Evolution of the Gains from Racial Desegregation in the US Marriage Market

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Abstract

Interracial marriages have increased in the US over the past several decades, but the trends differ across race, gender, and education groups. This suggests racial desegregation in the marriage market may not have improved marriage prospects of all groups. This paper studies why some groups have gained more from marital desegregation than others over the past four decades. To this end, I build a transferable utility matching model to define and estimate the welfare gains from marital desegregation by comparing the equilibrium rates of singlehood in the observed marriage market with those in a completely segregated marriage market. I find that among Blacks and Whites, college-educated men gained more than their female and lower-educated male counterparts. I implement a decomposition method based on quantitative comparative statics to examine the separate roles of the changes in population distribution and the changes in marital surplus from all types of race-education pairs. I find that the rise in the welfare gains for college-educated Black men is largely driven by the increase in the joint surplus from marriage with college-educated White women. Other Black men and women did not benefit as much from any change in the marital surplus. I also find that the rise in welfare gains for college-educated White men is mechanically driven by the increase in the number of Asian and Hispanic women in the marriage market. Simulation results suggest that progress toward racial integration in the marriage market would significantly reduce the rates of singlehood among Blacks.

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1 Introduction

Interracial marriage has been slowly but steadily increasing in the US since the *Loving v. Virginia* decision in 1967, which lifted all legal barriers to marrying across racial lines.[1] However, the trends are not the same for everyone. Among Blacks and non-Hispanic Whites (henceforth, Whites), men with four-year college degree (henceforth, college-educated) are more likely to marry out than their female counterparts and their lower-educated male counterparts. This suggests racial desegregation[2] in the marriage market may not have equally improved everyone's marriage prospect.

Interracial marriage matters, because not only is it a barometer of social integration by itself, but it also has a potential to transmit positive racial attitude to children.[3] Moreover, interracial marriage opens up more options of partners; if some groups face disproportionately high barriers to marrying out, it would limit their pool of potential partners. This is particularly concerning for Black women, who have a lack of marriageable men within their race due to the high incarceration rate of Black men (Charles and Luoh, 2010; Liu, 2020; Caucutt et al., 2021). Therefore, understanding why some groups marry out more than others is important to identify the barriers each group faces in the marriage market that may limit their marriage prospects.

It is challenging to distinguish what drives the differing interracial marriage rates, and there has been a lack of quantitative assessment on the drivers. Marriage rates depend on both the gains from marriage and the available pool of partners. On one hand, the interracial marriage rates may differ across groups due to differing gains from each type of marriage, which can be shaped by preferences or social stigmas that are prevalent in the market. Prior evidence reveals some explanations that are consistent with the interracial marriage trends. For example, women have stronger same-race preferences in dating than men (Fisman et al., 2008); higher educated people are more open to interracial marriage (Pew Research Center, 2017b); and Black women experience more social pressures and discrimination in marrying out than Black men do (Banks, 2012; Stewart, 2020). On the other hand, population supplies can also affect interracial marriage sorting, independent of preferences. For example, it may be that college-educated White and Black men marry out more simply because there are now increasingly more college-educated women than college-educated men, across all races, in the US marriage market. Therefore, without a clear framework, it is difficult to evaluate which of these two factors – the gains from marriage or the population distribution – plays a larger role in interracial marriage rates.

[1] As shown in Figure A1, the interracial marriage rate among married people aged 35-44 increased from 3% in 1980 to 11% in 2019.

[2] The term "desegregation" refers to the ending of the separation of two groups, usually referring to races. Hence, marital desegregation refers to the ending of all *legal* barriers to interracial marriage. Social or cultural barriers to marrying out can still exist in the desegregated world.

[3] Prior evidence shows that children of interracial couples have more social contact with racially diverse groups (Kalmijn, 2010) and are less likely to identify with an ancestry (Duncan and Trejo, 2011).

In this paper, I address three main questions. First, who has gained more from marital desegregation? Second, why do some groups gain more than others? Third, how would the rates of singlehood change if the marriage market becomes more racially integrated? To answer these questions, I build a transferable utility matching model in the spirit of Choo and Siow (2006) and estimate the model for each year using the Census data from 1980 to 2019. Using the estimated model, I first measure the welfare gains from marital desegregation by comparing the expected utilities in the (actual) desegregated marriage market and the expected utilities in the (counterfactual) completely segregated marriage market. Then, I implement a decomposition method to measure the contributions made by changes in population distribution and changes in marital surplus from all types of race-education pairs. Lastly, I perform counterfactual simulation to predict how progress towards complete racial integration[4] would affect the marriage rates across groups. I focus on explaining the gains for Blacks and Whites in this paper, because these two races have experienced the largest increases in interracial marriage rates over the past four decades.[5]

The benefits of the matching model are twofold. First, it identifies the marital surplus, which is the excess of the sum of utilities a couple gets when married together over what they get when remaining single. This value is distinguished from the mechanical population effects. To flexibly allow marital surplus to differ across race and education, I specify the type space used for matching as four major races (White, Black, Hispanic, Asian) interacted with four levels of educational attainment, which are (i) high school dropouts, (ii) high school graduates or GED, (iii) less than four years of college, (iv) four-year college degree or above. Second, the model yields a system of equilibrium matching functions, which provides a clear link between the equilibrium marriage patterns and the marital surplus as well as the population distribution. This system can capture the general equilibrium effects of the changes in population and the changes in marital surplus that happened in the US marriage market over the past four decades.[6]

I begin my analysis by estimating the matching model. The estimated maritial surplus matrices show several notable patterns. First, the joint surplus from interracial marriage among the college-educated increased over time across most pairs, suggesting that race has become less of a barrier to

[4] I define "complete racial integration" in the marriage market as a scenario where race is no longer a factor considered for marriage decision – i.e. there is no social cost of marrying across racial lines.

[5] The reasons I do not focus on explaining the gains for Hispanics and Asians are two-fold: First, interracial marriage rates have been fairly high (over 15%) for them even in 1980, and their propensities of marrying out did not increase over time as much as those of Blacks and Whites, as shown in Figure A2. Second, it is important to distinguish Hispanics and Asians by immigration status, as different generations of immigrants may have systematically different preferences for same-race marriage (Lichter et al., 2011; Furtado, 2015). It is outside the scope of this paper to examine welfare gains for Hispanics and Asians who exhibit markedly different interracial marriage trends and immigration trends from Blacks and Whites.

[6] Even a single change in population or in marital surplus can have equilibrium consequences. For example, Ahn (2022) shows that the changes in the cost of cross-border marriage between Taiwan and Vietnam, which were driven by the visa policy and the emergence of matchmaking firms, affected overall marriage patterns and intra-household allocations of *all* men and women.

marriage for the college-educated. In contrast, among the non-college-educated, the joint surplus from interracial marriage declined over time. Second, there is a wide variation in the value of interracial marriages depending on the gender and race of the spouse. Black-White marriages exhibit the lowest joint surplus among the interracial marriages involving a White spouse. Among Black-White marriages, marriages involving Black women have the lowest joint surplus. The values of interracial marriage among minority groups are lower than the values of interracial marriage involving a White spouse. These findings confirm that there is a wide variation in the barriers to interracial marriage.

Next, I characterize the welfare gains from marital desegregation using the matching model. These gains are defined as the expected utilities in the observed marriage market (with racial desegregation) minus the expected utilities in the counterfactual marriage market (with complete racial segregation). In the completely segregated market, everything else is the same as the observed world, *except* that the barriers to interracial marriage are set to be infinitely high for everyone. The counterfactual marriage patterns are constructed by setting the marital surpluses for all interracial marriages as infinitely negative and computing new equilibrium sorting patterns. Under the distributional assumption of Gumbel-distributed random preferences, the expected utilities in the framework are fully summarized by the probabilities of singlehood for each type. Hence, the welfare gains from marital racial integration essentially compare the equilibrium rates of singlehood between the racially desegregated market and the racially segregated market.

I find that among Black people, marital racial desegregation has improved the welfare of college-educated Black men the most. Even as early as 1980, college-educated Black men received positive welfare gains from marital desegregation, while their female counterparts and their lower-educated male counterparts received zero or negative gains. Over time, college-educated Black men have experienced a sharper increase in welfare gains than any other group among Blacks. In 2019, the magnitude of the welfare gains for college-educated Black men is substantial: marital desegregation has reduced the probability of being single for college-educated Black men by 17.5% compared to a complete segregation scenario. In contrast, Black women have not gained at all from marital racial desegregation, regardless of their education level, across all years. This shows that part of the reason behind low marriage rate of Black women[7] is that they face large barriers to marrying out.

Among White men, only college-educated men have experienced an increase in welfare gains over time, while non-college-educated men did not experience any rise in the gains. In 2019, the welfare gains for college-educated White men are translated into the reduction in the probability of being single by 8% compared to the scenario with complete segregation, while the correspond-

[7] As discussed in Caucutt et al. (2021), only 32% of Black women aged 25-54 were currently married in 2018, which is much lower than the marriage rate of White women (62%).

ing reduction is 2% for high school graduate White men. In contrast, all White women experienced similar rises in welfare gains over time, regardless of their education levels, from almost zero welfare gains in 1980. Overall, marital racial desegregation is welfare-improving for all White women in 2019; the probability of being single for White women is reduced by $3 \sim 5\%$ compared to a complete segregation scenario.

To understand why some groups gained more than others, I implement a decomposition method, for the first time in the marriage literature, to quantify the roles of various market-level changes, which include changes in population distribution and changes in joint surplus from all types of marriages. The idea for this method is to apply the implicit function theorem to a system of equilibrium matching functions. This captures the general equilibrium effects of each change in the model primitives on the welfare gains from marital desegregation for each type of man and woman. I use a fine-tuning method to link the four decades of changes in marital surplus and in population distribution to the changes in welfare gains. The estimated changes in welfare gains from this method closely match the changes from the data, thereby confirming the validity of this method. The main benefit of this method is that it successfully summarizes a large number of general equilibrium effects from various market-level changes that happened over a long time horizon.

I find that the population changes and the marital surplus changes made different impacts across groups. The rise in the college-educated Black men's welfare gains is largely driven by the increase in the joint surplus from the marriage with college-educated White women. Other Black men and women did not benefit as much from any change in the marital surplus. Notably, while college-educated Black women also gained from the increase in the joint surplus from the marriage with college-educated White men, this gain is less than half of what their male counterpart gained from the the marriage with White women. This finding suggests that the structure of marital surplus has evolved in a way that is only favorable to the most educated men among Blacks.

For Whites, I find that the rise in the welfare gains for college-educated White men is mechanically driven by the rise in the number of college-educated Asian and Hispanic women. For White high school graduate men, the decline in the joint surplus from marriage with Hispanic high school graduate women drove the overall decline in their welfare gains. College-educated White women did not gain at all from the population changes, which reflects that the imbalanced sex ratio among the college graduates, generated by the reversal of the college gender gap (Goldin et al., 2006), played an important role in driving the gender difference in welfare gains among White college graduates. Instead, White women, across all education levels, benefited from various equilibrium effects of the changes in the marital surplus.

Finally, I perform the counterfactual simulations to predict how the equilibrium rates of singlehood would change as the societal cost or barrier to marrying across racial lines decreases. I define “complete racial integration" as a scenario where race is no longer a factor considered for marriage matching. The result predicts that progress towards complete racial integration would significantly reduce the single rates of Black men and women. In contrast, racial integration would not affect the single rates of White men and women at all stages. This implies that the efforts toward improving racial relations could improve the marriage prospects for Blacks, who currently have low marriage rates and a high prevalence of single mothers, without harming the marriage prospects for Whites.

This paper contributes to several strands of literature. First, I contribute to the literature on the evolution of interracial marriage in the US. Existing literature examines interracial marriage trends across group using descriptive statistics (Fryer Jr., 2007; Pew Research Center, 2017b) and provides qualitative discussions. I add to this literature by providing quantitative assessments of the drivers for heterogeneous interracial marriage rates, using a structural matching model approach. Moreover, the measures of racial endogamy[8] have been developed in the previous literature (Fu and Heaton, 2008; Qian and Lichter, 2011), but they do not capture which group benefited from the declining tendency to marry within one's race. I provide the first estimates on who has gained more from marital desegregation and by how much.

Second, this paper contributes to the literature on equilibrium marriage sorting that uses a frictionless transferable utility (TU) framework. This paper is one of a few studies that investigate racial sorting using this framework. Since Choo and Siow (2006), who provided a benchmark model for empirically implementing the frictionless TU matching model with unobserved characteristics, this framework has been used and extended to study marriage sorting on various dimensions, including education (Chiappori et al., 2017, 2020), personality traits (Dupuy and Galichon, 2014), and income (Chiappori et al., 2022). Related to my paper, there has been a growing literature on interracial/interethnic marriages using the TU matching framework, including Ciscato and Weber (2020), who study the changes in multi-dimensional sorting in the US; Anderberg and Vickery (2021), who study the role of group density on racial sorting in the UK; and Adda et al. (2022), who study the role of legal status and cultural distance on intermarriage in Italy. However, none of these paper examines who has gained from desegregation in the US marriage market, and my paper is the first to define and estimate the welfare gain from marital desegregation for each demographic group using the TU matching model. Furthermore, this paper is the first to decompose the changes in equilibrium matching patterns into contributions by various market-level changes using the TU matching model.

Lastly, I add to the literature on the causes of the diverging patterns in marriage. As reviewed in Lundberg et al. (2016), marriage rates in the US have declined faster for high school graduates than college graduates and for Blacks than Whites. Most of the existing studies have examined the

[8]The term “racial endogamy" refers to the tendency of people to choose mates who are in the same racial group.

causes *within* each race, such as the rising incarceration of Black men (Charles and Luoh, 2010; Liu, 2020; Caucutt et al., 2021) and the decline in employment prospects for low-skilled male workers (Autor et al., 2019). I add to this literature by studying how different barriers to marrying *across* racial lines affect the probability of singlehood for different demographic groups.

The rest of the paper is organized as follows. Section 2 documents motivating trends that highlight difficulties in interpreting raw interracial marriage trends of Blacks and Whites. Section 3 describes the data and the sample selection for model estimation. Section 4 presents the matching model and explains the estimation of the marital surplus and expected utilities. Section 5 explains the method to measure the welfare gains from marital racial desegregation and presents the results. Section 6 presents the decomposition method and the results. Section 7 performs counterfactual simulations for complete racial integration. Section 8 concludes.

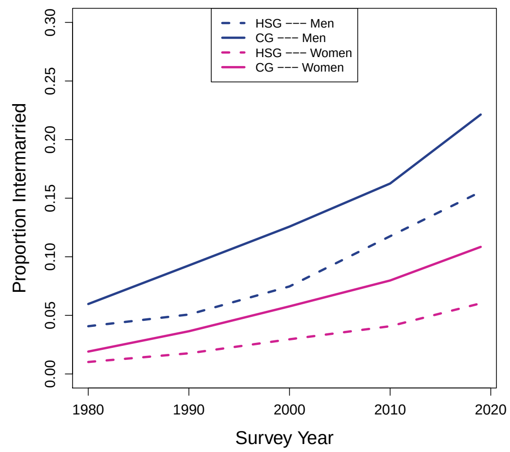
2 Motivating Trends

This section discusses the descriptive trends in interracial marriage and the trends in population distribution, which highlight the challenges in interpreting raw interracial marriage rates.

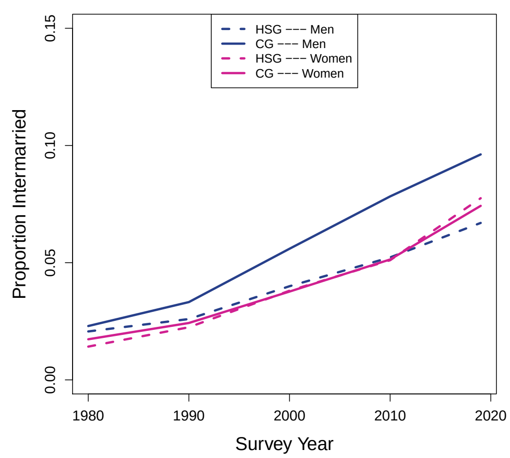
Interracial marriage rates have not increased in the same way across gender and education groups. Figure 1a shows that Black men with four-year college degree are more likely to marry out than their female counterparts and their high school graduate male counterparts. Notably, even high school graduate Black men are more likely to marry out than college-educated Black women. Similarly, Figure 1b shows that college-educated White men marry out more than their lower-educated male counterparts and their female counterparts. These differing trends reveal that racial desegregation in the marriage market may not have benefited everyone. For example, while Black women increasingly marry out more over time, Black men, especially the most educated ones, marry out even more. Therefore, it is unclear whether marital desegregation improved Black women's chances of marriage by giving them more options of partners or whether it worsened their chances by increasing competition for the college-educated Black men. Similar reasoning holds for White women, who marry out less than college-educated White men. This motivates the need for a formal measure of the welfare gain each group receives from marital desegregation, which I construct in Section 5.

It is also difficult to understand what drives these differing propensities of marrying out across groups. As emphasized throughout the literature on marriage markets (Chiappori and Salanié, 2016; Schwartz, 2013), marriage rates capture both the gains from marriage and the population distribution, and it is not straightforward to distinguish the two from marriage rates alone. Several changes in the US population may have affected interracial marriage rates, independent from the changes in the preferences or social stigma attached to interracial marriage. First, the US popula-

Figure 1: Interracial Marriage Rates Among Blacks and Whites



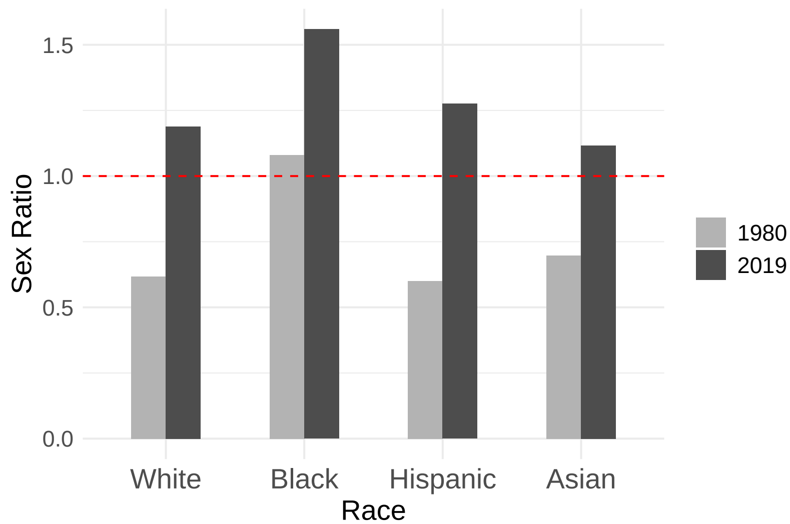
(a) Blacks



(b) Whites

Note: This figure shows the proportion of those who married out of their race among married individuals of the specified group aged 35-44 in each survey year. “HSG" refers to high school graduation or the equivalent GED. “CG" refers to the four-year college degree or above. Data sources for this figure are: 1960 5% sample Census, 1970 1% sample Census, 1980 5% sample Census, 1990 5% sample Census, 2000 5% sample Census, 2010 5% sample American Community Survey (2006-2010 5 year pooled sample), 2019 5% sample American Community Survey (2005-2019 5 year pooled sample). Survey weight is applied.

Figure 2: Female-to-Male Sex Ratio, Among 4-Year College Graduates, Age 35-44



Note: This figure shows sex ratio (female-to-male) among college graduates aged 35-44 for each race in 1980 and in 2019, respectively. Data sources for this figure are: 1980 5% sample Census microdata and 2019 5% sample American Community Survey (2015-2019 5-year pooled sample). Survey weight is applied.

tion has become more racially diverse due to the rise in Hispanic and Asian immigrants; Hispanic population has increased fourfold and Asian population has increased over sixfold since 1980 (Pew Research Center, 2017a, 2021). Therefore, the rise in interracial marriage could be a mechanical consequence of the rising Hispanic and Asian population.

Second, there has been a reversal of the gender gap in college education: while more men used to have college degree than women in the past, the opposite is true now. This is a well-documented trend (Goldin et al., 2006; Chiappori et al., 2009; Chuan and Zhang, 2022). What I additionally show in Figure 2 is that this reversal in the gender gap is true for *all races*. This implies that there are now a larger number of potential college-educated partners, across all races, for men than for women among college graduates. Therefore, the gender differences in interracial marriage shown in Figure 1 could be the mechanical consequence from the reversal of the gender gap in higher education, rather than the consequence from the gender differences in the gains from interracial marriage.

Motivated by these conjectures, I proceed by building a structural matching model in Section 4 that provides a clear framework to (i) measure the welfare gains from marital desegregation and to (ii) disentangle the effects of the structural marital gains from the mechanical effects of the population distribution.

3 Data

I begin by describing the data used for estimating the marriage matching model for each year spanning from 1980 to 2019. I use the US Decennial Census for years 1980, 1990, and 2000, and I use the 5-Year American Community Survey sample for the years 2010 and 2019, all of which are extracted from IPUMS (Ruggles et al., 2022).[9] The reason I start from year 1980 is because the census question on the Hispanic origin was added in 1980.[10] For the previous years, the Hispanic origin of each respondent is imputed by the IPUMS based on several criteria including one's and family's birthplace, surname, and family relationship, among others. However, it is problematic to use the imputed Hispanic variable, because interracial marriages involving Hispanics are not well-identified.[11]

I impose the following sample restriction for estimation. For *each* survey year, I first select the sample of currently married couples where wife is aged 35-44 and husband is aged 37-46. The lower bound of this age range is selected to exclude the ages that are too young so that people can marry in the future. The upper bound of the age range is selected to keep the age distribution across different calendar years comparable.[12] Two years of age gap between husband's age group and wife's age group reflects the fact that men tend to marry younger women, and the most common spousal age differences in the data are 1 and 2 years.[13] I focus on heterosexual married couples, because same-sex marriage had not been legalized nationwide until 2015.

To the married couple sample, I add the sample of never-married single men and women who are in the same corresponding age groups. I do not include divorced people in the single sample to abstract away from the issues of selection into divorce. Institutionalized individuals are excluded from the estimation sample, as they are unlikely to be participating in the marriage market. Never-married singles in the estimation sample include those living with unmarried partners.[14] As shown in the Table A2, cohabitation among the sample of singles has increased from 11.3% in 1980 to 24.6% in 2019. I later perform sensitivity checks in Appendix A.3 and confirm that excluding cohabiting singles do not affect the main results.

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[9] All data are 5-in-100 national sample of the population for the corresponding year.

[10] In fact, as discussed in O'Flaherty (2015), "Hispanic" only gained meaning around 1970.

[11] Specifically, the occurrences of marriages between non-Hispanic whites and Hispanics are recorded to be zero in 1960 and 1970. This is because spouse's race is one of the criteria to impute the Hispanic origin of individuals for 1960 and 1970 Decennial Censuses.

[12] The choice of age range 35-44 is common in the marriage literature (e.g. Chiappori et al. (2020), Bertrand et al. (2021)).

[13] Note that this age restriction rules out couples who have spouses outside the specified age range. For example, age 35 women married to age 33 men are excluded from the sample. This age restriction is necessary to properly estimate the marital surplus and perform counterfactual analyses, but it may be problematic as it arbitrarily rules out certain age pairs.

[14] From 1990 Census and onwards, "unmarried partner" living with the head of household can be identified.

I now describe the type spaces used for the estimation. I consider 4 races in my estimation, which are Non-Hispanic Whites, Black/African Americans, Hispanics, and Asians.[15] Hence, the type space for race is $\mathscr{R} \equiv \{White, Black, Hispanic, Asian\}$. I exclude other races, including mixed races because their sample size is too small; Appendix Table A1 shows that other races, which include Native Americans, Alaska Indians, and other races, are less than 1% of the population of interest each year; and people who reported to be mixed race, an option available from 2000 onward, make up less than 3% for each available year. For education type space, I consider four levels of educational attainment: $\mathscr{E} \equiv \{HSD, HSG, SC, CG\}$, where HSD: high school dropout, HSG: high school graduate or GED with no college education, SC: less than 4 years of college education, and CG: 4 years of college education or more. Hence, the full type spaces for the estimation are $\mathscr{M} = \mathscr{F} \equiv \mathscr{R} \times \mathscr{E}$, which consist of 16 different types. Specifically, $\mathscr{R} \times \mathscr{E} = \{WhiteHSD, WhiteHSG, WhiteSC, WhiteCG, \ldots, AsianHSD, AsianHSG, AsianSC, AsianCG\}$.

4 Marriage Matching Model

In this section, I present the matching model that serves as the building block for the analyses of marital racial integration throughout this paper. Building on Choo and Siow (2006), I construct a frictionless matching framework with perfectly transferable utility (TU) and random preferences to identify and estimate the structural gains from any race-education matching in the US marriage market.

4.1 The Setting

In this setting, each man or woman has two traits that are observed by the analyst: race and education.[16] Each man i belongs to a type $I = (R_i, E_i) \in \mathscr{M} \equiv \mathscr{R} \times \mathscr{E}$, where $\mathscr{R}$ and $\mathscr{E}$ denote the type spaces for race and for education, respectively. Similarly, each woman j belongs to a type $J = (R_j, E_j) \in \mathscr{F} \equiv \mathscr{R} \times \mathscr{E}$. In addition, each individual has other traits that are unobservable to the analyst but are observable to all men and women.[17]

A *matching* indicates who marries whom, including the choice for singlehood. I augment the type spaces for men and women to allow for singlehood: $\tilde{\mathscr{M}} := \mathscr{M} \cup \{\emptyset\}$ and $\tilde{\mathscr{F}} := \mathscr{F} \cup \{\emptyset\}$, where

[15] "Asian" include Chinese, Japanese, and other Asians or pacific islanders.

[16] I only focus on these two traits as they are likely to be determined before marriage. Other observable characteristics from the census data, such as the current wage and hours of work, are not used because they could be the outcomes that are endogenously determined by the marriage.

[17] The unobservable heterogeneity allows for richer matching patterns, which is otherwise not possible with a deterministic matching model. As discussed throughout the matching literature (Chiappori and Salanié, 2016; Chiappori, 2017; Galichon and Salanié, 2022), the uni-dimensional deterministic matching model gives a stark prediction that the matching is perfectly assortative, which is obviously unrealistic in the real world.

$\{\emptyset\}$ means no partner. Feasible matching must satisfy the population constraints. I denote n^I the number of type I men and m^J the number of type J women available in the marriage market. I also let μ^{IJ} denote the number of (I, J) marriages, $\mu^{I\emptyset}$ the number of single I men, $\mu^{\emptyset J}$ the number of single J women. The feasibility condition is the following:

$$n^I = \mu^{I\emptyset} + \sum_J \mu^{IJ} \quad \forall\, I \tag{1}$$

$$m^J = \mu^{\emptyset J} + \sum_I \mu^{IJ} \quad \forall\, J \tag{2}$$

In other words, the sum of the number of married and single must be equal to the total number of individuals in the marriage market by type and gender.

In a perfectly transferable utility framework, a matching also indicates how marital surplus z_{ij} generated by (i, j) marriage is divided between the couple. The joint marital surplus is interpreted as the total utilities man i and woman j get when married together minus the sum of utilities man i and woman j get when remaining single. It is expressed as a sum of two components:

$$z_{ij} = Z^{IJ} + \varepsilon_{ij} \tag{3}$$

where Z^{IJ} is a deterministic part of the surplus that depends only on observed types of spouses, and ε_{ij} is an idiosyncratic part of the surplus that reflects unobserved heterogeneity in marital preferences.

Similarly, the utility of singles is expressed as:

$$z_{i\emptyset} = Z^{I\emptyset} + \varepsilon_{i\emptyset}$$
$$z_{\emptyset j} = Z^{\emptyset J} + \varepsilon_{\emptyset j}$$

where $Z^{I\emptyset}$ and $Z^{\emptyset J}$ are normalized to zero.

A matching equilibrium is achieved when (i) no Mr. i or Ms. j who is currently married would rather be single and (ii) no Mr. i or Ms. j who are not currently married together would both rather be married together than remain in their current situation. This equilibrium condition results from stability, and the stable matching is generally unique (Gale and Shapley, 1962; Shapley and Shubik, 1971).

4.2 Identification of Marital Surplus and Expected Utility

As discussed in Choo and Siow (2006) and Chiappori and Salanié (2016), it is not possible to identify the joint surplus from marriage (Equation (3)) without further imposing a structure on the idiosyncratic terms. This is because the analyst cannot observe how people match based on unobserved traits. Following Choo and Siow (2006), I impose the separability assumption, which restricts matching on unobserved traits:

Assumption 1 (Separability). *The joint surplus from a marriage between a type I man and a type J woman is of the form*

$$z_{ij} = Z^{IJ} + \alpha_i^J + \beta_j^I. \tag{4}$$

This assumption allows for matching on unobservable traits, but conditional on the observed types of both spouses. For example, α_i^J reflects that a marriage between Mr. $i \in I$ and Ms. $j \in J$ may occur because Mr. i has unobservable traits (e.g. a certain hobby) that type J women value when choosing a partner. Moreover, α_i^J can also reflect that Mr i has idiosyncratic preferences for type J women. Similar implications hold for β_j^I. However, the separability assumption does not allow the matching on unobserved traits of both spouses. For example, this rules out matching that occurs because Mr. i has idiosyncratic preference for some unobserved traits of Ms. j.

The separability assumption leads to the following property, which is crucial for identification:

Proposition 1 (Choo and Siow, 2006; Chiappori et al., 2017). *For any stable matching, there exists values U^{IJ} and V^{IJ} satisfying the following property:*

- *Each man i will match with a woman of type J that maximizes $U^{IJ} + \alpha_i^J$ over $\tilde{\mathscr{F}}$*
- *Each woman j will match with a man of type I that maximizes $V^{IJ} + \beta_j^I$ over $\tilde{\mathscr{M}}$.*
- *$U^{I\emptyset}$ and $V^{\emptyset J}$ are normalized to be zero.*
- *$U^{IJ} + V^{IJ} = Z^{IJ}$ if the number of (I, J) match is non-zero.*

Proof. See Chiappori et al. (2017).

U^{IJ} (resp. V^{IJ}) can be interpreted as the husband's (resp. wife's) portion of the deterministic part of the joint marital surplus that is shared between the spouses. An important consequence of Proposition 1 is that the separability assumption simplifies the two-sided matching problem by turning it into a series of discrete choice problem. U^{IJ} can be obtained from a maximization of $U^{IJ}+\alpha_i^J$ over $\tilde{\mathscr{F}}$. Similarly, $V^{IJ} = Z^{IJ}-U^{IJ}$ can be obtained through a maximization of $Z^{IJ}-U^{IJ}+\beta_j^I$ over $\tilde{\mathscr{M}}$ given U^{IJ}. This leads to an identification of marital surplus Z^{IJ}.

Marital Surplus: Following a common practice in the literature, I assume that the unobserved heterogeneities α_i^J and β_j^I are distributed as standard type-I extreme values.[18] Then, solving the

[18] Galichon and Salanié (2022) show that any distributions for the random terms can be used to identify the marital surplus, as long as these distributions are known ex ante.

model in a standard way (Choo and Siow, 2006), I get the following formula for Z^{IJ} for husband's type I and wife's type J:

$$Z^{IJ} = ln\Big(\frac{(\mu^{IJ})^2}{\mu^{I\emptyset}\mu^{\emptyset J}}\Big) \tag{5}$$

Note that, unlike raw marriage rate, the above measure of marital surplus controls for the effects of demographic composition, by scaling the proportion of I, J marriages by the geometric average of the proportion of unmarrieds of those racial groups. Because the marital surplus is a function of number of couples and singles, it can easily be estimated using the empirical matching patterns in the data.

The notion of marital surplus can encompass any economic, social or other benefits associated with (I, J) marriage. Hence, Z^{IJ} for certain types of interracial marriage can have low values if there are high barriers to those types of marriage. What marital surplus *cannot* tell us is which party drives the value of the joint surplus. For example, if the marriage between a Black man and a White woman has a high value of joint surplus, we cannot distinguish whether it is because Black men value marriage with White women more or it is because White women value marriage with Black men more.

Expected utilities: Another object of interest, which will be used throughout this paper, is the type-specific expected utilities from the marriage market. This can also be easily identified and estimated in this framework. As shown in Choo and Siow (2006), the expected utility from the marriage market for male type I is the following:

$$\bar{u}^I = E\Big[\max_J(U^{IJ} + \alpha_i^J)\Big] = ln\Big(\sum_j exp(U^{IJ}) + 1\Big) = -ln(Pr(single \,|\, I)) \tag{6}$$

The above equation shows that the expected utility of type I men can be fully expressed by their probability of being single, which is a well-established property of assuming Gumbel distributed idiosyncratic terms in a discrete choice framework. A similar result applies to the female type J, and I denote $\bar{v}^J$ the expected utility of women of type J.

4.3 The System of Equilibrium Matching Functions

The matching model yields a system of equilibrium matching functions that link population distribution and marital surplus to equilibrium matching patterns. These functions allow counterfactual simulations and comparative statics, which will be used in Section 5 and Section 6.

To obtain this system of matching functions, I begin by re-arranging the marital surplus formula (Equation (5)) as the following:

$$\mu_t^{IJ} = exp\Big(\frac{Z^{IJ}}{2}\Big)\sqrt{\mu^{I\emptyset}\mu^{\emptyset J}} \tag{7}$$

Next, I plug the above expression into the feasibility constraints (Equations (1) and (2)) to obtain:

$$n^I = \mu^{I\emptyset} + \sum_J exp\Big(\frac{Z^{IJ}}{2}\Big)\sqrt{\mu^{I\emptyset}\mu^{\emptyset J}} \quad \forall\ I \tag{8}$$

$$m^J = \mu^{\emptyset J} + \sum_I exp\Big(\frac{Z^{IJ}}{2}\Big)\sqrt{\mu^{I\emptyset}\mu^{\emptyset J}} \quad \forall\ J \tag{9}$$

Let K be the total number of types for I and J, respectively. Then, Equations (8) and (9) define a system of $2K$ matching equations with $2K$ unknowns, which are the number of single men of each type ($\mu_t^{I\emptyset}$) and the number of single women of each type ($\mu_t^{\emptyset J}$) for all I, J.

The model primitives for this system are the vector of number of each type men (denoted as $\mathbf{n}$), the vector of number of each type women (denoted as $\mathbf{m}$), and the marital surplus matrix (denoted as $\mathbf{Z}$). The counterfactual simulations can be performed by deriving new equilibrium sorting patterns using the system of Equations (7), (8), and (9) with any counterfactual population distribution and/or marital surplus.

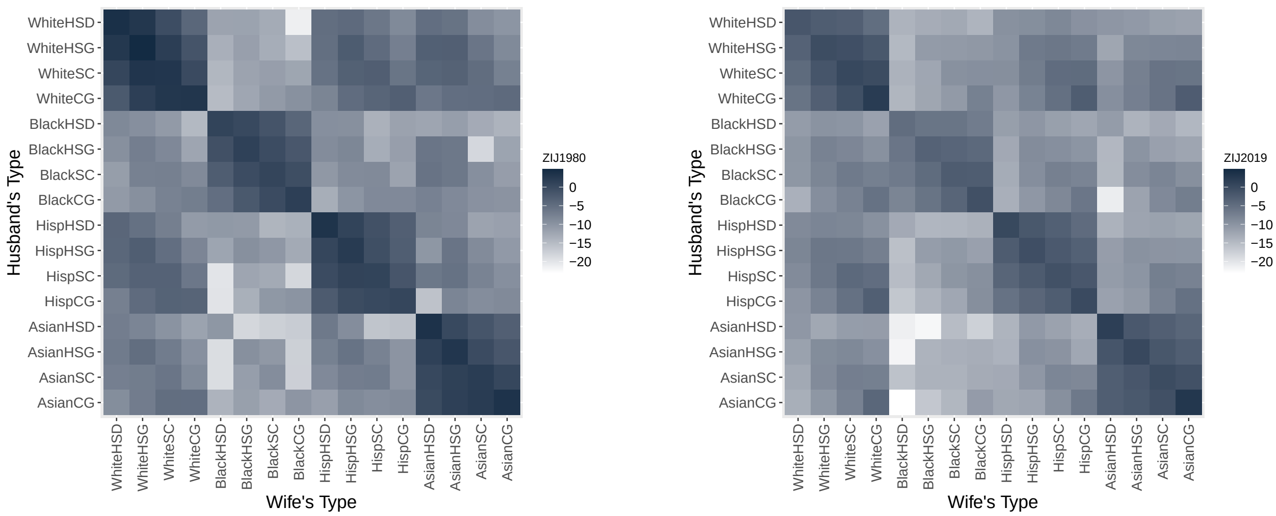
4.4 Descriptive Statistics: Estimated Marital Surplus

I estimate the marital surplus matrix for each year t using the empirical marriage patterns in the data. In this section, I provide descriptive statistics of the estimated marital surplus, which is denoted as $\hat{\mathbf{Z}}_t$. I document the evolutions of the value of interracial marriage across race and education groups.

In Figure 3, I plot the heatmaps of the marital surplus for the survey years 1980 and 2019.[19] Panel (a) confirms that the US marriage market was largely segregated by race in 1980. The values of marital surplus in 1980 are highest for the cells corresponding to the same-race marriages for all races. Panel (b) shows several patterns. First, compared to 1980, the values of marital surplus have generally gone down for most marriages in 2019, especially for the marriages involving lower-educated people. This reflects a well-known retreat from marriage in the US (Lundberg et al., 2016). Second, the marriage market is still largely segregated by race in 2019; the same-race marriages still exhibit highest values of marital surplus across all races. Third, within each block of interracial marriages, the values of marital surplus are highest for college graduates in 2019.

[19] Specific values of the marital surplus are presented in Appendix Figure A3 for year 1980 and Appendix Figure A4 for year 2019.

Figure 3: Estimated Marital Surplus $\hat{\mathbf{Z}}_t$, 1980 vs 2019



(a) 1980

(b) 2019

<u>Note:</u> This figure shows heatmap for estimated marital surplus $\hat{Z}_t^{IJ}$ for the survey year 1980 (Panel (a)) and the survey year 2019 (Panel (b)), respectively. I refers to husband's type (Row) and J refers to wife's type (Column). *HSD*: high school dropout, *HSG*: high school graduate with no college education, *SC*: less than 4 years of college education, and *CG*: 4 years of college education or more.

To better understand how the values of interracial marriage have changed compared to same-race marriage for different race and education groups, I report selected marital surplus for marriages and the corresponding changes in the marital surplus over the 1980-2019 period. Table 1 reports selected $\hat{Z}_t^{IJ}$ for marriages involving a White spouse, and Table 2 reports selected $\hat{Z}_t^{IJ}$ for marriages involving a Black spouse. When describing the changes in marital surplus below, I focus on the sign of the changes rather than on the magnitude of the changes. This is because marital surplus is a non-linear log function of number of marriages and singles as shown in Equation (5), which makes it difficult to directly compare the levels of the changes in $\hat{Z}_t^{IJ}$ across $\hat{Z}_t^{IJ}$s with differing starting values.

Table 1 reveals several implications about the evolution of the values of interracial marriage. First, it has become less costly to marry across racial lines for college graduates, but no evidence is shown for high school graduates. Panel A shows that the marital surpluses of all interracial marriages involving a White college-educated spouse have increased over the past four decades, while the value for the same-race marriage between White college-educated men and women has decreased. In contrast, Panel B shows that the marital surpluses of most interracial marriages, as well as the same-race marriage, have gone down for marriages involving a White high school graduate spouse. Notably, the values of interracial marriages between White high school graduates and Hispanic or Asian high school graduates have experienced a sharp decline over the past four decades. These different trends by education in the value of interracial marriage is consistent with the theory that achieved status like education is becoming increasingly more important than ascribed status like race in marriage (Kalmijn, 1991; Schwartz, 2013).

Table 1: Selected Marital Surplus Involving **White Spouse**

	Panel A: Marital surplus for CG couple	1980	2019	$\Delta^{2019-1980}$	**Panel B: Marital surplus for HSG couple**	1980	2019	$\Delta^{2019-1980}$
White-White	$Z^{WhiteCG,WhiteCG}$	3.01	2.13	**-0.88**	$Z^{WhiteHSG,WhiteHSG}$	4.74	-0.38	**-5.12**
White-Black	$Z^{WhiteCG,BlackCG}$	-9.58	-7.48	**+2.10**	$Z^{WhiteHSG,BlackHSG}$	-11.47	-10.84	**+0.63**
	$Z^{BlackCG,WhiteCG}$	-7.08	-5.55	**+1.53**	$Z^{BlackHSG,WhiteHSG}$	-7.21	-7.67	**-0.45**
White-Hispanic	$Z^{WhiteCG,HispCG}$	-3.01	-2.61	**+0.40**	$Z^{WhiteHSG,HispHSG}$	-2.46	-6.64	**-4.17**
	$Z^{HispCG,WhiteCG}$	-3.59	-2.92	**+0.67**	$Z^{HispHSG,WhiteHSG}$	-2.79	-6.24	**-3.44**
White-Asian	$Z^{WhiteCG,AsianCG}$	-4.5	-2.59	**+1.91**	$Z^{WhiteHSG,AsianHSG}$	-2.99	-8.43	**-5.44**
	$Z^{AsianCG,WhiteCG}$	-5.07	-4.07	**+1.00**	$Z^{AsianHSG,WhiteHSG}$	-5	-9.06	**-4.06**

Notes: This table reports selected marital surplus for marriages involving at least one White spouse. Panel A reports marital surplus for marriages where both spouses are college graduates. Panel B reports marital surplus for marriages where both spouses are high school graduates. For Z^{IJ}, I refers to husband's type and J refers to wife's type. $\Delta^{2019-1980}$ denotes the change in corresponding marital surplus from 1980 to 2019. CG refers to 4-year college graduates, and HSG refers to high school graduates or equivalent GED.

Second, the barriers of marrying across racial lines widely differ across gender and race. Black-White marriages exhibit the lowest value among the interracial marriages involving a White spouse. Among Black-White marriages, marriages between Black men and White women have higher values than marriages between White men and Black women, both in 1980 and in 2019. This shows that while there has been improvements over time, there are still barriers for Black-White marriages compared to other types of interracial marriages, especially for the marriages involving Black women. There is also a gender difference in White-Asian marriage: marriage between White men and Asian women have higher value than marriage between Asian men and White women. It is worth noting that these patterns of the marital surplus resemble the prior evidence on the racial preferences in the dating market. Both Hitsch et al. (2010) and Lin and Lundquist (2013) show using the data from dating applications that Black women and the Asian men are the groups that are least likely to send to or receive messages from dating candidates outside their race.

Similar to Table 1, Table 2 also shows that the most values of interracial marriage involving a Black spouse have only increased for the college-educated. The values of Black-Hispanic marriages and Black-Asian marriages are lower than the value of Black-White marriages in both years. This shows that there are higher social or cultural barriers to marry across races among minorities, even after accounting for their relatively small proportions in the population. The estimates also show that interracial marriages involving Black women have lower values than the interracial

Table 2: Selected Marital Surplus Involving **Black Spouse**

	Panel A: Marital surplus for CG couple				**Panel B: Marital surplus for HSG couple**			
		1980	2019	$\Delta^{2019-1980}$		1980	2019	$\Delta^{2019-1980}$
Black-Black	$Z^{BlackCG,BlackCG}$	1.63	-0.86	**-2.49**	$Z^{BlackHSG,BlackHSG}$	1.33	-3.4	**-4.74**
Black-White	$Z^{BlackCG,WhiteCG}$	-7.08	-5.55	**+1.53**	$Z^{BlackHSG,WhiteHSG}$	-7.21	-7.67	**-0.45**
	$Z^{WhiteCG,BlackCG}$	-9.58	-7.48	**+2.10**	$Z^{WhiteHSG,BlackHSG}$	-11.47	-10.84	**+0.63**
Black-Hispanic	$Z^{BlackCG,HispCG}$	-8.65	-6.1	**+2.55**	$Z^{BlackHSG,HispHSG}$	-8.3	-8.97	**-0.67**
	$Z^{HispCG,BlackCG}$	-9.99	-9.32	**+0.67**	$Z^{HispHSG,BlackHSG}$	-9.45	-11.12	**-1.67**
Black-Asian	$Z^{BlackCG,AsianCG}$	-9.9	-7.14	**+2.76**	$Z^{BlackHSG,AsianHSG}$	-6.35	-9.95	**-3.60**
	$Z^{AsianCG,BlackCG}$	-10.17	-10.96	**-0.79**	$Z^{AsianHSG,BlackHSG}$	-9.5	-13.87	**-4.38**

Notes: This table reports selected marital surplus for marriages involving at least one Black spouse. Panel A reports marital surplus for marriages where both spouses are college graduates. Panel B reports marital surplus for marriages where both spouses are high school graduates. For $Z^{I,J}$, I refers to husband's type and J refers to wife's type. $\Delta^{2019-1980}$ denotes the change in corresponding marital surplus from 1980 to 2019. CG refers to 4-year college graduates, and HSG refers to high school graduates or equivalent GED.

marriage involving Black men for all cases, which again confirms that there are higher barriers for Black women to marry out of their race than for Black men.

In the following sections, I investigate how these various changes in marital surplus over time have affected the excess utility that each group receives from the racially desegregated marriage market.

5 Measuring Gains from Marital Racial Desegregation

In this section, I measure the welfare gain from marital desegregation for each group. The goal is to understand the welfare implications of the heterogeneous trends of marrying out across groups shown in Figure 1. Specifically, I aim to understand whether racial desegregation has improved welfare even for the groups who marry out less than their counterparts, such as Black women and White women.

To this end, I construct a "racially segregated marriage market" using the matching model presented in Section 4. The idea is to construct a counterfactual world where everything is to be the same as the actual world, *except* for the barriers to interracial marriage, which are set to be infinitely high for everyone. The barriers, which I interchangeably refer to as the costs, include all social stigma or preferences attached to interracial marriage. All individuals re-optimize their marriage choices in this counterfactual marriage market, thereby resulting in new equilibrium matching patterns. Because the only differences from the actual market are the costs of interracial marriage, any deviation from the actual marriage patterns can be fully attributed to the changes in the costs of interracial marriage. Hence, I isolate the effects of marital racial desegregation by comparing the observed marriage patterns and the new equilibrium marriage patterns in a racially segregated marriage market.

Section 5.1 describes the estimation strategy in detail. Section 5.2 presents the results on the welfare gains from marital desegregation.

5.1 Estimation Strategy

Counterfactual simulation for complete segregation: I describe the steps to compute the counterfactual equilibrium marriage patterns for the scenario of complete racial segregation.

- **Step 1:** For each survey year t, I take the marital surplus matrix $\hat{\mathbf{Z}}_t$ that is estimated in Section 4.4.

- **Step 2:** For each $\hat{\mathbf{Z}}_t$, I replace $\hat{Z}_t^{IJ} = -\infty$ for all (I, J) that correspond to interracial marriage $(R_i \neq R_j)$. This is equivalent to setting the cost of interracial marriages to be infinitely high so that interracial marriages do not happen. Marital surpluses for all same-race marriage are set to be same as the estimated values in $\hat{\mathbf{Z}}_t$. Then, $\hat{\mathbf{Z}}_t^{Segregated}$ is the resulting counterfactual marital surplus matrix for complete racial segregation.

- **Step 3:** I compute the counterfactual marriage patterns for each survey year t, using $\hat{\mathbf{Z}}_t^{Segregated}$ and the actual population vectors $\mathbf{n}_t$ and $\mathbf{m}_t$,. This is done by applying the Iterative Projection Fitting Procedure (IPFP) on the system of matching functions represented by Equations (8) and (9) in Section 4.3.[20]

Through the above procedures, I obtain the counterfactual equilibrium numbers of single men and women of each type for each survey year. One remark is that complete racial segregation can be represented by *any* marital surplus matrix where the entries for interracial marriages have infinitely negative values. However, because the objective is to only capture the changes in sorting patterns due to changes in the value of interracial marriage, I choose the values of same-race marriages in $\hat{\mathbf{Z}}_t^{Segregated}$ to be same as their values in $\hat{\mathbf{Z}}_t$.

Welfare gains from marital desegregation: Using the counterfactual sorting patterns, I define and estimate "welfare gains from marital desegregation." This measure captures the excess utility each group receives in the actual marriage market over what they would get in a completely racially segregated marriage market. Although I do not directly quantify the costs of interracial marriage

[20] Galichon and Salanié (2022) explain that IPFP is an efficient and fast way to solve for the stable matching. This algorithm solves the system of equations defined by Equations (8) and (9) iteratively, starting from the vector of arbitrary guesses $\mu^I_{(0)}$ and $\mu^J_{(0)}$. An underlying idea behind this algorithm is that the average utilities ($\bar{u}^I$ and $\bar{v}^J$) of each type of men and women act as *prices* in the marriage market that equate demand and supply of partners. Hence, the algorithm adjusts the prices alternatively on each side of the market until it reaches the stable matching.

that may differ across groups, I implicitly account for the heterogeneous costs by comparing (i) the observed state with varying costs across individuals and (ii) the counterfactual state with the infinitely high costs of interracial marriage that are same for everyone. Hence, the welfare gains capture the differing evolution in the marital surplus for interracial marriage that are shown in Section 4.4.

Formally, welfare gains from marital desegregation for each man type I is defined as the following:

$$Gain^I_{m,t} = \underbrace{\bar{u}_t^{I,Desegregated}}_{Actual} - \underbrace{\bar{u}_t^{I,Segregated}}_{Counterfactual} \tag{10}$$

where $\bar{u}_t^{I,Desegregated}$ is the type-specific expected utility in the actual marriage market and $\bar{u}_t^{I,Segregated}$ is the corresponding expected utility in the racially segregated marriage market in year t. $Gain^I_{t,m}$ captures the *additional* expected utilities that type I men receive from the lowered cost of interracial marriage in year t.

Analogously, for each woman type J, the welfare gains from marital desegregation is:

$$Gain^J_{f,t} = \underbrace{\bar{v}_t^{J,Desegregated}}_{Actual} - \underbrace{\bar{v}_t^{J,Segregated}}_{Counterfactual} \tag{11}$$

As shown in Equation (6), the type-specific expected utilities are fully summarized by the probabilities of singlehood for each type under the assumption of Gumbel distributed stochastic terms. Therefore, the welfare gain essentially can be understood as the difference in the prevalence of singlehood between the actual world and the counterfactual world with complete racial segregation. For example, if fewer type I men remain single in the actual world than in the completely racially segregated world, it means that marital racial integration has increased the average welfare of type I men through an increase in the probabilities of marriage.

For ease of interpretation, I rescale the welfare gains to represent the percentage change in the single rate that would occur when the marriage market is completely segregated. To explain, note that $Gain^I_{m,t}$ can be re-written as:

$$\begin{aligned} Gain^I_{m,t} &= ln\Big(Pr(Single \mid I, t, Segregated)\Big) - ln\Big(Pr(Single \mid I, t, Desegregated)\Big) \\ &\approx \frac{Pr(Single \mid I, t, Segregated) - Pr(Single \mid I, t, Desegregated)}{Pr(Single \mid I, t, Desegregated)} \end{aligned}$$

Therefore, when multiplying the welfare gain by 100, the interpretation becomes the percentage change in the single rate of type I men that would occur if we move from desegregation to segregation in each year t.

Remark: This welfare gain measure is silent about the mechanisms through which marital desegregation operates. A positive utility gain *only captures* the fact that the single rate of a given type is lower in the desegregated world than in the perfectly segregated world. This gain *does not tell* anything about with whom those individuals marry more and what drives the gain. In Section 6, I investigate the specific mechanisms that drive the evolution of each welfare gain over time.

5.2 Results

I present the estimated welfare gains from marital desegregation in over the past four decades for Blacks and Whites in Figure 4. To facilitate the comparison of the magnitude of the welfare gains, I plot using the same scale on the y-axis for all groups. I discuss the results for each group below.

Results for Black men: In 1980, Black men had on average positive welfare gains from marital desegregation as shown in Figure 4a, even though these positive gains were not significantly different from zero. Over the years, the most educated Black men experienced the highest increase in welfare gains. In 2019, the magnitude of the welfare gains for college-educated Black men is substantial: in the absence of marital desegregation, the probability of being single would be on average 17.5% higher for the college-educated Black men.

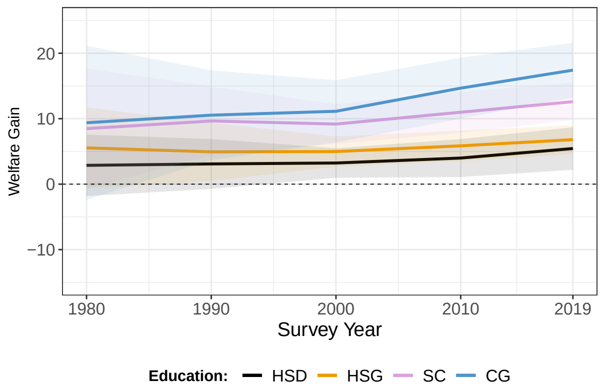
Results for Black women: In contrast, Figure 4b shows that Black women did not gain from marital desegregation across all years. While the welfare gain for college-educated Black women increased over time from the negative mean value in 1980, this increase is not as large as what college-educated Black men experienced.

This reveals a less-discussed aspect of the currently low marriage rates of Black women. Previous literature has focused on the explanations related to the within-race marriage market; the low marriage rate of Black women is typically attributed to the lack of marriageable Black men (Charles and Luoh, 2010; Liu, 2020; Caucutt et al., 2021). Figure 4b provides an explanation pertaining to the across-race marriage market. Black women do not benefit from marital desegregation at all, which further contributes to their currently low marriage rates.

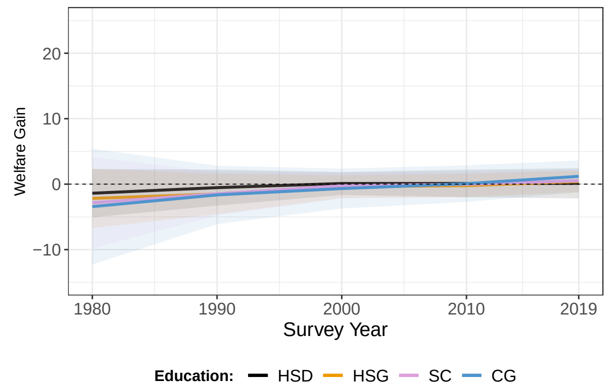
Results for White men: In 1980, all White men received on average positive welfare gains from marital racial desegregation as shown in Figure 4c, and these are statistically significant except for the high school dropouts. Over time, college-educated White men experienced a larger increase in welfare gains than other groups: in 2019, marital desegregation led to a reduction of in their probability of being single by 8% compared to the complete segregation scenario. In contrast, non-college-educated White men experienced a slight decrease in the welfare gains over time.

Results for White women: Welfare gains for White women show different patterns. While all White

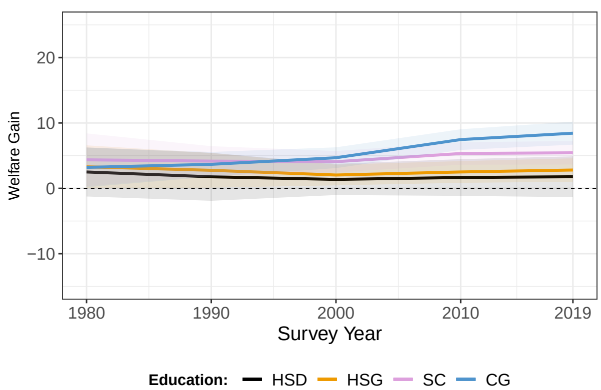
Figure 4: Type-Specific Welfare Gains from Marital Racial Integration



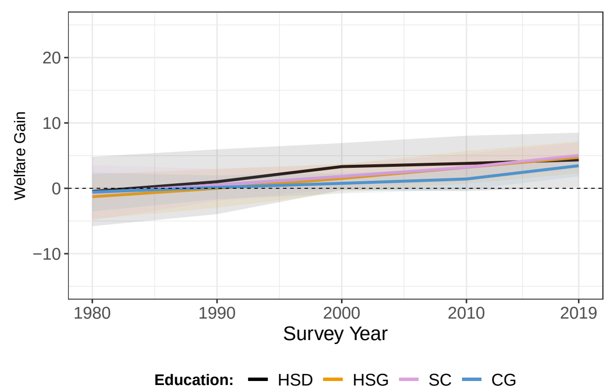
(a) Black Men



(b) Black Women



(c) White Men



(d) White Women

Note: These figures plot the welfare gain from marital desegregation as defined by Equation (10) and Equation (11) for each specified type of men and women. Data used to calculate the gains are: 1980-2000 Decennial Census, 2010 and 2019 5-Year ACS. I focus on age 37-46 men and age 35-44 women for each survey year. Further details on the sample restriction are described in Section 3. Shade for each line refers to the 95% confidence interval. Standard errors are calculated from the sampling variation in the data. *HSD*: high school dropout, *HSG*: high school graduate with no college education, *SC*: less than 4 years of college education, and *CG*: 4 years of college education or more.

women did not gain at all from marital racial desegregation in 1980, they increasingly gained more over time. Notably, there is no clear education difference in the trends, unlike the case of White men. In 2019, marital racial desegregation reduced the probability of being single for White women, across all education levels, by $3 \sim 5\%$ relative to the complete segregation scenario.

These results show that the welfare gains have not evolved in the same way even within each race. College-educated men benefited the most from racial desegregation in the recent years among Blacks and Whites. In the next section, I investigate why welfare gains from marital desegregation have evolved differently across groups.

6 Decomposition

The welfare gains from racial desegregation can be affected by both the population changes and the marital surplus changes. For example, the larger increase in the gains for college-educated Black men can be a byproduct of population changes that there are now more college-educated women than college-educated men in the marriage market, as shown in Figure 2. On the other hand, this can be affected by the increase in the value of interracial marriage among college-educated that is shown in Section 4.4.

I implement a decomposition method to measure the contributions of various market-level changes in population and in marital surplus on welfare gains for each type of men and women. Before describing the method, I first discuss the estimation challenges. Any single change in the population or in the structural value of marriage can affect the overall sorting patterns and the welfare of *all* men and women through the equilibrium channel. In the US marriage market, there have been a large number of changes over the past four decades; these include the changes in the number of each type of man and woman and the changes in the marital surplus for each type of marriage. Hence, it is challenging to summarize the equilibrium effects from such large number of different changes that happened in the marriage market over a long time horizon.

The decomposition method based on quantitative comparative statics successfully summarizes the general equilibrium effects from all changes in the marriage market. This method involves a combination of the implicit function theorem and a fine-tuning method to link four decades of changes in welfare gains to the four decades of changes in population and in marital surplus. The benefit of this method is that it can identify which changes played the biggest role in driving the change in welfare gains for each group. To the best of my knowledge, this decomposition has never been implemented in the marriage market literature.

Section 6.1 describes the method. Section 6.2 presents the decomposition results.

6.1 Overview of the Method

In this subsection, I provide a step-by-step description of the decomposition method.

Step 1: Implicit differentiation. The first step of this method is to use the implicit function theorem (IFT) on the system of equilibrium matching functions defined by Equations (8) and (9). The goal of this step is to measure how small changes in the model primitives affect the equilibrium number of single men and women of each type. To facilitate the application of IFT, I apply the following changes of variables: $\tilde{Z}_t^{IJ} = exp\left(\frac{Z_t^{IJ}}{2}\right)$ and $s_t^{I\emptyset} = \sqrt{\mu_t^{I\emptyset}}$ and $s_t^{\emptyset J} = \sqrt{\mu_t^{\emptyset J}}$. I use $\tilde{\boldsymbol{\theta}}_t = (\mathbf{n}_t, \mathbf{m}_t, \tilde{\mathbf{Z}}_t)$ to denote the complete set of model primitives.[21] Then, Equations (8) and (9) are re-written as:

$$F^I(\tilde{\boldsymbol{\theta}}_t) = (s_t^{I\emptyset})^2 + \sum_J \tilde{Z}_t^{IJ} s_t^{I\emptyset} s_t^{\emptyset J} - n_t^I = 0 \qquad \forall\, I \tag{12}$$

$$G^J(\tilde{\boldsymbol{\theta}}_t) = (s_t^{\emptyset J})^2 + \sum_I \tilde{Z}_t^{IJ} s_t^{I\emptyset} s_t^{\emptyset J} - m_t^J = 0 \qquad \forall\, J \tag{13}$$

Because each of $2K$ matching equations defined by Equations (12) and (13) is dependent on one another, I need to apply the IFT on the *whole* system in order to get partial derivatives of $s_t^{I\emptyset}$ for all I and $s_t^{\emptyset J}$ for all J with respect to each of the marginal changes in the model primitives $\tilde{\boldsymbol{\theta}}_t$.

For the brevity of notation, I use $\mathbf{s} = (s^{1\emptyset}, \dots, s^{K\emptyset}, s^{\emptyset 1}, \dots, s^{\emptyset K})$ to denote a vector of the square root of the number of singles of each type of individuals. I also denote $\mathbf{F}$ as a vector for F^I and $\mathbf{G}$ as a vector for G^J. Applying the IFT on the system of equations (12) and (13) leads to the following Jacobian matrix of partial derivatives of $\mathbf{s}_t$:

$$\left[\frac{\partial \mathbf{s}_t}{\partial \tilde{\boldsymbol{\theta}}_t}\right]_{(2K)\times(2K+K^2)} = -\begin{bmatrix} \dfrac{\partial \mathbf{F}}{\partial \mathbf{s}_t} \\[2ex] \dfrac{\partial \mathbf{G}}{\partial \mathbf{s}_t} \end{bmatrix}^{-1}_{(2K)\times(2K)} \begin{bmatrix} \dfrac{\partial \mathbf{F}}{\partial \tilde{\boldsymbol{\theta}}_t} \\[2ex] \dfrac{\partial \mathbf{G}}{\partial \tilde{\boldsymbol{\theta}}_t} \end{bmatrix}_{(2K)\times(2K+K^2)} \tag{14}$$

The full solution for the partial derivatives is presented in Appendix B.1. This Jacobian matrix summarizes how a small change in each model primitive $\tilde{\theta}_t^k$ affects the equilibrium number of single men and women of each type.

Step 2: Linking the Jacobian to welfare gains from marital desegregation. Because the welfare gain is a function of equilibrium single rates as shown in Equations (10) and (11), it is straightforward to use the Jacobian matrix (Equation (14)) to understand how a small change in each $\tilde{\theta}^k$ affects the welfare gains for each type of man and woman.

[21] Full expansion of $\tilde{\boldsymbol{\theta}}_t$ is $\tilde{\boldsymbol{\theta}}_t = \left(n_t^1, \dots, n_t^K, m_t^1, \dots, m_t^K, \tilde{Z}_t^{11}, \tilde{Z}_t^{12}, \dots, \tilde{Z}_t^{KK}\right)$. This vector has $2K + K^2$ components; because $K = 16$ in my setting, $\tilde{\boldsymbol{\theta}}_t$ has 288 components.

To demonstrate, consider the expected utility for type I men. Recall that $\bar{u}_t^I = -ln(Pr(single|I,t)) = -ln\left(\frac{\mu_t^{I\emptyset}}{n_t^I}\right)$ and $\mu_t^{I\emptyset} = (s_t^{I\emptyset})^2$. Then, the total differential of $\bar{u}_t^I$ is:

$$d\bar{u}_t^I = \frac{1}{n_t^I} dn^I - \frac{2}{s_t^{I\emptyset}} \left(\frac{\partial s_t^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}_t} d\tilde{\boldsymbol{\theta}}_t \right) \tag{15}$$

where the partial derivative $\frac{\partial s_t^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}_t}$ is from the Jacobian matrix shown in Equation (14). Because the welfare gain is the difference between the expected utilities for the actual world and for the completely segregated world, it can be expressed analogously using Equation (15).

Step 3: Fine-tuning method to link four decades of changes in $\tilde{\boldsymbol{\theta}}$. The implicit function theorem approach only applies to small changes in the model primitives. However, the objective is to understand the effects of four decades of changes in population and in marital surplus on welfare gains. Using the implicit function approach for such large changes may led to an incorrect decomposition.

To better approximate the effect of changes in model primitives on changes in welfare gains over the past four decades, I implement a fine-tuning method following Judd (1998). This method decomposes the large changes in the model primitives into a series of infinitesimal changes. Using this method, I evaluate the differentials for each infinitesimal change and update the approximation along the path of infinitesimal changes. I apply this method for each decade within the 1980-2019 period, based on available survey years.

To give a concrete example, I consider the changes from 1980 to 1990. I denote 1980 as $\tau = 0$ and 1990 as $\tau = 1$. Then $\tilde{\boldsymbol{\theta}}_0$ (resp. $\tilde{\boldsymbol{\theta}}_1$) is the vector of the values of model primitives in 1980 (resp. in 1990). Consider the homotopy:

$$\tilde{\boldsymbol{\theta}}_\tau = \tau \tilde{\boldsymbol{\theta}}_1 + (1-\tau)\tilde{\boldsymbol{\theta}}_0, \quad \tau \in [0,1]$$

which defines a series of intermediate values of the model primitives with interval $d\tau$ between observed values at $\tau = 0$ and $\tau = 1$. Because $\tilde{\boldsymbol{\theta}}_\tau$ is now a function of τ, Equation (15) is re-written as:

$$d\bar{u}_\tau^I = \frac{1}{n_\tau^I}(n_1^I - n_0^I) d\tau - \frac{2}{s_\tau^{I\emptyset}} \left(\frac{\partial s_\tau^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}_\tau} (\tilde{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_0) d\tau \right) \tag{16}$$

Using the above specification, I can use infinitesimal change $d\tau$ to evaluate and decompose the infinitesimal changes in $\bar{u}_t^I$. In practice, I specify $d\tau = 0.001$ when estimating Equation (16) for each decade. Summing $\Delta^{(\tau+d\tau)-\tau} \bar{u}_\tau^I$ over all $\tau \in [0,1]$ gives better approximation of $\Delta \bar{u}_t^I$ than directly using the observed 10-year changes of model primitives to evaluate Equation (15).

Application to the four decades of changes in welfare gains from racial desegregation is done analogously. Further details on how I perform this method are documented in Appendix B.2.

Step 4: Decomposition. The above steps lead to a linear decomposition of four decades of changes in welfare gains into contributions from changes in population and in marital surplus. Specifically, the welfare gain from marital desegregation of type I man between 1980 to 2019 are decomposed into the following:

$$\begin{aligned}\Delta^{2019-1980} Gain^I = & (Contribution\, by\, \Delta n^1) + \ldots + (Contribution\, by\, \Delta n^K) \\ & + (Contribution\, by\, \Delta m^1) + \ldots + (Contribution\, by\, \Delta m^K) \\ & + (Contribution\, by\, \Delta Z^{11}) + \ldots + (Contribution\, by\, \Delta Z^{KK}) \end{aligned} \tag{17}$$

Decomposition for type J women can be similarly expressed. As shown in Equation (17), this method can summarize a large number of the contributions from various market-level changes. Therefore, this allows me to identify which population changes or joint surplus changes have driven the welfare gain from marital desegregation for each group of men and women over the past four decades.

I evaluate the validity of the decomposition method by comparing the estimated changes in welfare gains from the data (using Equations (10) and (11)) and the estimated welfare gains using the IFT approach. The latter is simply the total sum of contributions for each group, as represented by the right-hand side of Equation (17). Table 3 shows that the estimates based on the method closely match those based on the data for each group of man and woman. This confirms the validity of the method.

Table 3: Data vs. IFT: 1980-2019 Changes in Welfare Gains from Marital Racial Desegregation

	Men		Women	
Type	Data	IFT	Data	IFT
WhiteHSD	-0.728	-0.730	4.826	4.827
WhiteHSG	-0.492	-0.494	6.018	6.021
WhiteSC	1.076	1.074	5.727	5.729
WhiteCG	5.231	5.230	4.020	4.021
BlackHSD	2.566	2.567	1.515	1.516
BlackHSG	1.254	1.255	2.575	2.576
BlackSC	4.114	4.115	3.366	3.366
BlackCG	8.028	8.027	4.635	4.637

Notes: This table reports the changes in (i) welfare gain of each group over the 1980-2019 period that is estimated from data and (ii) the corresponding changes estimated from the IFP method. “Data" column refers to the estimated change from the data, and “IFT" column refers to estimated changes using IFT according to Equation (17). $d\tau = 0.001$ is used when applying the fine-tuning method.

6.2 Decomposition Results

In this section, I start by documenting the contributions made by overall population changes and overall marital surplus changes. Recall that the welfare gain is interpreted as the percentage difference in the single rate that would occur in the absence of marital desegregation. Hence, $\Delta^{2019-1980} Gain$ in Table 4 represents the percentage point changes in these percentage differences over the 1980-2019 period.

Table 4 shows that the overall population changes and the overall marital surplus changes made different impacts across groups. For Blacks, the combined marital surplus changes had a positive and larger effect on the welfare gains than the combined population changes, across all groups. Notably, Black college-educated men have gained the most from the changes in marital surplus. The population changes only played a minor (and negative) role in the overall increase in welfare gains for Black college-educated men. This shows that the reversal of the gender gap in college education shown in Figure 2 did not drive the welfare gains for Black college-educated men.

A different picture emerges for Whites: the overall population changes and the overall marital surplus changes have opposite effects for different gender. For White men, the composite population changes made the positive contributions, whereas the composite marital surplus changes made the negative contributions to the welfare gains. For White high school graduate men, the large negative force from marital surplus changes completely offset the positive contributions from population changes. In contrast, White women benefited from the overall marital surplus changes, and these positive contributions are larger than the negative contributions from population changes.

In the following subsections, I document the specific changes that made the largest contributions to the welfare gain of each group.

Table 4: Decomposition of the 1980-2019 changes in the welfare gain from marital racial desegregation

		Total Contribution by the Changes in:	
Type	$\Delta^{2019-1980}Gain$	Population	Marital Surplus
BlackCG Men	8.0	-2.5	10.6
BlackHSG Men	1.3	-3.1	4.3
BlackCG Women	4.6	0.8	3.8
BlackHSG Women	2.6	1.0	1.6
WhiteCG Men	5.2	6.8	-1.6
WhiteHSG Men	-0.5	5.4	-5.8
WhiteCG Women	4.0	-1.2	5.2
WhiteHSG Women	6.0	-0.7	6.7

Notes: This table presents the decomposition of the 1980-2019 changes in the welfare gains from marital racial integration for the specified type of individuals. $\Delta^{2019-1980}Gains$ is the change in the welfare gains for the specified group over the 1980-2019 period. “Population" Column shows the summation of all contributions by changes in population over the 1980-2019 period. “Marital Surplus" Column shows the summation of all contributions by changes in marital surplus over the 1980-2019 period..

6.2.1 What drove the rise in welfare gains for Black college-educated men?

Table 5 presents top three positive and top five negative contributions from changes in marital surplus for college-educated Black men. The main finding is that $Z^{BlackCG,WhiteCG}$, the marital surplus for the marriage between Black college-educated men and White college-educated women, made the largest positive contribution. The magnitude of the contribution $Z^{BlackCG,WhiteCG}$ is substantial: it represents about 60% of the total change of their welfare gain. The contribution from $Z^{BlackCG,WhiteCG}$ is more than twice times larger than other top positive contributions. Top negative contributions only play a minor role.

Because $Z^{BlackCG,WhiteCG}$ has increased over time as shown in Table 2, the result reveals that the increase in the joint surplus from marriage between Black college-educated men and White college-educated women played a substantial role in driving up the welfare gains for Black college-educated men. It has to be noted that the matching model cannot distinguish precise reasons why the marital surplus for this pair has increased over time. However, this finding is consistent with

Table 5: Decomposition: Top three contribution from changes in **Z**, Black CG Men

		(1)	(2)	(3)
Contribution	Top (+)	4.7 $Z^{BlackCG,WhiteCG}$	1.8 $Z^{BlackCG,BlackSC}$	1.5 $Z^{BlackCG,HispCG}$
	Top (-)	-0.6 $Z^{WhiteCG,BlackCG}$	-0.6 $Z^{BlackSC,BlackCG}$	-0.4 $Z^{BlackCG,AsianHSD}$

Notes: This table presents the top three positive and negative contributions from marital surplus to the 1980-2019 changes in the welfare gains from marital desegregation for Black college graduate men. For marital surplus Z^{IJ}, I refers to husband's type and J refers to wife's type.

the story of less social stigma over time in marrying out among the college-educated. Moreover, the finding is also consistent with the significant advances in earnings of college-educated Black men, as documented in Bayer and Charles (2018), that made them more attractive partners to White college-educated women.

Other notable findings in Table 5 include the positive contribution from $Z^{BlackCG,BlackSC}$, which is the joint surplus from marriage between Black college-educated men and Black some-college women. Appendix Figure A5 shows that $Z^{BlackCG,BlackSC}$ has declined over time. Hence, this decomposition result suggests that Black college-educated men benefitted from having the options for different-race partners as the joint surplus from marriage with Black some-college women went down. Furthermore, Table 5 additionally shows that Black college-educated men benefitted from the increase in the joint surplus from marriage with Hispanic college-educated women.

6.2.2 Why did other groups within Blacks not gain as much?

Table 6 presents top three positive and top five negative contributions from changes in marital surplus for Black college-educated women. I find that none of the changes in Z^{IJ} had a comparably large positive impact on Black college-educated women's welfare gain, relative to the case of Black college-educated men. The largest positive contribution for Black college-educated women is from the change in $Z^{WhiteCG,BlackCG}$, the joint surplus from marriage between White college-educated men and Black college-educated women, which has increased over time as shown in Table 2. However, this impact is much lower than what Black college-educated men experienced from the rise in $Z^{BlackCG,WhiteCG}$ as shown in Table 5. These gender differences are consistent with the anecdotal evidence that Black women face higher barriers in marrying out than Black men due to social pressures (Banks, 2012) and discrimination in the dating market (Stewart, 2020).

I also find that none of the changes in the marital surplus had a large positive effect for Black high school graduate men and women, which are presented in Appendix Tables A3 and A4. These results confirm that among Blacks, the structural changes in the marital surplus have only been favorable to most educated Black men.

Table 6: Decomposition: Top three contribution from changes in **Z**, Black CG Women

		(1)	(2)	(3)
Contribution	Top (+)	2.0 $Z^{WhiteCG,BlackCG}$	1.9 $Z^{BlackCG,BlackCG}$	1.2 $Z^{BlackSC,BlackCG}$
	Top (-)	-1.0 $Z^{BlackCG,BlackSC}$	-0.9 $Z^{BlackCG,WhiteCG}$	-0.5 $Z^{BlackCG,BlackHSG}$

Notes: This table presents the top three positive and negative contributions from marital surplus to the 1980-2019 changes in the welfare gains from marital desegregation for Black college graduate women. For marital surplus Z^{IJ}, I refers to husband's type and J refers to wife's type.

6.2.3 What drove the rise in welfare gains for White college-educated men?

Recall that the increase in White college-educated men's welfare gain is driven by the overall population changes, as shown in Table 4. To understand which population changes played the largest role, I first decompose the population contribution into the ones made by (i) different-race female population, (ii) different-race male population, (iii) same-race female population, and (iv) same-race same male population. Table 7 shows that the positive contribution is mostly driven by the changes in different-race female population. This contribution is large enough to completely offset the negative contribution from the changes in different-race male population, who are the competitors to White men in the desegregated marriage market.

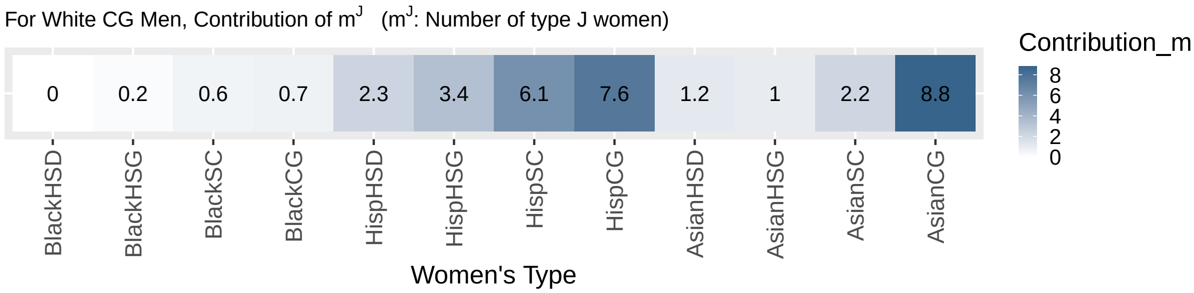
Table 7: Contributions from population changes to welfare gains for White CG men

	Contribution from Population			
	Diff-Race Female	Diff-Race Male	Same-Race Female	Same-Race Male
WhiteCG Men	34.1	-26.3	-5.0	4.0

Notes: This table presents the decomposition of the 1980-2019 changes in the welfare gains from marital desegregation for White CG men. This table focuses on the contributions from the population changes. Each column shows the summation of all contributions made by populations corresponding to the label.

Figure 5 further reveals that the rise in the number of Asian college-educated women and Hispanic college-educated women made the largest positive impact on White college-educated men's welfare gain over the past decades. The implication of these results is that the rise in the welfare gains over time for White college-educated men was a mechanical consequence from the increase in the college-educated Asian and Hispanic population, rather than from the increase in the value of interracial marriages.

Figure 5: Details on the contributions from the changes in different-race, different-sex population



Notes: This figure presents the decomposition of the 1980-2019 changes in the welfare gains from marital desegregation for White CG men. This figure focuses on the contributions from the changes in different-sex different-race population for White CG men. Each column shows the contribution made by the change in the corresponding population.

6.2.4 Why did the gains for White high school graduate men not increase over time?

One notable finding from Table 4 is that White high school graduate men had a large negative total contribution from the changes in marital surplus. To further investigate this, I present top three positive and negative contributions from the changes in marital surplus. Table 8 shows that the largest negative contribution is from the change in $Z^{WhiteHSG,HispHSG}$, which is the marital surplus between White high school graduate men and Hispanic high school graduate women. The magnitude of this contribution is more than twice time larger than other top contributing factors.

Table 8: Decomposition: Top three contributions from **Z**, White HSG Men

		(1)	(2)	(3)
Contribution	Top (+)	1.0 $Z^{WhiteHSG,WhiteHSG}$	0.8 $Z^{WhiteHSG,WhiteSC}$	0.8 $Z^{HispHSG,WhiteHSG}$
	Top (-)	-2.7 $Z^{WhiteHSG,HispHSG}$	-1.4 $Z^{WhiteSC,WhiteHSG}$	-1.0 $Z^{WhiteHSG,HispHSD}$

Notes: This table presents the top three positive and negative contributions from the 1980-2019 changes in marital surplus to the 1980-2019 changes in the welfare gains from marital desegregation for White HSG men. For marital surplus Z^{IJ}, I refers to husband's type and J refers to wife's type.

Recall that $Z^{WhiteHSG,HispHSG}$ has decreased over the past four decades as documented in Table 1. Hence, this result implies that the declining joint surplus from marriage with lower-educated Hispanic women is a part of the reason behind the lack of growth in welfare gains for lower-educated White men. This finding is consistent with Lichter et al. (2011) who suggest that the recent influx of new Hispanic immigrants has provided more same-race potential partners for

Hispanics, especially for the lower-educated people, so that it slowed the process of marital assimilation among lower-educated Hispanics.

6.2.5 What drove the rise in welfare gains for White women?

Lastly, I examine the drivers of the increases of welfare gains for White women. First, for White college-educated women, I decompose the population contribution into the ones made by (i) different-race male population, (ii) different-race female population, (iii) same-race male population, and (iv) same-race female population. Table 9 shows that, unlike the case of White college-educated men, White college-educated women did not benefit as much from the changes in different-race different-sex population. The positive contribution from the changes in the different-race male population is not large enough to completely offset the negative contribution from different-race female population, who are the competitors to White women in the desegregated marriage market.

Appendix Figure A12 further confirms that White college-educated women did not benefit as much from the increase in Asian and Hispanic college-educated population. This implies that the reversal of the gender gap in college education as documented in Section 2 explains the gender differences in the contribution made by population changes to the welfare gain.

Table 9: Contributions from population changes to welfare gains for White CG women

	Contribution from Population			
	Diff-Race Male	Diff-Race Female	Same-Race Male	Same-Race Female
WhiteCG Women	25.7	-26.8	-4.5	5.5

Notes: This table presents the decomposition of the 1980-2019 changes in the welfare gains from marital desegregation for White CG women. This table focuses on the contributions from the population changes. Each column shows the summation of all contributions made by populations corresponding to the label.

Why the changes in marital surplus have overall positive effects on White women's welfare gains is less clear. Further investigation in Appendix Figure A13 shows that there is no single large driver among the changes in marital surplus, but it is rather a combination of small equilibrium effects from multiple changes in the value of marriage that lead to the overall positive effects for White women.

7 Simulation: Complete Racial Integration

So far, I have shown how marital desegregation affected individual welfare and why the gains are different across groups. I now turn to a scenario of racial integration:[22] If there are less barriers to interracial marriage, how would it affect the probability of remaining single for each demographic group? I define *complete racial integration* as a scenario where race is no longer a factor considered in marriage matching. In this section, I perform counterfactual simulations to predict the impacts of progress towards the complete racial integration in the marriage market. I describe the estimation procedures below.

Constructing the marital surplus for complete integration: To construct a trajectory towards the complete integration, I first construct a marital surplus matrix for complete racial integration. Note that this is not straightforward because by definition, any matrix that does not depend on race of each spouse can reflect complete racial integration. In practice, I choose a marital surplus matrix that (i) only depends on education of both spouses and that (ii) minimizes the weighted Euclidean distance from the estimated $\hat{\mathbf{Z}}_t$.

For clarity, I rewrite the marital surplus Z_t^{IJ} as $Z_t^{(R_i,E_i),(R_j,E_j)}$, where R_i (resp. R_j) denotes husband's (resp. wife's) race and E_i (resp. E_j) denotes husband's (resp. wife's) education. Then, the marital surplus for each education pair is constructed as the weighted average of estimated $\hat{Z}_t^{IJ}$ from the data, conditional on education levels of both spouses:

$$\hat{Z}_t^{E_i,E_j} = \sum_{R_i,R_j} \widehat{Pr}(R_i,R_j|E_i,E_j,t)\hat{Z}_t^{(E_i,R_i),(E_j,R_j)} \tag{18}$$

The marital surplus matrix for complete integration, denoted by $\hat{\mathbf{Z}}_t^{Integrated}$, is constructed by simply replacing all $\hat{Z}_t^{(R_i,E_i),(R_j,E_j)}$ in $\hat{\mathbf{Z}}_t$ with the corresponding $\hat{Z}_t^{E_i,E_j}$. Figure 6 visualizes the differences between the counterfactual marital surplus and the actual marital surplus.

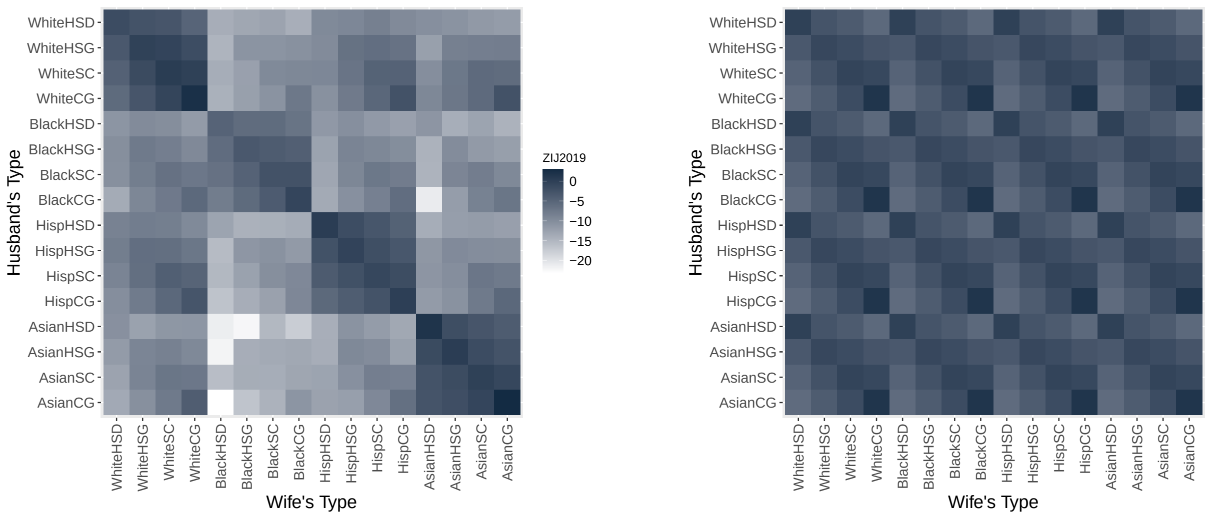
To construct a trajectory of progress towards the complete integration, I take the following convex combination of the marital surplus matrices:

$$\hat{\mathbf{Z}}_t^{Simulated}(p) = p\hat{\mathbf{Z}}_t^{Actual} + (1-p)\hat{\mathbf{Z}}_t^{Integrated} \tag{19}$$

where $p \in [0,1]$. This means that when p is closer to 0, the counterfactual marital surplus is closer to the actual marriage market. When p is closer to 1, the counterfactual marital surplus is closer to the case of complete racial integration.

[22]As discussed in O'Flaherty (2015), the term "integration" refers to a situation where groups of *equals* who cooperate with each other in mutually beneficial ways, which is different from the term "desegregation," which simply means removing legal barriers to intergroup contact.

Figure 6: Marital Surplus Matrix in 2019, Actual vs. Complete Integration



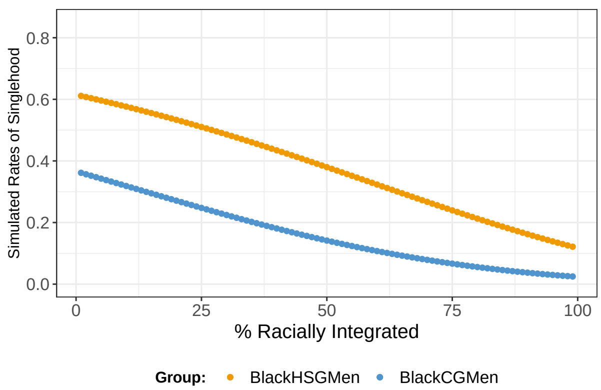
(a) Actual $\hat{\mathbf{Z}}_{2019}$ (b) Counterfactual $\hat{\mathbf{Z}}_{2019}^{Integrated}$

Note: This figure shows heatmap of the marital surplus Z_{2019}^{IJ} for the actual values that is estimated from data (Panel (a)) and the counterfactual values with complete integration (Panel (b)) and respectively. I refers to husband's type (Row) and J refers to wife's type (Column).

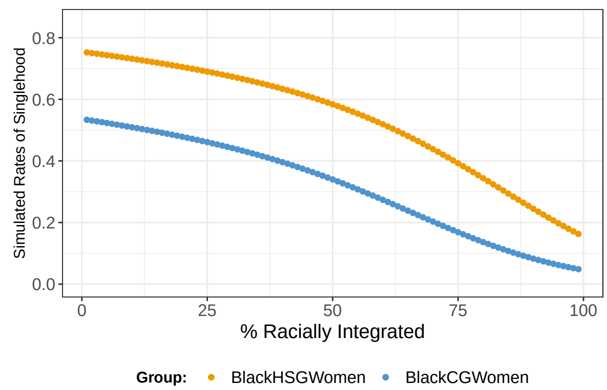
Computing the equilibrium rates of singlehood: For the survey year 2019, I compute the trajectory of the equilibrium single rates for each type of men and women, using the population vectors taken from data and the counterfactual marital surplus $\hat{\mathbf{Z}}_{2019}^{Simulated}(p)$ at each $p \in [0,1]$ with interval 0.01. When presenting the results, I rescale p so that it represents the percentage of progress towards complete integration in the marriage market.

Figure 7 shows the simulation results for each group of men and women. I find that progress towards the complete racial integration would reduce the singlehood among Black men and women. 50% of racial integration in the marriage market would reduce the single rate of Black high school graduate women by 17 percentage points and reduce the single rate of Black college-educated women by 20 percentage points. Further progress towards the racial integration not only closes the racial gap in marriage but also makes Blacks marry more than Whites. In comparison, I find that racial integration would not change much the rates of singlehood among Whites at all stages. These predictions show that racial integration would play an important role in improving the marriage prospects for Black men and women, who currently have low marriage rates, while not harming the marriage prospects for White men and women.

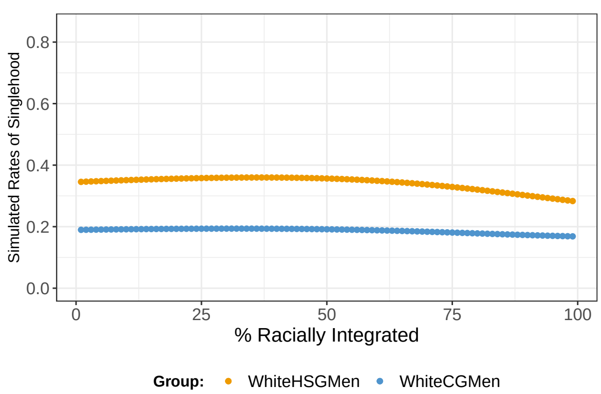
Figure 7: Simulated Rate of Singlehood for Varying Degree of Racial Integration



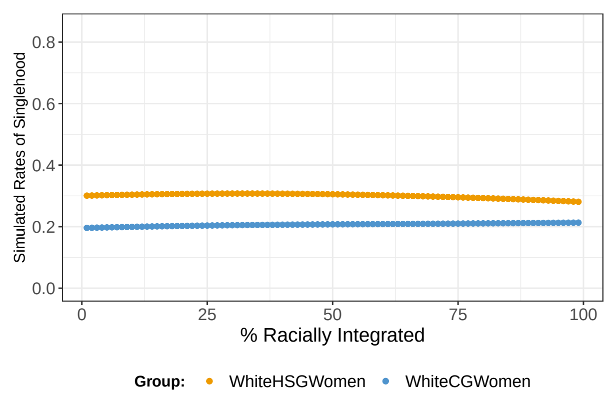
(a) Black Men



(b) Black Women



(c) White Men



(d) White Women

Note: This figure plots the simulated rate of singlehood at each % of racial integration (rescaled p) for the specified group. “% Racially Integrated" describes how close the counterfactual marital surplus is to the complete integration case. Estimation is done using the 2019 Census data. I focus on age 37-46 men and age 35-44 women. Further details on the sample restriction are described in Section 3.

8 Conclusion

This paper investigates the increases in interracial marriage rates for non-Hispanic Whites and Blacks in the US to understand: (i) who gained the most from marital desegregation, (ii) why some groups gain more than others, and (iii) how progress toward complete integration would affect marriage rates. The structural matching model enables me to define and estimate the welfare gains from marital desegregation. It also enables me to quantify the effects from various market-level changes on welfare gains across groups. This paper is the first to provide quantitative assessments to identify what drove differing welfare gains from marital desegregation across groups.

I show that marital racial desegregation has not equally improved everyone's welfare. Among Blacks, only college-educated Black men have substantially benefitted from marital racial desegregation. Notably, Black women did not gain at all from desegregation across all years. The rise in the welfare gains for college-educated Black men is largely explained by the increase in the joint surplus from marriage between college-educated Black men and college-educated White women. Other groups among Blacks did not benefit as much from the changing structure of marital surplus, which explains why their welfare gains lag behind that of college-educated Black men. Population changes did not play a substantial role in driving the changes in welfare gains for Black people.

Among Whites, college-educated White men have experienced a larger increase in welfare gain than their lower-educated male peers, and this is explained by the increase in the college-educated Asian and Hispanic female population. This reveals that the rise in the interracial marriage rates among college-educated White men is a mechanical consequence of population changes, rather than the increased gains from interracial marriage. Non-college-educated White men have not experienced any increase in the gains from racial desegregation over the past four decades, and this is partly explained by the declining gains from marriage between this group and non-college-educated Hispanic women. White women have experienced increases in welfare gains, regardless of education level, and these increases are driven by a combination of various equilibrium effects of the changes in marital surplus.

Simulation results show that progress towards racial integration would significantly increase the marriage rates among Blacks, without reducing the marriage rates of Whites. This finding suggests that the efforts toward improving racial relations could improve the marriage prospects of Blacks, who currently have low marriage rates and a high prevalence of single mothers.

My findings suggest two main avenues for future research. First, it is important to understand what drives the differing values of marital surplus across marriages. For example, the matching model cannot distinguish why the marital surplus between Black men and White women is higher than the marital surplus between White men and Black women. It would be helpful to further investigate whether these gender differences in marital surplus are affected by economic conditions, where people live, or other factors. Second, the question of which policies can promote interracial marriage needs to be further studied. Merlino et al. (2019) shows that greater racial diversity in high school increases the interracial dating as adults. It would be fruitful to study whether the policies that promote diversity in other settings, such as college, workplace, or residence, would increase interracial marriage and social integration.

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A Appendix: Tables and Figures

A.1 Additional Tables

Table A1: Percentage of Other Race and Mixed Races in Each Census Year, Female Aged 35-44, Male Aged 37-46

Year	Other Race	Mixed Race
1980	0.71%	N/A
1990	0.78%	N/A
2000	0.88%	2.12%
2010	0.92%	1.65%
2019	0.89%	2.55%

Notes: This table presents the proportion of people who reported Other Race (which includes “American Indian or Alaska Native" and “Other race") and Mixed Race among women aged 35-44 and men aged 37-46 for each survey year. A response option of mixed race was added from the 2000 census and onwards. Data sources for this table are: 1960 5% sample Census, 1970 1% sample Census, 1980 5% sample Census, 1990 5% sample Census, 2000 5% sample Census, 2010 5% sample American Community Survey (2006-2010 5 year pooled sample), 2019 5% sample American Community Survey (2005-2019 5 year pooled sample). Survey weight is applied.

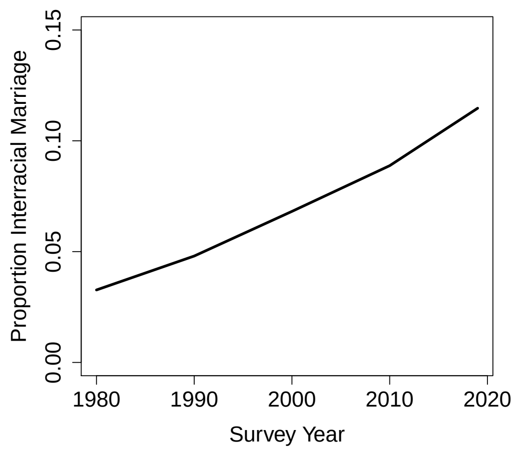
Table A2: Percentage of Never Married Singles who Cohabit in Each Census Year, Female Aged 35-44, Male Aged 37-46

Year	% Cohabiting
1990	11.3%
2000	17.7%
2010	21.3%
2019	24.6%

Notes: This table presents the proportion of respondents who reported to have cohabiting partners among never-married single women aged 35-44 and never-married single men aged 37-46 for each survey year. A response option for a cohabiting partner was added from the 1990 census and onwards. Data sources for this table are: 1990 5% sample Census, 2000 5% sample Census, 2010 5% sample American Community Survey (2006-2010 5 year pooled sample), 2019 5% sample American Community Survey (2005-2019 5 year pooled sample).

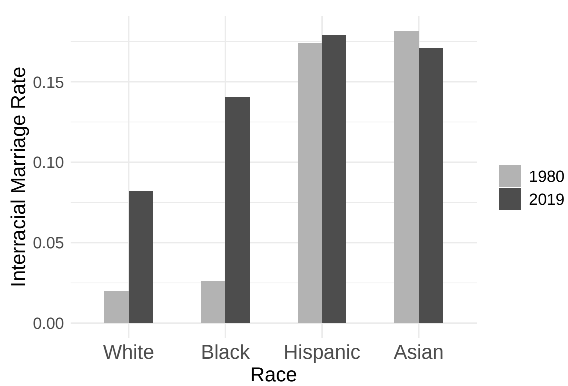
A.2 Additional Figures

Figure A1: Interracial Marriage Rate, Among Married, Age 35-44



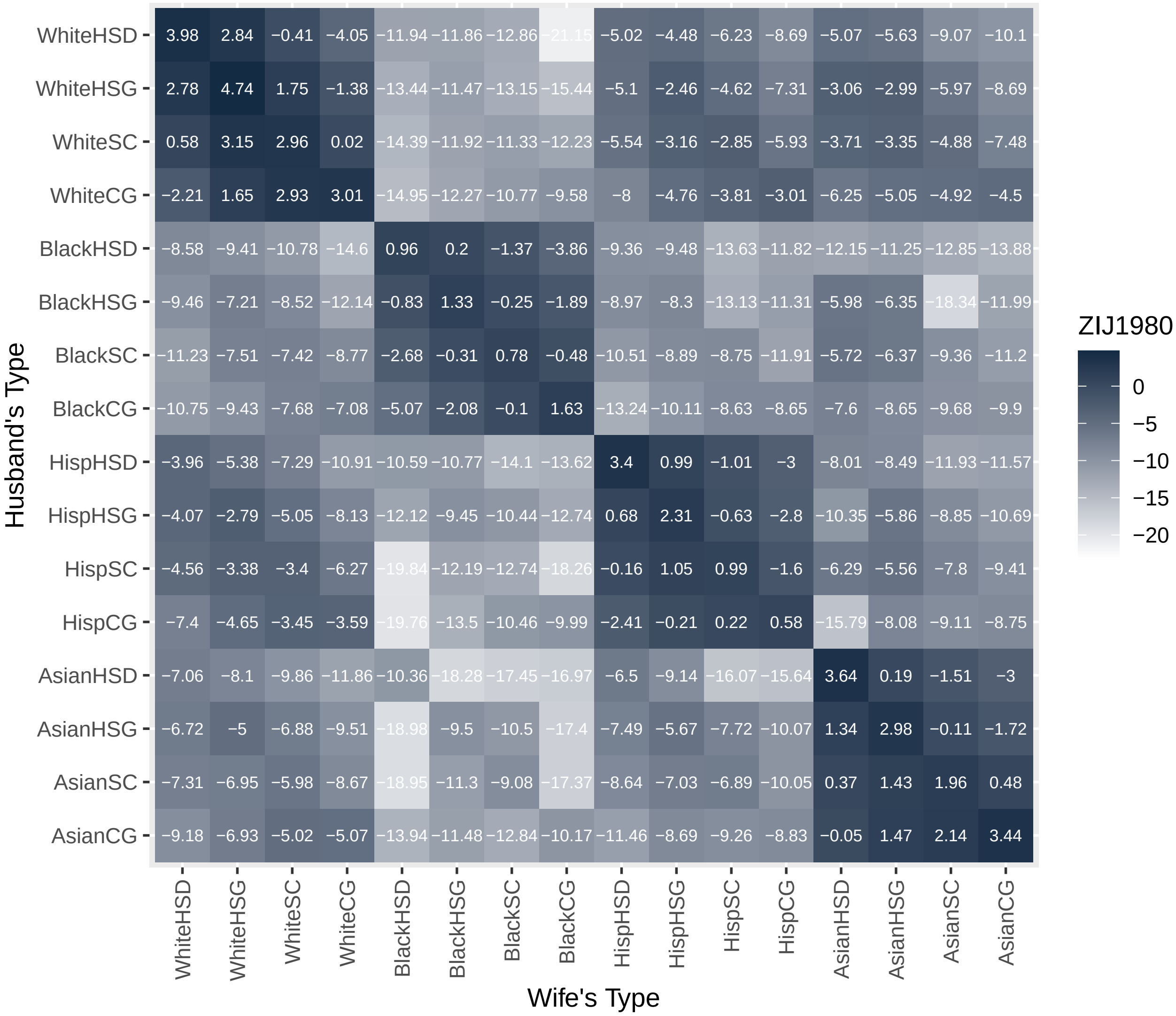
<u>Note:</u> This figure shows the proportion of interracial marriage among married men and women aged 35-44 for each survey year. Data sources for this figure are: 1960 5% sample Census, 1970 1% sample Census, 1980 5% sample Census, 1990 5% sample Census, 2000 5% sample Census, 2010 5% sample American Community Survey (2006-2010 5 year pooled sample), 2019 5% sample American Community Survey (2005-2019 5 year pooled sample). For Hispanics, 1960 and 1970 are excluded as the Hispanic identification is imputed by the IPUMS and does not properly capture the interracial marriage with non-Hispanic whites. Survey weight is applied.

Figure A2: Interracial Marriage For Each Race, Among Married, Age 35-44



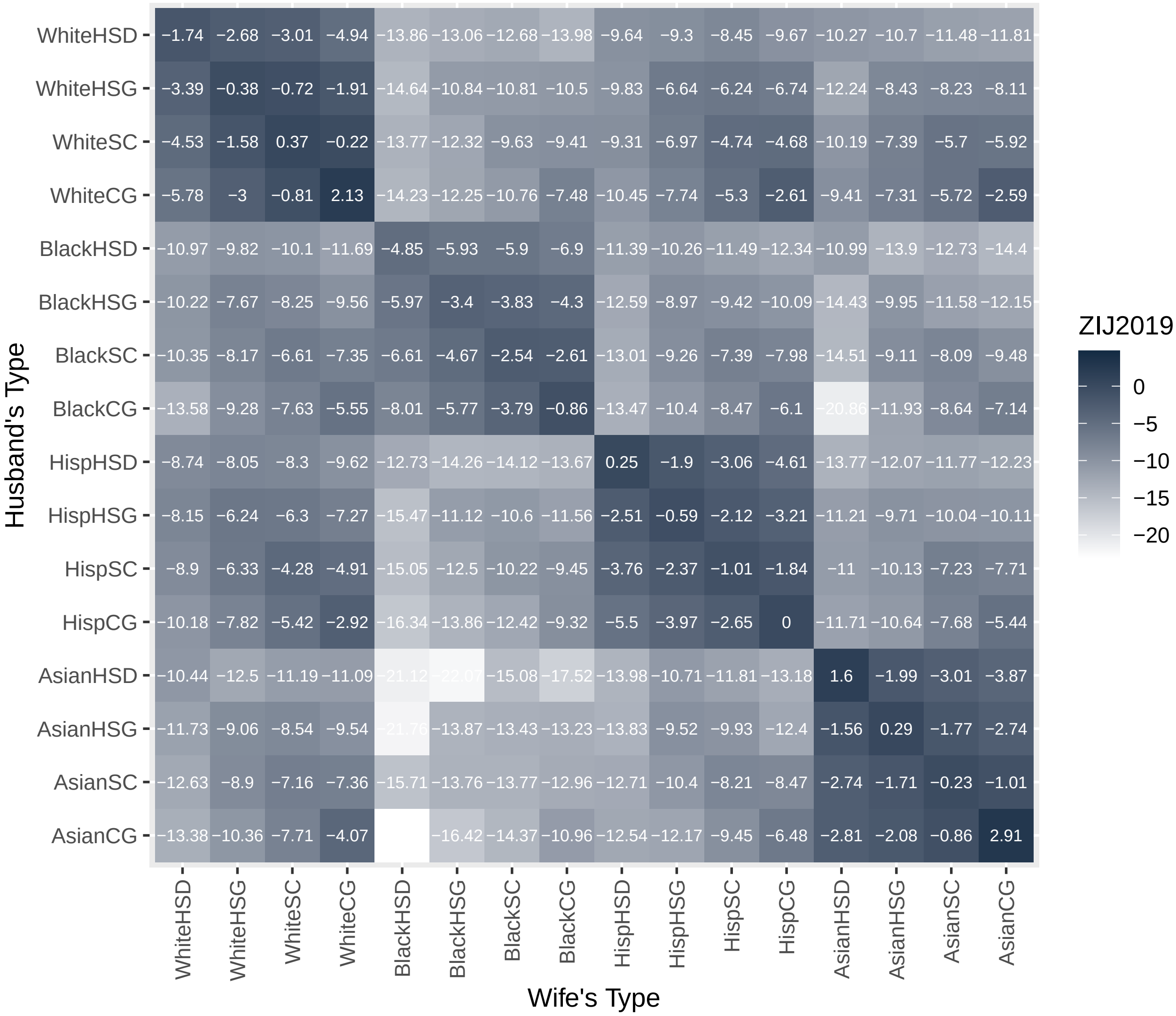
<u>Note:</u> This figure shows the proportion of those who married out of their race among married men and women aged 35-44 in 1980 and in 2019, respectively. Data sources for this figure are: 1980 5% sample Census microdata and 2019 5% sample American Community Survey (2015-2019 5-year pooled sample). Survey weight is applied.

Figure A3: Marital Surplus Z^{IJ}, 1980



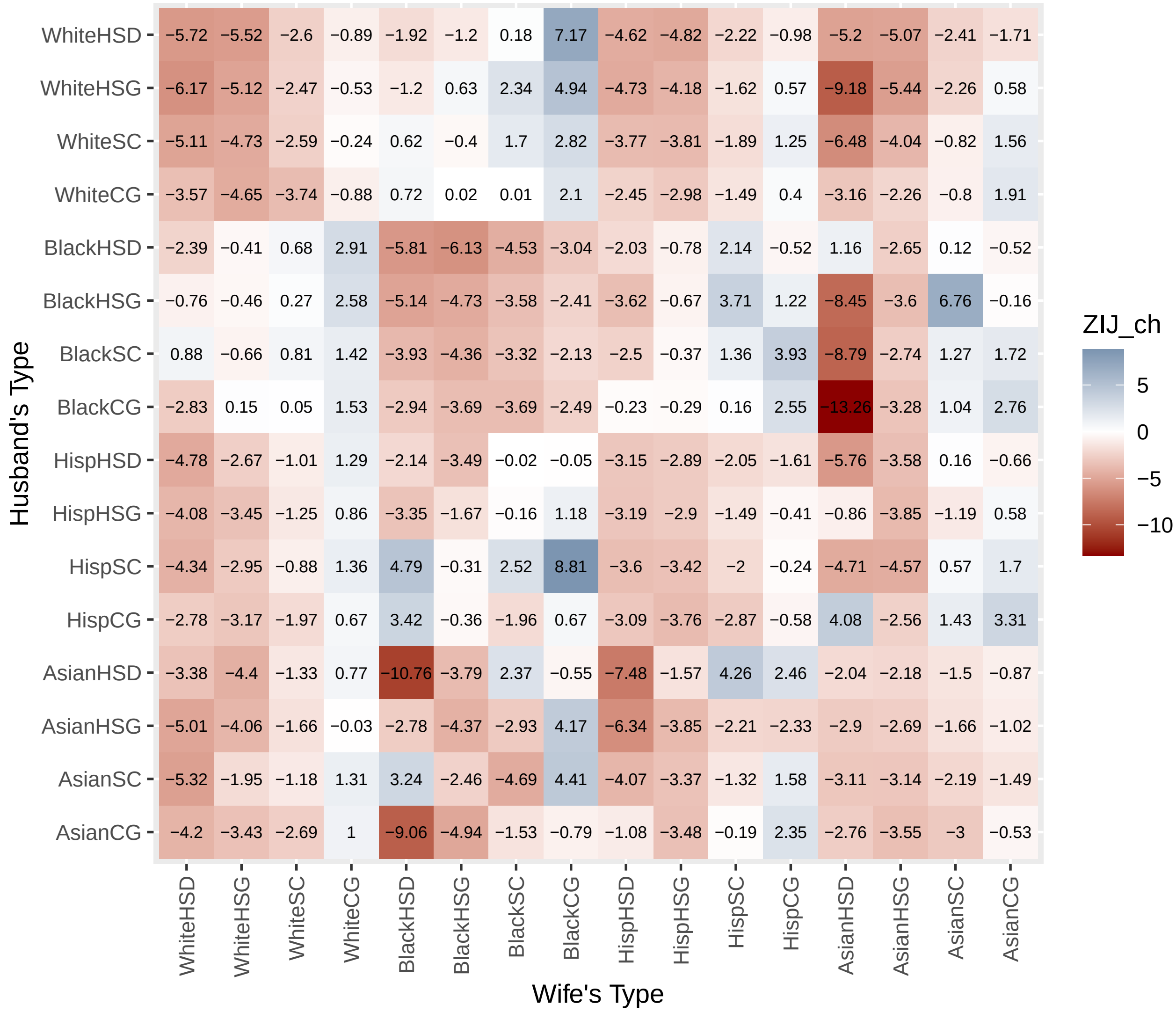
Note: This figure shows the heatmap for estimated marital surplus $\hat{Z}_t^{IJ}$ for the survey year 1980. Data used to estimate this matrix is described in Section 3. I refers to husband's type (Row) and J refers to wife's type (Column).

Figure A4: Marital Surplus Z^{IJ}, 2019



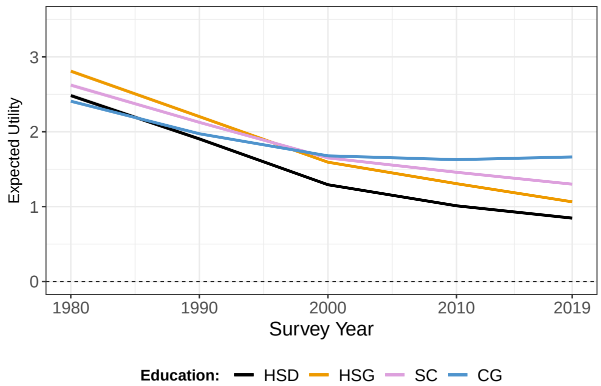
Note: This figure shows the heatmap for estimated marital surplus $\hat{Z}_t^{IJ}$ for the survey year 2019. Data used to estimate this matrix is described in Section 3. I refers to husband's type (Row) and J refers to wife's type (Column).

Figure A5: Changes in Marital Surplus Z^{IJ} from 1980 to 2019

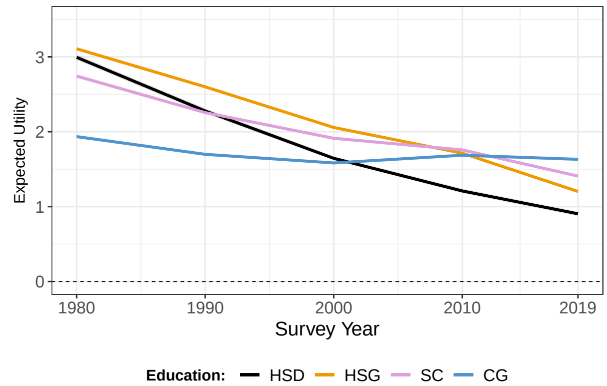


Note: This figure shows the heatmap for the change in estimated marital surplus $\hat{Z}_t^{IJ}$ from year 1980 to 2019. Data used to estimate this matrix is described in Section 3. I refers to husband's type (Row) and J refers to wife's type (Column). A note of caution is that the magnitudes of these differences are not comparable with each other. This is because the Z_t^{IJ} is a nonlinear log function as shown in Equation (5) so that we cannot compare the magnitudes of the changes in Z_t^{IJ} with different starting values. Hence, I only focus on the sign of each change, rather than on the magnitude of each change.

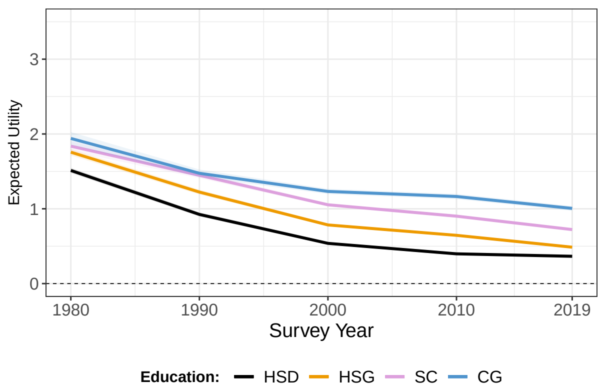
Figure A6: Trends in the Type-Specific Expected Utilities



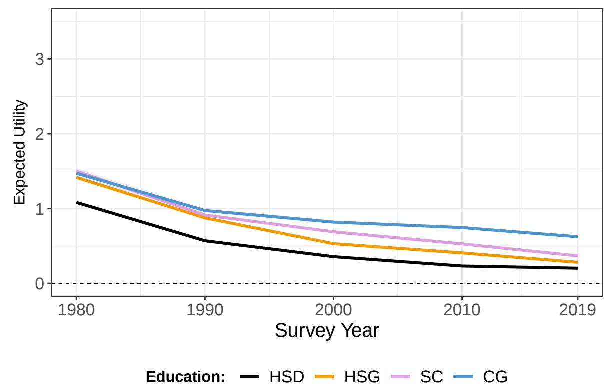
(a) White Men



(b) White Women



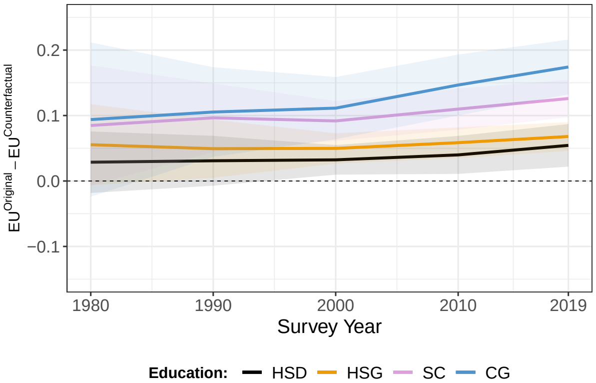
(c) Black Men



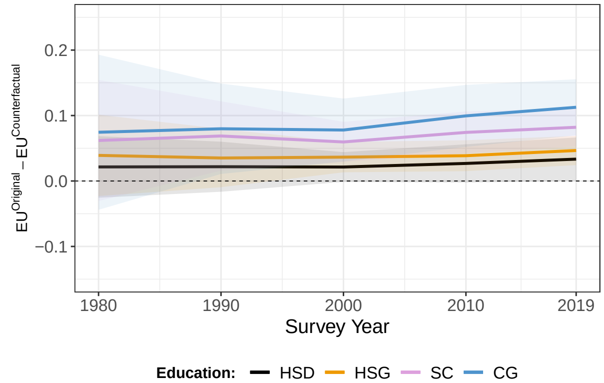
(d) Black Women

Note: These figures plot the individual expected utilities (Equation (6)) for each specified type of men and women. Data used to calculate the gains are: 1980-2000 Decennial Census, 2010 and 2019 5-Year ACS. I focus on age 37-46 men and age 35-44 women for each survey year. Further details on the sample restriction are described in Section 3. Shade for each line refers to the 95% confidence interval. Standard errors are calculated from the sampling variation in the data.

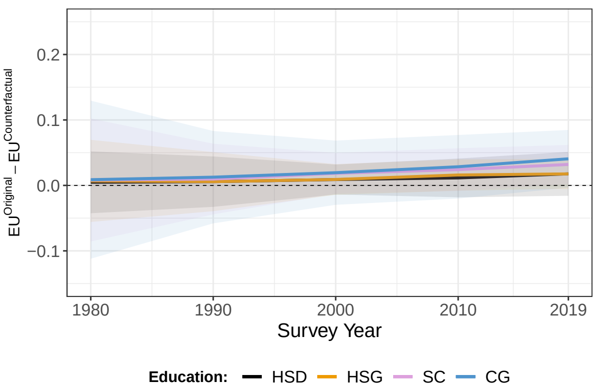
Figure A7: Type-Specific Utility Gains from Marital Desegregation, **Black Men**



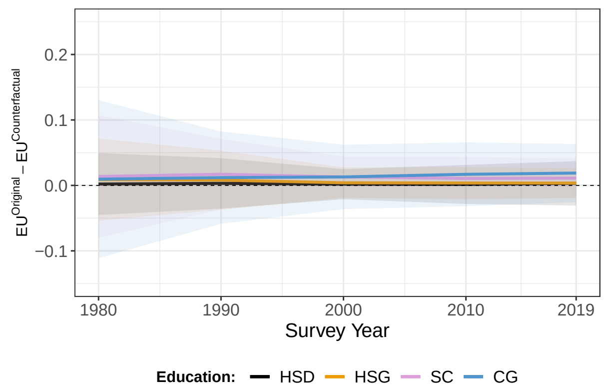
(a) Counterfactual: No Interracial



(b) Counterfactual: No Black-White



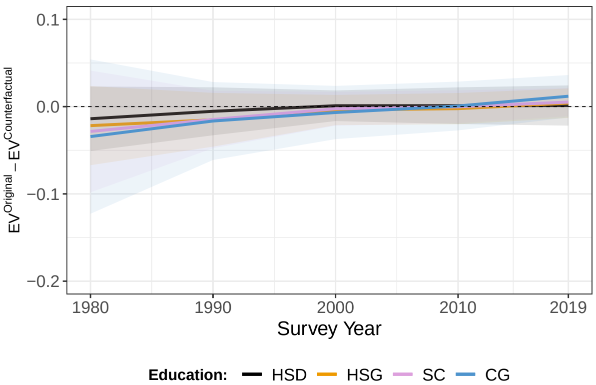
(c) Counterfactual: No Black-Hispanic



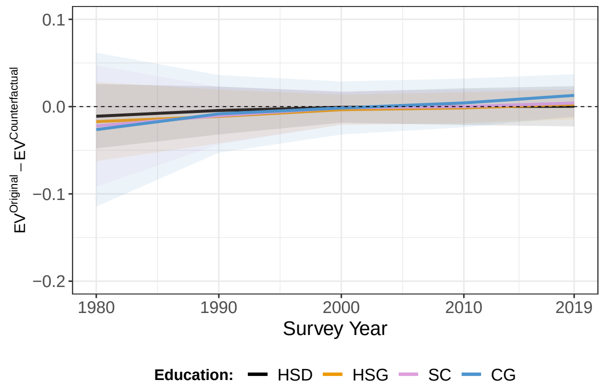
(d) Counterfactual: No Black-Asian

Note: These figures plot the welfare gains from marital desegregation as defined by Equation (10) and Equation (11) for each specified type. Each panel calculates the utility gains using a different counterfactual scenario as denoted by the label. Data used to calculate the gains are: 1980-2000 Decennial Census, 2010 and 2019 5-Year ACS. I focus on age 37-46 men and age 35-44 women for each survey year. Further details on the sample restriction are described in Section 3. Shade for each line refers to the 95% confidence interval. Standard errors are calculated from the sampling variation in the data.

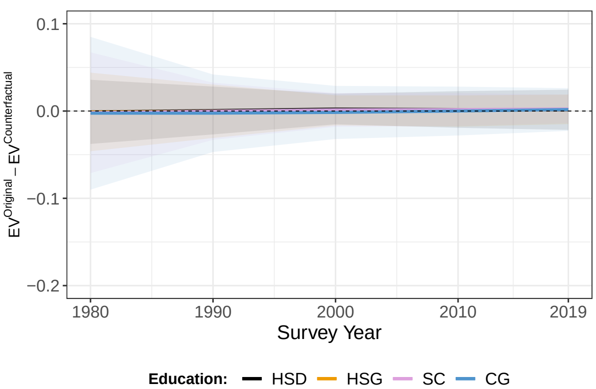
Figure A8: Type-Specific Utility Gains from Marital Desegregation, **Black Women**



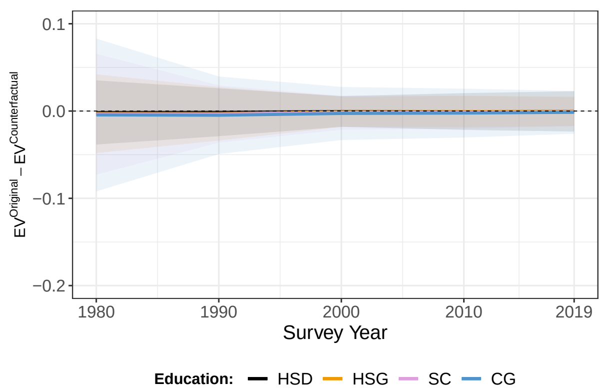
(a) Counterfactual: No Interracial



(b) Counterfactual: No Black-White



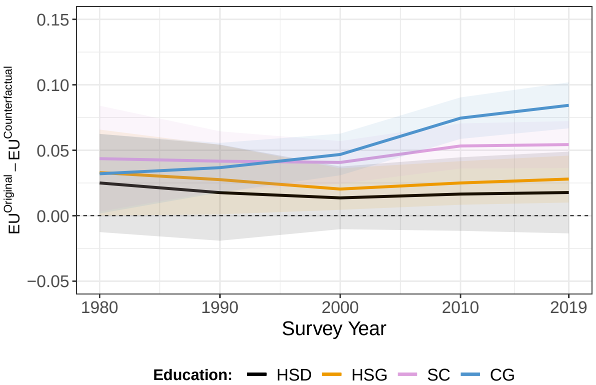
(c) Counterfactual: No Black-Hispanic



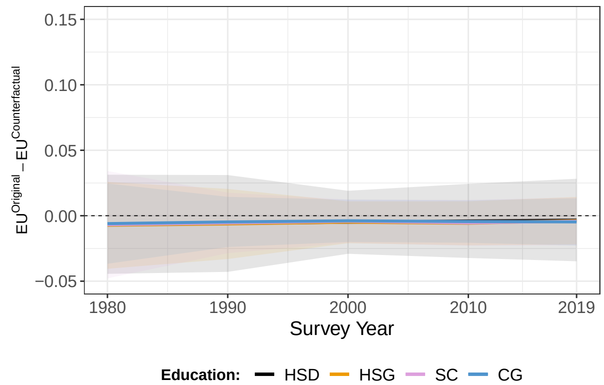
(d) Counterfactual: No Black-Asian

Note: These figures plot the welfare gains from marital desegregation as defined by Equation (10) and Equation (11) for each specified type. Each panel calculates the utility gains using a different counterfactual scenario as denoted by the label. Data used to calculate the gains are: 1980-2000 Decennial Census, 2010 and 2019 5-Year ACS. I focus on age 37-46 men and age 35-44 women for each survey year. Further details on the sample restriction are described in Section 3. Shade for each line refers to the 95% confidence interval. Standard errors are calculated from the sampling variation in the data.

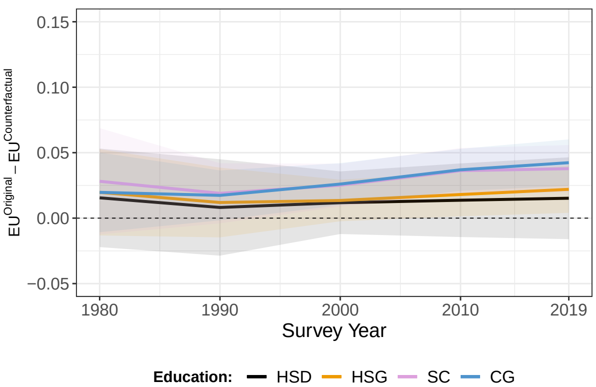
Figure A9: Type-Specific Utility Gains from Marital Desegregation, **White Men**



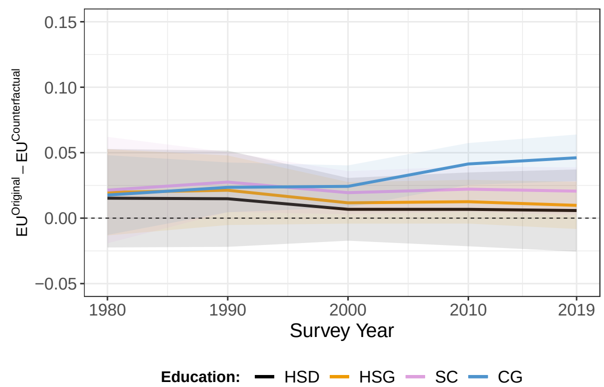
(a) Counterfactual: No Interracial Marriages



(b) Counterfactual: No White-Black Marriages



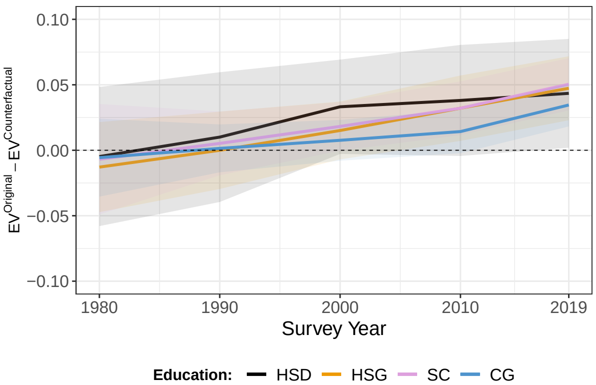
(c) Counterfactual: No White-Hispanic Marriages



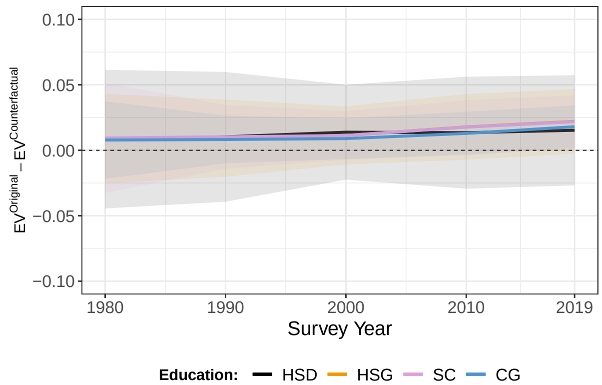
(d) Counterfactual: No White-Asian Marriages

Note: These figures plot the welfare gains from marital desegregation as defined by Equation (10) and Equation (11) for each specified type. Each panel calculates the utility gains using different counterfactuals. Data used to calculate the gains are: 1980-2000 Decennial Census, 2010 and 2019 5-Year ACS. I focus on age 37-46 men and age 35-44 women for each survey year. Further details on the sample restriction are described in Section 3. Shade for each line refers to the 95% confidence interval. Standard errors are calculated from the sampling variation in the data.

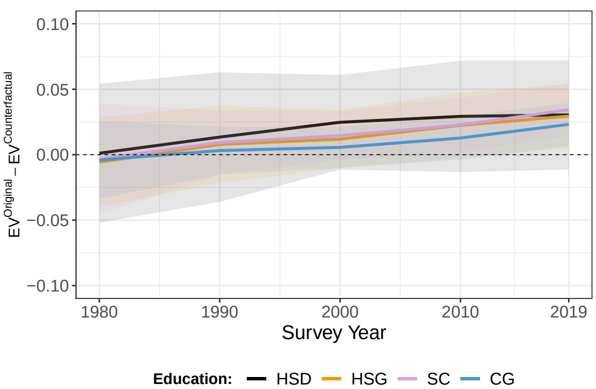
Figure A10: Type-Specific Utility Gains from Marital Desegregation, **White Women**



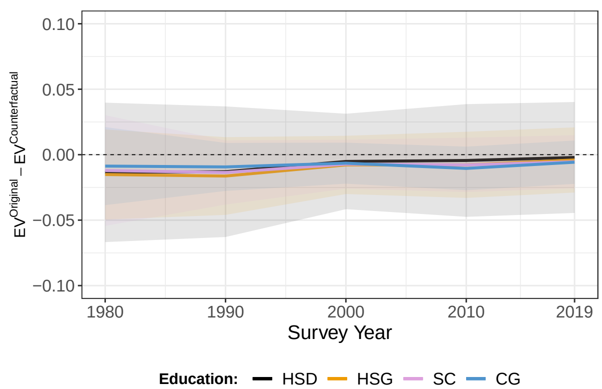
(a) Counterfactual: No Interracial



(b) Counterfactual: No White-Black



(c) Counterfactual: No White-Hispanic



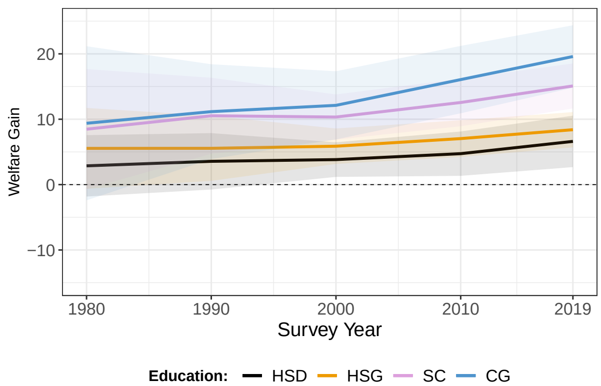
(d) Counterfactual: No White-Asian

Note: These figures plot the welfare gains from marital desegregation as defined by Equation (10) and Equation (11) for each specified type. Each panel calculates the utility gains using a different counterfactual scenario as denoted by the label. Data used to calculate the gains are: 1980-2000 Decennial Census, 2010 and 2019 5-Year ACS. I focus on age 37-46 men and age 35-44 women for each survey year. Further details on the sample restriction are described in Section 3. Shade for each line refers to the 95% confidence interval. Standard errors are calculated from the sampling variation in the data.

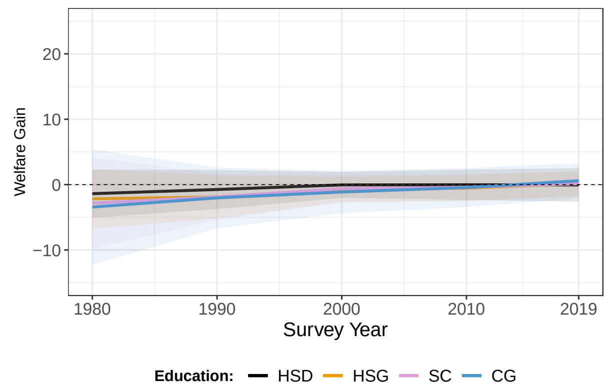
A.3 Sensitivity Check: Excluding Cohabiting Singles

As shown in Table A2, the proportion of never-married singles who cohabit with a partner has increased over time. To see how the cohabiting singles affect the results, I perform sensitivity analyses that exclude cohabiting singles from the single population. I re-estimate the welfare gains from marital desegregation for each group, which is presented in Figure A11. The results confirm that excluding cohabiting singles do not affect the results for welfare gain from marital desegregation.

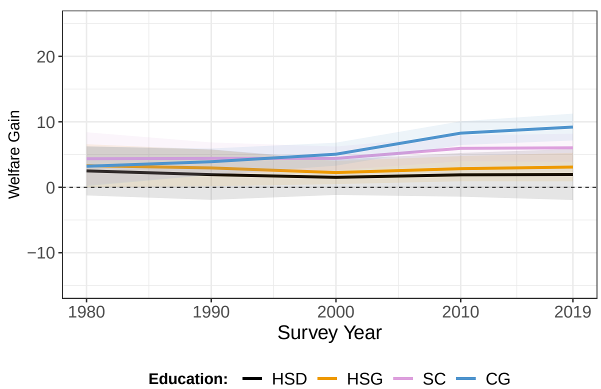
Figure A11: Type-Specific Welfare Gains from Marital Racial Integration, Excluding Cohabiting Singles



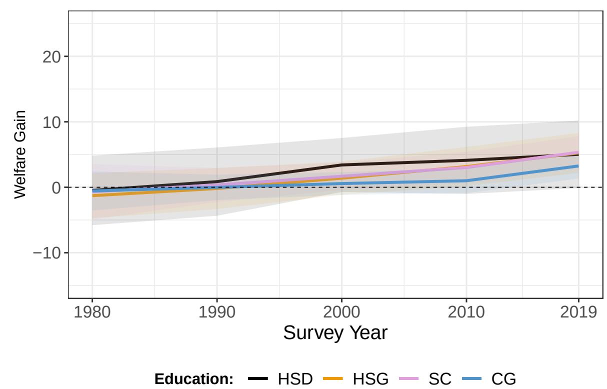
(a) Black Men



(b) Black Women



(c) White Men



(d) White Women

Note: These figures plot the welfare gains from marital desegregation as defined by Equation (10) and Equation (11) for each specified type of men and women. Data used to calculate the gains are: 1980-2000 Decennial Census, 2010 and 2019 5-Year ACS. I focus on age 37-46 men and age 35-44 women for each survey year. Further details on the sample restriction are described in Section 3. I exclude cohabiting singles from the estimation sample. Shade for each line refers to the 95% confidence interval. Standard errors are calculated from the sampling variation in the data.

B Appendix: Decomposition

B.1 Full Solution of IFT Partials

Full solution for the IFT partials: Full solution for the Jacobian matrix (Equation 14) is as follows:

$$\left[\frac{\partial \mathbf{s}}{\partial \tilde{\boldsymbol{\theta}}}\right]_{(2K)\times(2K+K^2)} = -\underbrace{\begin{bmatrix} \dfrac{\partial \mathbf{F}}{\partial \mathbf{s}} \\[2ex] \dfrac{\partial \mathbf{G}}{\partial \mathbf{s}} \end{bmatrix}^{-1}_{(2K)\times(2K)}}_{[A]} \underbrace{\begin{bmatrix} \dfrac{\partial \mathbf{F}}{\partial \tilde{\boldsymbol{\theta}}} \\[2ex] \dfrac{\partial \mathbf{G}}{\partial \tilde{\boldsymbol{\theta}}} \end{bmatrix}_{(2K)\times(2K+K^2)}}_{[B]}$$

where

$$[A] = \begin{bmatrix}
2s_{1\emptyset} + \sum_J \tilde{Z}_{1J} s_{\emptyset J} & 0 & \cdots & 0 & \tilde{Z}_{11} s_{1\emptyset} & \tilde{Z}_{12} s_{1\emptyset} & \cdots & \tilde{Z}_{1K} s_{1\emptyset} \\
0 & 2s_{2\emptyset} + \sum_J \tilde{Z}_{2J} s_{\emptyset J} & \cdots & 0 & \tilde{Z}_{21} s_{2\emptyset} & \tilde{Z}_{22} s_{2\emptyset} & \cdots & \tilde{Z}_{2K} s_{2\emptyset} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & \vdots \\
0 & 0 & \cdots & 2s_{K\emptyset} + \sum_J \tilde{Z}_{KJ} s_{\emptyset J} & \tilde{Z}_{K1} s_{K\emptyset} & \tilde{Z}_{K2} s_{K\emptyset} & \cdots & \tilde{Z}_{KK} s_{K\emptyset} \\
\tilde{Z}_{11} s_{\emptyset 1} & \tilde{Z}_{21} s_{\emptyset 1} & \cdots & \tilde{Z}_{K1} s_{\emptyset 1} & 2s_{\emptyset 1} + \sum_I \tilde{Z}_{I1} s_{I\emptyset} & 0 & \cdots & 0 \\
\tilde{Z}_{12} s_{\emptyset 2} & \tilde{Z}_{22} s_{\emptyset 2} & \cdots & \tilde{Z}_{K2} s_{\emptyset 2} & 0 & 2s_{\emptyset 2} + \sum_I \tilde{Z}_{I2} s_{I\emptyset} & \cdots & 0 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\
\tilde{Z}_{1K} s_{\emptyset K} & \tilde{Z}_{2K} s_{\emptyset K} & \cdots & \tilde{Z}_{KK} s_{\emptyset K} & 0 & 0 & \cdots & 2s_{\emptyset K} + \sum_I \tilde{Z}_{IK} s_{I\emptyset}
\end{bmatrix}^{-1}$$

and

$$[B] = \begin{bmatrix}
-1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & s_{1\emptyset} s_{\emptyset 1} & s_{1\emptyset} s_{\emptyset 2} & \cdots & s_{1\emptyset} s_{\emptyset K} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & -1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & s_{2\emptyset} s_{\emptyset 1} & s_{2\emptyset} s_{\emptyset 2} & \cdots & s_{2\emptyset} s_{\emptyset K} & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & & \vdots \\
0 & 0 & \cdots & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & s_{K\emptyset} s_{\emptyset 1} & s_{K\emptyset} s_{\emptyset 2} & \cdots & s_{K\emptyset} s_{\emptyset K} \\
0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & s_{1\emptyset} s_{\emptyset 1} & 0 & \cdots & 0 & s_{2\emptyset} s_{\emptyset 1} & 0 & \cdots & 0 & \cdots & s_{K\emptyset} s_{\emptyset 1} & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & -1 & \cdots & 0 & 0 & s_{1\emptyset} s_{\emptyset 2} & \cdots & 0 & 0 & s_{2\emptyset} s_{\emptyset 2} & \cdots & 0 & \cdots & 0 & s_{K\emptyset} s_{\emptyset 2} & \cdots & 0 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -1 & 0 & 0 & \cdots & s_{1\emptyset} s_{\emptyset K} & 0 & 0 & \cdots & s_{2\emptyset} s_{\emptyset K} & \cdots & 0 & 0 & \cdots & s_{K\emptyset} s_{\emptyset K}
\end{bmatrix}$$

Estimation of the Jacobian matrix is done by combining [A] and [B] using matrix multiplication.

B.2 Details on Decomposition Procedures

In this section, I describe the estimation steps to decompose the expected utility of type I men. The application to the welfare gain, which is a function of expected utilities, can be done analogously.

STEP 1: First, to link the change in the expected utility to the IFT partials, I take the total differential of the expected utility:

$$d\bar{u}^I = \frac{1}{n^I} dn^I - \frac{2}{s^{I\emptyset}} \Big(\underbrace{\frac{\partial s^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}}}_{From\ IFT} d\tilde{\boldsymbol{\theta}} \Big) \tag{20}$$

STEP 2: A naive way of expressing the changes in $\bar{u}^I$ from year 1980 to 2019 using Equation (15) is the following:

$$\Delta^{2019-1980} \bar{u}^I = \frac{1}{n^I} \Delta^{2019-1980} n^I - \frac{2}{s^{I\emptyset}} \Big(\frac{\partial s^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}} \Delta^{2019-1980} \tilde{\boldsymbol{\theta}} \Big)$$

where $\Delta^{2019-1980} y$ refers to change in y from 1980 to 2019. However, this is problematic because the implicit function theorem and the total differentials only give good approximations for *very small* changes in the model primitives. US has experienced large changes in population distribution over the past four decades. Moreover, marital surplus $\mathbf{Z}$ also has experienced changes over time. Hence, it is improper to use 40 years of changes to evaluate Equation (20).

A better, but still not ideal, approach is to divide the time period into smaller time periods based on available survey years. Because I use the census data with 10-year intervals, $\Delta^{2019-1980} \bar{u}^I_t$ can be decomposed into:

$$\Delta^{2019-1980} \bar{u}^I = \Delta^{1990-1980} \bar{u}^I + \Delta^{2000-1990} \bar{u}^I + \Delta^{2010-2000} \bar{u}^I + \Delta^{2019-2010} \bar{u}^I$$

However, changes in model primitives over each decade may still be considered large.

In order to better approximate the effect of changes in model primitives on $d\bar{u}^I_t$, I implement the homotopy method following Judd (1998). This method decomposes the large changes in the model primitives into a series of infinitesimal changes. I apply this method for each decade based on the available survey years: 1980 to 1990, 1990 to 2000, 2000 to 2010, and 2010 to 2019.

To give a concrete example, I consider the changes from 1980 to 1990. Let me denote 1980 as $\tau = 0$ and 1990 as $\tau = 1$. Then $\tilde{\boldsymbol{\theta}}_0$ (resp. $\tilde{\boldsymbol{\theta}}_1$) is the vector of the values of model primitives in 1980 (resp. in 1990). Then I consider the homotopy:

$$\tilde{\boldsymbol{\theta}}_\tau = \tau \tilde{\boldsymbol{\theta}}_1 + (1-\tau) \tilde{\boldsymbol{\theta}}_0, \quad \tau \in [0,1]$$

which defines a series of intermediate values of the model primitives with interval $d\tau$ between observed values at $\tau = 0$ and $\tau = 1$. Because $\tilde{\boldsymbol{\theta}}_\tau$ is now a function of τ as defined above, $d\tilde{\boldsymbol{\theta}}_\tau$ becomes $d\tilde{\boldsymbol{\theta}}_\tau = (\tilde{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_0)d\tau$. Then, applying the homotopy to Equation (20),

$$d\bar{u}^I = \frac{1}{n_\tau^I}(n_1^I - n_0^I)d\tau - \frac{2}{s^{I\emptyset}}\left(\left[\frac{\partial s^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}_\tau}\right]_\tau (\tilde{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_0)d\tau\right) \tag{21}$$

where $\left[\frac{\partial s^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}_t}\right]_\tau$ means that this partial is evaluated at each τ. Note that $s_\tau^{I\emptyset}$ is updated as τ progresses with interval $d\tau$.

I use Equation (21) to estimate $d\bar{u}^I$ for each decade and to decompose $d\bar{u}^I$ into contributions by change in each of the model primitives. With the homotopy method, I can use infinitesimal change dt to evaluate and decompose $\Delta^{(\tau+d\tau)-\tau}\bar{u}^I$ for $\tau \in [0,1]$. I specify $d\tau = 0.001$ when estimating Equation (21) for each decade. Summing $\Delta^{(\tau+d\tau)-\tau}\bar{u}^I$ over all $\tau \in [0,1]$ gives better approximation of $\Delta\bar{u}^I$ than using the observed 10-year changes of model primitives to evaluate Equation (20).

For a more concrete illustration, I describe in detail how I perform first few steps for this fine-tuning method:

- **STEP 2.1:** From $\tau = 0 \to \tau = 0.001$

  The goal is to estimate $\bar{u}_{0.001}^I$. Starting from $\bar{u}_0^I$,

  $$\bar{u}_{0.001}^I = \bar{u}_0^I + d\bar{u}_0^I$$

  Using the fine-tuning method, $d\bar{u}_0^I$ is expressed as:

  $$d\bar{u}_0^I = \frac{1}{n_0^I}(n_1^I - n_0^I)\cdot 0.001 - \frac{2}{s_0^{I\emptyset}}\frac{\partial s_0^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}_\tau}(\tilde{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_0)\cdot 0.001$$

  Note that $\frac{\partial s_0^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}_\tau}$ is a function of $s_0^{I\emptyset}$, $s_0^{\emptyset J}$, Z_0^{IJ}, all of which are evaluated at $\tau = 0$.

  In this step, I also need to compute $s_{0.001}^{I\emptyset}$ and $s_{0.001}^{\emptyset J}$, because these will be used in the next step. For example,

  $$\begin{aligned} s_{0.001}^{I\emptyset} &= s_0^{I\emptyset} + ds_0^{I\emptyset} \\ &= s_0^{I\emptyset} + \frac{\partial s_0^{I\emptyset}}{\partial \tilde{\boldsymbol{\theta}}_\tau}(\tilde{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_0)\cdot 0.001 \end{aligned}$$

- **STEP 2.2:** From $\tau = 0.001 \to \tau = 0.002$.

The goal is to estimate $\bar{u}^I_{0.002}$. Starting from $\bar{u}^I_{0.001}$,

$$\bar{u}^I_{0.002} = \bar{u}^I_{0.001} + d\bar{u}^I_{0.001}$$

Using the fine-tuning method, $d\bar{u}^I_{0.001}$ is expressed as:

$$d\bar{u}^I_{0.001} = \frac{1}{n^I_{0.001}}(n^I_1 - n^I_0)\cdot 0.001 - \frac{2}{s^{I\emptyset}_{0.001}}\frac{\partial s^{I\emptyset}_{0.001}}{\partial\tilde{\boldsymbol{\theta}}_\tau}(\tilde{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_0)\cdot 0.001$$

where $n^I_{0.001} = 0.001 n^I_1 + 0.999 n^I_0$.

Note that $\frac{\partial s^{I\emptyset}_{0.001}}{\partial\tilde{\boldsymbol{\theta}}_\tau}$ is a function of $s^{I\emptyset}_{0.001}$, $s^{\emptyset J}_{0.001}$, and $Z^{IJ}_{0.001}$. I have already estimated $s^{I\emptyset}_{0.001}$ and $s^{\emptyset J}_{0.001}$ from the previous step, and $Z^{IJ}_{0.001} = 0.001 Z^{IJ}_1 + 0.999 Z^{IJ}_0$.

In this step, I also need to compute $s^{I\emptyset}_{0.002}$ and $s^{\emptyset J}_{0.002}$, because these will be used in the next step. For example,

$$\begin{aligned} s^{I\emptyset}_{0.002} &= s^{I\emptyset}_{0.001} + d s^{I\emptyset}_{0.001} \\ &= s^{I\emptyset}_{0.001} + \frac{\partial s^{I\emptyset}_{0.001}}{\partial\tilde{\boldsymbol{\theta}}_\tau}(\tilde{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_0)\cdot 0.001 \end{aligned}$$

- **STEP 2.3 and above:** The rest of the estimation proceeds analogously until τ reaches 1.

STEP 3: I now explain how to decompose the changes from 1980 to 2019 in individual expected utilities $\bar{u}^I$ into contributions by each model primitive. As an example, let's consider how $\Delta^{1990-1980}\bar{u}^I$ is estimated according to Equation (21):

$$\Delta^{1990-1980}\bar{u}^I = \sum_{\tau\in[0,1], d\tau=0.001} \frac{1}{n^I_\tau}(n^I_1 - n^I_0)d\tau - \frac{2}{s^{I\emptyset}}\left(\left[\frac{\partial s^{I\emptyset}}{\partial\tilde{\boldsymbol{\theta}}_\tau}\right]_\tau (\tilde{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_0)d\tau\right)$$

where $\tau = 0$ refers to year 1980 and $\tau = 1$ refers to year 1990.

Because $\Delta^{1990-1980}\bar{u}^I$ is a linear function in $(\tilde{\theta}_1 - \tilde{\theta}_0)$, it can be linearly decomposed into parts that are attributed to each model primitive θ^k.[23] I call this the **contribution** of θ^k to $\Delta^{1990-1980}\bar{u}^I$. The contribution of θ^k is essentially the change in θ^k from 1980 to 1990 multiplied by a multiplier that measures how sensitive $\bar{u}^I$ is with respect to the change in θ^k. Because summing up all contributions of the model primitives leads to $\Delta^{1990-1980}\bar{u}^I$, each contribution can be thought of as a portion of the changes in the expected utilities that is attributed to θ^k. In order to decompose

[23] For example, the part of $\Delta^{1990-1980}\bar{u}^I$ that is contributed by the number of $WhiteHSG$ women is $\sum_{\tau\in[0,1],d\tau=0.001} -\frac{2}{s^{I\emptyset}}\left(\left[\frac{\partial s^{I\emptyset}}{\partial m^{WhiteHSG}_\tau}\right]_\tau (m^{WhiteHSG}_1 - m^{WhiteHSG}_0)d\tau\right)$.

changes in $\bar{u}^I$ over a longer time frame from 1980 to 2019, I simply sum up all four decade-by-decade contributions of each model primitive.

While I only described the decomposition steps for $\bar{u}^I$ for the illustration purpose, the decomposition for the welfare gains, which is $\bar{u}^{I,actual} - \bar{u}^{I,counterfactual}$, is straightforward.

B.3 More decomposition results

Table A3: Decomposition: Top three contribution from changes in **Z**, Black HSG Men

		(1)	(2)	(3)
Contribution	Top (+)	1.1 $Z^{WhiteHSG,WhiteHSG}$	0.7 $Z^{BlackHSG,WhiteCG}$	0.6 $Z^{WhiteSC,WhiteHSG}$
	Top (-)	-0.6 $Z^{BlackHSG,AsianHSD}$	-0.4 $Z^{BlackHSG,WhiteHSG}$	-0.4 $Z^{BlackHSG,HispHSD}$

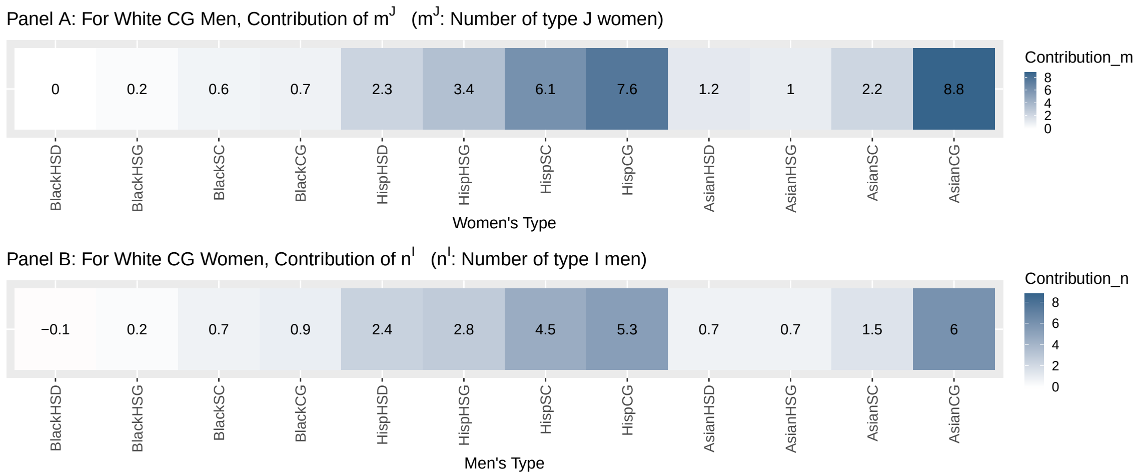
Notes: This table presents the top three positive and negative contributions from marital surplus to the 1980-2019 changes in the welfare gains from marital desegregation for Black HSG men. For marital surplus Z^{IJ}, I refers to husband's type and J refers to wife's type.

Table A4: Decomposition: Top three contribution from changes in **Z**, Black HSG Women

		(1)	(2)	(3)
Contribution	Top (+)	1.5 $Z^{BlackHSG,BlackHSG}$	1.2 $Z^{BlackSC,BlackHSG}$	0.7 $Z^{BlackHSD,BlackHSG}$
	Top (-)	-0.2 $Z^{WhiteCG,WhiteSC}$	-0.2 $Z^{BlackHSG,BlackHSD}$	-0.2 $Z^{BlackSC,BlackSC}$

Notes: This table presents the top three positive and negative contributions from marital surplus to the 1980-2019 changes in the welfare gains from marital desegregation for Black HSG women. For marital surplus Z^{IJ}, I refers to husband's type and J refers to wife's type.

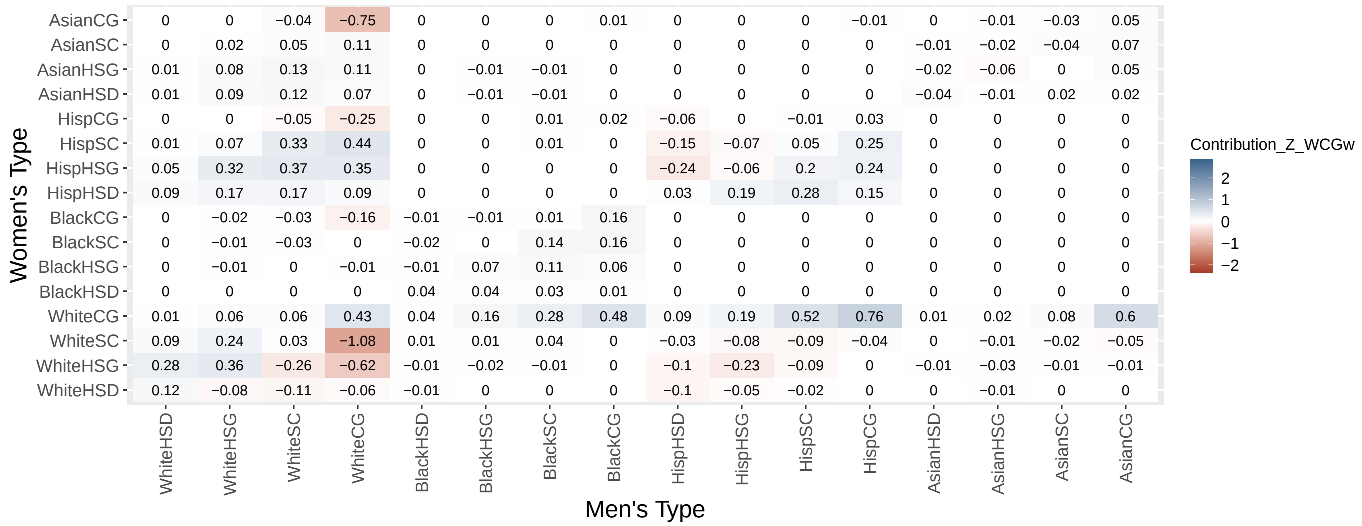
Figure A12: Contribution of Each Population Primitive to the 1980-2019 Change in Individual Expected Utility, For White CG Men and White CG Women



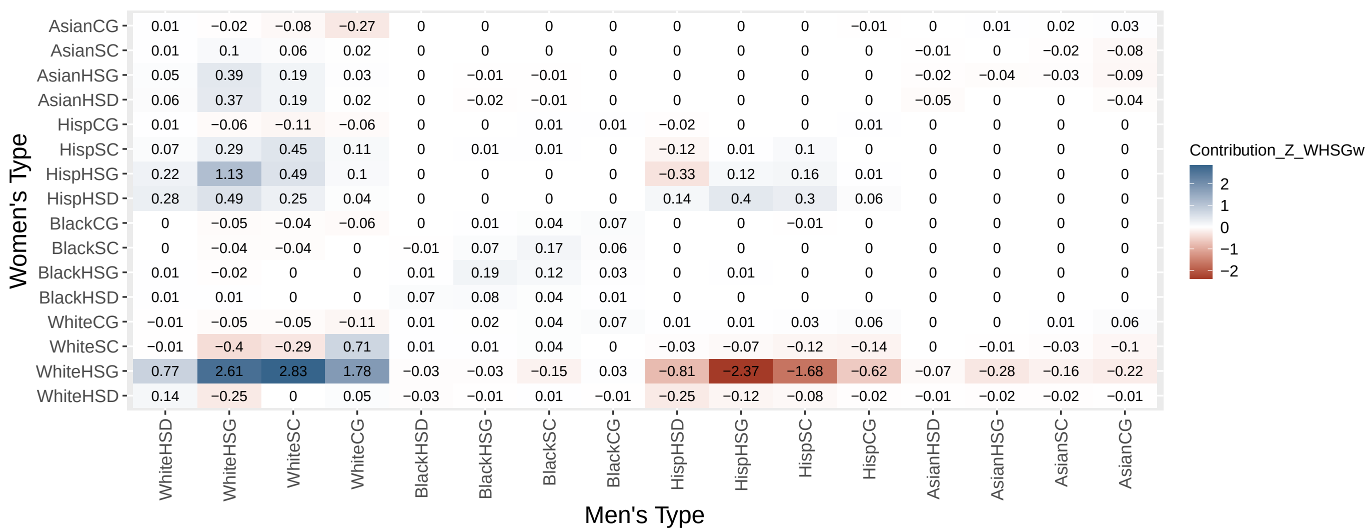
Note: Panel A presents the decomposition results for white CG men's changes in the utility gains from marital racial segregation over time. Specifically, Panel A only presents the decomposition results regarding female population model primitives. Panel B presents the decomposition results for white CG women's changes in the utility gains from marital racial segregation over time. Specifically, Panel A only presents the decomposition results regarding male population model primitives.

Figure A13: Contribution of Each Marital Surplus Primitive to the 1980-2019 Change in Individual Expected Utility, For White CG Women and White HSG Women

Panel A: For White CG Women, Contribution of Z^{IJ} (Z^{IJ}: Marital Surplus for (I,J) match)

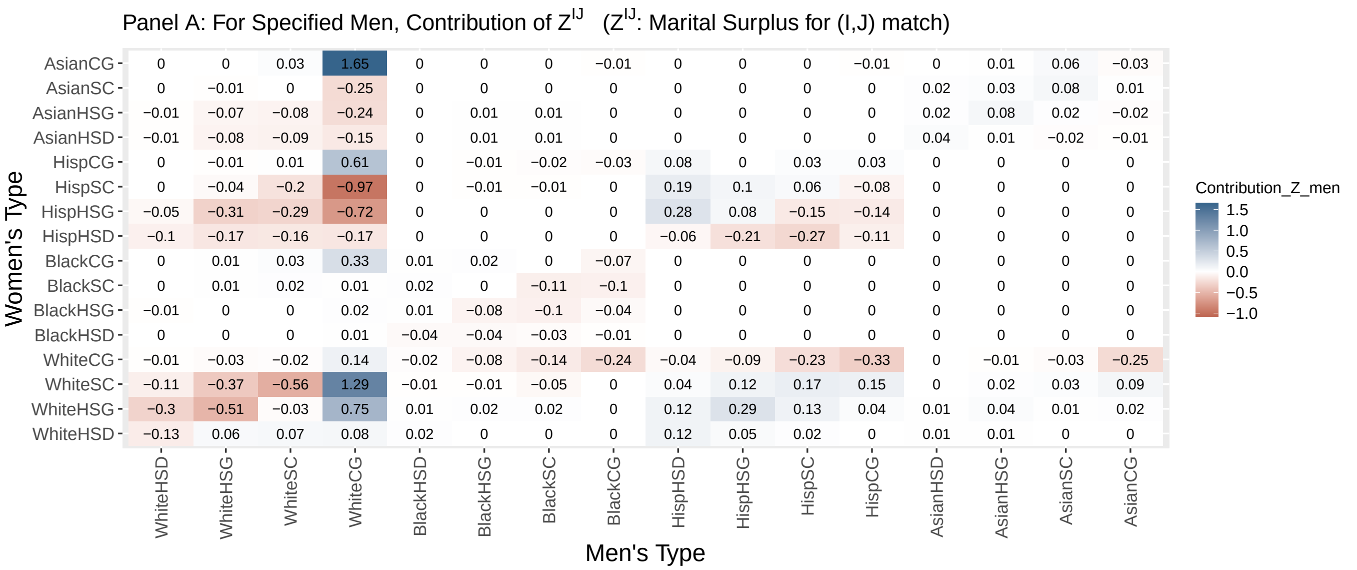


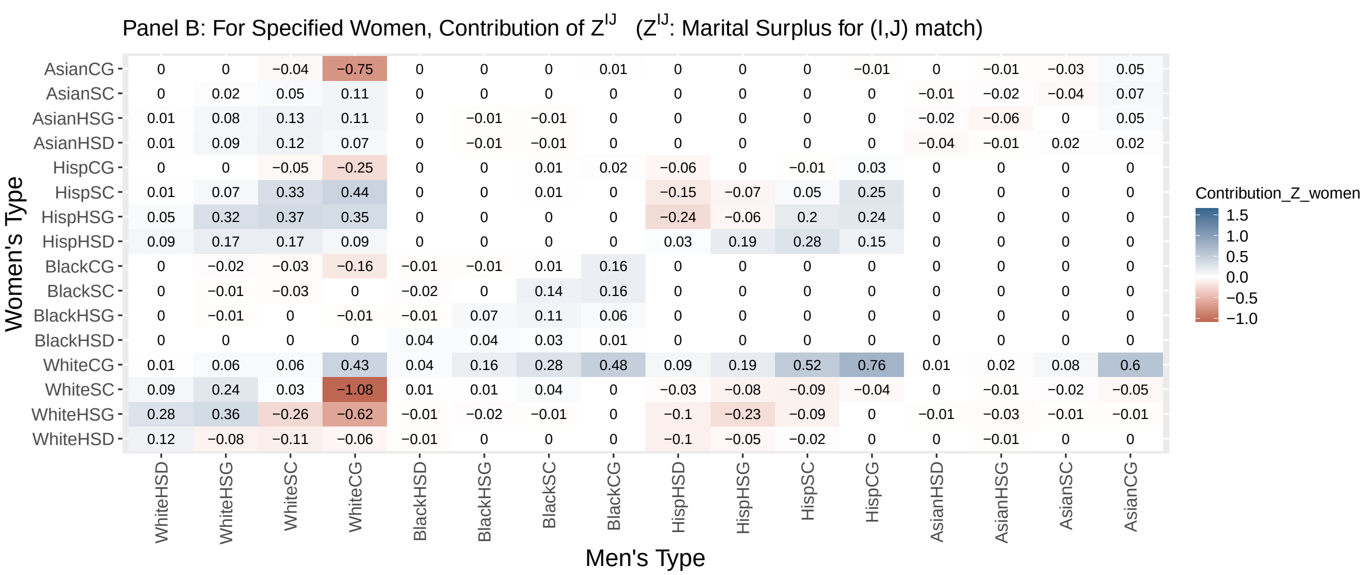
Panel B: For White HSG Women, Contribution of Z^{IJ} (Z^{IJ}: Marital Surplus for (I,J) match)



Note: Panel A presents the decomposition results for White CG women's changes in the utility gains from marital racial segregation over time. Panel B presents the decomposition results for White HSG women's changes in the utility gains from marital racial segregation over time. Only the contributions made by marital surplus are presented.

Figure A14: Contribution of Each Marital Surplus Primitive to the 1980-2019 Change in Individual Expected Utility, For White CG





Note: Panel A presents the decomposition results for White CG men's changes in the utility gains from marital racial segregation over time. Panel B presents the decomposition results for White CG women's changes in the utility gains from marital racial segregation over time. Only the contributions made by marital surplus are presented.