Theorem 1 [Associativity of _@_ (assoc@)]

$(L1 @ L2) @ L3 = L1 @ (L2 @ L3)$

Proof of Theorem 1 by structural induction on L1.

fresh constant

- Elt. E $\rightarrow$ e
- List $\rightarrow$ $l1, l2, l3$

I. Base Case

what to show is $(nil @ l2) @ l3 = nil @ (l2 @ l3)$

$(nil @ l2) @ l3 \rightarrow l2 @ l3$ by (@1)

$nil @ (l2 @ l3) \rightarrow l2 @ l3$ by (@1)

II. Induction Case

what to show is $((e|l1) @ l2) @ l3 = (e|l1) @ (l2 @ l3)$

Induction Hypothesis

$(l1 @ L2) @ L3 = l1 @ (L2 @ l3)$ .... (IH)

$((e|l1) @ l2) @ l3$

$\rightarrow (e|(l1 @ l2)) @ l3$ by (@2)

$\rightarrow e|((l1 @ l2) @ l3)$ by (@2)

$\rightarrow e|(l1 @ (l2 @ l3))$ by (IH)

$(e|l1) @ (l2 @ l3)$

$\rightarrow e|l1 @ (l2 @ l3))$

End of Proof of Theorem 1

Theorem 2 [Correctness of a Tail recursive reverse (ctrr)]

$rev1(LI) = rev2(LI)$

Proof of Theorem 2 By structural induction on LI

fresh constant

- Elt.E $\rightarrow$ e
- List $\rightarrow$ $l1$

I. Base case

what to show is $rev1(nil) = rev2(nil)$

$\underline{rev1(nil)}$
$\rightarrow nil$ by (r1-1)

$rev2(nil)$
$\rightarrow \underline{sr2(nil, nil)}$ by (r2)
$\rightarrow nil$ by (sr2-1)

II. Induction case

what to show is $rev1(e \mid l1) = rev2(e \mid l1)$

∘ Induction Hypothesis

$rev1(l1) = rev2(l2)$ ... (IH)

$\underline{rev1(e \mid l1)}$
$\rightarrow \underline{rev1(l1)} @ (e \mid nil)$ by (r1-2)
$\rightarrow \underline{rev2(l1)} @ (e \mid nil)$ by (IH)
$\rightarrow sr2(l1, nil) @ (e \mid nil)$ by (r2)

$\underline{rev2(e \mid l1)}$
$\rightarrow \underline{sr2(e \mid l1, nil)}$ by (r2)
$\rightarrow sr2(l1, e \mid nil)$ by (r2-2)

sr2(l1, nil) @ (e | nil) と sr2(l1, e | nil) の書き換えがこれ以上できなくなったため、lemma (補題) が必要。候補の補題としては以下が挙げられる。

$$sr2(L1, E|nil) = sr2(L1, nil) @ (E|nil)$$

この候補は lemma としては具体的すぎるため、より一般化した以下のものを lemma として使用する。

Lemma:

$$sr2(L1, E2 | L2) = sr2(L1, nil) @ (E2 | L2) \quad \cdots (P\text{-}sr2)$$

<u>sr2(l1, e | nil)</u>

→ sr2(l1, nil) @ (e | nil)　　by (P-sr2)

<u>End of Proof of Theorem 2</u>

Lemma I [A property of sr2 (p-sr2)]

$$sr2(L1,\ E2 \mid L2) = sr2(L1,\ nil) \ @ \ (E2 \mid L2)$$

Proof of Lemma I By structural induction on $L1$

fresh constant.

- Elt, E $\rightarrow$ e, $e2$
- List $\rightarrow$ $l1$, $l2$

I. Base case

what to show is $sr2(nil,\ e2 \mid l2) = sr2(nil,\ nil) \ @ \ (e2 \mid l2)$.

$sr2(nil,\ e2 \mid l2)$

$\rightarrow e2 \mid l2$ by (sr2-1)

$sr2(nil,\ nil) \ @ \ (e2 \mid l2)$

$\rightarrow nil \ @ \ (e2 \mid l2)$ by (sr2-1)

$\rightarrow e2 \mid l2$ by (@1)

II. Induction case

what to show is $sr2(e \mid l1,\ e2 \mid l2) = sr2(e \mid l1,\ nil) \ @ \ (e2 \mid l2)$

Induction Hypothesis

$sr2(l1,\ E2 \mid L2) = sr2(l1,\ nil) \ @ \ (E2 \mid L2)$ ... (IH)

$sr2(e \mid l1,\ e2 \mid l2)$

$\rightarrow sr2(l1,\ e \mid e2 \mid l2)$ by (sr2-2)

$\rightarrow sr2(l1,\ nil) \ @ \ (e \mid e2 \mid l2)$ by (IH)

$\underline{sr2(e \mid l1, nil)}$ @ $(e2 \mid l2)$

$\rightarrow \underline{sr2(l1, e \mid nil)}$ @ $(e2 \mid l2)$ by (sr2-2)

$\rightarrow \underline{(sr2(l1, nil)}$ @ $(e2 \mid l2)$ by (IH)

$\rightarrow sr2(l1, nil)$ @ $\underline{((e \mid nil) @ (e2 \mid l2))}$ by (assoc@)

$\rightarrow sr2(l1, nil)$ @ $(e \mid (\underline{nil @ (e2 \mid l2)}))$ by (@2)

$\rightarrow sr2(l1, nil)$ @ $(e \mid e2 \mid l2)$ by (@1)

End of Proof of Lemma 1

Theorem 3 [Reverse of reverse (rev.rev)]

$rev1(rev1(L)) = L$

Proof of theorem 3 by structural Induction

Fresh constan:

$Elt, E \rightarrow e$

$List \rightarrow \ell$

I. Base case

what to show is: $rev1(rev1(nil)) = nil.$

$rev1(nil) = nil \quad \cdots (H-1)$

$rev1(E|L1) = rev1(L1) @ (E|nil) \quad \cdots (H-2)$

$rev1(\underline{rev1(nil)})$

$\rightarrow rev1(nil)$ by (H-1)

$\rightarrow nil$ by (H-1)

II. Induction case

what to show is: $rev1(rev1(e|\ell)) = e|\ell$

Induction Hypothesis:

$rev1(rev1(\ell)) = \ell \quad \cdots (IH)$

$rev1(\underline{rev1(e|\ell)})$

$\rightarrow \underline{rev1(rev1(\ell) @ (e|nil))}$ by (H-2)

ここで Lemma を導入する.

Lemma:

$rev1(L1 @ L2) = rev1(L2) @ rev1(L1) \quad \cdots (lemma)$

$\rightarrow rev1(e|nil) @ \underline{rev1(rev1(\ell))}$ by (lemma)

$\rightarrow \underline{rev1(e|nil)} @ \ell$ by (IH)

$\rightarrow (\underline{\mathrm{rev1}(\mathrm{nil})}) @ (e \mid \mathrm{nil})) @ \ell$ by (r1-2)

$\rightarrow (\underline{\mathrm{nil} @ (e \mid \mathrm{nil})}) @ \ell$ by (r1-1)

$\rightarrow \underline{(e \mid \mathrm{nil}) @ \ell}$ by (@1)

$\rightarrow e \mid \underline{(\mathrm{nil} @ \ell)}$ by (@2)

$\rightarrow e \mid \ell$ by (@1)

End of proof of Theorem 3.

Lemma : [Reverse of Concatenation (rev-concat)]

$$\mathrm{rev1}(L1 @ L2) = \mathrm{rev1}(L2) @ \mathrm{rev1}(L1)$$

Proof of Lemma by Structural Induction

Fresh Constant

- Elt.E $\to e$
- List $\to \ell$

Induction Hypothesis

$$\mathrm{rev1}(\ell @ L2) = \mathrm{rev1}(L2) @ \mathrm{rev1}(\ell) \quad \cdots \quad (IH)$$

I. Base case.

what to show is $\mathrm{rev1}(nil @ L2) = \mathrm{rev1}(L2) @ \mathrm{rev1}(nil)$

$\mathrm{rev1}(nil @ L2)$

$\to \mathrm{rev1}(L2)$ by (@1)

$\mathrm{rev1}(L2) @ \mathrm{rev1}(nil)$

$\to \mathrm{rev1}(L2) @ nil$ by (r1-1)

$\to \mathrm{rev1}(L2)$ by (@1)

II. Induction Case :

what to show is $\mathrm{rev1}((e | \ell) @ L2) = \mathrm{rev1}(L2) @ \mathrm{rev1}(e | \ell)$

$\mathrm{rev1}((e | \ell) @ L2)$

$\to \mathrm{rev1}(e | (\ell @ L2))$ by (@2)

$\to \mathrm{rev1}(\ell @ L2) @ (e | nil)$ by (r1-2)

$\to (\mathrm{rev1}(L2) @ \mathrm{rev1}(\ell)) @ (e | nil)$ by (IH)

$\mathrm{rev1}(L2) @ \mathrm{rev1}(e | \ell)$

$\to \mathrm{rev1}(L2) @ (\mathrm{rev1}(\ell) @ (e | nil))$ by (r1-2)

$\to (\mathrm{rev1}(L2) @ \mathrm{rev1}(\ell)) @ (e | nil)$ by (assoc@)

End of proof of Lemma