Internals - 03

~~(18CS41)~~

$\boxed{18MAT41}$

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(1a.) Let X be the random variable for the random values $x_1 = 0$ $x_2 = 1$ $x_3 = 2$ $x_4 = 3$ $x_5 = 4$ $x_6 = 5$ and the given respective probabilities are,

$P(X = x_1) = P(0) = k$

$P(X = x_2) = P(1) = 5k$

$P(X = x_3) = P(2) = 10k$

$P(X = x_4) = P(3) = 10k$

$P(X = x_5) = P(4) = 5k$

$P(X = x_6) = P(5) = k$

$\sum_{i=1}^{6} P(X = x_i) = 1$

$\Rightarrow k + 5k + 10k + 10k + 5k + k = 1$

$\Rightarrow 32k = 1$

$\Rightarrow k = \frac{1}{32}$

$P(x \leq 1) = P(0) + P(1)$

$\Rightarrow P(x \leq 1) = 1k + 5k$

$\Rightarrow P(x \leq 1) = 6k = \frac{6}{32}$ $= 0.1875$ //.

$\Rightarrow P(0 \leq x < 3) = P(0) + P(1) + P(3)$

$= 16k$

$k = \frac{16}{32}$ $= 0.5$ //.

(12).

given, $x_1 = 2$, $x_2 = 5$ $x_3 = 7$

$y_1 = 3$ $y_2 = 4$ $y_3 = 5$

And the probabilities are,

$p_{11} = 1/6$ $p_{12} = 1/6$ $p_{13} = 1/6$

$p_{21} = 1/12$ $p_{22} = 1/12$ $p_{23} = 1/12$

$p_{31} = 1/12$ $p_{32} = 1/12$ $p_{33} = 1/12$

The joint distribution table is as follows.

x \ y	3	4	5	$f(x_i)$
2	1/6	1/6	1/6	1/2
5	1/12	1/12	1/12	1/4
7	1/12	1/12	1/12	1/4
$g(y_i)$	1/3	1/3	1/3	1

The marginal distributions of x and y are

x_i	2	5	7
$f(x_i)$	1/2	1/4	1/4

y_i	3	4	5
$g(y_i)$	1/3	1/3	1/3

$\mu_x = E(x) = \sum_i x_i f(x_i)$

$\Rightarrow \mu_x = (2 \times 1/2) + (5 \times 1/4) + (7 \times 1/4)$

$\Rightarrow \mu_x = 4$

$\mu_y = E(y) = \sum_j y_j f(y_j)$

$\Rightarrow \mu_y = (3 \times 1/3) + (4 \times 1/3) + (5 \times 1/3)$

$\Rightarrow \mu_y = 4$

$E(XY) = \sum_{i,j} x_i y_j p_{ij}$

$\Rightarrow E(XY) = (2 \times 3 \times 1/6) + (2 \times 4 \times 1/6) + (2 \times 5 \times 1/6)$

$+ (5 \times 3 \times 1/12) + (5 \times 4 \times 1/12) + (5 \times 5 \times 1/2)$

$+ (7 \times 3 \times 1/12) + (7 \times 4 \times 1/12) + (7 \times 5 \times 1/12)$

$= 16$

$\therefore \operatorname{cov}(X,Y) = E(XY) - \mu_x \mu_y$

$\Rightarrow \operatorname{cov}(X,Y) = 16 - 4 \times 4$

$\operatorname{cov}(X,Y) = 0$

(3a). consider $\frac{1}{z^2-4} = \frac{1}{(z-2)(z+2)}$

$\frac{1}{(z-2)(z+2)} = \frac{A}{z-2} + \frac{B}{z+2}$

$\Rightarrow 1 = A(z+2) + B(z-2)$

put $z=2$: $1 = A(4)$ $A = 1/4$

$z=-2$: $1 = B(-4)$ $B = -1/4$

$\therefore \frac{1}{z^2-4} = \frac{1}{4}\frac{1}{z-2} - \frac{1}{4}\frac{1}{z+2}$

$$\therefore \int_C \frac{dz}{z^2-4} = \frac{1}{4}\int_C \frac{dz}{z-2} - \frac{1}{4}\int_C \frac{dz}{z-(-2)} \quad \text{---} (1)$$

(a) $C: |z| = 1$ $z = a = 2$ and -2 both of them lie outside

Hence by Cauchy's theorem $\int_C \frac{dz}{z^2-4} = 0$

(b) $C: |z| = 3$: $z = a = 2$ and -2 lies inside the circle Also in each of the integrals as in the RHS of eqn (1) is

$f(z) = 1$

Applying Cauchy's integral formula,

$$\int_C \frac{f(z)}{z-a}\, dz = 2\pi i f(a)$$

$$\int_C \frac{dz}{z-2} = 2\pi i f(2) = 2\pi i(1) = 2\pi i$$

$$\int_C \frac{dz}{z+2} = 2\pi i f(-2) = 2\pi(1) = 2\pi i$$

$$\therefore (1) \Rightarrow \int_C \frac{dz}{z^2-4} = \frac{1}{4}(2\pi i) - \frac{1}{4}(2\pi i) = 0$$

$$\therefore \int_C \frac{dz}{z^2-4} = 0 \,//..$$

(3b) Consider, $\frac{1}{(z+1)(z-2)} = \frac{A}{z+1} + \frac{B}{z-2}$

$\Rightarrow 1 = A(z-2) + B(z+1)$

put $z = 2$ $\quad 1 = B(3)$ $\quad \therefore B = 1/3$

$z = -1$ $\quad 1 = A(-3)$ $\quad \therefore A = -1/3$

$$\therefore \frac{1}{(z+1)(z+2)} = -\frac{1}{3}\frac{1}{z+1} + \frac{1}{3}\frac{1}{z-2}$$

$$\therefore \frac{e^{2z}}{(z+1)(z-2)} = \frac{1}{3}\left[\frac{e^{2z}}{z+2} - \frac{e^{2z}}{z+1}\right]$$

$$\int_c \frac{e^{2z}}{(z+1)(z-2)} = \frac{1}{3}\left[\int_c \frac{e^{2z}}{z-2}\,dz - \int_c \frac{e^{2z}}{z+1}\,dz\right] \quad \text{—(1)}$$

Here the points $z=a=-1$ and 2 lies inside $|z|=3$

and we have cauchy's integral formula.

$$\int_c \frac{f(z)}{z-a}\,dz = 2\pi i f(a)$$

Taking $f(z) = e^{2z}$ and $a = 2, -1$ we get

$$\int_c \frac{e^{2z}}{z-2}\,dz = 2\pi i f(2) = 2\pi i e^4$$

and $$\int_c \frac{e^{2z}}{z+1}\,dz = 2\pi i f(-1) = 2\pi i e^{-2} = \frac{2\pi i}{e^2}$$

Substituting these in Eq (1)

$$\int_c \frac{e^{2z}}{(z+1)(z-2)}\,dz = \frac{1}{3}\left[2\pi i e^4 - \frac{2\pi i}{e^2}\right]$$

$$= \frac{2\pi i}{3}\left[e^4 - \frac{1}{e^2}\right] //.$$