Assignment 2 for csc446

Yingchao Yu V00830514

1\. (a) $f(x) = \begin{cases} x & x \le \frac{c}{2} \\ -x + c & x > \frac{c}{2} \end{cases}$

(b) $F(x) = \begin{cases} \frac{1}{2}x^2 & x \le \frac{c}{2} \\ -\frac{1}{2}x^2 + cx - \frac{1}{4}c^2 = \frac{c^2}{4} - \frac{(c-x)^2}{2} & x > \frac{c}{2} \end{cases}$

(c) $F(c) = 1 \Rightarrow \frac{c^2}{4} = 1 \Rightarrow c = 2$

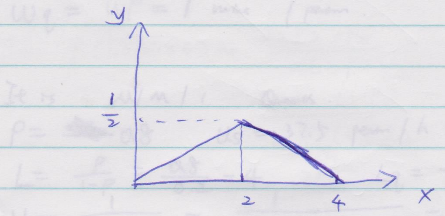
(d) $F(1.5) - F(0.5) = \left(1 - \frac{0.25}{2}\right) - \frac{0.25}{2} = 0.75$

(e) $E(x) = \int_0^1 x^2\,dx + \int_1^2 (-x+2)\,x\,dx$
$= \frac{1}{3} - \frac{7}{3} + 3 = 1$

$E(x^2) = \int_0^1 x^3\,dx + \int_1^2 (-x+2)x^2\,dx$
$= \frac{1}{4} + \frac{14}{3} - \frac{15}{4} = \frac{7}{6}$

$V(x) = \cancel{E^2(x) - E(x^2) =}\ E(x^2) - E^2(x) = \frac{1}{6}$

f



2. Since $E(x+y) = E(x)+E(y)$, it is obviously that $E(x_1+x_2+\cdots x_n) = E(x_1)+E(x_2)+\cdots E(x_n)$

Thus, $E(y) = E(x_1)+E(x_2)+\cdots E(x_n)$

$= \eta+\eta+\cdots\eta$

$= n\eta$

$E(y^2) = E(x_1x_1+x_1x_2+\cdots x_n\cdot x_n)$

$= E(x_1^2)+E(x_2^2)+\cdots E(x_n^2) + \cancel{E(x_ix_j)} + \overbrace{\sum E(x_ix_j)}^{n^2-n} \quad (i\neq j)$

$E^2(y) = E(x_1)\cdot E(x_1) + E(x_1)\cdot E(x_2)+\cdots E^2(x_n) = E^2(x_1)+E^2(x_2)+\cdots E^2(x_n) + \cancel{E(x_ix_j)} + \overbrace{\sum E(x_ix_j}^{n^2-n} \quad (i\neq j)$

$E(y^2) - E^2(y) = \left(E(x_1^2) - E^2(x_1)\right) + \left(E(x_2^2) - E^2(x_2)\right) + \cdots \left[E(x_n^2) - E^2(x_n)\right]$

$= \sigma^2+\sigma^2+\cdots\sigma^2$

$= n\sigma^2$

3. We get $\lambda_1 = 10$ $\lambda_2 = 20$ $u_1 = 30$, $\lambda_s = 30$

a. $\frac{\lambda_s}{u_s} = 0.8 \Rightarrow u_s = 37.5$ person/h $\Rightarrow$ ~~Ser~~ Avg service time $= \frac{60}{37.5} = 1.6$ mins

b. It ~~is~~ is M/M/1 Queues.

$\rho = \frac{\lambda_1}{u_1} = \frac{1}{3}$, $L = \frac{\rho}{1-\rho} = 0.5$ $L_q = \frac{\rho^2}{1-\rho} = \frac{1}{6}$

$W = \frac{1}{u_1(1-\rho)} = \frac{1}{30p/h \cdot \frac{2}{3}} = \frac{1}{20p/h} = 0.05\ h/p = 3$ mins /person

$W_q = W\rho = 1$ mins /person.

c. It is M/M/1 Queues.

$\rho = 0.8$ $u_s = 37.5$ person/h

$L = \frac{\rho}{1-\rho} = \frac{0.8}{0.2} = 4$ $L_q = \frac{\rho^2}{1-\rho} = 3.2$

$W = \frac{1}{u_s(1-\rho)} = \frac{1}{37.5\ \text{person/h} \cdot 0.2} = 8$ mins/ person

$W_q = W\rho = 6.4$ mins / person.

d. $W_{total\,1} = 3$ mins/p + 8 mins /p $= 11$ mins / person

$L_{total\,1} = 0.5 + 4 \times \frac{1}{3} = \frac{1}{2} + \frac{4}{3} \doteq 1.83$ person

$W_{total\,2} = 8$ mins / person.

5. $\lambda = 0.5$ customer / min

$\sigma^2 = 1.5^2 = 2.25$

$\frac{1}{u} = 1.5$

$\rho = \frac{\lambda}{u} = 0.75$

This is M/G/1 Queues,

Thus, $L = \rho + \frac{\rho^2(1+\sigma^2 u^2)}{2(1-\rho)}$

$= 3$ customer

$L_Q = L - \rho = 2.25$ customer

$W = \frac{1}{u} + \frac{\lambda(1/u^2+\sigma^2)}{2(1-\rho)}$

$= 1.5 + 4.5 = 6$ min/customer

$W_q = 6 - 1.5 = 4.5$ mins / customer

$Cost = W_q \cdot C = 5000 \times 4.5/60 = 375\ \$$

6. ① M/M/2 Queue::

when c=2, $P_0 = \left(1+2\rho + (2\rho)^2 \cdot \frac{1}{2} \cdot \frac{1}{1-\rho}\right)^{-1}$

$= \left(\frac{1+\rho}{1-\rho}\right)^{-1}$

$= \frac{1-\rho}{1+\rho}$

Thus. $P(L(\infty) \geq c) = \frac{(2\rho)^2 \cdot P_0}{2(1-\rho)}$

$= \frac{4\rho^2 \cdot (1-\rho)}{(1+\rho)(1-\rho) \cdot 2}$

$= \frac{2\rho^2}{1+\rho}$

$L_1 = 2\rho + \frac{2\rho^3}{(1-\rho)(1+\rho)}$

$= \frac{2\rho}{1-\rho^2}$

$L_{Q_1} = L_1 - 2\rho = \frac{2\rho}{1-\rho^2} - 2\rho = \frac{2\rho^3}{1-\rho^2}$

$W_1 = \frac{L_1}{2\lambda} = \frac{\rho}{\lambda(1-\rho^2)}$ $\quad W_{q_1} = \frac{\rho}{\lambda(1-\rho^2)} - \frac{1}{u}$

$\rho_1 = \frac{2\lambda}{2 \cdot u} = \frac{\lambda}{u}$

② M/M/1 ~~—~~ Queue: 2 of it.

$P_2 = \frac{\lambda}{u}$, $L_2 = \frac{\rho}{1-\rho} \cdot 2 = \frac{2\rho}{1-\rho}$ $L_{Q_2} = \frac{\rho^2}{1-\rho} \cdot 2 = \frac{2\rho^2}{1-\rho}$

$W_2 = \frac{1}{u(1-\rho)}$ $W_{Q_2} = \frac{\rho}{u(1-\rho)}$

Comparing ① and ②:

$\rho_1 = \rho_2 = \frac{\lambda}{u}$

$L_1 = \frac{2\rho}{(1-\rho)(1+\rho)} < \frac{2\rho}{1-\rho} = L_2$ Since $(1+\rho) > 1$

$L_{Q_1} = \frac{2\rho^3}{1-\rho} < \frac{2\rho^2}{1-\rho} = L_{Q_2}$

$W_1 = \frac{\rho}{\lambda(1-\rho^2)} = \frac{1}{u(1-\rho^2)} = \frac{1}{u(1-\rho)(1+\rho)} < \frac{1}{u(1-\rho)} = W_2$

$W_{Q_1} = \frac{\rho^2}{u(1-\rho)(1+\rho)} = \frac{\rho}{u(1-\rho)} \cdot \frac{\rho}{1+\rho} < \frac{\rho}{u(1-\rho)} = W_{Q_2}$

Thus. M/M/2 costs less, which is better.

7. mean $= 0.2 \times 3 + 0.7 \times 7 + 0.1 \times 12 = 6.7$ mins.

$u = \frac{60}{6.7} = \frac{600}{67}$ cars / hr $\lambda = 34$ cars / hr

$\rho = \frac{\lambda}{4u} = \frac{1139}{1200} = \cancel{0.949}\ 0.95$ $a = 4\rho = \frac{1139}{300} \doteq 3.80$

~~$P_0 = 0.019$~~

when $c = 4$, $P_0 = 0.019$ $P_N = 0.139$

$L_{Q_1} = 0.865$, $\lambda_{e_1} = \lambda(1-P_N) = 29.274$ cars/hr $= 0.488$ cars/min

$W_{Q_1} = \frac{L_{Q_1}}{\lambda_{e_1}} = 1.773$

when $c = 5$. $P_0 = 0.022$ $P_N = 0.084$

$L_{Q_2} = 0.2789$, $\lambda_{e_2} = \lambda(1-P_N) = 31.144$ cars/hr $= 0.519$ cars/min

$W_{Q_2} = \frac{L_{Q_2}}{\lambda_{e_2}} = 0.538$

Since Wq2< Wq1 and Lq2<Lq1 , then Setting a new stalling will be better.

8. $\lambda = 1$ $\quad u = \frac{4}{3}$ $\quad \rho = \frac{\lambda}{u} = \frac{3}{4}$

a. mean $= 3 \times 0.25 = 0.75$ h / ~~pe~~ person

$V(x) = \sigma^2 = 3 \times 0.25^2 = 0.1875$

$$\cancel{L_Q = \frac{\rho^2(1+\sigma^2 u^2)}{2(1-\rho)} = 1.5}$$

b. Since $mean^2 = 9 \times 0.25^2 \neq \sigma^2$, it's not.

c. $$L_Q = \frac{\rho^2(1+\sigma^2 u^2)}{2(1-\rho)} = 1.5$$

d. $$w = \frac{1}{u} + \frac{\lambda(1/u^2 + \sigma^2)}{2(1-\rho)} = 2.25$$

9. a. $u_1 = u_2 = 3$ items/hr $\quad u_s = 8$ items/hr

$\lambda = 5$ items/hr

when $10\% x \le 1 \Rightarrow x \le 10$,

$\lambda_{real} = 5 + 0.1x = x \Rightarrow x = \frac{50}{9}$ which is ≤ 10.

Thus, $\lambda_{real} = \frac{50}{9}$ $\quad u_{total} = 6$ items/h

$\rho = \frac{\lambda_{real}}{u_{total}} = \frac{25}{27}$

$$W_Q = \cancel{\frac{\rho^2}{u(1-\rho^2)} = \frac{625}{624} - 1} \; \frac{2\rho}{\lambda(1-\rho^2)} - \frac{1}{u} = 2.0$$

Since $\lambda_{real} < u_{total} \Rightarrow \lambda_s = \lambda_{real} = \frac{50}{9}$.

$\rho = \frac{\lambda_s}{u_s} = \frac{25}{36}$

$$W_Q = \frac{\rho}{u(1-\rho)} = \frac{25}{88} \doteq 0.284$$

b. $\lambda_{max} + 0.1x = x$ $\quad$ and $\quad x \le 6$

$\lambda_{max} = 0.9x$

$\lambda_{max} \le 5.4 \Rightarrow \lambda_{max} = 5.4$ items/hr

10. $L = \sum_{k=0}^{\infty} k(1-\rho)\rho^k$

$= \lim_{k\to\infty} (1-\rho)(\rho^1 + 2\rho^2 + 3\rho^3 + \cdots k\rho^k)$

$= \lim_{k\to\infty} (1-\rho)\left(\frac{\rho^1(1-\rho^k)}{1-\rho} + \frac{\rho^2(1-\rho^{k-1})}{1-\rho} + \cdots \frac{\rho^k(1-\rho^0)}{1-\rho} \right)$

$= \lim_{k\to\infty} (1-\rho)\left(\frac{\rho^1 - \rho^{k+1} + \rho^2 - \rho^{k+1} + \rho^3 - \rho^{k+1} + \cdots \rho^k - \rho^{k+1}}{1-\rho} \right)$

$= \lim_{k\to\infty} \frac{\rho\cdot(1-\rho^k)}{1-\rho} - k\rho^{k+1}$

$= \frac{\rho}{1-\rho}$