,
- unless it is a high-utility or infrequent activity (e.g., doctor appointment).
- e.g. [Arentze et al., 2000]

Research question: can we combine the two?

	Econometric	Rule-based
Micro-economic theory	X	—
Parameter inference	X	—
Testing/validation	X	—
Joint decisions	—	X
Complex rules	—	X
Complex constraints	—	X

Combinatorial choices

Mathematical optimization

- Each individual is solving a combinatorial optimization problem.
- Decisions: see the long list before...
- Objective function: utility (to be maximized).
- Constraints: complex rules.

[Pougala et al., 2023]

Challenges

- Stochasticity: random utility $\rightarrow$ rely on simulation.
- Large number of variables and constraints $\rightarrow$ decomposition methods.
- Interacting individuals (households, social groups) $\rightarrow$ this talk.
- Time horizon $\rightarrow$ future work.

Outline

Choice model as an optimization problem

Travel demand: activity based models

Model

Graph-based model

Illustrations and results

Social groups

We consider a social group N of agents that cooperate and desire to maximize their aggregated utility.

- Coordination, joint activities.
- Group decision making
- Service to the group, maintenance.
- Resource constraints.
- Escorting.

Objective function: utility of the group

- Function of the utility of each member. But which function?
- Lack of consensus in the literature.
- Additive: the (weighted) sum of the utility of each member.
- Autocratic: the utility of the "strongest" member.
- Egalitarian: the utility of the "weakest" member.
- Important for our framework: must be easy to linearize.

Constraints

Coordinated activities

- a is an activity that must be performed by all members of the group.
- Dining out.
- Family gathering.
- Sport events.

Constraints

Distributed activities

- a is an activity that must be performed for the group.
- Maintenance.
- Grocery shopping.
- Meal preparation.
- Accounting of the sport club.

Constraints

Resource constraints

- One car per household.
- One meeting room in a shared office space.
- Modeling approach: treat the resource as an individual.
- "The car is a member of the family".
- It is associated with "activities" and a schedule.
- We can then introduce "coordinated activities" constraints.

Constraints

Escorting a child to school

- Specific instance of a resource constraint.
- The person escorting becomes a resource.
- As individuals and resources are modeled in the same way, coordinated activities constraints can be applied.

Space

Discrete and finite set L of locations.

For each (ℓ, ℓ'):

- $M_n^{\ell\ell'}$: available modes for agent n.
- $\rho_{\ell\ell' m}$: travel cost of the trip with mode m.
- $d_{\ell\ell' m}$: travel time of the trip with mode m.

Assumption: travel time and cost are exogenous.

Activities: notations

Set A_n of potential activities for each agent n.

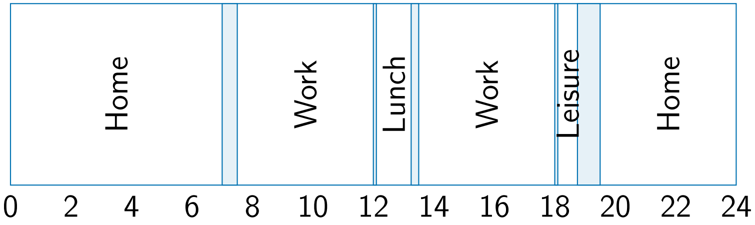
For each activity a:

- L_a: set of possible **locations** for a
- $c_{a\ell}$: cost of a at location ℓ
- $[\gamma^-_{a\ell}, \gamma^+_{a\ell}]$: opening hours for a at location ℓ
- $\tau^{\min}_a$ & $\tau^{\max}_a$: min & max duration of a.
- C_a: maximum capacity for a.
- N_a: set of required agents for a.

Activities: further assumptions

- **Start and end at home:** The first activity (dawn) and the last activity (dusk) always take place at the agent's home.
- **Group of activities:**
  - Some activity groups (e.g., shopping) must be performed at least a specified number of times over the planning horizon.
  - Examples: shopping, domestic tasks, sport, etc.

Scheduling



Utility function

Collective decisions $\Rightarrow$ maximize the **utility of the group**

$$U = \sum_{n \in N} U_n = \sum_{n \in N} \left(\sum_{a \in A_n} U_a^n + \xi_{an} + \sum_{\ell, \ell' \in L} \sum_{m \in M} U_{\ell \ell' m}^n + \xi_{\ell \ell' mn} \right)$$

- U_a^n: reward + joint activity reward - deviation from the prefered schedule - cost
- $U_{\ell \ell' m}^n$: joint travel utility (travel cost, travel time, etc.), usually negative.
- ξ_{an} and $\xi_{\ell \ell' mn}$: random term with a known distribution

Utility function

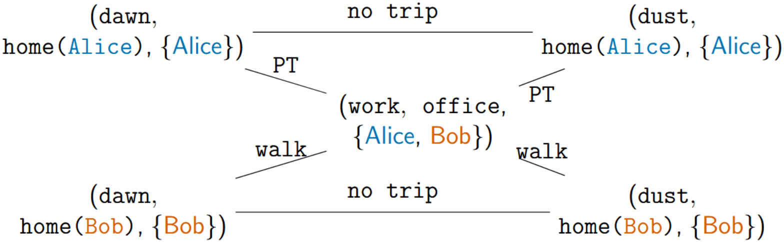
Error terms

- Rely on simulation.
- Draw ξ_{anr} and $\xi_{\ell\ell' mn}$, $r = 1, \ldots, R$.
- Optimization problem for each r.
- Utility: U_{anr}.

Outline

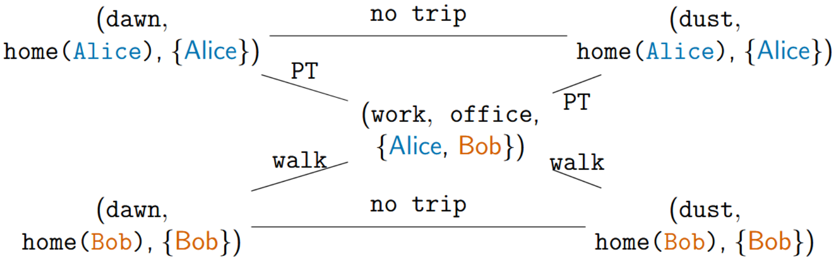
Graph-based modeling approach

- Formulation as a shortest path problem in a graph $G = (V, E)$ with additional constraints.



- Vertices V: triplet $v =$ (activity a_v, location ℓ_v, subgroup of agents S_v)
  $\rightarrow$ also encoding C_a and N_a
- Arcs E: transition of agents between activities
  $\rightarrow$ labeled with the transport mode

Graph-based modeling approach



- **One dawn**(n)**-dusk**(n) **path in** $G \Leftrightarrow$ **One sequence of activities/trips for an agent** n
- Problem reformulation: find one path per agent under time-consistency, combinatorial and budget constraints.

Variables

- Graph variables
  - $z_e^n \in \{0, 1\}$ — equals 1 if agent n travels along arc e
  - $w_v \in \{0, 1\}$ — equals 1 if vertex v is part of the path for all agents in S_v
- Time variables for each vertex v
  - $x_v \in \mathbb{R}_+$ — starting time of activity a_v
  - $\tau_v \in \mathbb{R}_+$ — duration of activity a_v

  These apply to all agents in S_v at location ℓ_v.

Constraints

1. **flow** constraints
   - path definition

2. **combinatorial** constraints
   - eligibility to pass through a vertex
   - group consistency
   - location uniqueness
   - group of activities

3. **time-consistency** constraints
   - schedule consistency
   - full time period covered
   - opening hours
   - duration bounds

4. a **budget** constraint

Constraints

- **flow conservation** constraints: dawn(n)-dusk(n) path definition

$$\sum_{e\in\delta^+(v)} z_e^n = \sum_{e\in\delta^-(v)} z_e^n \qquad \forall v \in V \quad \forall n \in N$$

$$\sum_{e\in\delta^+(\text{dawn}(n))} z_e^n = 1 \qquad \forall n \in N$$

$$\sum_{e\in\delta^-(\text{dusk}(n))} z_e^n = 1 \qquad \forall n \in N$$

Constraints

- **combinatorial** constraints
  - Group consistency

$$w_v = \sum_{e \in \delta^+(v)} z_e^n \qquad \forall v \in V \quad \forall n \in S_v$$

  - Eligibility

$$z_e^n = 0 \qquad \forall e = (u, v) \in E \quad \forall n \notin N_u \cap N_v$$

  - Group of activities

$$\sum_{v \in V :\, a_v \in G_k} w_v \geqslant n_k \qquad \forall k \in K$$

  - Location uniqueness

$$w_v + w_{v'} \leqslant 1 \qquad \forall v, v' \in V \text{ s.t. } a_v = a_{v'}, S_v = S_{v'}, \ell_v \neq \ell_{v'}$$

Constraints

- **time-consistency** constraints

$$
\begin{aligned}
& x_v \geqslant x_u + \tau_u + d_{uv} - T(1 - z_e^n) && \forall e = (u, v) \in E \quad \forall n \in N \\
& x_v \leqslant x_u + \tau_u + d_{uv} + T(1 - z_e^n) && \forall e = (u, v) \in E \quad \forall n \in N \\
& \gamma^-_{a_v, \ell_v} w_v \leqslant x_v \leqslant \gamma^+_{a_v, \ell_v} + T(1 - w_v) && \forall v \in V \\
& \tau^{\min}_{a_v} w_v \leqslant \tau_v \leqslant T(1 - w_v) && \forall v \in V \\
& \sum_{v \in V \,:\, n \in S_v} \tau_v + \sum_{e \in E} d_e z_e^n = T && \forall n \in N
\end{aligned}
$$

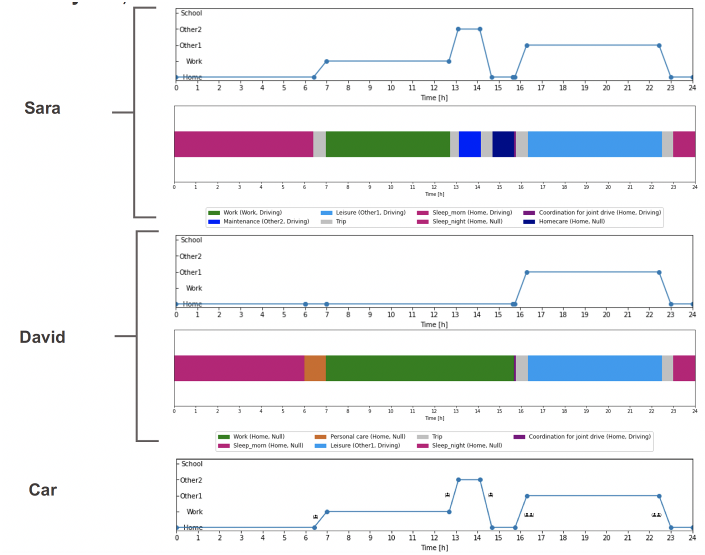
Additional constraints

- a **budget** constraint

$$\sum_{v \in V\,:\, n \in S_v} c_{a_v \ell_v} w_v + \sum_{e \in E} \rho_e z_e^n \leqslant B \qquad \forall n \in N$$

Outline

Car as a resource



To Tennis or Not to Tennis

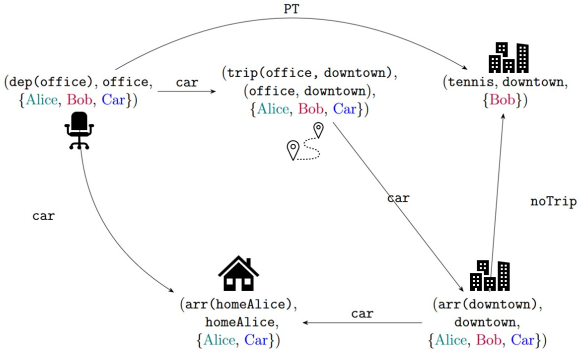
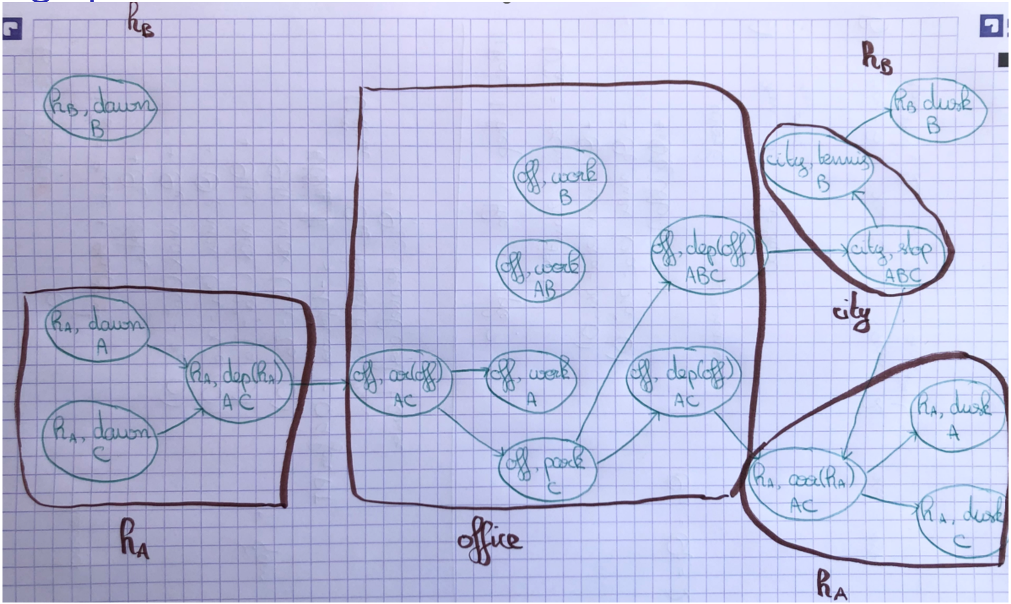


Figure: Example of ride sharing modeling

Example

- Alice (A) and Bob (B): two colleagues
- Alice has a car.
- Bob has another activity: tennis.

Full graph



hB
hB, dawn
B
hB
hB dusk
B
city, tennis
B
off, work
B
off, work
AB
off, dep(off)
ABC
city, stop
ABC
city
hA, dawn
A
hA, dep(hA)
AC
hA, dawn
C
off, arr(off)
AC
off, work
A
off, dep(off)
AC
off, park
C
hA, dusk
A
hA, arr(hA)
AC
hA, dusk
C
hA
office
hA

Hypotheses

- Alice and Bob derive a social reward by working together.
- Alice prefers to work in the afternoon.
- Bob can only play tennis between 4pm and 7pm.
- The trip from the office to the tennis takes much more time with public transport than with the car.

Different scenarios

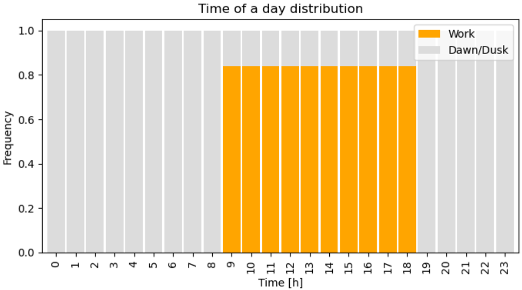
- If Alice and Bob work together, without the car, Bob can't go to tennis.

- If he arrives at work early, he can go to tennis, but he doesn't work with Alice.

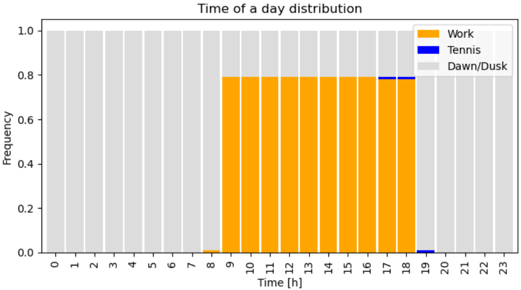
- If Alice and Bob work together and Alice comes by car, B can go to tennis by car with Alice.

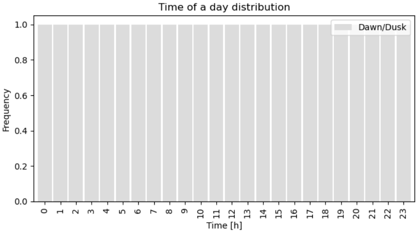
Simulation: From isolated individuals. . .



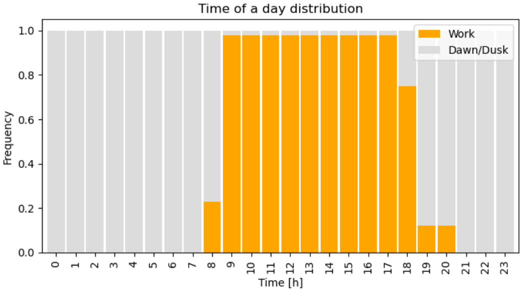
Alice



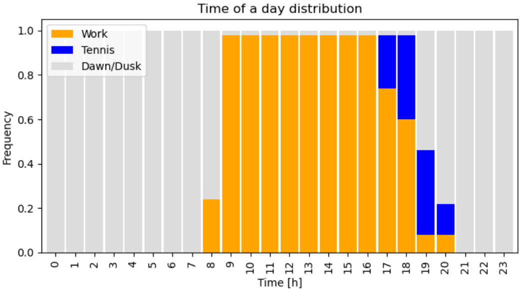
Bob



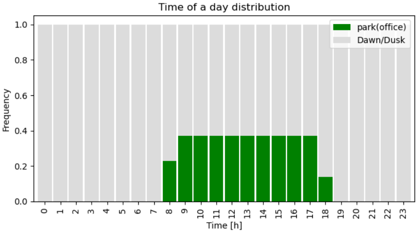
Simulation: ...to social groups



Alice



Bob



Conclusions

It works!

- Handles complex activity and schedule choices.
- Integrates behavioral and operational constraints.
- Enables realistic, data-driven simulations.

What's next?

- **Flexibility** is the key strength of the framework.
- **Scalability** remains a major challenge (time, activities).
- **Simulation cost** is high — need for efficient algorithms.
- **Connections** with vehicle routing problems suggest decomposition strategies.
- **Inference** could benefit from Bayesian approaches.

Summary

- **Goal:** develop operational combinatorial choice models, such as activity-based models.
- **Approach:** integrate econometric modeling with rule-based logic.
- **Methodology:** leverage operations research, mathematical optimization and simulation.
- Simulation of activity schedule: [Pougala et al., 2022a].
- Application with the Swiss Railways: [Manser et al., 2021].
- Estimation of the parameters: [Pougala et al., 2022b].
- Household interactions: [Rezvany et al., 2023], [Rezvany et al., 2024].
- Main advantage of the framework: flexibility.

Combinatorial choices

Main philosophy

Leverage the power of modern combinatorial optimization to model complex choice behavior.

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