Theorem 1 [Associativity of _@_ (assoc@)]

$(L1 @ L2) @ L3 = L1 @ (L2 @ L3)$

Proof of Theorem 1 by structural induction on L1.

fresh constant

- Elt. E $\rightarrow$ e
- List $\rightarrow$ l1, l2, l3

I. Base Case

what to show is $(nil @ l2) @ l3 = nil @ (l2 @ l3)$

$(nil @ l2) @ l3 \rightarrow l2 @ l3$ by (@1)

$nil @ (l2 @ l3) \rightarrow l2 @ l3$ by (@1)

II. Induction Case

what to show is $((e|l1) @ l2) @ l3 = (e|l1) @ (l2 @ l3)$

Induction Hypothesis

$(l1 @ L2) @ L3 = l1 @ (L2 @ l3)$ .... (IH)

$((e|l1) @ l2) @ l3$

$\rightarrow (e|(l1 @ l2)) @ l3$ by (@2)

$\rightarrow e|((l1 @ l2) @ l3)$ by (@2)

$\rightarrow e|(l1 @ (l2 @ l3))$ by (IH)

$(e|l1) @ (l2 @ l3)$

$\rightarrow e|l1 @ (l2 @ l3))$

End of Proof of Theorem 1

Theorem 2 [Correctness of a Tail recursive reverse (ctrr)]

rev1(L1) = rev2(L1)

Proof of Theorem 2 By structural induction on L1

fresh constant

- Elt.E → e
- List → l1

I. Base case

what to show is rev1(nil) = rev2(nil)

rev1(nil)
→ nil by (r1-1)

rev2(nil)
→ sr2(nil, nil) by (r2)
→ nil by (sr2-1)

II. Induction case

what to show is rev1(e|l1) = rev2(e|l1)

- Induction Hypothesis

rev1(l1) = rev2(l2) ... (IH)

rev1(e|l1)
→ rev1(l1) @ (e|nil) by (r1-2)
→ rev2(l1) @ (e|nil) by (IH)
→ sr2(l1, nil) @ (e|nil) by (r2)

rev2(e|l1)
→ sr2(e|l1, nil) by (r2)
→ sr2(l1, e|nil) by (r2-2)

sr2(l1, nil) @ (e|nil) と sr2(l1, e|nil) の書き換えがこれ以上できなくなったため、lemma(補題)が必要。候補の補題としては以下が挙げられる。

$$sr2(L1, E|nil) = sr2(L1, nil) @ (E|nil)$$

この候補はlemmaとしては具体的すぎるため、より一般化した以下のものをlemmaとして使用する。

Lemma:

$$sr2(L1, E2 | L2) = sr2(L1, nil) @ (E2 | L2) \quad \cdots (P\text{-}sr2)$$

$\underline{sr2(l1, e|nil)}$

$\rightarrow sr2(l1, nil) @ (e|nil)$ by (P-sr2)

End of Proof of Theorem 2

Lemma I [A property of sr2 (p-sr2)]

$sr2(L1, E2 | L2) = sr2(L1, nil) @ (E2 | L2)$

Proof of Lemma I By structural induction on L1

fresh constant.

- Elt, E → e, e2
- List → l1, l2

I. Base case

what to show is $sr2(nil, e2 | l2) = sr2(nil, nil) @ (e2 | l2)$.

$sr2(nil, e2 | l2)$

$\to e2 | l2$ by (sr2-1)

$sr2(nil, nil) @ (e2 | l2)$

$\to nil @ (e2 | l2)$ by (sr2-1)

$\to e2 | l2$ by (@1)

II. Induction case

what to show is $sr2(e | l1, e2 | l2) = sr2(e | l1, nil) @ (e2 | l2)$

Induction Hypothesis

$sr2(l1, E2 | L2) = sr2(l1, nil) @ (E2 | L2)$ ... (IH)

$sr2(e | l1, e2 | l2)$

$\to sr2(l1, e | e2 | l2)$ by (sr2-2)

$\to sr2(l1, nil) @ (e | e2 | l2)$ by (IH)

$\underline{sr2(e \mid l1, nil)}$ @ $(e2 \mid l2)$

$\to \underline{sr2(l1, e \mid nil)}$ @ $(e2 \mid l2)$ by (sr2-2)

$\to \underline{(sr2(l1, nil)}$ @ $(e2 \mid l2)$ by (Ift)

$\to sr2(l1, nil)$ @ $\underline{((e \mid nil) \text{ @ } (e2 \mid l2))}$ by (assoc@)

$\to sr2(l1, nil)$ @ $(e \mid (\underline{nil \text{ @ } (e2 \mid l2)}))$ by (@2)

$\to sr2(l1, nil)$ @ $(e \mid e2 \mid l2)$ by (@1)

End of Proof of Lemma 1

Theorem 3 [Reverse of reverse (rev.rev)]

$rev1(rev1(L)) = L$

Proof of theorem 3 by structural Induction

Fresh constan:

$Elt, E \to e$

$List \to \ell$

I. Base case

what to show is: $rev1(rev1(nil)) = nil.$

$rev1(\underline{rev1(nil)})$

$\to rev1(nil)$ by (r1-1)

$\to nil$ by (r1-1)

II. Induction Case

what to show is: $rev1(rev1(e|\ell)) = e|\ell$

Induction Hypothesis:

$rev1(rev1(\ell)) = \ell$ ... (IH)

$rev1(\underline{rev1(e|\ell)})$

$\to \underline{rev1(rev1(\ell) @ (e|nil))}$ by (r1-2)

ここでLemmaを導入する.

Lemma:

$rev1(L1 @ L2) = rev1(L2) @ rev1(L1)$ ... (lemma)

$\to rev1(e|nil) @ \underline{rev1(rev1(\ell))}$ by (lemma)

$\to \underline{rev1(e|nil)} @ \ell$ by (IH)

$\to (\underline{\mathrm{rev1}(nil)}) @ (e \mid nil)) @ \ell$ by (r1-2)

$\to (\underline{nil @ (e \mid nil)}) @ \ell$ by (r1-1)

$\to \underline{(e \mid nil) @ \ell}$ by (@1)

$\to e \mid \underline{(nil @ \ell)}$ by (@2)

$\to e \mid \ell$ by (@1)

<u>End of proof of Theorem 3</u>.