Lecture Notes for Discrete Maths

Emulie Chhor

May 2, 2021

1 Introduction

The Study of Discrete Maths can be divided in a few sections:

1. Logic and Quantifiers
2. Set Theory
3. Proofs
4. Number Theory and Cryptography
5. Induction and Recursion
6. Counting and Combinatorics
7. Discrete Probability
8. Graph Theory and Trees

2 Logic and Quantifiers

2.1 Why do we care about logic?

Learning about logic is useful for several reasons. First of all, studying logic is the backbone of all mathematical proofs done in advance maths. Furthermore, logic can also be applied in other areas. For instance, in computer science, we can use propositional logic to simplify digital circuits. In philosophy, we use logic to make sound argument.

2.2 Meta for Logic

The chapter of logic is structured as follows:

1. Define some definition/properties
2. Translating Statement to Mathematical Statement using these symbols
3. More Problems using these logical operators

2.3 Propositional Logic

The goal of this section is to determine wether a statement is true or false using truth table or propositional logic.

We also want to translate sentence into a propositional statement or with its quantifiers.

2.3.1 What is a Statement?

A statement is a proposition that can either be true or false, but not both. It links propositional variables with logical operators. A statement can be atomic or compound depending on the number of ideas it connects.

2.3.2 Logical Operators

As stated earlier, there are a few logical operators that can be used to link propositional variables together. Each of these operators are associated with a truth table that tells us the truth value of a proposition based on the value of its propositional variables.

Here is a list of the logical Operators:

1. Conjonction
2. Disjunction (inclusive or)
3. Exclusive Or
4. Negation
5. Implication
6. Biconditional

TODO: Add Truth Table for logical Operators

2.3.3 Propositional Equivalence

When we search for propositional equivalence, we want to determine wether or not two or more statement have the same propositional value, regardless of the value of the propositional variables. To determiner propositional equivalence, we use truth table and determine if there is a tautology ie if both truth table are the same.

2.3.4 Translating Sentences into Propositional Statement

2.4 Quantifiers

Another important concept in logic is quantifiers. Quantifiers tells us wether a proposition holds true for all elements or some elements.

The most used quantifiers are the following:

1. Universal Quantifier
2. Existential Quantifier
3. Uniqueness Quantifier

2.4.1 Negating a quantifier

2.4.2 Translating Statement in Statement with Quantifiers

2.5 Inference

The third concept in logic is the notion of inference. Using some premisses, we want to determine wether a conclusion holds true or not using inference rules.

There are a few inference rules:

1. Modus Ponens
2. Modus Tollens
3. Hypothetical Syllogism
4. Disjunctive Syllogism
5. Addition
6. Simplification
7. Conjunction
8. Resolution

2.6 Proofs Techniques

When we want to prove a statement, there is a few proofs techniques we can use. Depending on the structure of the theorems, we may want to use one method over another.

Here is a list of the proofs techniques:

1. Direct Proofs
2. Contraposition
3. Contradiction
4. Induction

2.6.1 More Tips

Althought most of the proofs follows the structure mentionned above, we can use additional tricks to make our proofs a little easier

1. Exhaustive Proofs and Proofs by Cases
2. Without Loss Of Generality
3. Counter-Example
4. Existence Proofs

3 Set Theory

4 Number Theory and Cryptography

5 Induction and Recursion

6 Counting and Combinatorics

7 Graph Theory and Trees