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Internals - 02

M4 (18MAT41)

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1EN18CS097
CSE 'A' Sec

1a. Let $w = \frac{az+b}{cz+d}$

$z=1, w=i \Rightarrow i = \frac{a+b}{c+d}$

$a+b = ic+id \Rightarrow a+b-ic-id=0$ —— ①

$z=i, w=0, 0 = \frac{ai+b}{ci+d}$

$ai+b$ —— ②

$z=-1, w=-i \Rightarrow -i = \frac{-a+b}{-c+d}$ —— ③

$-a+b = ic-id \Rightarrow -a+b-ic+id=0$ —— ③

Now eqn ① + ③

$\cancel{a}+b-ic-\cancel{id}=0$

$-\cancel{a}+b-ic+\cancel{id}=0$

$2b-2ic=0$

$b-ic=0$ —— ④

Eqn ② & ④ we write in the form,

$ai+b+0c=0$

$0a+b-ic=0$

Applying the rule of cross x ln we have,

$$\frac{a}{\begin{vmatrix} 1 & 0 \\ 1 & -i \end{vmatrix}} = \frac{-b}{\begin{vmatrix} i & 0 \\ 0 & -i \end{vmatrix}} = \frac{c}{\begin{vmatrix} i & 1 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{a}{-i} = \frac{-b}{-i^2} = \frac{c}{i} \Rightarrow \frac{a}{-i} = \frac{b}{-1} = \frac{c}{i} = k$$

$a = -ik$, $b = -k$, $c = ik$

Eqn ① $\Rightarrow$

$$-ik + k - i^2 k - di = 0 \Rightarrow -ik - k + k - id = 0$$

$$-i(d+k) = 0$$

$$d = -k$$

Eqn ②

$$w = \frac{-ikz - k}{ikz - k} = \frac{-k(1+iz)}{-k(1-iz)}$$

Thus $w = \frac{1+iz}{1-iz}$ //.

1b.

$$w = z + \frac{1}{z}$$

$$\Rightarrow \frac{dw}{dz} = 1 - \frac{1}{z^2}$$

$$\Rightarrow \frac{dw}{dz} = 0 \text{ \& } z = \pm 1$$

$\therefore$ ~~The~~ Transformation $w = z + \frac{1}{z}$ is not conformal ~~conformed~~ at $z = \pm 1$ & is analytic at every other point of the z-plane.

Let $z = re^{i\theta}$ & $w = u + iv$

$$w = u + iv = re^{i\theta} + \frac{1}{r} e^{-i\theta}$$

$$(u + iv) = \left(r + \frac{1}{r}\right) \cos\theta + i\left(r - \frac{1}{r}\right) \sin\theta$$

$\Rightarrow u \cancel{=}$

$\Rightarrow u = \left(r + \frac{1}{r}\right)\cos\theta$ & $v = \left(r - \frac{1}{r}\right)\sin\theta$,

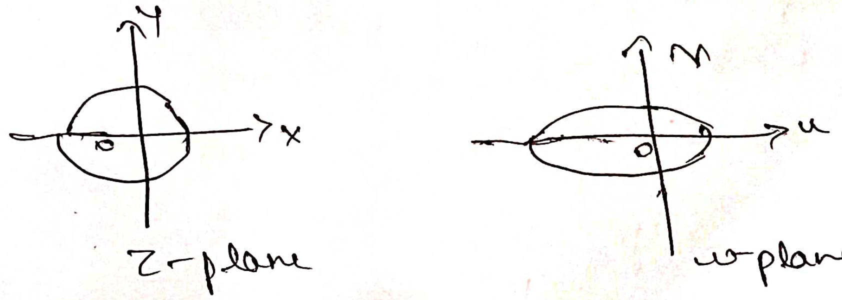
Case 1 :- Eliminating 'θ' b/w these eq^n we get

$$\Rightarrow \frac{u^2}{\left(r+\frac{1}{r}\right)^2} + \frac{v^2}{\left(r-\frac{1}{r}\right)^2} = 1 \quad \text{—— (2)}$$

Let $r = k_1 (\neq 1)$, where k_1 is a constant. This represents a circle centred at origin in the z-plane.

Then eq^n (2) $\Rightarrow$
$$\frac{u^2}{\left(k_1+\frac{1}{k_1}\right)^2} + \frac{v^2}{\left(k_1-\frac{1}{k_1}\right)^2} = 1$$

This represents an ellipse having centre at origin in the w-plane.



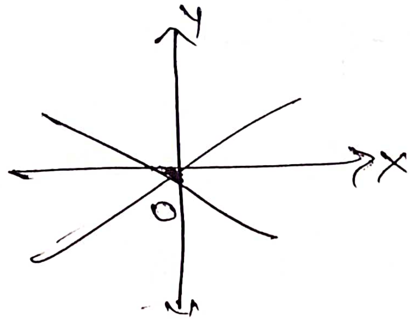
Case 2: eliminating from eq (1), we get

$$\frac{u^2}{\cos^2\theta} - \frac{v^2}{\sin^2\theta} = 4 \quad \text{—— (3)}$$

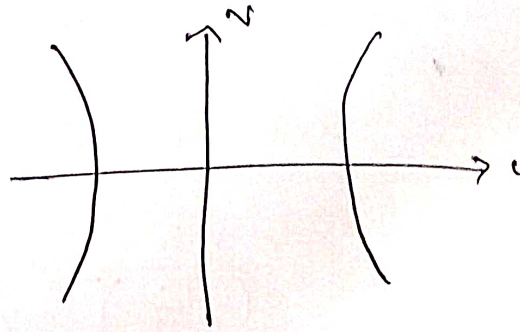
Let $\theta = k_2$, where k_2 a constant, This represents a radial lines in the z-plane.

Then eq (3) $\Rightarrow \frac{u^2}{\cos^2 k_2} - \frac{v^2}{\sin^2 k_2} = 4$

This represents a hyperbola having centre at origin in the w-plane.



z-plane



w-plane

Thus, the transformation $z = \frac{1}{z}$ transform circles with centre origin to ellipse having centre at origin & the radial lines to hyperbola having centre at origin.

3a. The normal eqn for fitting the straight line $y = ax + b$ are

$\Sigma y = a\Sigma x + nb$

$\Sigma xy = a\Sigma x^2 + b\Sigma x$

x	y	xy	x^2
1	1	1	1
3	2	6	9
4	4	16	16
6	4	24	36
8	5	40	64
9	7	63	81
11	8	88	121
14	9	126	196

$\Sigma x = 56$ $\Sigma y = 40$ $\Sigma yx = 364$ $\Sigma x^2 = 524$

the normal eqn becomes

$56a + 8b = 40$ & $524a + 56b = 364$

$\therefore a = 0.63 \approx 0.64$, $b = 0.54 \approx 0.55$

$\therefore$ Thus by substituting these values in $y = ax+b$ we obtain the eqn ,

$$y = 0.64x + 0.55 //.$$

3b, The normal eqns associated with $y = ax^2 + bx + c \rightarrow$ (A) are as follows.

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc \text{ ——— } (1)$$
$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x \text{ ——— } (2)$$
$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + \Sigma c x^2 \text{ ——— } (3)$$

x	y	xy	x^2	x^2y	x^3	x^4
1	10	10	1	10	1	1
2	12	24	4	48	8	16
3	13	39	9	117	27	81
4	16	64	16	256	64	256
5	19	95	25	475	125	625

$\Sigma x = 15$ $\Sigma y = 70$ $\Sigma xy = 232$ $\Sigma x^2y = 906$ $\Sigma x^3 = 225$ $\Sigma x^4 = 979$

$\Sigma x^2 = 55$

Eqn (1) $\Rightarrow$ $70 = 55a + 15b + 5c$

Eqn (2) $\Rightarrow$ $232 = 225a + 55b + 15c$

Eqn (3) $\Rightarrow$ $906 = 979a + 225b + 55c$

$\therefore a = 0.2857 \approx 0.29$, $b = 0.4856 \approx 0.49$, $c = 9.4$ //.

Thus the required second parabola is

$y = 0.29x^2 + 0.49x + 9.4$ Also at $x = 6$

$y = 22.78$ //.