Midterm Examination for the Algorithms and Data Structures II Course

(July 28, 2020, Lecture Theatre, 5-6 periods)

Name: Daiki Kurisu **Group:** • **IT-CMV** (Markov) **Student ID No.:** s1260174
• **SE-DE** (Yen)

Problem 1. (20 points)

Suppose you have algorithms with the five running times listed below. (Assume these are the exact running times.) **How many times slower** does each of the algorithms get when you a) double the input size ($n \to 2n$), or b) increase the input size by one ($n \to n+1$)?

1. n^2 2. n^3 3. $100n^2$ 4. $n\log n$ 5. 2^n

a) (10 points) 1. 4 2. 8 3. 4 4. 2 5. 2^n

b) (10 points) 1. 1 2. 1 3. 1 4. 1 5. 2

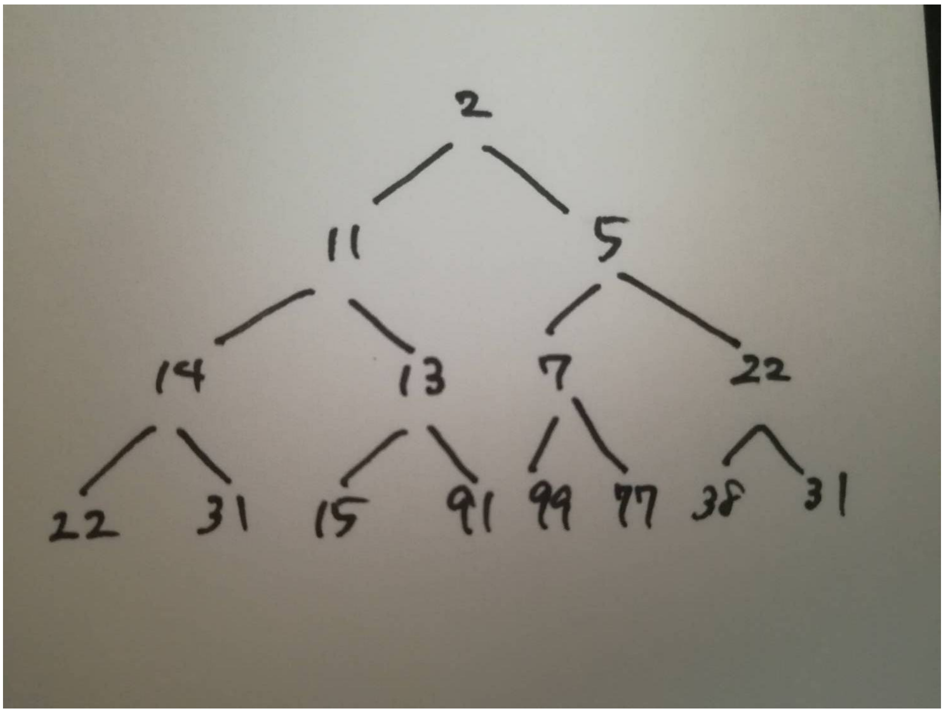
Note. When appropriate, give an approximate answer assuming $n \to \infty$. (For example, $1 + 1/n \approx 1$)

Problem 2. (10 points)

A **minimum heap** is a heap in which the key of each node is bigger than or equal to the key of its parent. Given the following array of numbers

{11, 22, 38, 2, 13, 77, 5, 14, 31, 15, 91, 99, 7, 22, 31},

construct a **minimum heap** using bottom up heap construction algorithm. Show your work.



<u>Problem 3.</u> (30 points)
Consider a graph G defined by the following adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

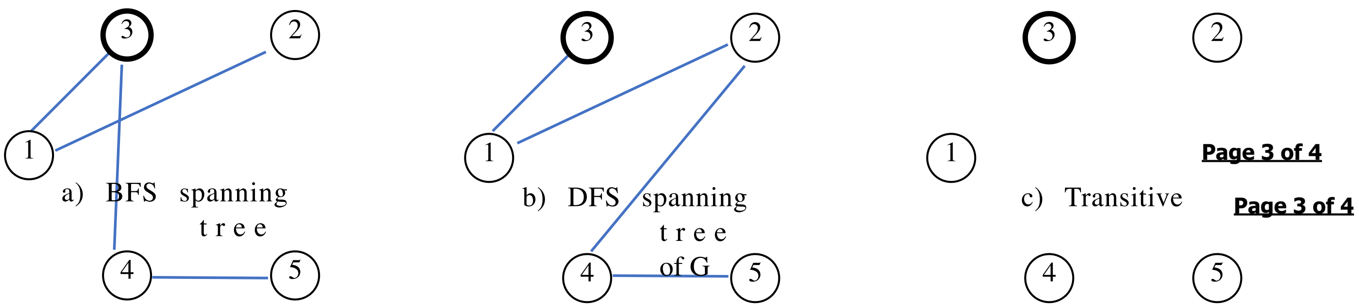
a) (10 points) In what sequence (order) vertices of this graph will be visited by the **Breath-First-Search** (BFS) algorithm starting from node 3? Use the BFS algorithm to find a spanning tree of G with <u>node **3**</u> as root and show it below.

Vertex sequence: 3 -> 1 -> 4 -> 2 -> 5……………………………………………………..

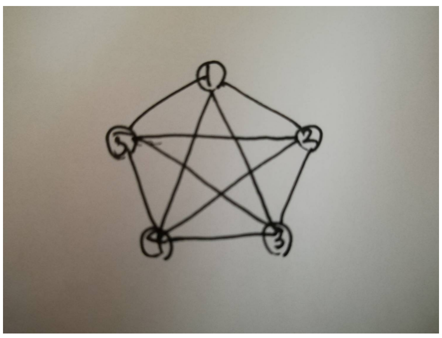
b) (10 points) In what sequence (order) vertices of this graph will be visited by the **Depth-First-Search** (DFS) algorithm starting from node 3? Use the DFS algorithm to find a spanning tree of G with <u>node **3**</u> as root and show it below.

Vertex sequence: 3 -> 1 -> 2 -> 4 -> 5……………………………………………………..

c) (10 points) Show the **transitive closure** of G.



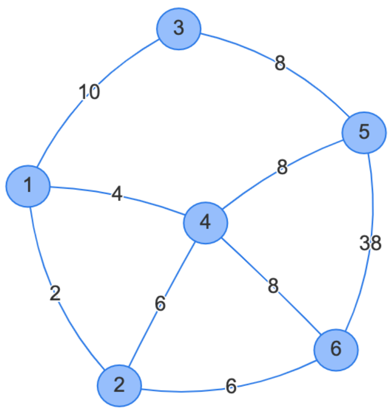
(c) Transitive



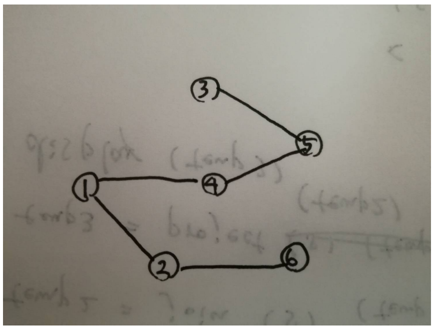
<u>Problem 4.</u> (40 points)

Consider a weighted graph G defined by the following distance matrix:

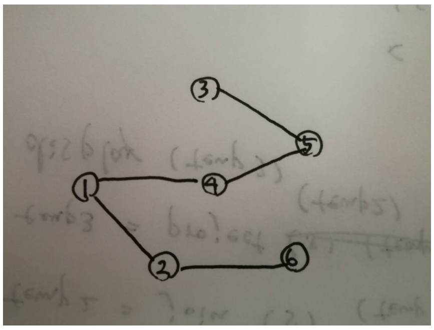
$$D = \begin{pmatrix} 0 & 2 & 10 & 4 & \infty & \infty \\ 2 & 0 & \infty & 6 & \infty & 6 \\ 10 & \infty & 0 & \infty & 8 & \infty \\ 4 & 6 & \infty & 0 & 8 & 8 \\ \infty & \infty & 8 & 8 & 0 & 38 \\ \infty & 6 & \infty & 8 & 38 & 0 \end{pmatrix}$$



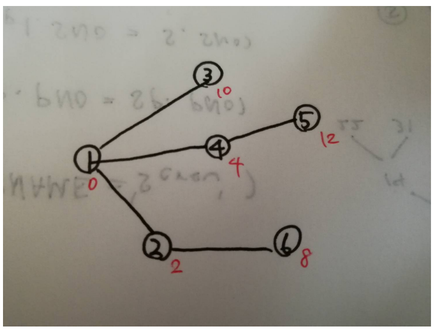
a) (10 points) Use the Prim's algorithm to find the **minimum** spanning tree of G with node 1 as root. (Put the answer in the figure below.)



b) (10 points) Use the Kruskal's algorithm to find the **minimum** spanning tree of G. (Put the answer in the figure below.)



c) (10 points) Use the Dijkstra's algorithm to find the **shortest paths** from node 1 to all the other nodes in the graph G. (Put the answer in the figure below and show the lengths of all the shortest paths.)



d) (10 points) Use the Floyd's algorithm to find the **all pairs shortest paths** of *G*. (Put the answer in the table below.)

	1	2	3	4	5	6
1	0	2	10	4	12	8
2	2	0	12	6	14	6
3	10	12	0	14	8	18
4	4	6	14	0	8	8
5	12	14	8	8	0	16
6	8	6	18	8	16	0