M4 Unit-Test-01

(18MAT41)

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CSE 'A' sec
4th sem.

1) S.T $w = z + e^z$ is analytic and find $\frac{dw}{dz}$

> By data $w = z + e^z$

$$u + iv = (x + iy) + e^{(x+iy)}$$

$$= (x + iy) + e^x \cdot e^{iy}$$

$$= (x + iy) + e^x(\cos y + i \sin y)$$

$$u + iv = (x + e^x \cos y) + i(y + e^x \sin y)$$

$u = x + e^x \cos y$ $\quad v = y + e^x \sin y$

$u_x = 1 + e^x \cos y$ $\quad v_y = 1 + e^x \sin y$

$u_y = -e^x \sin y$ $\quad v_x = e^x \sin y$

we observe that C-R Eqn's in the ~~cartes~~ Cartesian form $u_x = v_y$ and $v_x = -u_y$ are satisfied.

Thus, $w = z + e^z$ is analytic.

Also we have; $\frac{dw}{dz} = f'(z) = u_x + iv_x$

i.e. $\frac{dw}{dz} = (1 + e^x \cos y) + i(e^x \sin y)$

$$= 1 + e^x(\cos y + i \sin y) = 1 + e^x e^{iy}$$

$$= 1 + e^{x+iy}$$

Since $z = x + iy$, $\frac{dw}{dz} = 1 + e^z$ //.

Q2 Find $P(x \le 1)$, $P(x > 1)$ and $P(-1 < x \le 2)$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $P(x)$ | $k$ | $2k$ | $3k$ | $4k$ | $3k$ | $2k$ | $k$ |

> We must have $p(x) \ge 0$ for all $x$ and $\sum p(x) = 1$. The first condition is satisfied if $k \ge 0$ and the second condition requires that, $k + 2k + 3k + 4k + 3k + 2k + k = 1$

or $16k = 1$ $\therefore k = 1/16$

The discrete / finite probability distribution is as follows.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $P(x)$ | $1/16$ | $2/16$ | $3/16$ | $4/16$ | $3/16$ | $2/16$ | $1/16$ |

Mean $\mu = \sum x\, p(x) = \frac{1}{16}(-3-4-3+0+3+4+3)$

$= 0$

Variance $v = \sum (x-\mu)^2 p(x)$

$v = \frac{1}{16}(9+8+3+0+3+8+9) = \frac{40}{16}$

$= 5/2$ //.

Thus $k = 1/16$, Mean $= 0$ and $S.D = \sqrt{5/2}$ //.

Also, $P(x \le 1) = P(-3) + P(-2) + P(-1) + P(0) + P(1) = 13/16$ //.

$P(x > 1) = P(2) + P(3) = 3/16$ //.

$P(-1 < x \le 2) = P(0) + P(1) + P(2) = 9/16$ //.

M4 - unit Test - 02

(18MATH1)

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(1) find the bilinear transformation which map points $z = 1, i, -1$ unto $w = i, 0, -i$

> Let $w = \frac{az+b}{cz+d}$ $\quad z = 1, w = i$.

$$\Rightarrow i = \frac{a+b}{c+d}$$

$a+b = ic+id \Rightarrow a+b-ic-id = 0$ — (1)

$z = i, w = 0 \Rightarrow 0 = \frac{ai+b}{ci+d}$

$ai+b \longrightarrow$ (2)

$z = -1, w = -i \Rightarrow -i = \frac{-a+b}{-c+d}$

$-a+b = ic-id \Rightarrow -a+b-ic+id = 0$ — (3)

Now, eqn (1) + (3)

$\cancel{a}+b-ic-\cancel{id} = 0$
$-\cancel{a}+b-ic+\cancel{id} = 0$

$2b-2ic = 0$

$\Rightarrow b-ic = 0$ — (4)

Eq (2) & (4) we write in the form,

$ai+b+0c = 0$

$0a+b-ic = 0$

Applying the rule of cross x^n

$$\frac{a}{\begin{vmatrix}1 & 0\\ 1 & -i\end{vmatrix}} = \frac{-b}{\begin{vmatrix}i & 0\\ 0 & -i\end{vmatrix}} = \frac{c}{\begin{vmatrix}i & 1\\ 0 & 1\end{vmatrix}}$$

$$\frac{a}{-i} = \frac{-b}{-i^2} = \frac{c}{i} \Rightarrow \frac{a}{i} = \frac{b}{-1} = \frac{c}{i} = k$$

$a = -ik \quad b = -k \quad c = ik$

Eqn (1) $-ik+k-i^2k-di = 0 \Rightarrow -ik-\cancel{k}+\cancel{k}-id = 0$

$\boxed{d = -k}$

$w = \dfrac{-ikz - k}{ikz - k} = \dfrac{-\cancel{k}(1+iz)}{-\cancel{k}(1-iz)}$ $\therefore w = \dfrac{1+iz}{1-iz}$ //.

Q2) Fit a curve of the form $y = ae^{bx}$ for following data.

| | | | | | |
|---|---|---|---|---|---|
| x | 77 | 100 | 185 | 239 | 285 |
| y | 2.4 | 3.4 | 7.0 | 11.1 | 19.6 |

> $Y = A + bx$

$\Sigma Y = nA + b\Sigma x \longrightarrow (1)$ and $\Sigma xY = A\Sigma x + b\Sigma x^2$ — (1)

$Y = \log_e$ , $A \log_e a$

| x | y | $Y = \log_e y$ | xY | $x^2$ |
|---|---|---|---|---|
| 77 | 2.4 | 0.8754 | 67.405 | 5929 |
| 100 | 3.4 | 1.2237 | 123.37 | 10000 |
| 185 | 7.0 | 1.9459 | 359.991 | 34225 |
| 239 | 11.1 | 2.4069 | 575.249 | 57121 |
| 285 | 19.6 | 2.973 | 848.017 | 81225 |
| $\Sigma x = 886$ | $\Sigma y = 43.5$ | $\Sigma Y = 9.4274$ | $\Sigma xY = 1973.03$ | $\Sigma x^2 = 188500$ |

$Y = ax^b \Rightarrow$ taking $\log_{10}$ on both side

$\log y = \log_{10}(ax^b) = \log_{10} a + \log_{10} x^b$

$\log_{10} y = \log_{10} a + b \log_{10} x$

$Y = A + bx$

$\Sigma Y = nA + b\Sigma x$ and $\Sigma xY = A\Sigma x + b\Sigma x^2$

(1) $\Rightarrow 9.4274 = 5A + 886b$

(2) $\Rightarrow 1973.03 = 886A + 188500b$

$A = 0.1838$ $b = 9.6028 \times 10^{-3}$

$A = \log_e a$ , $a = e^A$ $a = e^{0.1838}$

$a = 1.2017$ $b = 9.6028 \times 10^{-3}$

$\boxed{Y = (1.2017)e^{9.6028 \times 10^{-3} x}}$

M4- unit - test - 03

(18MAT41)

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(1) Ans:- probability distribution is valid if $p(x) \geq 0$ and $\Sigma p(x) = 1$

$\therefore k \geq 0$ and $k + 5k + 10k + 5k + k = 1$

$32k = 1$

(i) $\Rightarrow \boxed{k = 1/32}$

(ii) $\Rightarrow P(X \leq 1) = P(0) + P(1)$

| $x = x_i$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $p(x)$ | $1/32$ | $5/32$ | $10/32$ | | $5/32$ | $1/32$ |

$P(x \leq 1) = P(0) + P(1)$

$= 1/32 + 5/32 = 6/32 = 0.1876$

(ii) $\Rightarrow \boxed{P(x \leq 1) = 0.1876}$

(iii) $P(0 \leq x \leq 3) = P(0) + P(1) + P(2) = 1/32 + 5/32 + 10/32$

$= 16/32 = 0.5$ %.

(02) Ans:- we shall first resolve $\dfrac{1}{(z+1)^2(z-2)}$ into partial fractions.

let, $\dfrac{1}{(z+1)^2(z-2)} = \dfrac{A}{(z+1)} + \dfrac{B}{(z+1)^2} + \dfrac{C}{(z-2)}$

or $1 = A(z+1)(z-2) + B(z-2) + C(z+1)^2$

put $z = -1$, $\therefore B = -1/3$

$z = 2$, $\therefore C = 1/9$

$z = 0$, $\therefore A = -1/9$

Now, $$\frac{1}{(z+1)^2(z-2)} = -\frac{1}{9}\frac{1}{z+1} - \frac{1}{3}\frac{1}{(z+1)^2} + \frac{1}{9}\left(\frac{1}{z-2}\right)$$

multiplying by $e^{2z}$ and integrating w.r.t $z$ over $C$ we have,

$$\int_c \frac{e^{2z}}{(z+1)^2(z-2)}dz = \frac{-1}{9}\int_c \frac{e^{2z}}{(z+1)}dz - \frac{1}{3}\int_c \frac{e^{2z}}{(z+1)^2}dz + \frac{1}{9}\int_c \frac{e^{2z}}{z-2}dz \quad \text{--- (1)}$$

The points $z = a = -1$, $z = a = 2$ lies outside the circle $|z| = 3$

$\therefore$ Cauchy's Integral formula $\Rightarrow$

$$\int_c \frac{f(z)}{(z-a)}dz = 2\pi i f(a) \;\&\; \int_c \frac{f(z)}{(z-a)^{n+1}}dz = \frac{2\pi i}{n!} f^{(n)}(a)$$

Taking $f(z) = e^{2z}$ we obtain $f'(z) = 2e^{2z}$

Now, $$\int_c \frac{e^{2z}}{z+1}dz = \int_c \frac{e^{2z}}{z-(-1)}dz = 2\pi i f(-1) = 2\pi i e^{-2}$$

$$= \frac{2\pi i}{e^2}$$

$$\int_c \frac{e^{2z}}{(z+1)^2} = 2\pi i(2e^{-2}) = \frac{4\pi i}{e^2}$$

Also $$\int_c \frac{e^{2z}}{z-2}dz = 2\pi i f(2) = 2\pi i e^4$$

Substituting these results in the RHS of (1) we obtain

$$\int_c \frac{e^{2z}}{(z+1)^2(z-2)}dz = \frac{2\pi i}{9}\left(e^4 - \frac{7}{e^2}\right)$$