Internals - 01

Complex Analysis, probability and statistical Distribution.

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(1)

By data $w = z + e^z$

$u + vi = (x + iy) + (e^{x+iy})$

$(x + iy) + e^x e^{iy}$ $\qquad e^{iy} = \cos y + i \sin y$

$= (x + iy) + e^x (\cos y + i \sin y)$

$u = x + e^x \cos y$, $v = y + e^x \sin y$

$u_x = 1 + e^x \cos y$ $\qquad v_x = \cancel{\ } e^x \sin y$

$u_y = -e^x \sin y$ $\qquad v_y = 1 + e^x \cos y$

C-R eqn in the cartesian form $u_x = v_y$ & $v_x = -u_y$ are satisfied. $\therefore$ $w = z + e^z$ is analytic.

Now, $\frac{dw}{dz} = f'(z)$ $\cancel{u_x + iv_x}$.

$f'(z) = u_x + iv_x$

i.e $\frac{dw}{dz} = (1 + e^x \cos y) + i (e^x \sin y)$

$= 1 + e^x (\cos y + i \sin y)$

$= 1 + e^x e^{iy}$

$= 1 + e^{x + iy}$

$z = x + iy$ $\qquad \therefore \frac{dw}{dz} = 1 + e^z$ //.

1b. Statement:- If $f(z) = f(re^{i\theta}) = u(r,\theta) + i\,v(r,\theta)$ is analytic at a point z, then there exist four continuous first order partial derivatives.

$$\frac{\partial u}{\partial r}, \frac{\partial u}{\partial \theta}, \frac{\partial v}{\partial r}, \frac{\partial v}{\partial \theta}$$ and satisfy the equations

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \quad \& \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Proof:- if $z = re^{i\theta}$

then $f'(z) = \dfrac{f(z+\delta z) - f(z)}{\delta z}$ exists and is unique.

In the polar form $f(z) = u(r,\theta) + i\,v(r,\theta)$

and let δz be the increment in z corresponding to $\delta r, \delta\theta$ in r, θ.

$$f'(z) = \lim_{\delta z \to 0} \frac{[u(r+\delta r, \theta+\delta\theta) + i\,v(r+\delta r, \theta+\delta\theta)] - [u(r,\theta) + i\,v(r,\theta)]}{\delta z}$$

$$f'(z) = \lim_{\delta z \to 0} \frac{u(r+\delta r, \theta+\delta\theta) - u(r,\theta)}{\delta z}$$

$$+ i \lim_{\delta z \to 0} \frac{v(r+\delta r, \theta+\delta\theta) - v(r,\theta)}{\delta z} \quad \text{--- (1)}$$

consider $z = re^{i\theta}$.

$$\delta z = \frac{\partial z}{\partial r}\delta r + \frac{\partial z}{\partial \theta}\delta\theta$$

$$= \frac{\partial}{\partial r}(re^{i\theta}) + \frac{\partial}{\partial \theta}(re^{i\theta})\delta\theta$$

i.e $\delta z = e^{i\theta}\delta r + i r e^{i\theta}\delta\theta$ //.

Since δz tends to zero, we have the following two possibilities.

Case (i)

Let $\delta\theta = 0$ so that $\delta z = e^{i\theta}\delta r$, $\delta z \to 0$ imply $\delta r \to 0$

Now Eq^n (1) $\Rightarrow$

$$f'(z) = \lim_{\delta r \to 0} \frac{u(r+\delta r, \theta) - u(r,\theta)}{e^{i\theta}\delta r} + i \lim_{\delta r \to 0} \frac{v(r+\delta r,\theta) - v(r,\theta)}{e^{i\theta}\delta r}$$

i.e $f'(z) = e^{-i\theta}\left[\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right] \longrightarrow (2)$

Case (2) :- Let $\delta r = 0$ so that $\delta z = i r e^{i\theta}\delta\theta$ & $\delta z \to 0$ imply $\delta\theta \to 0$

Now Eq^n (1) $\Rightarrow$

$$f'(z) = \lim_{\delta\theta \to 0} \frac{u(r,\theta+\delta\theta) - u(r,\theta)}{r i e^{i\theta}\delta\theta} + i \lim_{\delta\theta \to 0} \frac{v(r,\theta+\delta\theta) - v(r,\theta)}{i r e^{i\theta}\delta\theta}$$

$$= \frac{1}{i r e^{i\theta}}\left[\lim_{\delta\theta \to 0} \frac{u(r,\theta+\delta\theta) - u(r,\theta)}{\delta\theta} + i \lim_{\delta\theta \to 0} \frac{v(r,\theta+\delta\theta) - v(r,\theta)}{\delta\theta}\right]$$

$$f'(z) = \frac{1}{i r e^{i\theta}}\left[\frac{\partial u}{\partial\theta} + i\frac{\partial v}{\partial\theta}\right]$$

$$= \frac{1}{r e^{i\theta}}\left[\frac{1}{i}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial\theta}\right]$$

$$f'(z) = \frac{1}{r e^{i\theta}}\left[-i\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial\theta}\right] = e^{-i\theta}\left[\frac{-i}{r}\frac{\partial u}{\partial\theta} + \frac{\partial v}{r\,\partial\theta}\right]$$

$$\therefore f'(z) = e^{-i\theta}\left[\frac{1}{r}\frac{\partial v}{\partial r} - \frac{i}{r}\frac{\partial u}{\partial\theta}\right] \longrightarrow (3)$$

Equating RHS of Eqn ② & ③ we have,

$$e^{-i\theta}\left[\frac{\partial u}{\partial r}+i\frac{\partial v}{\partial r}\right]=e^{-i\theta}\left[\frac{1}{r}\frac{\partial v}{\partial \theta}-\frac{i}{r}\frac{\partial u}{\partial \theta}\right]$$

$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta} \quad \text{and} \quad \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$$

(3a)

~~if k = 0 and~~

(3b)

we have mean $(\mu)=np$ and σ (S.D) $=\sqrt{npq}$ for a binomial distribution.

By data $np=2.5$

& $\sqrt{npq}=\sqrt{1.875}$

$\therefore npq=1.875$ $\qquad 2.5q=1.875 \quad \therefore q=0.75$

$p=1-q=0.25$

$np=2.5$, we have $(0.25)n=2.5$

$$\boxed{n=10}$$

Let x denotes the no of correctly answered ans.

$$P(x)={}^nC_x\, p^x q^{n-x}$$

$$={}^{10}C_x\left(1/4\right)^x\left(3/4\right)^{10-x}$$

$$p(x)=\frac{1}{4^{10}}\left[{}^{10}C_x(3)^{10-x}\right].$$

Since the estimate is needed for 4096 students we have,

$$4096\, p(x) = \frac{4096}{4^{10}} \left[{}^{10}C_x (3)^{10-x} \right]$$

$$= \frac{2^{12}}{2^{20}} \left[{}^{10}C_x (3)^{10-x} \right]$$

i.e $4096\, p(x) = \frac{1}{256} \left[{}^{10}C_x (3)^{10-x} \right]$

(i) we have to find

$f(8) + f(9) + f(10)$

$$= \frac{1}{256} \left[{}^{10}C_8\, 3^2 + {}^{10}C_9\, 3 + 1 \right]$$

$$= \frac{1}{256} \left[9 \cdot {}^{10}C_2 + 3 \cdot {}^{10}C_1 + 1 \right]$$

$$= \frac{1}{256} (436) = 1.703 \simeq 2$$

No. of students correctly answering 8 or more q^ns is 2

(ii) we have to find $f(0) + f(1) + f(2)$

$$= \frac{1}{256} \left[{}^{10}C_2\, 3^8 + {}^{10}C_1\, 3^9 + 3^{10} \right]$$

$$= \frac{3^8}{256} \left[45 + 30 + 9 \right] = \frac{3^8}{256} (84)$$

$$= 2152.8$$

$$\simeq 2153$$

No. of students correctly answering less than 2 questions is 2153.

(iii) we have to find $f(5)$

$$= \frac{1}{256}\left[{}^{10}C_5\, 3^5\right] = 239.2$$

≈ 239 //..

$\therefore$ No. of students correctly answering exactly 5 questions is 239 //..

(3a) we must have $p(x) \geq 0$ for all x and $\Sigma p(x) = 1$. The first condition is satisfied if $k \geq 0$. and second condition is requiring that $k + 2k + 3k + 4k + 3k + 2k + k = 1$

or $16k = 1$

$$\boxed{\therefore k = \frac{1}{16}}$$

The discrete / finite probability distribution is as follows

x	-3	-2	-1	0	1	2	3
$P(x)$	$1/16$	$2/16$	$3/16$	$4/16$	$3/16$	$2/16$	$1/16$

$$\mu = \Sigma x\, p(x) = \frac{1}{16}(-3 - 4 - 3 + 0 + 3 + 4 + 3) = 0$$

$$V = \Sigma (x - \mu)^2 . P(x)$$

$$V = 1/16\ (9 + 8 + 3 + 0 + 3 + 8 + 9) = \frac{40}{16} = 5/2$$

$\therefore k = 1/16 \quad \mu = 0 \quad \&\ S.D\ (\sigma) = \sqrt{5/2}$

Now, $p(x \leq 1) = p(-3) + p(-2) + p(-1) + p(0) + p(1)$

$= 13/16$

$p(x > 1) = p(2) + p(3) = 9/16$

$P(-1 < x \leq 2) = P(0) + P(1) + P(2) = 9/16$ //.