M4 (18MAT41)

Assignment - 02

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① Find the Equation of the best fitting straight line for the following data (i)

x	1	2	3	4	5
y	14	13	9	5	2

→ The normal equations for the fitting the straight line

$y = ax + b$ ——— ⊛

$a\Sigma x + nb = \Sigma y$ ---- ① $a\Sigma x^2 + b\Sigma x = \Sigma xy$ ---- ②

Here $n = 5$

x	y	xy	x^2
1	14	14	1
2	13	26	4
3	9	27	9
4	5	20	16
5	2	10	25
$\Sigma x = 15$	$\Sigma y = 43$	$\Sigma xy = 97$	$\Sigma x^2 = 55$

∴ Eqn ① & ② becomes

$15a + 5b = 43$

$55a + 15b = 97$

$a = -3.2$

$b = 18.2$

∴ Eqn ⊛ becomes

$y = -3.2x + 18.2$

$y = 18.2 - 3.2x$ //.

(ii)

x	1	2	3	4	5	6	7
y	80	90	92	83	94	99	92

Let $y = ax + b$ ..... ⊛

$a\Sigma x + nb = \Sigma y$ ——— ①

$a\Sigma x^2 + b\Sigma x = \Sigma xy$ ——— ②

Here $n = 7$

Eqn ① & ② becomes

$28a + 7b = 630$

$140a + 28b = 2576$

x	y	xy	x^2
1	80	80	1
2	90	180	4
3	92	276	9
4	83	332	16
5	94	470	25
6	99	594	36
7	92	644	49
$\Sigma x = 28$	$\Sigma y = 630$	$\Sigma xy = 2576$	$\Sigma x^2 = 140$

$a = 2$

$b = 82$

Eq^n (*) becomes

$y = 2x + 82$ //.

(2) Fit a second degree parabola to the following data

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

> The normal equations associated with $y = ax^2 + bx + c$ —— (*)

$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$ —— (1)

$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$ —— (2)   Here $n = 7$

$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$ —— (3)

x	y	xy	x^2	x^2y	x^3	x^4
1.0	1.1	1.1	1	1.1	1	1
1.5	1.3	1.95	2.25	2.925	3.37	5.0625
2.0	1.6	3.2	4	6.4	8	16
2.5	2.0	5	6.25	12.5	15.625	39.0625
3.0	2.7	8.1	9	24.3	27	81
3.5	3.4	11.9	12.25	41.65	42.875	150.0625
4.0	4.1	16.4	16	65.6	64	256
$\Sigma x = 17.5$	$\Sigma y = 16.2$	$\Sigma xy = 47.65$	$\Sigma x^2 = 50.75$	$\Sigma x^2y = 154.47$	$\Sigma x^3 = 161.87$	$\Sigma x^4 = 548.1875$

$\therefore$ Eq^n (1), (2) and (3) becomes,

$16.2 = 50.75a + 17.5b + 7c$

$47.65 = 161.875a + 50.75b + 17.5c$

$154.475 = 548.1875 + 161.875b + 50.75c$

$a = 0.2428 \quad b = -0.1926 \quad c = 1.0357$

$\therefore$ Eq^n (*) becomes

$y = 0.2428x^2 - 0.1928x + 1.0357$ //.

(3) Fit a curve of the form $y = ae^{bx}$ for the data,

x	0	2	4
y	5.02	10	31.64

> Consider $y = ae^{bx}$ —— (*)

The normal eq^{ns} are as follows,

$\Sigma y = na + b\Sigma x$ —— (1) Here $n=3$

$\Sigma xy = A\Sigma x + b\Sigma x^2$ —— (2)

x	y	$Y = \log_e y$	xY	x^2
0	5.02	1.6134	0	0
2	10	2.3025	4.605	4
4	31.62	3.4537	13.8148	16
$\Sigma x = 6$		$\Sigma Y = 7.3696$	$\Sigma xY = 18.41$	$\Sigma x^2 = 20$

Eq^n (1) and (2) becomes

$7.3696 = 3A + 6b$

$18.4198 = 6A + 20b$

$A = 1.5363 \quad b = 0.46$

$\log_e a = A \Rightarrow a = e^b \Rightarrow a = e^{1.5363} \quad \therefore a = 4.6473$

$\therefore$ Eqⁿ (*) becomes

$$y = 4.6473 e^{0.46x}$$

(2) Fit a curve of the form $y = ax^b$ for the data

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.1	6.8	7.5

> $y = ax^b$ ——— (*)

$y = A + bx$

$\Sigma y = nA + b\Sigma x$ ——— (1) Here $x = \log_{10} x$

$\Sigma xy = A\Sigma x + b\Sigma x^2$ ——— (2) , $y = \log_{10} y$

$n = 6$

x	y	$y = \log_{10} y$	$x = \log_{10} x$	xy	x^2
1	2.98	0.4742	0	0	0
2	4.26	0.6294	0.3010	0.1894	0.0906
3	5.21	0.7168	0.4771	0.3419	0.2276
4	6.1	0.7853	0.6020	0.4727	0.3624
5	6.8	0.8325	0.6989	0.5818	0.4884
6	7.5	0.8750	0.7781	0.6808	0.6054
		$\Sigma y = 4.3132$	$\Sigma x = 2.8571$	$\Sigma xy = 2.2666$	$\Sigma x^2 = 1.7744$

Eqⁿ (1) & (2) becomes,

$4.3132 = 6A + 2.8571b$

$2.2666 = 2.8571A + 1.7744b$

$\therefore A = 0.4741$, $b = 0.5139$

$A = \log_{10} a$

$a = \text{antilog}_{10} A$

$a = \text{antilog}_{10}(0.4741)$ $a = 2.9792$ $\therefore$ Eqⁿ (*) becomes

$y = 2.972 x^{0.5139}$ //.

(5) S.T If 'θ' is the angle b/w the lines of regression, then $\tan\theta = \frac{\sigma x \sigma y}{\sigma x^2 + \sigma y^2}\left(\frac{1-r^2}{r}\right)$

> W.K.T

If θ is acute, the angle b/w the lines

$y = m_1 x_1 + c_1$ is given by

$y = m_2 x + c_1$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \quad \text{———} (*)$$

we have lines of regression,

$$y - \bar{y} = r\frac{\sigma y}{\sigma x}(x - \bar{x}) \quad \text{———} (1)$$

$$x - \bar{x} = r\frac{\sigma x}{\sigma y}(y - \bar{y}) \quad \text{———} (2)$$

we will recreate the Eqn (2) in the form of Eqn (1)

$$\frac{1}{x - \bar{x}} = \frac{\sigma y}{r\sigma x}\frac{1}{y - \bar{y}}$$

$$\therefore (y - \bar{y}) = \frac{\sigma y}{r\sigma x}(x - \bar{x}) \quad \text{———} (3)$$

Slopes of (1) & (3) are respectively given by

$m_1 = \frac{r\sigma y}{\sigma x}$ and $m_2 = \frac{\sigma y}{r\sigma x}$

Substitute in Eqn (*)

$$\tan\theta = \frac{\frac{\sigma y}{r\sigma x} - \frac{r\sigma y}{\sigma x}}{1 + \left(\frac{\cancel{r}\sigma y}{\sigma x}\cdot\frac{\sigma y}{\cancel{r}\sigma x}\right)}$$

$$\tan\theta = \frac{\frac{\sigma y}{\sigma x}\left(\frac{1}{r} - r\right)}{1 + \frac{\sigma y^2}{\sigma x^2}}$$

$$\tan\theta = \frac{\frac{\sigma y}{\sigma \cancel{x}} \cdot \sigma \cancel{x^2}\left(\frac{1-r^2}{r}\right)}{\sigma x^2 + \sigma y^2}$$

$$\tan\theta = \frac{\sigma x \, \sigma y}{\sigma x^2 + \sigma y^2}\left(\frac{1-r^2}{r}\right) \quad //..$$

Hence proved //..