Lecture3 Exercises

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Exercise ①

hd (0 :: 1 :: nil)

→ 0 by (hd 2)

tl (0 :: 1 :: nil)

→ 1 :: nil by (Tl 2)

$[2 .. 5]$

$\rightarrow$ if $2 > 5$ then $\{nil\}$ else $\{2 \mid [2+1 .. 5]\}$ by ([_..._])

$\rightarrow$ if false then $\{nil\}$ else $\{2 \mid [2+1 .. 5]\}$ by (>)

$\rightarrow 2 \mid [2+1 .. 5]$ by (if2)

$\rightarrow 2 \mid [3 .. 5]$ by (+)

$\rightarrow 2 \mid$ if $3 > 5$ then $\{nil\}$ else $\{3 \mid [3+1 .. 5]\}$ by ([_..._])

$\rightarrow 2 \mid$ if false then $\{nil\}$ else $\{3 \mid [3+1 .. 5]\}$ by (>)

$\rightarrow 2 \mid 3 \mid [3+1 .. 5]\}$ by (if2)

$\rightarrow 2 \mid 3 \mid [4 .. 5]\}$ by (+)

$\rightarrow 2 \mid 3 \mid$ if $4 > 5$ then $\{nil\}$ else $\{4 \mid 4+1 .. 5\}$ by ([_..._])

$\rightarrow 2 \mid 3 \mid$ if false then $\{nil\}$ else $\{4 \mid [4+1 .. 5]\}$ by (>)

$\rightarrow 2 \mid 3 \mid 4 \mid [4+1 .. 5]$ by (if2)

$\rightarrow 2 \mid 3 \mid 4 \mid [5 .. 5]$ by (+)

$\rightarrow 2 \mid 3 \mid 4 \mid$ if $5 > 5$ then $\{nil\}$ else $\{5 \mid [5+1 .. 5]\}$ by ([_..._])

$\rightarrow 2 \mid 3 \mid 4 \mid$ if false then $\{nil\}$ else $\{5 \mid [5+1 .. 5]\}$ by (>)

$\rightarrow 2 \mid 3 \mid 4 \mid 5 \mid [5+1 .. 5]$ by (if2)

$\rightarrow 2 \mid 3 \mid 4 \mid 5 \mid [6 .. 5]$ by (+)

→ 2|3|4|5|[6..5] by (+)

→ 2|3|4|5| if 6>5 then {nil} else {6|[6+1..5]} by ([..._])

→ 2|3|4|5| if true then {nil}
else {6|[6+1..5]} by (>)

→ 2|3|4|5|nil by (if1)

終了!

Exercise ②

gcd(24, 36)

→ gcd(36, 24 rem 36) by (gcd2)

→ gcd(36, 24) by (rem)

→ gcd(24, 36 rem 24) by(gcd2)

→ gcd(24, 12) by (rem)

→ gcd(12, 12 rem 24) by (gcd2)

→ gcd(12, 12) by (rem)

→ gcd(12, 12 rem 12) by (gcd2)

→ gcd(12, 0) by (gcd1)

→ 12 by (gcd1)

Exercise (3)

fact (5)

→ 5 * fact (P5) by (fact 2)

→ 5 * fact (4) by (P)

→ 5 * (4 * fact (P4)) by (fact 2)

→ 5 * (4 * (3 * fact (P3))) by (fact 2)

→ 5 * (4 * (3 * (2 * fact (P2)))) by (fact 2)

→ 5 * (4 * (3 * (2 * fact (1)))) by (P)

→ 5 * (4 * (3 * (2 * fact (P1)))) by (fact 2)

→ 5 * (4 * (3 * (2 * fact (0)))) by (P)

→ 5 * (4 * (3 * (2 * (1 * 1)))) by (fact 1)

→ 5 * (4 * (3 * (2 * 1))) by (*)

→ 5 * (4 * (3 * 2)) by (*)

→ 5 * (4 * 6) by (*)

→ 5 * 24 by (*)

→ 120 by (*)

Exercise 4

ordc - fact (5)

→ g(5, 1) by (ordc-fact 2)

→ cond (5 > 1, g(5, 2*1) * g(sd(5,1), 2*1), 5) by (g)

→ cond(true, g(5, 2*1) * g(sd(5, 1), 2*1), 5) by (>)

→ g(5, 2*1) * g(sd(5,1), 2*1) by (cond 1)

→ g(5, 2*1) * g(4, 2*1) by(sd)

→ g(5, 2*1) * g(4, 2) by (*)

→ g(5, 2) * g(4, 2) by (*)

→ g(5, 2) * cond (4>2, g(4, 2*2) *
g(sd(4,2), 2*2), 4) by (g)

→ g(5,2) * cond (True, g(4, 2*2) *
g(sd(4,2), 2*2), 4) by (>)

→ g(5, 2) * (g(4, 2*2) * g(sd(4,2), 2*2)) by (cond 1)

→ g(5, 2) * (g(4, 2*2) * g(2, 2*2)) by (sd)

→ g(5, 2) * (g(4, 4) * g(2, 2*2)) by (*)

→ g(5,2) * (g(4,4) * g(2, 4)) by (*)

→ g(5,2) * (g(4,4) * cond (2>4, g(2, 2*4) * g(sd(2,4), 2*4), 2)) by (g)

→ g(5,2) * (g(4,4) *
cond (false, g(2, 2*4) * g(sd(2,4), 2*4), 2)) by (>)

→ g(5,2) * (g(4,4) * cond(false, g(2, 2*4) * g(sol(2,4), 2*4), 2)) by (>)

→ g(5,2) * (g(4,4) * 2) by (cond 2)

→ g(5,2) * (cond(4>4, g(4, 2*4) * g(sol(4,4), 2*4), 4) * 2) by (g)

→ g(5,2) * (cond(false, g(4, 2*4) * g(sol(4,4), 2*4), 4) * 2) by (>)

→ g(5,2) * (4 * 2) by (cond 2)

→ g(5,2) * 8 by (*)

→ cond(5>2, g(5, 2*2) * g(sol(5,2), 2*2), 5) * 8 by (g)

→ cond(true, g(5, 2*2) * g(sol(5,2), 2*2), 5) * 8 by (>)

→ (g(5, 2*2) * g(sol(5,2), 2*2)) * 8 by (cond 1)

→ (g(5, 4) * g(sol(5,2), 2*2)) * 8 by (*)

→ (g(5,4) * g(sol(5,2), 4)) * 8 by (*)

→ (g(5,4) * g(3,4)) * 8 by (sol)

→ (g(5,4) * cond(3>4, g(3, 2*4) * g(sol(3,4), 2*4), 3)) * 8 by (g)

→ (g(5,4) * cond(false, g(3, 2*4) * g(sol(3,4), 2*4), 3)) * 8 by (>)

→ g(5,4) * 3 * 8 by (cond 2)

→ g(5,4) * 24 by (*)

→ cond(5>4, g(5, 2*4) * g(sol(5,4), 2*4), 5)) * 24 by (g)

→ cond (true , g(5, 2*4) * g(sd(5, 4), 2*4), 5) *24 by (g)

→ (g(5, 2*4) * g(sd(5,4), 2*4)) * 24

→ (g(5, 8) * g(sd(5,4), 2*4)) * 24 by (*)

→ (g(5, 8) * g(sd(5,4), 8)) * 24 by (*)

→ (g(5,8) * g(1, 8)) * 24 by (sd)

→ (g(5,8) * cond(1>8, g(1, 2*8) * g(sd(2,8), 2*8), 1)) * 24 by (g)

→ (g(5,8) * cond(false, g(1, 2*8) * g(sd(2,8), 2*8), 1)) * 24 by (>)

→ g(5,8) * 1 * 24 by (cond2)

→ cond(5>8, g(5, 2*8) * g(sd(5,8), 2*8), 5) * 1 * 24 (g)

→ cond(false, g(5, 2*8) * g(sd(5,8), 2*8), 5) * 1 * 24 by (>)

→ (5 * 1) * 24 by (cond2)

→ 5 * 24 by (*)

→ 120 by (*)

끝!

Exercise (5)

qsort (2 | 1 | 0 | 3 | 4 | nil)

→ partition (2, 1 | 0 | 3 | 4 | nil, nil, nil) by (qsort3)

この式自体が partition 2 の定義で

→ if 1 < 2 then { partition (2, 0 | 3 | 4 | nil, 1 | nil, nil) }
else { partition (2, 0 | 3 | 4 | nil, nil, 1 | nil) } by (partition2)

次に > の部分が true or False で

→ if true then { partition (2, 0 | 3 | 4 | nil, 1 | nil, nil) }
else { partition (2, 0 | 3 | 4 | nil, nil, 1 | nil) } by (<)

置き換えられるため その場所が redex となる.

True の方が評価されるので、then の後の式が次に来る.

→ partition (2, 0 | 3 | 4 | nil, 1 | nil, nil) by if1

→ if 0 < 2 then { partition (2, 3 | 4 | nil, 0 | 1 | nil, nil) }
else { partition (2, 3 | 4 | nil, 1 | nil, 0 | nil) } by (partition2)

→ if true then { partition (2, 3 | 4 | nil, 0 | 1 | nil, nil) }
else { partition (2, 3 | 4 | nil, 1 | nil, 0 | nil) } by (<)

→ partition (2, 3 | 4 | nil, 0 | 1 | nil, nil) by (if1)

→ if 3 < 2 then { partition (2, 4 | nil, 3 | 0 | 1 | nil, nil) }
else { partition (2, 4 | nil, 0 | 1 | nil, 3 | nil) } by (partition 2)

→ if false then { partition (2, 4 | nil, 3 | 0 | 1 | nil, nil) }
else { partition (2, 4 | nil, 0 | 1 | nil, 3 | nil) } by (<)

→ partition (2, 4 | nil, 0 | 1 | nil, 3 | nil) by (if2)

→ if 4 < 2 then { partition (2, nil, 4 | 0 | 1 | nil, 3 | nil) }
else { partition (2, nil, 0 | 1 | nil, 4 | 3 | nil) } by (partition2)

→ if false then § partition (2, nil, 4|0||nil, 3|nil) §
else § partition (2, nil, 0||nil, 4|3|nil) § by (<)

→ partition (2, nil, 0||nil, 4|3|nil) by (if 2)

→ qsort (0||nil) @ (2| qsort (4|3|nil) by (partition)

→ partition(0, ||nil, nil, nil) @
(2| qsort(4|3|nil)) by (qsort 3)

eg nil @ L2 = L2 (@1)
eg (x|L) @ L2
= x|(L@L2) (@2)

→ (if 1<0 then § partition(0, nil, ||nil, nil)
else § partition (0, nil, nil, ||nil)
@ (2| qsort(4|3|nil)) by (partition2)

→ (if false then § partition (0, nil, 1|nil, nil)
else § partition (0, nil, nil, 1|nil)
@ (2| qsort (4|3|nil)) by (<)

→ partition (0, nil, nil, 1|nil) @ (2| qsort (4|3|nil)) by (if 2)

→ qsort(nil) @ (0| qsort (1|nil) @ (2| qsort (4|3|nil)) by (partition 1)

→ nil @ (0| qsort (1|nil) @ (2| qsort (4|3|nil)) by (qsort 1)

→ nil @ (0||nil) @ (2| qsort (4|3|nil)) by (qsort 2)

→ (0|1|nil) @ (2| qsort (4|3|nil)) by (@1)

→ (0|1|nil) @ (2| partition (4, 3|nil, nil, nil) by (qsort 3)

→ (0|1|nil) @ (2| (if 3<4 then § partition (4, nil, 3|nil, nil) §
else § partition (4, nil, nil, 3|nil) § by (partition 2)

→ (0|1|nil) @ (2|(if true then { partition(4, nil, 3|nil, nil)}
else { partition(4, nil, nil, 3|nil} by(<)

→ (0|1|nil) @ (2|(partition(4, nil, 3|nil, nil)) by(if 1)

→ (0|1|nil) @ (2|(qsort(3|nil) @ (4|qsort(nil)) by(partition)

→ (0|1|nil) @ (2|3|nil) @ (4|qsort(nil)) by(qsort2)

→ (0|1|nil) @ (2|3|nil) @ (4|nil) by(qsort1)

→ 0|((1|nil) @ (2|3|nil)) @ (4|nil) by(@2)

→ (0|1|(nil @ (2|3|nil)) @ (4|nil) by(@2)

→ 0|1|2|3|nil) @ (4|nil) by(@1)

→ 0|(1|2|3|nil) @ (4|nil) by(@2)

→ 0|1|(2|3|nil) @ (4|nil) by(@2)

→ 0|1|2|(3|nil) @ (4|nil) by(@2)

→ 0|1|2|3|(nil) @ (4|nil) by(@2)

→ 0|1|2|3|4|nil by(@1)

終わり