Lecture Notes for Advanced Linear Algebra - Macauley

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1 Introduction

1. Vector Spaces
2. Linear Maps
3. MultiLinear Forms
4. Spectral Theory
5. Inner Product Spaces
6. Self-Adjoint Mappings
7. Positive Linear Maps

2 Vector spaces

2.1 Vector spaces and linearity

2.2 Spanning, independence, and bases

2.3 Direct sums and products

2.4 Quotient spaces

2.5 Dual vector spaces

2.6 Annihilators

3 Linear maps

3.1 Rank and nullity

3.2 Applications of the rank-nullity theorem

3.3 Algebra of linear maps

3.4 The four subspaces

3.5 The transpose of a linear map

3.6 Matrices

3.7 Change of basis

4 Multilinear forms

4.1 Determinant prerequesites

4.2 Symmetric and skew-symmetric multilinear forms

4.3 Alternating multilinear forms

4.4 Determinants of linear maps

4.5 The determinant and trace of a matrix

4.6 Minors and cofactors

4.7 Tensors

5 Spectral theory

5.1 Eigenvalues and eigenvectors

5.2 The Cayley-Hamilton theorem

5.3 Generalized eigenvectors

5.4 Invariant subspaces

5.5 The spectral theorem

5.6 Generalized eigenspaces

5.7 Jordan canonical form

5.8 Generalized eigenvectors of differential operators

5.9 Rational canonical form

6 Inner product spaces