FINTECH 545 HW2

YUQI WANG

yw596

PROBLEM 1

There are three methods of return calculation, assuming $r_t \sim N(0, \sigma^2)$ and P_{t-1}:

1. Classical Brownian Motion

$$P_t = P_{t-1} + r_t$$
$$P_t \sim N(P_{t-1}, \sigma^2)$$

2. Arithmetic Return System

$$P_t = P_{t-1}(1 + r_t)$$
$$P_t \sim N(P_{t-1}, (P_{t-1}\sigma)^2)$$

3. Log Return or Geometric Brownian Motion

$$P_t = P_{t-1}e^{r_t}$$
$$P_t \sim LN(\mu + \ln(P_{t-1}), \sigma^2) \text{ and } \mu = 0$$
$$P_t \sim LN(\ln(P_{t-1}), \sigma^2)$$
$$E[P_t] = e^{\ln(P_{t-1})+\frac{1}{2}\sigma^2} = P_{t-1}e^{\frac{1}{2}\sigma^2}$$
$$Std[P_t] = e^{\ln(P_{t-1})+\frac{1}{2}\sigma^2}\sqrt{e^{\sigma^2} - 1} = P_{t-1}e^{\frac{1}{2}\sigma^2}\sqrt{e^{\sigma^2} - 1}$$

Simulating each return equation 1000 times respectively, assuming $P_{t-1} = 0$, $\sigma = 0.2$:

	Expected $E[P_t]$	Observed $E[P_t]$	Expected $Std[P_t]$	Observed $Std[P_t]$
Brownian	100	100.0057	0.2	0.1999
Arithmetic	100	100.5661	20	19.9970
Geometric	102.0201	102.5957	20.6098	20.6799

The observed mean and standard deviation match the expectations.

PROBLEM 2

Given the current price of META is $299.08, calculate VaR using different models:

	%5 $VaR
Normal distribution	16.2055
Normal distribution with ew-var(λ = 0.94)	9.4649
MLE fitted T distribution	12.9007
Fitted AR(1) model	16.2207
Historic Simulation	11.8089

Normal Distribution:

This method assumes a symmetric, bell-shaped curve. However, financial returns often exhibit fat tails and excess kurtosis, which may not be well-captured by a normal distribution. The higher VaR might be a result of underestimating extreme events.

Normal Distribution with Exponentially Weighted Variance:

The exponentially weighted variance attempts to give more weight to recent observations. This could be useful in capturing changing volatility over time. The lower VaR suggests that recent observations have a greater impact on the variance.

MLE Fitted T Distribution:

The t-distribution is more robust to outliers and fat tails compared to the normal distribution. The moderate VaR value might be a result of the heavier tails in the t-distribution.

Fitted AR(1) Model:

The autoregressive model (AR(1)) incorporates the serial correlation in the data. If there's a strong autocorrelation, it can lead to higher VaR values, as it captures the persistence of returns.

Historic Simulation:

Historic simulation uses past data to estimate the VaR. It captures the historical distribution of returns. The moderate VaR might be a result of the method considering a range of historical scenarios.

In summary, the differences in VaR values are reasonable given the assumptions and characteristics of each method. It's important to note that the choice of model and method depends on the underlying characteristics of the financial data.

PROBLEM 3

I use 3 methods mentioned in class to calculate 5% $VaR of the Portfolio.

Port/Method	PV	Normal Monte Carlo $VaR	Delta Normal $VaR	Historical $VaR
A	1089316.16	14178.28	15446.10	17065.30
B	574542.41	7621.99	8094.60	10983.46
C	1387409.51	16338.76	18194.04	22186.52
Total	3051268.07	35018.05	39008.47	47618.78

Discussion:

1. Normal Monte Carlo Method:
   Method Description: Utilizes a Monte Carlo simulation assuming normal distribution of stock returns to generate multiple scenarios and calculates the portfolio's value at risk.
   Results: Provided a quantitative estimate of potential losses under normal market conditions.
   This method takes into account the non-linear relationship between stock prices and portfolio value. The lowest VaR might be due to its ability to model a wide range of market conditions.

2. Delta Normal Method:
   Method Description: Linear approximation based on the delta, representing the sensitivity of the portfolio value to small changes in stock prices.
   Results: Offers a quick and straightforward approach but relies on the assumption of normality and linear relationships between stock prices and portfolio value.
   This linear approximation may not fully capture the complexities of the portfolio, especially during extreme market movements.

3. Historical VaR:
   Method Description: Employs historical data to directly observe past portfolio value changes and calculates the VaR based on the historical distribution.

Results: Captures the impact of actual market behaviors but assumes future market conditions will resemble the past.
The highest VaR result may be attributed to its empirical realism, capturing extreme events and tail risks present in historical data. However, during periods of significant market changes or structural shifts, relying solely on historical observations might overstate potential risks.

In summary, the choice of VaR model can significantly impact risk estimates. It's essential to consider the assumptions and characteristics of each method, and often a combination of models can provide a more comprehensive risk assessment.