1. $T(1) = 1,\ T(n) = 4T(\frac{n}{2}) + n$

$SM \Rightarrow T(n) \leq n^2 = O(n^2)$

$IA: T(1) = 1 \leq 1^2 = O(n^2)$

IV: Die Behauptung gilt auch für $2n$

$IS: T(2n) = 4T(\frac{2n}{2}) + 2n = 4T(n) + 2n$

$\overset{IV}{\leq} 4n^2 + 2n = O(n^2)$

2. $T(1) = 1,\ T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

$IM \Rightarrow T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

$= 2(2T(\frac{n}{16}) + \sqrt{\frac{n}{4}}) + \sqrt{n}$

$= 2(2(2T(\frac{n}{64}) + \sqrt{\frac{n}{16}}) + \sqrt{\frac{n}{4}}) + \sqrt{n}$

$\vdots$

$= 2^i T(\frac{n}{4^i}) + \sum_{k=0}^{i-1} 2^k \sqrt{\frac{n}{4^k}}$

$= 2^i T(\frac{n}{4^i}) + \sqrt{n} \cdot \sum_{k=0}^{i-1} \frac{2^k}{\sqrt{4^k}}$

$= 2^i T(\frac{n}{4^i}) + \sqrt{n} \cdot \sum_{k=0}^{i-1} \frac{2^k}{2^k}$

$= 2^i T(\frac{n}{4^i}) + \sqrt{n} \cdot \sum_{k=0}^{i-1} 1$

Rekursionsbasis bei $\frac{n}{4^i} = 1$ erreicht: setze $i = \log_4 n$

$T(n) = 2^{\log_4 n} \cdot T(\frac{n}{4^{\log_4 n}}) + \sqrt{n} \cdot \sum_{k=0}^{\log_4 n - 1} 1$

$= 2^{\log_4 n} T(\frac{n}{n}) + \sqrt{n} \cdot (\log_4 n)$

$= 2^{\log_4 n} + \sqrt{n} \log_4 n$

$\overset{NR}{=} 2^{\frac{\log_2 n}{2}} + \sqrt{n} \log_4 n$ $\qquad$ $NR: \log_4 n = \frac{\log_2 n}{\log_2 4} = \frac{\log_2 n}{2}$

$= 2^{\log_2 n^{\frac{1}{2}}} + \sqrt{n} \log_4 n$

$= n^{\frac{1}{2}} + \sqrt{n} \log_4 n$

$= \sqrt{n} + \sqrt{n} \log_4 n = \Theta(\sqrt{n} \log n)$

3. $T(1) = 1,\ T(2) = 1,\ T(3) = 1,\ T(n) = 2T(n-1) + n^2$

$SM \Rightarrow T(n) \leq 2^n = O(2^n)$

$IA: T(1) = 1 \leq 2^1,\ T(2) = 1 \leq 2^2,\ T(3) = 1 \leq 2^3 = O(2^n)$

IV: Die Behauptung gilt für $n-1$

$IS: T(n) = 2T(n-1) + n^2 \overset{IV}{\leq} 2 \cdot 2^n + n^2 = O(2^n)$