2.2 Theoretical background of an HTS bearing

In this section, we describe the physics of an HTS bearing.

2.2.1 Rotational loss

Even though there is no physical contact between a rotor and stator, several physical mechanisms contribute to the friction of an HTS bearing, including hysteresis friction and eddy current friction.

Hysteresis loss

Time varying magnetic field in a type II superconductor creates hysteresis loss. Bean [30] describes that the energy loss due to the hysteresis loss scales as

$$\Delta E \propto \frac{(\Delta B)^3}{J_c}, \tag{2.1}$$

where ΔB is the peak-to-peak variation of the time varying magnetic field in a superconductor, and J_c is the critical current of a superconductor.

In a context of HTS bearings, ΔB arises from azimuthal inhomogeneity of the magnetic field about the axis of rotation due to imperfection in a fabrication process of a single ring-shape rotor. When a ring magnet consists of segmented magnets in azimuth, the joint between adjacent magnets also creates the azimuthal inhomogeneity of the magnetic field. As a rotor magnet rotates, this spatial magnetic field inhomogeneity becomes time varying magnetic field with respect to the stationary HTS tile. When the rotor wobbles during its rotation, the radial-, vertical-, or tilt-mode of the magnet vibration also creates time varying magnetic field in a superconductor.

The magnetic field variation ΔB depends on the quality of the magnet and the relative distance between the rotor magnet and the array of HTS tiles. When the magnetization of a rotor magnet has a temperature dependence, the temperature of the rotor magnet also affects the magnitude of ΔB.

The hysteresis loss is inversely proportional to critical current J_c. A superconductor has higher critical current as temperature of the superconductor decreases.

Therefore, the COF is expected to decreases as temperature of the HTS tiles decreases. Zeiberger et al. reported that the critical current of a bulk YBCO sample increases by factor of $20 \sim 30$ from 77 K to 4 K when the externally applied magnetic field is in the range of $0 - 1$ T [31].

The energy loss due to the hysteresis is related to the deceleration as

$$\begin{aligned} \frac{\Delta E}{\Delta t} &= \tau_D \omega \\ &= I_m \alpha \omega \\ \therefore \quad \alpha &= \Delta E \frac{I_m}{2\pi}, \end{aligned} \tag{2.2}$$

where I_m is the moment of inertia of a rotor, and ω and α is angular speed and angular acceleration, respectively. Equation 2.2 shows that the hysteresis loss appears to the deceleration as frequency independent term.

Eddy current loss

Any time varying magnetic field ΔB in an electrically conductive material induces an EMF and therefore eddy current. This eddy current dissipates as Joule heat. This process of eddy current loss contributes as friction of a rotating magnet. The energy loss in unit time due the eddy current [32, 33] is

$$P = Fv \tag{2.3}$$

$$\propto \sigma(\Delta B)^2 \omega^2. \tag{2.4}$$

Therefore, with the same argument as Equation 2.2, the eddy current loss contributes to the deceleration as an angular speed, ω, dependent term. Sources of the time varying magnetic field are the same as the case we discuss for the hysteresis loss. One additional source of the time varying magnetic field is the trapped magnetic field in the HTS tiles. The stator HTS is not a continuous ring, but an array of HTS tiles. Therefore, the field that is trapped in each tile has higher concentration of the flux at the center of the tile and the trapped magnetic field decays as approaching to the edge of the HTS tile. As the rotor magnet rotates above the array of HTS tiles, the spatially fixed magnetic field with respect to each HTS tile becomes time varying field in the rotor magnet and creates friction due to the eddy current interaction.

The eddy current loss is proportional to the electrical conductivity of surrounding metals. Equation 2.4 indicates that it is best to construct hardware with non-electrically conductive material, such as G-10 and vespel.

When the operational temperature of the HTS bearing changes from LN_2 temperature to LHe temperature, the electrical conductivity of material changes correspondingly. The electrical conductivity of metals tends to increase as the temperature of metal increases. For aluminum 6061, the resistance ratio, $\frac{\rho(77K)}{\rho(4K)}$, is ~ 1.2, and OFHC copper has $\frac{\rho(77K)}{\rho(4K)} \sim 23$ [34]. An OFHC copper is often used to maximize the thermal conductivity at cryogenic temperature. When the OFHC copper is used around the HTS bearing, the eddy current loss is expected to increase as the operational temperature decreases.

On the other hand, the eddy current loss due to the finite electrical conductivity of the rotating magnet may cause not only to increase the COF, but also to increase the temperature of the levitating magnet. This is because the rotor is thermally isolated except through radiative heat exchange. When NdFeB is used as a rotor magnet, the electrical conductivity of NdFeB at room temperature is about 5 order of magnitude lower than that of Al at the same temperature. This effect adds the heat input to the levitating rotor magnet in addition to the absorption from the optical load.

The eddy current loss is a frequency dependent loss, and therefore the contribution to the COF may be small when the rotation frequency is low.

Coefficient of friction

Hull et al. [35] proposed to quantify the coefficient of friction (COF) of an HTS bearing as the ratio of drag force F_D to lift force F_L as

$$COF = \frac{F_D}{F_L} \tag{2.5}$$

$$= \alpha \frac{I}{MgR_D} \tag{2.6}$$

The drag force is $F_D = \tau_D/R_D$, where R_D is the outer radius of the rotor and τ_D is drag torque due to friction. The drag torque is calculated from the measured angular deceleration as $\tau_D = I_m\alpha$, where I_m is the total moment of inertia of a rotor and α ($\alpha < 0$) is the acceleration of a rotor magnet. The lift force is $F_L = Mg$, where M is the mass of a rotor and g is the deceleration of gravity.

2.2.2 Stiffness

The stiffness of a levitating HTS bearing is quantified by a spring constant due to the analogy of a spring system. Hull shows the analytical relationship of the spring constant to the superconducting levitating system by using the frozen flux model [24]. The derivation assumes a dipole magnet that levitates above an infinite plane of a type II superconductor.

When a dipole with the magnetization m is placed at the distance z above a type II superconductor with FC, the flux due to the Meisner effect and trapped flux at a pinning center can be treated as two images, diamagnetic image and frozen image. The diamagnetic image is a mirror image of the levitating dipole at the distance z from the interface of the superconductor. The frozen image in the superconductor appears at the same location as the diamagnetic image, but the direction of the magnetization differs by 180 degrees from the magnetization of the diamagnetic image.

We treat these two images as external magnetic field sources and calculate the magnetic interaction between the external field and the real dipole above superconductor. We label these two images as $\vec{B}^{dia}$ and $\vec{B}^{frozen}$. If we assume that the magnetization direction of the levitating dipole is normal to the plane of a superconductor as $\vec{m} = m\hat{z}$, the spring constant can be written as

$$\begin{aligned} k_z &= -\frac{\partial F_z}{\partial z} \\ &= -m(4\frac{\partial^2 B_z^{dia}}{\partial z^2} + \frac{\partial^2 B_z^{frozen}}{\partial z^2}) \qquad (2.7) \\ k_x &= -\frac{\partial F_x}{\partial x} \\ &= -m\frac{\partial^2 B_x^{frozen}}{\partial x \partial z} \qquad (2.8) \end{aligned}$$

$$
\begin{aligned}
k_y &= -\frac{\partial F_y}{\partial y} \\
&= -m\frac{\partial^2 B_y^{frozen}}{\partial y \partial z}.
\end{aligned}
\tag{2.9}
$$

If we substitute the explicit expression of the magnetic field B, such as for a dipole, it is easy to show that the three spring constants relate as

$$
k_x + k_y - k_z = 0. \tag{2.10}
$$

This is analogue to Earnshaw's theorem of a type II superconductor when the superconductor is field-cooled. Equation 2.10 shows that the spring constants, k_x, k_y, k_z, can be all positive values simultaneously. Therefore, the field cooled levitating magnet is stable in all three directions. When a ring shape magnet is levitating above an array of HTS tiles, a levitating element has a symmetry in the x and y directions, and therefore $k_x = k_y \equiv k_r$.

$$
2k_r - k_z = 0. \tag{2.11}
$$

When there is a spring constant to each degree of freedom, there is a corresponding natural frequency as

$$
f_z = 2\pi\sqrt{\frac{k_z}{M}} \tag{2.12}
$$

$$
f_r = 2\pi\sqrt{\frac{k_r}{M}}, \tag{2.13}
$$

where M is the total mass of a rotor. When rotation frequency of the rotor coincides with a natural frequency, two frequencies resonate. As a consequence, the rotor wobbles unstably and the COF increases [23]. It is important to design the stiffness of the HTS bearing such that the natural frequency is away from the rotation frequency.

Equations 2.7, 2.8, 2.9 show that the spring constant is proportional to m^2 and the magnetic field geometry. Therefore, a HTS bearing becomes stiffer when a stronger magnet is used. Also, a special configuration of a magnet is proposed to increase the second derivative of the magnetic field and therefore to increase the stiffness with a given magnetization (e.g. see [36]). These conclusions indicate that the stiffness of an HTS bearing does not depend on temperature unless the magnetic properties of the levitating magnet depends on temperature.

2.2.3 Damping property

When a levitating magnet is forced to displace either in the radial or vertical directions from its equilibrium position instantaneously, the magnet oscillates. The oscillation decays as a function of time due to a damping property of the HTS bearing.

The mechanism of the damping is the same as the energy loss in the COF, the hysteresis loss and the eddy current loss. The advantage of the damping property of the HTS bearing is that the damping becomes stronger when the displacement of the rotor magnet from its equilibrium position becomes bigger. This is because the hysteresis loss and the eddy current loss are both as a function of magnetic field variation, $(\Delta B)^3$ and $(\Delta B)^2$, respectively. The damping property tends to be dominated by the hysteresis loss because the rotor magnet and the HTS tiles are in close proximity.

When the operational temperature changes from LN_2 to LHe temperature, the primary concern is the reduction of the damping property due to the hysteresis loss in the superconductor. When we consider the COF due to the hysteresis loss, the COF decreases as the temperature of the HTS decreases. On the other hand, the damping decreases as the temperature of HTS decreases.

2.3 Hardware of prototype SMB

The magnet and the HWP are the rotor of a magnetic bearing that is levitated above a ring of YBCO HTS materials. The sintered NdFeB magnet has an inside radius of 2.54 cm, an outside radius of 3.56 cm, thickness of 1.2 cm, and mass of 0.2 kg. It is magnetized in the axial direction and has a remnance of $\sim 11 \times 10^3$ gauss and an energy product of 30×10^6 gauss-oersted. The moments of inertia of the HWP and magnet are 83 and 1910 gr$\cdot$cm^2, respectively. A HWP holder, made of Delrin and with a gear at its outer circumference, holds the magnet/HWP combination together and is part of the rotor, see Figure 2.1.

The magnet is held at an appropriate distance above a ring of HTS tiles which consists of 12 pieces of melt-textured YBCO [35]. The distance is a free parameter and is typically between 4 to 10 mm. Two clamps, each resembling a plier, hold the rotor in place during the cool-down of the system. A vacuum rotary feed-through that is mounted outside of the cryostat rotates a shaft and a pair of cams is used to open and close the clamps. The HTS tiles and the clamps are mounted on a G-10 board.

Rotation of the rotor is achieved by means of a half-gear that is driven by a second vacuum feed-through. During cool-down the half-gear and the gear at the outer circumference of the rotor are engaged. Once the system has cooled to 4 K, the clamps are opened and the half-gear is turned. If the need arises to re-rotate the rotor, the half-gear can be slowly engaged with the rotor, and the process repeats.

We tested all the mechanical components of the HTS bearing at liquid nitrogen, liquid helium and intermediate temperatures. Measurements between 4.2 and 77 K were conducted in a liquid He cryostat (LHC) in which the stator was mounted on a 0.9 cm thick copper cold-plate. The temperature of one of the HTSs is monitored by calibrated silicon diode and carbon resistor thermometers attached to a point on its perimeter at mid-height. The temperature of the HTS was controlled with heating resistors. The rotation rate was measured by an LED and a photodiode together with a reflective tape attached on the rotor.