Entry Deterrence in Procurement Auctions*

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Abstract

Firms have incentives to alter competitors' beliefs about their entry to deter others from entering the market. They may achieve this objective by disclosing their intent to enter. We study procurement auctions conducted by Montana Department of Transportation, where a designated online Q&A forum serves as an entry disclosure device. We specify and estimate a model of procurement auctions with costly entry, in which firms have the option to disclose entry. We find that disclosure deters entry from others, and disclosure is beneficial for a firm if they can disclose at an early period. Overall, the availability of disclosure device decreases the auctioneer's payment by 6.3%, while increasing the winner's construction costs by 4.5% and decreasing the total entry costs by 11.1%.

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1 Introduction

When multiple firms contemplate entering a market, there may not be sufficient capacity for the market to profitably accommodate all potential entrants. Even if every firm prefers to be an entrant ex-ante, some firms ultimately enter while others stay out. In such an environment, belief about others' entry is crucial. As discussed in Farrell (1987), if a firm can influence the beliefs of other firms regarding its intent to enter, it may compel those other firms to reconsider their own entry decisions. For instance, once all potential entrants believe that a given firm will enter the market, this can benefit the firm since the other firms may then be less inclined to enter.

In attempting to influence the beliefs of rival firms and deter their entry, it is common for firms to publicly announce one's intent to enter the market. For example, a firm may make a pre-announcement on releasing new products for this purpose. In the early 1990s, Microsoft was accused of making product pre-announcements just for the purpose of deterring competitors from entering. The district court judge noted that "Microsoft could unfairly hold onto this [dominant] position with aggressive pre-announcements of new products in the face of the introduction of possibly superior competitive products." [1] While strategic entry deterrence through disclosure raises concerns from an antitrust perspective, there is a notable lack of empirical research quantifying this effect.

In this paper, we investigate how entry disclosure affects auction outcomes by studying procurement auctions conducted by the Montana Department of Transportation (MDOT). A notable feature of the auctions that we study is that there is a designated online forum on the web page of MDOT where potential bidders can ask clarifying questions about the project being let. The questions posted by the potential bidders, the identity of the firm asking questions, the time the question was posted, as well as the responses posted by MDOT, are all publicly accessible information. Since asking questions on the forum typically requires a bidder to have invested some time in reviewing the project plan, posting a question on the online forum serves as an entry disclosure. Indeed, over 99% of the questions are posted by actual entrants. By linking the activity on the forum to actual entry and bidding behavior in the auction, we

[1] The ruling of Judge Stanley Sporkin in Civil Action No. 94-1564 (United States of America v.s. Microsoft Corporation 1995).

study the effect of entry disclosure on equilibrium entry, bidder profits, and procurer surplus.

To understand how participating firms perceive the Q&A forum, we have held an interview with the participating firms. Their responses indicate that the firms indeed perceive that questions are posted in a strategic manner and are not always intended to gather information about the project:

> "*There is always a strategical consideration to the questions we ask and is not solely determined by us needing the information. It can be gamesmanship with the other bidders.*"

Moreover, their claim indicates that they take the questions as a *credible* signal for a firm entering the auction:

> "*It's safe to assume that contractors would not be asking questions unless they are going to bid the project.*"

These claims support the idea of considering the Q&A forum as a disclosure device, which forms the foundation of the paper.

In our setup, entry disclosure has two distinct and competing effects. First, as previously noted, a firm can alter opponents' beliefs through disclosure, thereby reducing their expected profits from entry, which consequently leads to less entry from other firms. On the other hand, a key feature of our setup is that the set of entrants remains unknown at the time of bidding, while the firms that have disclosed entry will be participating in bidding. This uncertainty regarding the set of entrants generates a different force from entry deterrence. Knowing that a firm would certainly be bidding due to disclosure, the other firms that do enter may bid more aggressively against the firm, compared to the case where the firm remains silent about their entry status. Hence, entry disclosure through posting questions may be detrimental for the firms who have disclosed entry.

To understand the trade-off between the two competing effects of entry disclosure, we construct and estimate a model of a procurement auction with selective entry, wherein firms can post questions on a Q&A forum that serves as an entry disclosure device. Our model consists of two stages: (i) entry and disclosure; and (ii) bidding. In the first stage, firms arrive sequentially at the market. They make entry decisions based on their private entry costs and information available on the Q&A forum, i.e., the firms are aware of who has disclosed entry. Upon entry, the firm draw their construction cost and may choose to post a question on the forum, thereby disclosing its entry. The second stage occurs after all the entry decisions have been finalized, and entrants submit their bids simultaneously, while considering all disclosures and their own private construction costs. The bidding procedure is a first-price sealed bid auction, and the bidder with the lowest bid wins. The effect of introduction of the Q&A forum on auction outcomes is ambiguous and thus an empirical question. Using the estimates, we consider how the auction outcomes change by alternative platform designs regarding transmission of information about entry.

We establish three key patterns that highlight the economic forces of the posted questions, i.e., entry disclosure. First, we document that presence of a question on the forum is associated with a lower entry probability among bidders. This is a pattern we would see if disclosures do deter entry. Second, the strongest opponents' bid is weaker for the firms who disclose early than: (i) those who never disclose; or (ii) those who disclose late. If an early entry disclosure has a strong effect of entry deterrence, dominating its effect on inviting in more aggressive bids, we would see this pattern (i). Since late disclosures are expected to have weaker effects on entry deterrence than early disclosures in a sequential entry setting, we would expect to see pattern (ii). Third, entrants place stronger bids when they face more questions. If firms' bidding behavior respond to the information on the Q&A forum, we would expect this pattern.

We also document patterns that do *not* match alternative forces that could be in play. One alternative force that could explain the first pattern presented above is difference in quality of the proposals across auctions. If the presence of a question acts as a proxy for the proposal's quality, we would similarly see the first pattern that this proxy is associated with a lower entry probability among bidders. However, we find that that bids are more stronger in auctions in which questions are present. This pattern goes against this alternative explanation because we would naturally expect a weaker bid if the proposal has low quality. Therefore, we believe that this force is not the first order factor that drives our data.

We show that the firms' primitives are non-parametrically identified from firms' entry, disclosure, and bidding behaviors. The primitives we wish to recover are the distribution of firms' entry timing, entry costs, distribution of costs of entry disclosure, i.e., posting questions, and construction costs. The main challenge in identification is that entry timing is only observed for the firms who post questions. In the first step, we recover the construction costs and their distribution, following Guerre et al. (2000). Next, we recover a firm X's belief on the evolution of the history conditional on X entering at a fixed time point t. If firm X discloses, this object is directly identified from the observed patterns. However, since we do not observe the entry timing for those who do not disclose, we cannot identify firm X's belief directly from the data when they do not disclose. Add a reasonable intuition By considering the expected value from the auction stage and the belief on the evolution of disclosure history, we are able to determine the value of disclosure at each history. The knowledge of values of disclosure at histories allows us to identify the distribution of disclosure costs/benefits. Finally, we provide results for the identification of the distributions of entry cost and entry timing. For this part, we begin by providing an argument on identifying the density of entering AND then disclosing, conditional on a disclosure history. This can be shown by considering the duration of time until the next disclosure happens. To map this conditional density to a simple form of our primitives when the current history includes n disclosures ($n \geq 1$), we conduct an inductive argument on the number of disclosures, n. This form is a product of the pdf of entry timing and the pdf of entry costs. Given this form, by comparing histories with same disclosure value, we can identify the distribution of entry timing. Finally, given the identified objects, we can identify the distribution of entry costs. [Is there a better intuition? Maybe too long.]

Given our estimates, we can quantify the value of disclosure. First, we show that disclosure is beneficial for the bidder at the beginning of the entry period, but becomes detrimental toward the end. For a bidder with a median construction cost, the value of disclosure is XX% of the estimated project cost at the beginning, while the loss from disclosure is XX% at the end. The intuition behind this finding is that if a bidder enters early and discloses, they can deter entry from others, despite the fact that remaining entrants may bid more aggressively. In our scenario, the deterrence effect predominates. However, if a bidder enters late and discloses, the force of entry deterrence diminishes since there will not be many potential entrants left on the sideline. Consequently, the aggressive bidding from other entrants results in harm for the late-disclosing bidder. Next, stronger bidders who have smaller construction costs have larger values for disclosure. At the beginning of the entry period, the value of disclosure is XX% for a bidder whose cost is at the XX-th percentile , while the value is XX% for a bidder at XX-th percentile. This result shows that entry disclosure also acts as a signal for being a strong bidder.

[STILL CONSIDERING WHICH ONES TO PUT IN THE PAPER, ADD DISCUSSIONS ON EFFICIENCY regarding excess entry]: In our counterfactual experiment, we compare equilibrium auction outcomes under three alternative scenarios: (i) shutdown of the forum, where bidders cannot send out any signals to others; (ii) limiting the number of time points at which disclosures become public; and (iii) shutting down the forum while allowing the auctioneer to send a public signal based on the number of entrants. In scenario (i), we find that auctioneer's cost decreases by XX%, relative to the current platform. This implies that the effect of disclosure on entry deterrence dominates the effect on the entrants' bids. Hence, under our setting, availability of disclosure in the current form has negative consequences for the auctioneer. In scenario (ii), we find ???. This practice allows us to consider how disclosure affects bids from the entrants, while limiting the entry deterrence effect. redAND then ??? Finally, in scenario (iii), in contrast to the cases we have considered so far where the bidders get to exchange information, we allow the auctioneer to gather information about entry and then send out a signal. We find a huge decrease in the auctioneer's cost, a XX% reduction. Together, these results indicate that the impact of entry disclosure is substantial in this market. More broadly, how information about bidders is transmitted must be carefully considered when auctioneers design markets.

Related Literature The paper contributes to three literatures—the literature on strategic entry deterrence; the literature on selective entry into auctions; and the literature on empirical applications of information design.

The paper provides an empirical equilibrium analysis to test how strategic entry deterrence can affect market outcomes. While a significant amount of theoretical work has been carried out, e.g., Dixit (1979), Milgrom & Roberts (1982), and Farrell (1987), empirical work on this question is still limited. Goolsbee & Syverson (2008) and Sweeting et al. (2020) study how limit pricing by the incumbent affects entry behavior in the airline market. Morton (2000) studies the effect of incumbent's advertisement, and Ellison & Ellison (2011) studies strategic investment to deter entry in the pharmaceutical market. Ely & Hossain (2009) studies the effects of early period bidding in online auctions. Although they find a similar result to our paper that early period bidding deters entry but causes more aggressive bidding from the entrants, there are two important distinctions. First, Ely & Hossain (2009) tests for such effect by experimentally placing bids, while our analysis analyzes the effect of entry disclosure, which arises as an equilibrium outcome. Next, they study a second-price auction setup, while ours is a first-price auction. In second-price auctions, more aggressive bidding due to entry disclosure is not a pattern we would expect under a private-value framework, since bidding their own value would be an undominated strategy for the bidders. In contrast, entry disclosure may cause more aggressive bidding from others under our setup, sealed-bid first-price auctions with private values.

The paper also relates to the literature on selective entry into auctions. Ye (2007) and Quint & Hendricks (2018) theoretically studies indicative bidding; De Silva et al. (2008) studies the effect of releasing information about seller's valuation on bidding in procurement auctions; Krasnokutskaya & Seim (2011) studies how the introduction of bid preference program affects firms entry and bid decisions; and Gentry & Li (2014) studies non-parametric identification of an auction game with selective entry. The paper also studies a setting where there is selective entry, but is the first to study how entry disclosure can deter entry from others in first-price auctions with costly entry.

Finally, our paper also relates to the literature on empirical applications of information design. Vatter (2024) studies optimal scoring design in Medicare Advantage. Some papers have studied applying the idea of Bayes Correlated Equilibrium (BCE) (Bergemann & Morris 2013, 2016) to an empirical setting. Magnolfi & Roncoroni (2023), Hara et al. (2024), Gualdani & Sinha (2023), and Syrgkanis et al. (2021) study estimation of static games, dynamic games, discrete choice models, and auctions respectively, when the information structure is unknown. This approach would allow us to estimate the set of outcomes that could arise from the case where the auctioneer has access to the bidders' private types (construction costs). However, it is more natural to consider a case where the auctioneer only has access to bidders' entry. In this case, the optimal signal would take a form of recommending a bid schedule (a map from types to bids) (Bergemann & Morris 2019). We focus on a class of signals, a public binary signal, to provide a lower bound for what the auctioneer can attain through providing information in their favorable way.

2 Institutional Background and Data

2.1 Institutional Background

We describe the letting process of procurement auctions let by the Montana Department of Transporation (MDOT). MDOT uses sealed-bid first price auctions to award construction projects. The set of firms who participate in bidding will not get disclosed until the final result is revealed.

MDOT advertises projects three to four weeks prior to the bidding date. The project advertisement contains a detailed specification of what the project entails. On the same day as the advertisement, a Q&A forum opens up on MDOT's website. On this forum, firms can post questions about the project, and MDOT provides answers to the posted questions. Posted questions become publicly observable typically on the day it gets posted, subject to a review by the MDOT. Answers from MDOT are provided within two days in most cases. The forum shows the time at which the question got posted, the company's name, contact person, question, and answer to the question. FIGURE XX presents a screenshot of the forum. The forum closes three days before the bidding window closes. While other public procurement auctions also accept questions from the firms, the unique feature here is that this forum gets continuously updated along with identity of the firms who posted questions and a timestamp. Content of questions

To participate in bidding on a project, firms must prepare documents, which they are required to submit along with the bid, and engage in negotiations with subcontractors. Document preparation and these negotiations are a costly process, since it takes time and effort. Therefore, entry into auctions is costly. [2]

2.2 Data

Our data covers projects auctioned between January 2017 and December 2022. For each auction, the data include the description of projects, location, the engineer's es-

[2]Costliness of entry into procurement auctions have been pointed out in the literature (e.g., Li & Zheng (2009)).

timate of the total cost of the project, and the bids along with the identity of the firms. Moreover, our data include information from the Q&A forum: posted question, MDOT's answer, identity of the firm who posted the question, and the time question got posted. 592 projects were advertised during the sample period, while we focus on 434 projects whose construction reports were available, which allow us to identify the type of construction of the projects.

Table XX presents summary statistics of the auctions. The median engineer's estimate is around \$1.30 million, while the median winning bid is around \$1.22 million. In what follows, we will normalize the bids by the engineer's estimate. The median normalized winning bid is 3.5% lower than the engineer's estimate. MDOT may reject all the bids and 16 auctions experienced a rejection.[3] On average, we have three entrants. We define potential entrants as a firm who has at least once entered into an auction within the same district × type of construction pair during the sample period. A typical auction has 12 potential entrants. We see some variety in the types of projects, where the most popular type is projects on overlay (18%).[4] Projects are mostly equally spread across districts, while Great Falls has the largest share (26%).[5]

3 Model

In this section, we develop a model of a procurement auction with costly entry and the option to disclose entry. An auctioneer procures a project and holds a first-price auction. There are N potential bidders who may participate in bidding. We denote the set of potential bidders as $\mathcal{N} = \{1, \cdots, N\}$.

The model consists of two stages: (i) entry and disclosure; and (ii) bidding. In the first stage, firms sequentially arrive at the market randomly without knowing others' arrival timing. When they arrive at the market, firms observe the disclosures that have been made, make decisions on entry, and decide whether to disclose if they enter. After the first stage is completed, the entrants move on to the second stage, which is about

[3]If bids are rejected, the project may get revised and advertised in a future date.

[4]We follow the categorization of types of construction provided in the construction reports provided by MDOT. Some projects fall under multiple categories and if so we assign the project to the more popular type.

[5]We split the state into five districts, following the coverage of five MDOT district offices. See https://www.mdt.mt.gov/contact/organization/districts.aspx

Table 1: Summary Statistics

	Mean	Standard deviation	10th percentile	Median	90th percentile
Engineer's estimate ($000)	2,949	4,315	144	1,297	8,597
Lowest bid ($000)	3,022	4,702	154	1,225	8,382
Lowest bid / Engineer's estimate	1.021	0.314	0.750	0.965	1.320
#Entrants	2.82	1.50	1	3	5
#Potential entrants	12.44	5.62	4	12	20
#Questions	0.83	0.97	0	1	2
Type of projects	N	percent			
Bridge construction	51	11.8			
Overlay	78	18.0			
Reconstruction	46	10.6			
Safety	67	15.4			
Others	192	44.2			
Districts	N	percent			
Missoula	94	21.7			
Butte	76	17.5			
Great Falls	113	26.0			
Glendive	73	16.8			
Billings	78	18.0			

Note: Total number of projects is 434. There were 5 auctions without an entrant.

bidding. Firms observe the entire history on disclosures, and place bids simultaneously.

First stage An auction is announced and the Q&A forum, i.e., the disclosure device, becomes available at $t = 0$. The disclosure device closes at $t = T$, while disclosures will still be observable after its closure. Each potential bidder $i \in \mathcal{N}$ draws $\tau_i \in [0, T]$ from distribution F_τ, the time at which bidder i decides whether or not to participate in the auction. At $t = \tau_i$, firm i arrives at the market, observes the disclosure history h^{τ_i}, and draw their entry cost c_i^E from distribution F_E. Disclosure history h^t is public information and records the time at which questions are posted as well as the the identities of those who post, up to time t. We denote the set of all time-t histories as $\mathcal{H}^t$. The entry cost, c_i^E includes the cost of inspecting the project plan, assessing required material and labor for the project, negotiating with the subcontractors, and arriving at a cost estimate. Firm i may enter the auction by paying the entry cost c_i^E or stay out without paying anything. We denote the firm i's strategy on entry as: $\chi_{i,\tau}$: $\chi_{i,\tau}(h^\tau, c_i^E) \mapsto a_i^E \in \{0, 1\}$.

If firm i enters, at the same time $t = \tau_i$, i draws their construction cost c_i, and an opportunity to disclose entry arises. With probability p^Q, firm i faces a need to disclose and always discloses without paying any additional cost. While inspecting the project plan, there may be issues that prevent the firms from making progress in the process. This part reflects such case and assume that it happens with probability p^Q. With the other probability $1-p^Q$, firm i may engage in costly disclosures by paying a disclosure cost c_i^Q, which follows a distribution F_Q. This may be thought of as a cost to find an appropriate question, while it may also be a reputation cost. Disclosures can only be made by a firm who has entered. We denote the firm i's strategy on disclosure as: $\iota_{i,\tau}(h^\tau, c_i, c_i^Q) \mapsto a_i^Q \in \{0,1\}$. If firm i discloses, disclosure history h^τ gets updated accordingly.

Second stage After the forum closes at $t = T$, entrants, the firms who have entered, participate in bidding. The auction format is a sealed-bid first price auction. Before the firms place their bids, the entrants observe the entire disclosure history h^T. Given h^T, the entrants place their bids b_i simultaneously. We denote the firm i's strategy on bidding as: $b_i(h^T, c_i) \mapsto \mathbb{R}$. Firm i's payoff π_i is:

$$\pi_i = (b_i - c_i)\mathbb{1}\{i \text{ wins}\} - c_i^E a_i^E - c_i^Q a_i^Q.$$

Assumption on firms' types As described above, each firm i's types are characterized by the tuple $(\tau_i, c_i^E, c_i^Q, c_i)$. We assume that these four random variables are mutually independent, and draws across firms are independent and identically distributed.

Equilibrium We consider Perfect Bayesian Equilibrium of the game presented above. Equilibrium consists of firms' strategy profile $(\chi_{i,\tau}, \iota_{i,\tau}, b_i)$ such that:

1. firm i enters ($a_i^E = 1$) if and only if their expected profit from entry exceeds their entry cost c_i^E
$$\mathbb{E}\left[\pi_i | h^{\tau_i}, a_i^E = 1\right] > c_i^E$$
2. firm i costly discloses ($a_i^Q = 1$) if and only if their expected gain from entry ex-

ceeds their disclosure cost c_i^E

$$\mathbb{E}\big[\pi_i|h^{\tau_i}, a_i^E=1, a_i^Q=1\big]-\mathbb{E}\big[\pi_i|h^{\tau_i}, a_i^E=1, a_i^Q=0\big]>c_i^Q$$

3. firm i bids b_i that maximizes their expected profit conditional on the entire disclosure history h^T and construction cost c_i

$$b_i=\arg\max_b\,(b-c_i)\Pr(i\text{ wins}|h^T, b)$$

and consistent beliefs given the strategy profile. If there are multiple equilibria, we assume that one equilibrium is chosen and played.

4 Identification

In this section, we provide a discussion on identification of the model.

Step 1. Construction costs c_i and its distribution F_{c_i} First, we aim to identify the construction costs c_i of each bidder and its distribution F_{c_i}. The argument follows Guerre et al. (2000). We make an assumption on bidders' strategies as follows:

Assumption 1. *Firmi's bidding strategy is strictly increasing in their construction costs c_i, conditional on the entire disclosure history h^T.*

For each public history h^T, the bidder i's problem at the bidding stage is to maximize their expected value $V_i(h^T, c_i)$:

$$V_i(h^T, c_i)=\max_b(b-c_i)G_{-i}(b|h^T), \tag{4.1}$$

where $G_{-i}(b|h^T)$ denotes the distribution of the lowest rival bid, $\wedge\mathbf{b}_{-i}$, conditional on h^T. Note that $G_{-i}(\cdot|h^T)$ is nonparametrically identified, and hence $F_c(c_i|h^T)$ is identified. Moreover, $F_{c_i}(c)$ is identified by pooling across all realizations of h^T:

$$F_{c_i}(c)=\Pr(c_i\leq c)=\int F_{c_i}(c|h^T)dF_{\mathscr{H}^T}(h^T). \tag{4.2}$$

Note that the right-hand side of (4.2) is the probability that c_i is less than c without conditioning on h^T (but conditional on i bidding on the auction). The distribution, $F_{\mathscr{H}^T}$, is the distribution of time-T history h^T in $\mathscr{H}^T$.

Step 2. Beliefs on history evolution $h^{\tau_i} \to h^T$ Next, we identify the belief of firm i on time-T history h^T conditional on history at entry timing h^{τ_i} and disclosure a_i^Q. We denote such belief as $\mu_i(h^T | h^\tau, \tau_i = \tau, a_i^Q)$.

When firm i discloses, their belief is directly identified from the data. However, when firm i does not disclose, their belief cannot be directly identified from data because there is a selection issue due to the fact that their entry timing is not observed.

Assume symmetry: For the sake of simplicity, we provide an argument for the symmetric case. The proof that allows for asymmetry is given in the Appendix XX. The key idea of the proof is to create a simple mapping from observed evolution of disclosure histories to a firm's belief.

No assumption on symmetry: will move to appendix

Suppose that under h^τ, bidders $j_1, \ldots, j_J$ signals before i each at $\tau_{j_1}, \ldots, \tau_{j_J}$ ($\tau_{j_1} < \cdots < \tau_{j_J}$) , and for the rest of bidders $k_1, \ldots, k_K$, their signals are yet to be observed. Take τ_k such that $\tau_{j_m} < \tau_k < \tau$ holds for all m. Let

$$A_i^t(h, c_i) \equiv F_E^i(V_i^{\prime t}(h)) F_Q(\Delta v_i(h, c_i)) \tag{4.3}$$

We consider the following density P:

$$\begin{aligned} P &= \Pr\big(j_m \text{ signals at } \tau_{j_m}\ \forall m, i \text{ signals at } \tau, k_n \text{ does not signal before } \tau\ \forall n,\ \vec{c}_j, c_i\big) \\ &= \prod_m f_\tau^{j_m}(\tau_{j_m}) A_{j_m}^{\tau_{j_m}}(h^{\tau_{j_m}}(\tau_{j_1}, \ldots, \tau_{j_J}), c_{j_m}) f_{c_{j_m}}(c_{j_m}) \\ &\times \prod_n \left\{ 1 - F_\tau^{k_n}(\tau) + \int_0^\infty \int_0^\tau f_\tau^{k_n}(t) \Big(1 - A_{k_n}^t(h_t(\tau_{j_1}, \ldots, \tau_{j_J}), c_{k_n})\Big) f_{c_{k_n}}(c_{k_n}) dt\, dc_{k_n} \right\} \\ &\times f_\tau^i(\tau) A_i^\tau(h^\tau(\tau_{j_1}, \ldots, \tau_{j_J}), c_i) f_{c_i}(c_i) \end{aligned} \tag{4.4}$$

We consider the following density Q:

$$
\begin{aligned}
Q &= \Pr\big(j_m \text{ signals at } \tau_{j_m}\ \forall m, i \text{ does not signal before } \tau, k_n \text{ does not signal before } \tau\ \forall n,\ \vec{c}_j, c_i\big)\\
&= \prod_m f_\tau^{j_m}(\tau_{j_m}) A_{j_m}^{\tau_{j_m}}(h^{\tau_{j_m}}(\tau_{j_1},\dots,\tau_{j_J}), c_{j_m}) f_{c_{j_m}}(c_{j_m})\\
&\times \prod_n \left\{1 - F_\tau^{k_n}(\tau) + \int_0^\infty \int_0^\tau f_\tau^{k_n}(t)\Big(1 - A_{k_n}^t(h_t(\tau_{j_1},\dots,\tau_{j_J}), c_{k_n})\Big) f_{c_{k_n}}(c_{k_n}) d\,t\,d\,c_{k_n}\right\}\\
&\times \left\{1 - F_\tau^i(\tau) + \int_0^\infty \int_0^\tau f_\tau^i(t)\big(1 - A_i^t(h_t(\tau_{j_1},\dots,\tau_{j_J}), c_i)\big) f_{c_i}(c_i) d\,t\,d\,c_i\right\}
\end{aligned}
\tag{4.5}
$$

Taking the ratio between these two densities gives us:

$$
P/Q = \frac{f_\tau^i(\tau) A_i^\tau(h^\tau(\tau_{j_1},\dots,\tau_{j_J}), c_i) f_{c_i}(c_i)}{\left\{1 - F_\tau^i(\tau) + \int_0^\infty \int_0^\tau f_\tau^i(t)\big(1 - A_i^t(h_t(\tau_{j_1},\dots,\tau_{j_J}), c_i)\big) f_{c_i}(c_i) d\,t\,d\,c_i\right\}}
\tag{4.6}
$$

Exploiting the relation that

$$
\frac{\partial\big(1 - F_\tau^i(\tau) + \int_0^\infty \int_0^\tau f_\tau^i(t)\big(1 - A_i^t(h^\tau, c_i)\big) f_c(c_i) d\,t\,d\,c_i\big)}{\partial \tau} = \int_0^\infty f_\tau^i(\tau) A_i^\tau(h^\tau(\tau_{j_1},\dots,\tau_{j_J}), c_i) f_c(c_i) d\,c_i,
$$

the function

$$
\begin{aligned}
\Gamma_i(\tau; h^\tau = (\tau_{j_1},\dots,\tau_{j_J})) &= 1 - F_\tau^i(\tau) + \int_0^\infty \int_0^\tau f_\tau^i(t)\big(1 - A_i^t(h^\tau, c_i)\big) f_c(c_i) d\,t\,d\,c_i\\
&= 1 - \int_0^\infty \int_0^\tau f_\tau^i(t) A_i^t(h^\tau, c_i) f_c(c_i) d\,t\,d\,c_i
\end{aligned}
$$

is identified up to scale for all $\tau \in [\tau_{j_J}, T]$. Since $\Gamma_i(0; h^\tau = \phi)$=1 holds, $\Gamma_i(\tau; h^\tau = \phi)$ is identified. Therefore, $f_\tau^i(\tau) A_i^\tau(h^\tau = \phi, c_i) f_c(c_i)$ is identified for all $\tau \in [0, T]$.

Now, given that $f_\tau^i(\tau) A_i^\tau(h^\tau = \phi, c_i) f_c(c_i)$ is identified, $\Gamma_i(\tau; h^\tau = \tau)$ is identified. As a result, $\Gamma_i(t; h^t = \tau)$ $(t \geq \tau)$ such that h^t includes one disclosure at τ is identified. By induction on the number of disclosures made, repeating this argument will allow us to identify $\Gamma_i(t; h^\tau)$ for all histories h^τ. Note that $f_\tau^i(\tau) A_i^\tau(h^\tau, c_i) f_c(c_i)$ is also identified for all h^τ.

Let

$$
\begin{aligned}
R = & \prod_m f_\tau^{j_m}(\tau_{j_m}) A_{j_m}^{\tau_{j_m}}(h^{\tau_{j_m}}(\tau_{j_1}, \ldots, \tau_{j_{m-1}}), c_{j_m}) f_{c_{j_m}}(c_{j_m}) \\
& \times \prod_p f_\tau^{l_p}(\tau_{l_p}) A_{l_p}^{\tau_{l_p}}(h^{\tau_{l_p}}(\tau_{j_1}, \ldots, \tau_{j_J}, \tau_{l_1}, \ldots, \tau_{l_{p-1}}), c_{l_p}) f_{c_{l_p}}(c_{l_p}) \\
& \times \prod_n \Gamma_{k_n}(T; h^\tau = (\tau_{j_1}, \ldots, \tau_{j_J}, \tau_{l_1}, \ldots, \tau_{l_L}))
\end{aligned}
$$

and

$$
\begin{aligned}
S = & \prod_m f_\tau^{j_m}(\tau_{j_m}) A_{j_m}^{\tau_{j_m}}(h^{\tau_{j_m}}(\tau_{j_1}, \ldots, \tau_{j_{m-1}}), c_{j_m}) f_{c_{j_m}}(c_{j_m}) \\
& \times \prod_p \Gamma_{l_p}(\tau; h^\tau = (\tau_{j_1}, \ldots, \tau_{j_J})) \\
& \times \prod_n \Gamma_{k_n}(\tau; h^\tau = (\tau_{j_1}, \ldots, \tau_{j_J})).
\end{aligned}
$$

Since all the objects that appear in R and S are identified, R and S are identified.

Belief of i can be written as:

$$\Pr_i(h^T | h^\tau, \tau_i = \tau, A_i^Q = 0) = R/S$$

and since R and S are identified, this is also identified.

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Step 3. Value of disclosure We argue that the value of disclosure is identified. The expected value, $v_i^1(h^\tau, c_i)$, from posting a question at time τ and history h^τ when i's construction cost is c_i is simply

$$v_i^1(h^\tau, c_i) = \int V_i(h^T, c_i)\mu_i(h^T | h^\tau, \tau_i = \tau, a_i^Q = 1) d h^T, \tag{4.7}$$

where $V_i(h^T, c_i)$ is the value from the bidding stage and is given by expression (4.1). For each h^T, the expected value of bidder i with cost realization c_i is $V_i(h^T, c_i)$. We integrate $V_i(h^T, c_i)$ using the beliefs on distribution of possible time-T histories, $\mu_i(h^T | h^\tau, \tau_i = \tau, a_i^Q)$, to obtain the expected value. Similarly, the expected value, $v_i^0(h^\tau, c_i)$, from not posting a question at time τ and history h^τ when construction cost is c_i is

$$v_i^0(h^\tau, c_i) = \int V_i(h^T, c_i)\mu_i(h^T|h^\tau, \tau_i = \tau, a_i^Q = 0)dh^T. \tag{4.8}$$

Note that $V_i(h^\tau, c_i)$ is identified for all h^τ and c_i. Moreover, μ_i has been identified in Step 2. Hence, the right-hand side of equations (4.7) and (4.8) are all identified and $v_1(h^\tau, c_i)$ and $v_0(h^\tau, c_i)$ are both identified. We let $\Delta v(h^\tau, c_i)$ denote the value of disclosure, $\Delta v_i(h^\tau, c_i) \equiv v_i^1(h^\tau, c_i) - v_i^0(h^\tau, c_i)$. Therefore, the value of disclosure is identified.

Step 4. Probability of forced disclosure p^Q and distribution of disclosure costs F_Q
First, we make the following assumption:

Assumption 2. *Support of disclosure values is $[\underline{v^D}, \overline{v^D}]$. Firms always disclose at the upper bound of the disclosure value: $F_Q(\overline{v^D}) = 1$.*

The variation we exploit here is the difference in disclosure values across firms with different construction costs but those who are facing the same disclosure history. When a firm is not forced to disclose, the decision to post a question is given by the following expression:

$$\begin{cases} \iota_{i,\tau}(h^\tau, c_i, c_i^Q) = 1 & \text{if } \Delta v_i(h^\tau, c_i) \geq c_i^Q \\ \iota_{i,\tau}(h^\tau, c_i, c_i^Q) = 0 & \text{if otherwise} \end{cases} \tag{4.9}$$

The expected value of the auction $\tilde{v}(h^\tau, c_i)$ at time h^τ and costs c_i is:

$$\tilde{v}(h^\tau, c_i) = p^Q v_i^1(h^\tau, c_i) + (1 - p^Q)\mathbb{E}_{F^Q}\left[\max\left\{v_i^0(h^\tau, c_i), v_i^1(h^\tau, c_i) - c_i^Q\right\}\right]. \tag{4.10}$$

The first term corresponds to the case where firm i is forced to disclose. The first term inside the expectation bracket is the expected value from not disclosing, and the second term is the expected value from disclosing. The expected value of entry, $v_i(h^\tau)$ is then

$$v_i(h^\tau) = \mathbb{E}_{F_{c_i}}[\tilde{v}(h^\tau, c_i)]. \tag{4.11}$$

The decision to enter is given by the following expression:

$$\begin{cases} \chi_{i,\tau}(h^\tau, c_i^E) = 1 & \text{if } v_i(h^\tau) \geq c_i^E \\ \chi_{i,\tau}(h^\tau, c_i^E) = 0 & \text{if otherwise} \end{cases} \tag{4.12}$$

Fix $v', v'' \in \mathbb{R}$. Recalling that the expected gain from disclosure, $\Delta v(h^\tau, c_i)$, is identified for all h^τ and c_i. Now let us take c_i' and c_i'' appropriately so that $\Delta v(h^\tau, c_i') = v'$ and $\Delta v(h^\tau, c_i'') = v''$ for some h^τ. The density that a type c_i' discloses at h^τ is as follows:

$$\begin{aligned} f\left(a_i^Q = 1, \tau_i = \tau, h^\tau, c_i'\right) &= f_{\mathscr{H}^\tau}(h^\tau | c_i', \tau_i = \tau)\, f_\tau(\tau)\, f_{c_i}(c_i')\, F_E(v_i(h^\tau))\, \left(p^Q + (1-p^Q)F_Q(v')\right) \\ &= f_{\mathscr{H}^\tau}(h^\tau | \tau_i = \tau)\, f_\tau(\tau)\, f_{c_i}(c_i')\, F_E(v_i(h^\tau))\, \tilde{F}_Q(v') \end{aligned} \tag{4.13}$$

where $f_{\mathscr{H}^\tau}$, f_τ and f_c are the density of $F_{\mathscr{H}^\tau}$, F_τ and F_c, respectively. Also, we denote $\tilde{F}_Q(v) = \left(p^Q + (1-p^Q)F_Q(v)\right)$. The first term on the right-hand side of (4.13) is the probability that event h^τ occurs conditional on τ_i being equal to τ, and the second term is the probability that τ_i is equal to τ. The third term is the probability that the cost draw is c_i'. The fourth term corresponds to the entry probability. Finally, The last term is the probability of disclosure ($a_i^Q = 1$) conditional on h^τ and $c_i = c_i'$, which is equivalent to the probability that (i) firm is forced to disclose; or (ii) firm is not forced and the cost of posting a question, c_i^Q, is lower than v', i.e., $c_i^Q \leq \Delta v_i(h^\tau, c_i') (= v')$.

Similarly, the density that a type c_i'' discloses at h^τ is as follows:

$$\Pr\left(a_i^Q = 1, \tau_i = \tau, h^\tau, c_i''\right) = f_{\mathscr{H}^\tau}(h^\tau | \tau_i = \tau)\, f_\tau(\tau)\, f_{c_i}(c_i'')\, F_E(v_i(h^\tau))\, \tilde{F}_Q(v'') \tag{4.14}$$

Because construction costs c_i are identified for all entrants from Step 1, the left-hand side of expressions (4.13) and (4.14) are both identified. Moreover, $f_c(c_i')$ and $f_c(c_i'')$ are both identified because $F_{c_i}(c)$ is identified. Hence, from the ratio of expressions (4.13) and (4.14), we identify $\tilde{F}_Q(v')/\tilde{F}_Q(v'')$. Because F_Q is a distribution, p^Q and F_Q are identified.[6]

[6] If $\tilde{F}_Q(v')/\tilde{F}_Q(v'')$ is identified for all $v', v'' \in \mathbb{R}$, it implies that $\tilde{F}_Q(v)$ is identified up to a constant, say, $\tilde{F}_Q(0)$. This is because we can express $\tilde{F}_Q(v)$ as follows: $\tilde{F}_Q(v) = \tilde{F}_Q(0)\left(\tilde{F}_Q(v)/\tilde{F}_Q(0)\right)$, where the ratio $\left(\tilde{F}_Q(v)/\tilde{F}_Q(0)\right)$ is identified. There is a unique value of $\tilde{F}_Q(0)$ such that $\lim_{v \to \overline{v^D}} F_Q(v) = 1$.

Step 5. Value of entry As we have seen in (4.15), value of entry $v_i(h^\tau)$ can be expressed as:

$$\begin{aligned} v_i(h^\tau) &= \mathbb{E}_{F_{c_i}}[\tilde{v}(h^\tau, c_i)] \\ &= \int \tilde{v}(h^\tau, c_i) dF_{c_i}(c_i) \\ &= \int\int p^Q v_i^1(h^\tau, c_i) + (1-p^Q)\mathbb{E}_{F^Q}\left[\max\left\{v_i^0(h^\tau, c_i), v_i^1(h^\tau, c_i) - c_i^Q\right\}\right] dF_Q dF_{c_i} \end{aligned}$$

Since all the objects that appear in this expression are identified objects, value of entry $v_i(h^\tau)$ is also identified.

Step 6. Distribution of entry costs F_E and entry timing F_τ In this final step, we aim to identify the final two distributions: entry cost and entry timing. The idea is to exploit variation in value of entry and value of disclosure across firms facing different disclosure histories.

Suppose that under h^τ, bidders $j_1, \ldots, j_J$ signals before i each at $\tau_{j_1}, \ldots, \tau_{j_J}$ ($\tau_{j_1} < \cdots < \tau_{j_J}$) , and for the rest of bidders $k_1, \ldots, k_K$, their signals are yet to be observed. Take τ_k such that $\tau_{j_m} < \tau_k < \tau$ holds for all m. Let

$$A_i^t(h, c_i) \equiv F_E^i(V_i^t(h)) F_Q(\Delta v_i(h, c_i)) \tag{4.15}$$

We consider the following density P:

$$\begin{aligned} P &= \Pr\Big(j_m \text{ signals at } \tau_{j_m}\ \forall m, i \text{ signals at } \tau, k_n \text{ does not signal before } \tau\ \forall n,\ \vec{c}_j, c_i\Big) \\ &= \prod_m f_\tau^{j_m}(\tau_{j_m}) A_{j_m}^{\tau_{j_m}}(h^{\tau_{j_m}}(\tau_{j_1}, \ldots, \tau_{j_J}), c_{j_m}) f_{c_{j_m}}(c_{j_m}) \\ &\times \prod_n \left\{1 - F_\tau^{k_n}(\tau) + \int_0^\infty \int_0^\tau f_\tau^{k_n}(t)\Big(1 - A_{k_n}^t(h_t(\tau_{j_1}, \ldots, \tau_{j_J}), c_{k_n})\Big) f_{c_{k_n}}(c_{k_n}) dt\, dc_{k_n}\right\} \\ &\times f_\tau^i(\tau) A_i^\tau(h^\tau(\tau_{j_1}, \ldots, \tau_{j_J}), c_i) f_{c_i}(c_i) \end{aligned} \tag{4.16}$$

We consider the following density Q:

$$
\begin{aligned}
Q &= \Pr\big(j_m \text{ signals at } \tau_{j_m}\ \forall m, i \text{ does not signal before } \tau, k_n \text{ does not signal before } \tau\ \forall n,\ \vec{c}_j, c_i\big) \\
&= \prod_m f_\tau^{j_m}(\tau_{j_m}) A_{j_m}^{\tau_{j_m}}(h^{\tau_{j_m}}(\tau_{j_1},\ldots,\tau_{j_J}), c_{j_m}) f_{c_{j_m}}(c_{j_m}) \\
&\times \prod_n \left\{1 - F_\tau^{k_n}(\tau) + \int_0^\infty \int_0^\tau f_\tau^{k_n}(t)\Big(1 - A_{k_n}^t(h_t(\tau_{j_1},\ldots,\tau_{j_J}), c_{k_n})\Big) f_{c_{k_n}}(c_{k_n}) d\,t\, d\,c_{k_n}\right\} \\
&\times \left\{1 - F_\tau^i(\tau) + \int_0^\infty \int_0^\tau f_\tau^i(t)\big(1 - A_i^t(h_t(\tau_{j_1},\ldots,\tau_{j_J}), c_i)\big) f_{c_i}(c_i) d\,t\, d\,c_i\right\}
\end{aligned}
\tag{4.17}
$$

Taking the ratio between these two densities gives us:

$$
P/Q = \frac{f_\tau^i(\tau) A_i^\tau(h^\tau(\tau_{j_1},\ldots,\tau_{j_J}), c_i) f_{c_i}(c_i)}{\left\{1 - F_\tau^i(\tau) + \int_0^\infty \int_0^\tau f_\tau^i(t)\big(1 - A_i^t(h_t(\tau_{j_1},\ldots,\tau_{j_J}), c_i)\big) f_{c_i}(c_i) d\,t\, d\,c_i\right\}}
\tag{4.18}
$$

Exploiting the relation that

$$
\frac{\partial(1 - F_\tau^i(\tau) + \int_0^\infty \int_0^\tau f_\tau^i(t)\big(1 - A_i^t(h^\tau, c_i)\big) f_c(c_i) d\,t\, d\,c_i)}{\partial \tau} = \int_0^\infty f_\tau^i(\tau) A_i^\tau(h^\tau(\tau_{j_1},\ldots,\tau_{j_J}), c_i) f_c(c_i) d\,c_i,
$$

the function $\Gamma_i(\tau; h^\tau = (\tau_{j_1},\ldots,\tau_{j_J})) = 1 - F_\tau^i(\tau) + \int_0^\infty \int_0^\tau f_\tau^i(t)\big(1 - A_i^t(h^\tau, c_i)\big) f_c(c_i) d\,t\, d\,c_i$ is identified up to scale for all $\tau \in [\tau_{j_J}, T]$. And thus

$$
\begin{aligned}
\frac{\partial \Gamma_i}{\partial \tau}(\tau; (\tau_{j_1},\ldots,\tau_{j_J})) &= \int_0^\infty f_\tau^i(\tau) A_i^\tau(h^\tau(\tau_{j_1},\ldots,\tau_{j_J}), c_i) f_c(c_i) d\,c_i \\
&= f_\tau^i(\tau) F_E^i(V_i^\tau(h^\tau(\tau_{j_1},\ldots,\tau_{j_J}))) \int_0^\infty F_Q(\Delta v_i(h^\tau(\tau_{j_1},\ldots,\tau_{j_J}), c_i)) f_c(c_i) d\,c_i
\end{aligned}
$$

is identified up to scale. Since $\Gamma_i(0; h^\tau = \phi)$=1 holds, $\Gamma_i(\tau; h^\tau = \phi)$ is identified. Therefore, $\frac{\partial \Gamma_i}{\partial \tau}(\tau; h^\tau = (\phi))$ is identified for all $\tau \in [0, T]$. Since F_Q is identified, $f_\tau^i(\tau) F_E^i(V_i^\tau(h^\tau = \phi))$ is identified for all $\tau \in [0, T]$.

Now, given that $f_\tau^i(\tau) F_E^i(V_i^\tau(\phi))$ is identified, $\Gamma_i(\tau; h^\tau = \tau)$ is identified. As a result, $f_\tau^i(\tau) F_E^i(V_i^\tau(h^\tau))$ such that h^τ includes one disclosure is identified. By induction on the number of disclosures made, repeating this argument will allow us to identify $f_\tau^i(\tau) F_E^i(V_i^\tau(h^\tau))$ for all histories h^τ.

In order to identify f_τ, fix $\tau', \tau'' \in [0, T]$ and take $h^{\tau'}$ and $h^{\tau''}$ appropriately so that $v_i(h^{\tau'}) = v_i(h^{\tau''}) = v$ for some constant $v \in \mathbb{R}$. We identify the ratio, $f_\tau(\tau')/f_\tau(\tau'')$ from the ratio of $f_\tau^i(\tau)F_E^i(V_i^\tau(h^\tau))$ and $f_\tau^i(\tau')F_E^i(V_i^{\tau'}(h^{\tau'}))$. Since F^τ is a distribution, F^τ is identified.

Finally, since F^τ and $f_\tau^i(\tau)F_E^i(V_i^\tau(h^\tau))$ are identified, F^E is also identified.

5 Estimation

In this section, we provide an outline of the estimation procedure, which closely follows the identification argument.

5.1 Parametric assumptions

Although we have provided a non-parametric identification result, we impose parametric assumptions to take our model to data. First, we assume that firms are *ex-ante* symmetric conditional on auction-level characteristics: all the firms share the same distribution for entry timing, entry costs, disclosure costs, and construction costs if the auction is the same construction type and from the same district.

In what follows, we will set $T = 1$. We assume that the distribution of entry timing follows a Beta distribution with two shape parameters α_τ and β_τ. Next, we make the following assumption on entry costs. With probability p^E, each firm gets a chance to consider whether they would enter an auction, while a firm always stays out with the other probability $1 - p^E$. This reflects the fact that firms may face various constraints, such as running other projects. When they consider entering, they draw an entry cost c_i^E, which is from a truncated normal distribution on $[0, \infty)$ with parameters μ_E and σ_E. Here, we parameterize $mu_E = X_a \beta^E + \alpha^E$, where X_a is the logarithm of number of potential entrants. Finally, we assume that the distribution of disclosure costs F_Q follows a truncated normal distribution with parameters mu_Q and σ_Q. Note that we have also assumed that firms are in a position where they must disclose with probability p^Q.

5.2 Estimation procedure

We estimate our parameters in 4? steps. In the first step, we start by estimating the construction costs for each entrant and the distribution of such costs, exploiting the bidding results. Next, we estimate firms' beliefs on how disclosure history evolves over time, conditional on their disclosure action. Given the estimates from the bidding stage and estimated beliefs on disclosure history, we turn to the estimation on the value of disclosure and entry. Finally, using the obtained estimates, we estimate our model primitives via maximum likelihood.

Step 1. Construction costs c_i To account for the fact that some bids ultimately get rejected, we assume that there is a secret reserve price p^r and it follows a log-normal distribution. To estimate the construction costs c_i, we exploit the optimality of the bids as in Guerre et al. (2000). Construction cost c_i when the bid is b_i is estimated exploiting the first order condition for bidding:

$$c_i = b_i - \frac{1 - G_{-i}(b)}{g_{-i}(b)} \tag{5.1}$$

where G_{-i} is the CDF of the lowest bid among the opponents and g_{-i} is the corresponding pdf.[7]

We assume that G_{-i} follows a log-normal distribution log-normal (μ_b, σ_b) with:

$$\mu_b = \mathbf{X}_{\mathbf{i}}^{\mu_{\mathbf{b}}} \beta^{\mu_{\mathbf{b}}}, \qquad \sigma_b = \mathbf{X}_{\mathbf{i}}^{\sigma_{\mathbf{b}}} \beta^{\sigma_{\mathbf{b}}},$$

where $X_i^{\mu_b}$ includes a dummy for whether i disclosed, time at which i disclosed, number of others' disclosures, number of potential bidders, construction type dummies, and district dummies. For the variance, $X_i^{\sigma_b}$ includes number of others' disclosures, number of potential bidders, construction type dummies, and district dummies. We estimate $(\beta^{\mu_b}, \sigma^{\mu_b})$ via maximum likelihood. Once we obtain the estimates for the distribution G_{-i}, we exploit (5.1) and estimate construction costs for each entrant.

Step 2. Belief on the evolution of disclosure history Closely following the identification argument, we start by estimating the observed evolution of disclosure histories. Let us note here again that the observed evolution of disclosure histories and the beliefs on the histories are different. We parameterize the distribution of the length of

[7] This will be the minimum of the opponents' bid and the secret reserve price.

time between the n-th and $n+1$-th disclosure as follows (time between $t=0$ and the first disclosure will be also included as case $n=0$):

$$\Pr(\tau^{n+1}-\tau^{n}\leq x)=\frac{\Phi((x-\mu_t)/\sigma_t)-\Phi(-\mu_t/\sigma_t)}{1-\Phi(-\mu_t/\sigma_t)}$$

if the $(n+1)$-th disclosure does not exists and

$$\Pr((n+1)\text{-th disclosure does not exist})=\frac{1-\Phi(((1-\tau^{n})-\mu_t)/\sigma_t)}{1-\Phi(-\mu_t/\sigma_t)}$$

where timing of the n-th disclosure is given by τ^n. [8] The parameters (μ_t,σ_t) are characterized as:

$$\mu_t=\mathbf{X}^{\mu_t}\beta^{\mu_t},\qquad \sigma_t=\mathbf{X}^{\sigma_t}\beta^{\sigma_t},$$

where X^{μ_t} includes n-th disclosure timing τ^n, number of disclosures n, log of (number of firms who have not disclosed yet +1), construction type dummies, and district dummies. For the variance, X^{σ_t} includes number of disclosures n, and log of (number of firms who have not disclosed yet +1). We estimate $(\beta^{\mu_t},\sigma^{\mu_t})$ via maximum likelihood.

We turn to the estimation of the beliefs of the firms. First, we estimate the belief of a firm when the firm discloses at some time τ facing history h^τ. In order to estimate this object, we simulate the evolution of the disclosure history, using the distribution of the length of time between disclosures we have estimated above. We denote the estimated belief as: $\hat{\mu}_i(h^T|h^\tau,\tau_i=\tau,a_i^Q=1)$.

Next, we estimate the firm's belief when the firm enters but does not disclose at some time τ facing history h^τ. Suppose that the latest disclosure before τ is at τ^n. If there was none, let $\tau^n=0$. We exploit the following relationship:

$$\begin{aligned}&\Pr(\tau^{n+1}>t \text{ or } (n+1)\text{-th disclosure does not exist})^{(M-1)/M}\\&=\Pr_i(\tau^{n+1}>t \text{ or } (n+1)\text{-th disclosure does not exist})\end{aligned}$$

where M is the number of potential entrants who have not disclosed at τ. This relationship allows us to back out the distribution of the time between disclosures from firm i's perspective. We simulate the evolution of disclosure history using i's belief on

[8] Here, we are implicitly assuming that the decision of disclosures depend on the number of disclosures up to that time and the timing of the most recent disclosure.

the timing of disclosures. We denote the estimated belief as: $\hat{\mu}_i(h^T|h^\tau, \tau_i = \tau, a_i^Q = 0)$.

Step 3. Value of disclosure In this step, we aim to obtain an estimate for the value of disclosures. First, we start by estimating the value from the bidding stage $V(h^T, c)$. In what follows, all the estimated objects are estimated for all type of construction × district pairs. We estimate this value by:

$$\hat{V}_i(h^T, c) = \max_b (b - c)(1 - \hat{G}_{-i}(b)).$$

Next, we estimate the value with and without disclosure, $v^1(h^\tau, c)$ and $v^0(h^\tau, c)$. This value is estimated by:

$$\hat{v}^j(h^\tau, c) = \sum_{h^T} \hat{V}_i(h^T, c)\, \hat{\mu}_i(h^T|h^\tau, \tau_i = \tau, a_i^Q = j)$$

for $j = 0, 1$. Then, value of disclosure $\Delta v(h^\tau, c)$ can be estimated as:

$$\widehat{\Delta v}(h^\tau, c) = \hat{v}^1(h^\tau, c) - \hat{v}^0(h^\tau, c).$$

Step 4. Model primitives In our final step, we estimate our model primitives, distribution of entry timing F_τ, entry costs F_E and p^E, and disclosure costs F_Q and p^Q using entry and disclosure data. Note that if have an estimate $\hat{F}^Q$ for F_Q, value of entry $v(h^\tau)$ can be estimated as:

$$\begin{aligned}
\hat{v}(h^\tau) &= \sum_c \int p^Q v^1(h^\tau, c) + (1 - p^Q)\mathbb{E}_{F^Q}\left[\max\left\{v^0(h^\tau, c), v^1(h^\tau, c) - c^Q\right\}\right] d\hat{F}_Q \\
&= \sum_c \Big[\big(p^Q + (1 - p^Q) F_Q(\max\{v^1(h^\tau, c) - v^0(h^\tau, c), 0\})\big)\, v^1(h^\tau, c) \\
&\quad + (1 - p^Q)\big(1 - F_Q(\max\{v^1(h^\tau, c) - v^0(h^\tau, c), 0\})\big)\, v^0(h^\tau, c) \\
&\quad - \int_0^{\max\{v^1(h^\tau, c) - v^0(h^\tau, c), 0\}} c^Q \, d\hat{F}_Q \Big].
\end{aligned}$$

where summation is taken over the estimated costs c.

Using this expression, we estimate the model primitives via maximum likelihood.

Suppose agents $i_1,\dots,i_I$ does not enter, agents $j_1,\dots,j_J$ enters but does not disclose, and agents $k_1,\dots,k_K$ enters and discloses, under some history h^T. The likelihood function for observing this history is:

$$\begin{aligned}
&= \prod_l \int f_c(c_{i_l})dc \int (1-F_E(v(h^t)))dt \\
&\times \prod_m f(c_{j_m}) \int f_\tau(t) F_E(v(h^t))(1-F_Q(\Delta v(h^t, c_{j_m})))dt \\
&\times \prod_n f(c_{k_n}) f(\tau_{k_n}) F_E(v(h^{\tau_{k_n}})) F_Q(\Delta v(h^{\tau_{k_n}}, c_{k_n}))
\end{aligned}$$

6 Estimation Results

This section discusses the results from the estimation of the parameters in the model.

6.1 Parameter Estimates

Table 2 presents the estimation results for the model parameters. Figure 1 shows the CDF of firms' arrival timing. Firms are more likely to arrive at the latter half of the entry period, and 70% of the firms arrive at the latter half. The median arrival time is 0.71, which corresponds to around a week before the forum closes.

Figure 2 shows the relationship between values of entry and entry probability for the case where we have 12 potential bidders, which is the median size of entrants' pool. Firms consider entry with 28% probability when there are 12 potential bidders. This probability decreases with the number of potential bidders. Median size of entry cost is 3.4% of the engineer's estimate. Our estimate of entry costs is comparable with the numbers obtained in the literature (Bajari et al. 2010, Krasnokutskaya & Seim 2011).

Figure 3 shows the relationship between values of disclosure and disclosure probability. Firms gets in need for posting a question so that they always disclose with 27% probability, which means that firms disclose with this probability if disclosure does harm to them. As value of disclosure increases, disclosure probability also increases. For example, when the value of disclosure is 1% of the estimated cost, firms disclose with 31% probability.

Table 2: Estimated parameters of the model

Distribution	c_E: Truncated Normal on $[0,\infty)$ c_Q: Truncated Normal on $[0,\infty)$ τ: Beta	
	Estimate	S.E.
Entry		
Prob. of considering entry: p^E		
Const.	0.851	0.146
ln(# Pot bidder)	-0.231	0.093
μ_E	-2.926	0.120
σ_E	0.383	0.182
Disclosure		
Prob. of always disclosing: p^Q	0.268	0.139
μ_Q	-2.416	0.501
σ_Q	0.642	1.102
Timing		
α_τ	1.227	0.314
β_τ	0.661	0.227

Note: Table presents estimates of the model parameters. Standard errors are calculated using 100 bootstrap draws, with sampling at the auction level.

We summarize the estimated distributions of construction costs in Table 3. We report the median, 25-th and 75-th percentiles as a fraction of the engineer's estimate for each construction type. The median cost is estimated to be 74–89% of engineer's estimate. Overall, projects related to overlay/reconstruction have higher construction costs than the other project types.

Table 4 shows estimation results on the distribution of opponents' best bids. Note that it is the opponents' best bid that determines one's profits in a first price auction. First, opponents' best bid becomes stronger as a firm faces more disclosures (questions). This reflects the fact that disclosures are made by the *actual* entrants. Next, the results suggest that making disclosures at earlier periods weakens opponents' best bid, if we hold others' disclosure behavior fixed. To understand the impact, we compare two cases: (i) Firm X discloses at $t = 0$, while no other firm discloses; and (ii) No firm discloses. If firm X places the median bid ($b_i = 1.03$), firm X's winning probability is 11.4 p.p. higher under case (i) for an auction on an overlay project in the Missoula district. On the other hand, the estimates suggest that a last minute disclosure

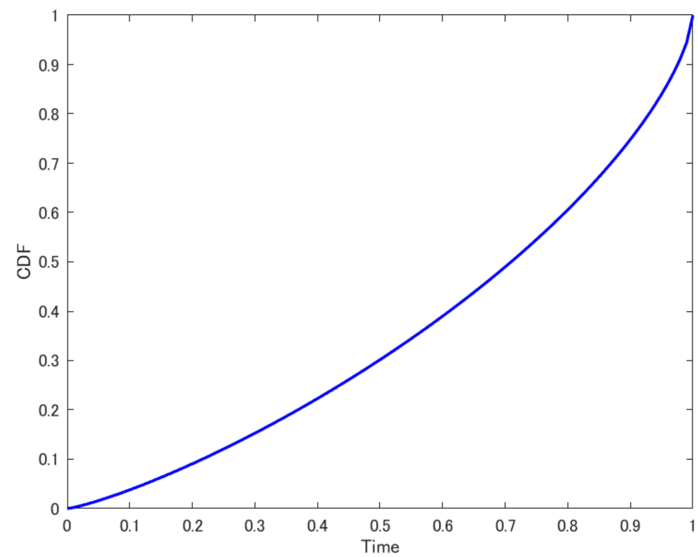

Figure 1: CDF of arrival timing

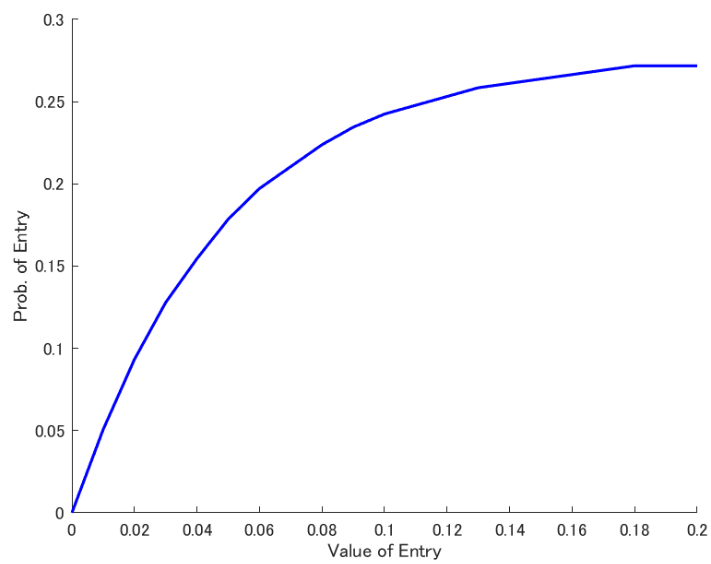

Figure 2: Value of entry and entry probability

Notes: This figure shows the relationship between values of entry and entry probability for the case where we have 12 potential bidders, which is the median size of entrants' pool.

Table 3: Distribution of Construction Costs: by Construction Types

Construction Types	25-th percentile	Median	75-th percentile
Bridge	0.38	0.75	1.02
Overlay	0.74	0.88	1.01
Reconstruction	0.72	0.89	1.11
Safety	0.50	0.74	0.99
Others	0.56	0.74	0.97

Note: Total number of projects is 434. There were 5 auctions without an entrant. The presented numbers are fractions of the engineer's estimate.

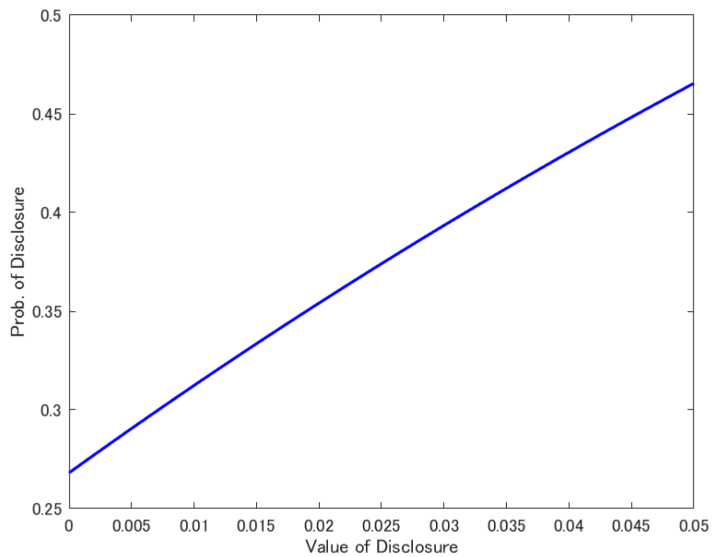

Figure 3: Value of disclosure and disclosure probability

Notes: This figure shows the relationship between value of disclosure and disclosure probability.

would strengthen opponents' best bid, though the effect is not significant. These results are in line with our discussion on the trade-off of entry disclosure. As we have discussed, firms may decrease the number of entrants by disclosing, while the bids from other entrants may become more aggressive. The results suggest that the former effect dominates for disclosures made during earlier periods, while the latter dominates when disclosures are made at the last minute, at a time close to $t = 1$. These results do not incorporate the fact that firms' disclosures may affect others' disclosure behaviors. Next, we consider the value of disclosures, taking the evolution of disclosure history into account and allowing the firm to optimize their bids. In what follows, the figures

Table 4: Distribution of opponents' best bid: Log-normal

Variables	Estimate	S.E.
μ		
Constant	-0.023	0.067
Asked	0.044	0.027
Asked $\times\tau$	-0.045	0.033
ln(# Pot. Bidders)	-0.017	0.034
# Q from others	-0.037	0.010
Type:		
Overlay	0.066	0.030
Safety	0.018	0.050
Bridge	0.207	0.054
Recons	0.102	0.044
Others	Reference	
District:		
Missoula	Reference	
Butte	-0.027	0.028
Great Falls	0.050	0.029
Glendive	0.035	0.032
Billings	-0.022	0.043
$\log\sigma$		
Constant	-1.232	0.144
ln(# Pot. Bidders)	-0.018	0.082
# Q from others	-0.207	0.039
Type:		
Overlay	-0.596	0.100
Safety	0.043	0.134
Bridge	0.274	0.140
Recons	-0.240	0.139
Others	Reference	
District:		
Missoula	Reference	
Butte	-0.148	0.092
Great Falls	-0.149	0.091
Glendive	-0.028	0.094
Billings	0.320	0.123

Note: This table presents estimated parameters of the distribution of opponents' best bid. The opponents' best bid is defined as the minimum of the opponents' bid and the secret reserve price.

we present in this section will based on the auctions that are on overlay projects from the Missoula district, which is the mode for the number of auctions across (type of construction, district)-pairs and has the median number of potential bidders, 12.

Figure 4 shows the value of disclosure by time and firms' construction costs. The values shown in the figure are for the case where there is no disclosure up to the corresponding time. This result suggests that the value of disclosure decreases over time. For a bidder with median construction cost ($c = 0.86$), the value is 1.5% of the engineer's estimate at $t = 0$. As we move to a later period, the value decreases to 0.8% at $t = 0.5$, and it turns to a loss of 0.04% at $t = 1$. This decreasing pattern is observed across different values of construction costs. At a given timing, we see that value of disclosure is decreasing in construction costs except for the cases where the timing is close to the end $t = 1$. For example, at $t = 0$, firms whose construction cost is at the 25-percentile ($c = 0.75$) would gain 2.1% of the engineer's estimate, while 1.5% at the median cost ($c = 0.86$) and 0.7% at the 75-percentile cost ($c = 0.98$). This pattern suggests that the stronger entrants with low construction costs are more likely to disclose in most cases. As a result, the disclosures will also carry information about the strength of the bidders, not only information about firms' entry. The entrants who have disclosed are more likely to be a strong bidder with low construction costs. Finally, the value of disclosure at $t = 1$ turns out to be negative. At the very end of the entry period, entry disclosure cannot deter entry from others and makes the other entrants to bid more aggressively. Therefore, firms who disclose at the last minute would incur a loss.

Next, we consider the value of entry for the firms. Figure 5 shows the value of entry by time and number of disclosure. The values shown in the figure are for the case where the most recent disclosure is made at $t = 0$ if there is one. Value of entry increases as we move to a later period if we hold the number of disclosures fixed. If there are no disclosures, value of entry is 9.8% of the engineer's estimate at $t = 0$, while it increases up to 10.5% at $t = 1$. As we would expect, information that there is no disclosures up to their arrival time is more valuable at a later period. At a fixed time point, having more disclosures decreases firms' value of entry. Having one disclosure decreases a firm's entry value by 1.3–1.4 % of the engineer's estimate, relative to the case where there is no disclosure. This corresponds to a 4–5% decrease in entry probability. Having another disclosure further decreases entry value by 1.1–1.2% of the engineer's estimate. Again, This corresponds to a 5–6% decrease in entry probability.

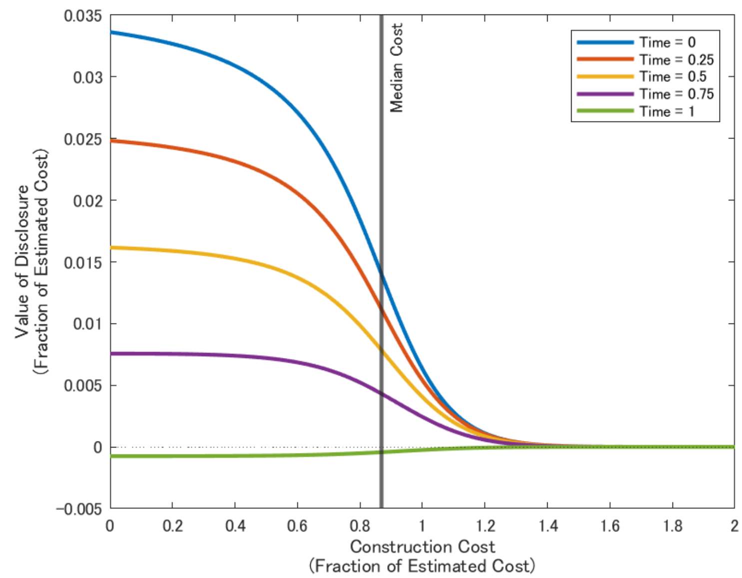

Figure 4: Value of disclosure and disclosure probability

Notes: This figure shows the relationship between values of disclosure and disclosure probability. The results are based on auctions on overlay projects from the Missoula district. The values are for the case where there is no disclosure up to the corresponding time.

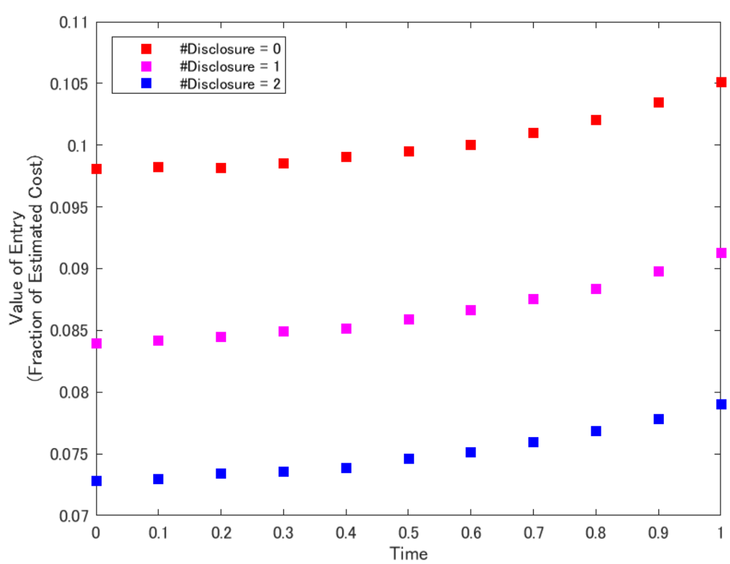

Figure 5: Value of entry

Notes: This figure shows the relationship between values of entry and entry timing, across different numbers of disclosures. The results are based on auctions on overlay projects from the Missoula district. The values are for the case where the most recent disclosure is made at $t = 0$ if there is one.

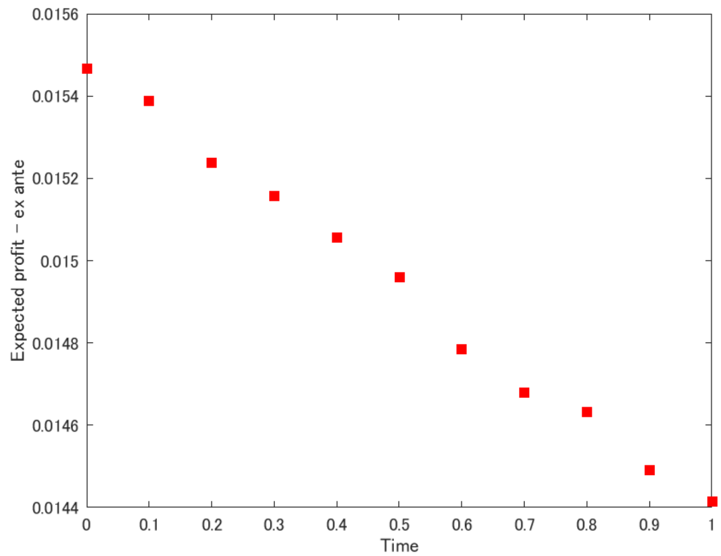

Figure 6: *Ex-ante* value of arrival timing

Notes: This figure shows the values of arrival time, from an *ex-ante* perspective. The results are based on auctions on overlay projects from the Missoula district.

Finally, we consider the value of arrival timing. Figure 6 presents values of entry from an *ex-ante* perspective. The value presented is the value of arrival time *unconditional* on whether firms enter. Arriving at $t = 0$ has an *ex-ante* value of 1.55% of the engineer's estimate, while arriving at $t = 1$ has 7% lower value, 1.44% of the engineer's estimate. There are two effects in play that alters the values of arrival timing. Arriving early allows a firm to enter into the auction and deter others' entry by making a disclosure. However, the firm will face a larger uncertainty in the number of entrants. If a firm arrives late, the firm has an informational advantage from the available disclosures so that they may be able to stay away from an auction that would be inefficient for them to enter. In our setup, arriving early turns out to be valuable, which means that the possibility of entry deterrence through disclosures has an impact that dominates the informational gains from arriving late.

7 Counterfactual Analysis

In this section, we use our model and estimates to evaluate the performance of the current platform design, the Q&A forum, relative to alternative platform designs. We simulate how auction outcomes, such as the auctioneer's payment conditional on the project getting allocated, winner's construction cost, and entry behavior, would change under different designs of how questions would be treated. To understand the role of entry disclosure, we run three counterfactuals, summarize in Table **??**. Let us again emphasize that entry disclosure impacts the auction outcomes through two channels: (i) entry deterrence – entry disclosure lowers value of entry for those who arrive at the market after the disclosure is made and thus may deter entry from those firms; and (ii) additional information at the bidding stage – a firm's disclosure may make other entrants bid more aggressively.

Table 5: Description of the Counterfactuals

Counterfactual	Description	Entry Deterrence	Additional Info at Bid
(0) Shutdown	Q&A never becomes public		
(1) Last minute disclosure	Q&A revealed publicly at $t = 1$ No info provided during $t \in [0, 1)$		✓
(2) Status quo	Current Q&A forum	✓	✓

The first counterfactual, (0) Shutdown, corresponds to the case where we shut down the Q&A forum. Under this case, the firms will privately communicate with the auctioneer if they have any questions about the project, and those questions never become public. Therefore, the option to disclose entry is not available, and there would be no entry deterrence nor additional information provided at the bidding stage. The next counterfactual, (1) Last minute disclosure, corresponds to the case where Q&A forum becomes public after $t = T (= 1)$ but before the bidding window closes. Under this case, firms can still ask questions but their entry would get disclosed after the entry period ends. This means that firms' entry disclosures do not deter entry, while additional information about firms' entry status is provided to the market. Our final counterfactual, (2) Status quo, corresponds to the case where we have the current Q&A forum held by MDOT. Since the disclosures become immediately available after getting posted, they may deter entry from others. Moreover, disclosures provide information at the bidding stage to the entrants. Note that our empirical estimates suggest that stronger entrants who have lower construction costs are more likely to disclose. This result means that disclosures provide information on entry as well as information on the strength of the entrants. In what follows, we simulate outcomes for auctions on overlay projects, which is the most popular type across auctions let by MDOT.

7.1 Last Minute Disclosure

First, we discuss the auction outcomes under counterfactual (1), where the Q&A forum becomes public after $t = 1$. In the equilibrium we have estimated, no firm engages in costly disclosures. All the disclosure made here are from firms who are in need for questions due to exogenous reasons, and as a result 27% of the entrants end up disclosing. Figure 7 provides estimated changes in auction outcomes relative to our benchmark case (0) Shutdown. Moving from a no disclosure environment (0) to counterfactual (1), auctioneer's payment decreases by 2.8%. This corresponds to a decrease by $36,000 for a median-sized project. On the other hand, we see a loss in efficiency in terms of the winner's construction cost. Winner's construction cost increases by 1.0%, corresponding to a $8,700 increase for a median-sized project. In terms of entry, we see a decrease in the number of entrants by 1.3% (0.04 entrants) and 3.8% decrease in the total entry cost.

Through disclosures, firms are giving up their information rents since their own entry is originally private information for them. In this counterfactual, some entrants are forced to give up their information rents since they are in need for asking a question through the forum. As a result, auctioneer's payment goes down. Although we see a decrease in the number of entrants in equilibrium, which would move the payment upwards, this effect from the information rent dominates.

Another factor that plays an important role here is asymmetry among the bidders. Note that in our benchmark case, entrants are in a symmetric position since there is no additional information for them. As long as the firms employ monotone and symmetric strategies, the winner is the firm with the lowest construction cost. This observation does not hold in our counterfactual case (1). Although firms are *ex-ante* symmetric, entrants would be placed in two different positions: disclosed and not disclosed. For example, let us consider a case where there are two entrants X and Y: firm X have disclosed entry, while Y have not. From X's perspective, there is still uncertainty about Y's entry. However, Y is certain about X's entry. As a result, Y will employ a stronger bidding strategy than X's strategy. Therefore, there would be some cases where Y wins even if Y has a weaker type, a larger construction cost, than X, resulting in inefficiency

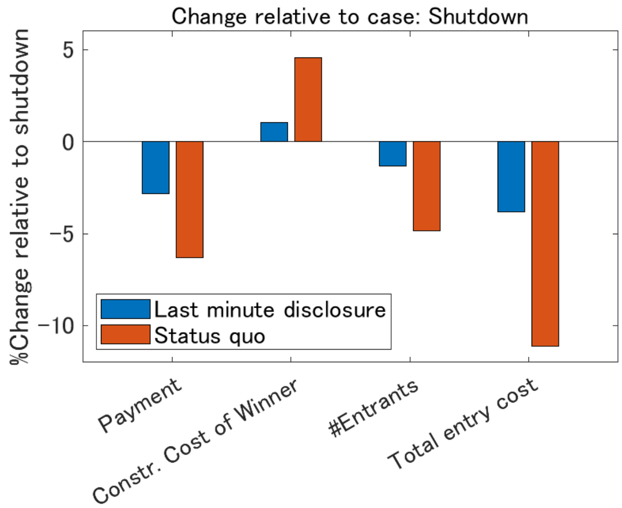

Figure 7: Changes in auction outcomes under different counterfactuals

Notes: Figure shows auction outcomes under different counterfactuals. The first two bars show the change in auctioneer's payment from scenario (0) to counterfactuals (1) and (2), respectively. The next two bars show changes in winner's construction cost. The rest of the bars show changes in the number of entrants and total entry costs.

in terms of the winner's construction cost. Moreover, we see a decrease in the number of entrants in equilibrium, which further pushes up the winner's construction cost.

We see a decrease in number of entrants and total entry cost. Since we see a decrease in auctioneer's payment and an increase in winner's construction cost, value of entry goes down in equilibrium. As a result, number of entrants goes down. While this decrease in the number of entrants would counteract against the decrease in payments, this force is not large enough to flip the sign. Finally, note that total entry cost decreases by a larger fraction than the number of entrants because the firms who are marginal here are the firms who have the largest entry costs among the entrants.

7.2 Status quo

Next, we describe the auction outcomes under counterfactual (2), where we are in the status quo with the current Q&A forum. As we have seen in Section 6, disclosure is more valuable in earlier periods than later periods. It turns out that one-thirds of the firms who enter during the first half of the entry period discloses, and about one-fifth of those disclosure are made strategically by paying a disclosure cost. Moreover, we have shown that a disclosure tend to be more valuable for the firms with low construction cost. We find that, overall, the firms who disclose have 1.5% lower construction cost than the firms who do not disclose. Therefore, disclosures also act as a signal for strength of the firms.

The estimated changes in auction outcomes in counterfactual (2), status quo, relative to our benchmark case, (0) Shutdown, is presented in Figure 7. By introducing the Q&A forum, corresponding to moving from our benchmark (0) to counterfactual (2), auctioneer's payment decreases by 6.3%, which is a larger decrease than counterfactual (1). This corresponds to a decrease by $82,000 for a median-sized project. As in counterfactual (1), we see a loss in efficiency in terms of the winner's construction cost relative to our benchmark scenario (0). Winner's construction cost increases by 4.5%, corresponding to a $38,000 increase for a median-sized project. This change is again larger than the change in counterfactual (1). We also see a decrease in the number of entrants by 4.9% (0.15 entrants) and 11.1% decrease in the total entry cost.

Disclosure carries information in two dimensions in this counterfactual. In addition to its information on entry, it contains information on firms' strength as well. This is because stronger firms are more likely to disclose their entry status. Therefore, to deter entry from others, firms are giving up their information rents that stems from their private information on entry status *and* construction cost. This additional component of information further moves the auctioneer's payment down.

Entrants would be placed into an asymmetric position in this counterfactual as well. Let us consider the same example, there are two entrants X and Y: firm X have disclosed entry, while Y have not. From X's perspective, there is still uncertainty about Y's entry. Moreover, X now knows that Y would be a relatively weaker firm since disclosure is more likely to be made by stronger firms. On the other hand, Y is certain about X's entry and holds a belief that X is a relatively stronger firm. As a result, X is tempted to place a weaker bid than the case where X has no information, while Y is tempted to place a stronger bid. Therefore, this environment creates a larger gap in these firms' strategies, which gives the firm with larger construction cost more chances to win the auction. As a result, we see a larger efficiency loss in the winner's construction cost than in counterfactual (1).

We see a large decrease in number of entrants and total entry cost. Relative to counterfactual (1), we see a further decrease in auctioneer's payment and an increase in winner's construction cost. Therefore, value of entry further goes down in equilibrium. As in counterfactual (1), the force that pushes the payment upwards stemming from the decrease in the number of entrants is not strong enough to flip this pattern.

In summary, the availability of this entry disclosure device, the Q&A forum, forces the firms to engage in deterring others' entry through disclosures. The key component in this setup is that firms differ in a new type dimension, which is the arrival time. Firms try to take advantage of this new type dimension – when they are assigned a strong type, i.e., get to arrive early, they can deter entry from others through an entry disclosure. As a result, we see a loss in efficiency in terms of the winner's construction cost, since the costs are now not the only dimension of firms' type that affects their strategies at the bidding stage. Moreover, because firms give up their information rents through disclosures to exploit their advantage in arrival time, auctioneer's payment goes down. Bernheim (1984) has pointed out that the effect of the possibility of strategic entry deterrence on market concentration is ambiguous in a setup where firms sequentially arrive at the market. In our setup, the entry disclosure device decreases the firms' expected profit from entry and as a result, we see less entrants.

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