Central limit theorem

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Abstract

Statistics project for the course of statistics of Cybersecurity at Sapienza. I choose the Central limit theorem and in particular i concentrate my analysis on history, motivation,intuition and all the math details behind the theorem

1 Introduction

2 Historical context

2.1 Discovery of the Normal curve

The normal curve is perhaps the most important probability graph in all of statistics. Its formula is shown here with a familiar picture. The "e" in the formula is the irrational number we use as the base for natural logarithms. μ and σ are the mean and standard deviation of the curve Formula THAATH I NEED TO REWERITE

2.2 Abram De Moivre Discoveries

De Moivre pioneered the development of analytic geometry and the theory of probability by expanding upon the work of his predecessors, particularly Christiaan Huygens and several members of the Bernoulli family. He also produced the second textbook on probability theory.He was a consultant for gamblers, in fact he derive the formula trying to solving a gambling problem whose solution depended on finding the sum of the terms of a binomial distribution. Later work,especially by Gauss about 1800, used the normal distribution to describe the pattern of random measurement error in observational data. Neither man used the name "normal curve." That expression did not appear until the 1870s. The normal curve formula appears in mathematics as a limiting case of what would happen if you had an infinite number of data points. To prove mathematically that some theoretical distribution is actually normal you have to be familiar with the idea of limits, with adding up more and more smaller and smaller terms. This process is a fundamental component of calculus. So it's not surprising that the formula first appeared at the same time the basic ideas of calculus were being developed in England and Europe in the late 17th and early 18th centuries. The normal curve formula first appeared in a paper by DeMoivre in 1733. He lived in England, having left France when he was about 20 years old. Many French Protestants, the Huguenots, left France when the King canceled the Edict of Nantes which had given them civil rights. In England DeMoivre became a good friend of Isaac Newton and other prominent mathematicians. He wrote the 1733 paper in Latin, and in 1738 he translated it himself into English for the 2nd edition of his book, The Doctrine of Chances, one of the first textbooks on probability. (The first edition had been published in 1718.) In DeMoivre's work the normal curve formula did not look like it does now, in particular because there was no notation then for e. and there was no general sense of standard deviation, which is represented by σ in today's equation.

2.2.1 Why did DeMoivre do it? What problem was he working on?

he is dealing with "Problems of Chance," that he wants to see how likely it is that an "experiment" will produce a given outcome. Note that he credits the Bernoulli brothers with prior work – but they just didn't do quite enough. The core problem for DeMoivre is to find the sum of "several" terms in a binomial expansion. He wanted a shortcut because the problem was "so laborious." DeMoivre wanted to avoid having to add up all these coefficients. He needed to describe the general shape of the distribution of the values on a line of coefficients without having to compute each one. We can see what happens with a few graphs. A clear example of this problem can be seen in:

n	Expansion of $(a+b))^n$	Coefficients	Sum 2^n
1	a+b	1 1	2
2	$a^2+2ab+b^2$	1 2 1	4
3	$a^3+3a^2b+3ab^2+b^3$	1 3 3 1	8
4	—	1 4 6 4 1	16
5	—	1 5 10 10 5 1	32
-	etc.	etc.	etc.

Table 1: Coefficients table

Imagine that you want to find the sum of several terms in one line, say the middle two terms in line for n = 5. We quickly see that 10 + 10 = 20. But what if you want to find the sum of the middle 10 terms in the line where n = 100? A problem like this could easily come up in a game of chance. This is what he meant by "laborious."

DeMoivre wanted to avoid having to add up all these coefficients. A solution is to describe the general shape of the distribution of the values on a line of coefficients without having to compute each one. We can see(as he noticed) that as number of event increase distribution approached a smooth curve.

ex. Lets start with a binomial distribution for two event

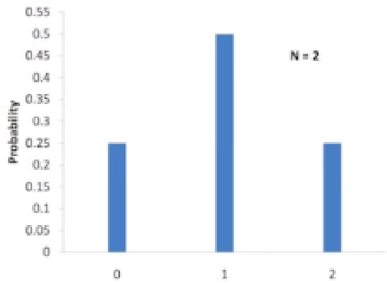

Figure 1: Binomial distribution of two event.

Then adding more event

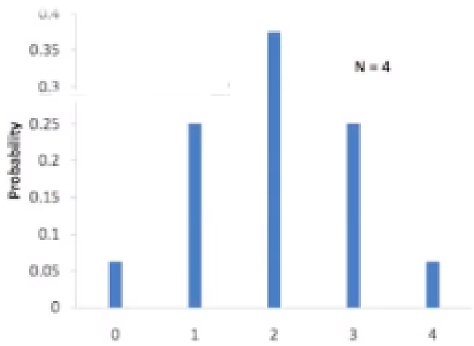

Figure 2: Binomial distribution of 4 event.

Then locking for a more crowded situation

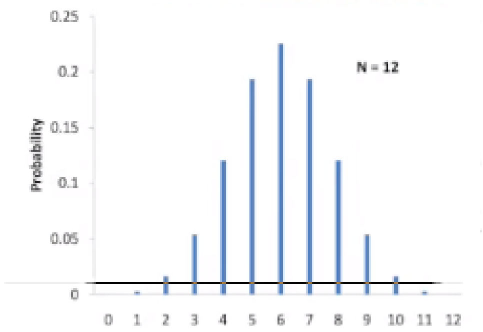

Figure 3: Binomial distribution of 12 event.

De Moivre understand the important of this result and try to find a way to write this curve infact if you can replace the binomial coefficient graph that is made up of lots of bars by a smooth curve, then instead of having to add lots of individual numbers you can just find the area under the curve, which is exactly one of calculus's strengths. You can see that as n increases the graphs look more and more like a bell-curve. DeMoivre began by considering the expansion of $(1+1)^n$. This expression comes up naturally in analyzing a coin toss with equal probabilities of heads and tails. He focused on the ratio of the middle term to the sum of all the coefficients, 2^n . DeMoivre wanted to know what happens to the ratio when n gets very large. It is informative to see how a 17th century mathematician has to deal with such a problem. He had "a dozen years or more" ago found that the ratio of the middle term to the sum can be expressed, using modern parentheses, as $2A(n+1)^n/n^n\sqrt{n-1}$

2.3 Gauss

Gauss about 1800, used the normal distribution to describe the pattern of random measurement error in observational data. Neither man used the name "normal curve." That expression did not appear until the 1870s.

3 The Central Limit theorem

3.1 Prove

The standard version of the central limit theorem was first proved by the French mathematician Pierre-Simon Laplace in 1810, states that the sum or average of an infinite sequence of independent and identically distributed random variables, when suitably rescaled, tends to a normal distribution. Fourteen years later the French mathematician Siméon-Denis Poisson began a continuing process of improvement and generalization. Laplace and his contemporaries were interested in the theorem primarily because of its importance in repeated measurements of the same quantity. If the individual measurements could be viewed as approximately independent and identically distributed, then their mean could be approximated by a normal distribution.

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First you have to upload the image file from your computer using the upload link in the file-tree menu. Then use the includegraphics command to include it in your document. Use the figure environment and the caption command to add a number and a caption to your figure. See the code for Figure 4 in this section for an example.

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Figure 4: This frog was uploaded via the file-tree menu.

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Item	Quantity
Widgets	42
Gadgets	13

Table 2: An example table.

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You can make lists with automatic numbering …

1. Like this,
2. and like this.

…or bullet points …

- Like this,
- and like this.

3.7 How to write Mathematics

LaTeX is great at typesetting mathematics. Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_{i}^{n} X_i$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.

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References

[Gre93] George D. Greenwade. The Comprehensive Tex Archive Network (CTAN). *TUGBoat*, 14(3):342–351, 1993.