CSC 446 Project

Supermarket Queuing Simulation Research

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- **Problem Description:**

Nowadays, with the constant development, there are more supermarkets in the towns than before. In this competitive environment, what operators of supermarkets consider is to profit efficiently. Cashier plays an important role to improve efficiency and attract more customers. What customers want is not only the good quality and cheap price of product but also the service quality of the supermarkets. It is obviously that people do not want to wait in the long queue in supermarkets, as a consequence they may give up buying products. However, adding more cashier counter means more investment, which may cause phenomenon of idle resource and waste. Therefore, the management of queue system of cashier is the key part to meet customers' need and expand income reasonably. Thus we need to simulate the situation such that we can get an optimal solution.

Problem mapping:

Model for project :

Generally, there are three models in the supermarkets: M/M/1, M/G/1 and M/M/C/N.

The definition of queuing models:

1. The definition of M/M/1:

   M/M/1 indicates a single-server that has unlimited queue capacity and an infinite population model. The interarrivals and service times are exponentially distributed.

2. The definition of M/G/1:

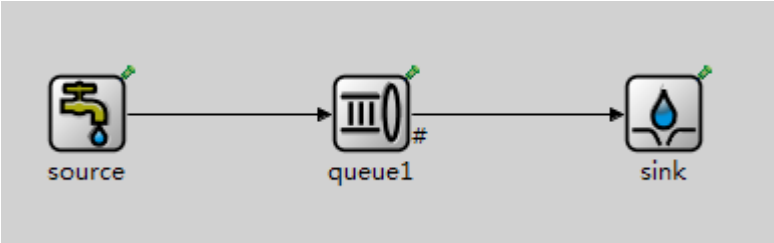
   M/M/1 indicates a single-server that has unlimited queue capacity and an infinite population model. The interarrivals is exponentially distributed and the service times is uniformly distributed.

3. The definition of M/M/C/N:

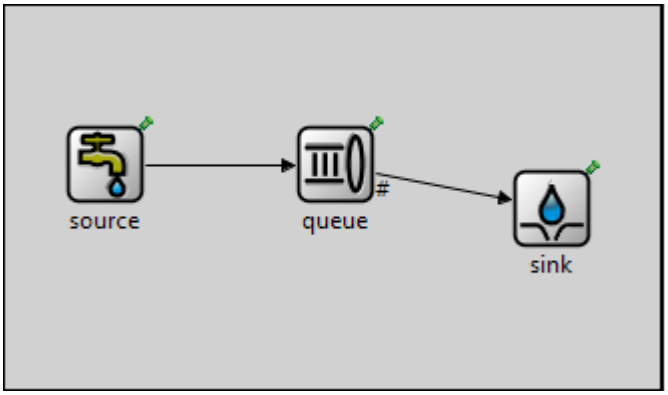
   M/M/1 indicates c channels operating in parallel; Limited Capacity ($N \geq c$), when an arrival occurs and the system is full, that arrival is turned away. Service times are exponentially distributed.

- **The mapping of problem:**

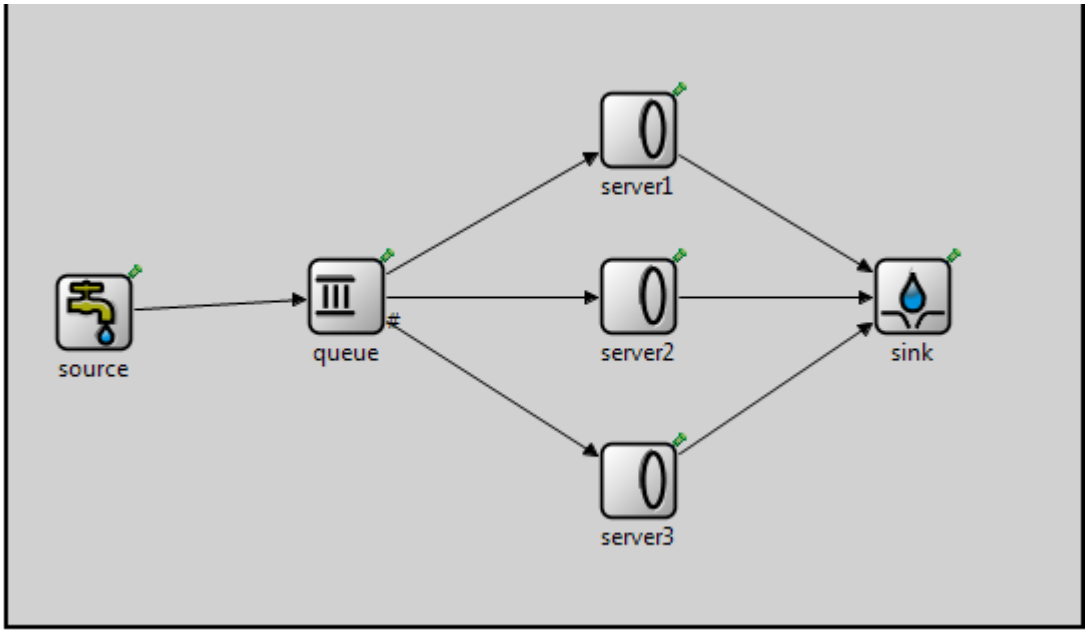
1. **M/M/1:**



2. **M/G/1:**



3. **M/M/3/20:**



Simulation goals and parameters:

Simulation goals:

The simulation goals are to simulate the supermarket queuing system in three models such that we can analysis the queuing system in the supermarkets to improve service quality and expand profits in the future.

Simulation parameters:

1. M/M/1:

   Total job number =100000

   Mean service time =exponential(1 seconds)

   Mean arrival time = exponential({10 seconds, 5 seconds, 2.5 seconds, 1.66 seconds, 1.25 seconds})

2. M/G/1:

   Total job number =100000

   Mean service time =uniform(0,2)s

   Mean arrival time = exponential{10 seconds, 5 seconds, 2.5 seconds, 1.66 seconds, 1.25 seconds}

3. M/M/3/20:

   Total job number =100000

Mean service time (1-3 server) =exponential(1 seconds)

Mean arrival time = exponential({3.3333 seconds, 1.6667 seconds, 0.8333 seconds, 0.5556 seconds, 0.4167 seconds})

● Methodology:

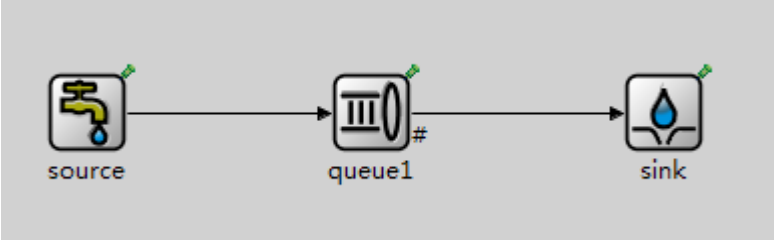
➢ Simulation tools:

OMNET++:

OMNeT++ is an extensible, modular, component-based C++ simulation library and framework, primarily for building network simulators.

➢ Setup the simulations:

1. Create a new omnet++ project named ”csc446project”
2. Add queueinglib to its project reference
3. Map the flow chart (example) as below:



4. Create initialization file “omnetpp.ini” to set parameters and environment. Such as the exponential serve mean time and arrival mean time and repeat times.

5. Run configurations. Add * to run number such that program can run all

50 situations of simulation.

➢ **Collect the stats:**

1. Create a file "csc446.anf" which is the analysis file for simulation results.

2. Export those data to excel.

● Analysis:

➢ **Analysis of M/M/1:**

▪ Statistic of 50 runs and 5 utilizations :

This table contains: result data of L, Lq, W, Wq, rou , the confidence intervals , the actual service mean and the actual interarrivals mean and average stats across ten different runs using different initial seeds.

	NO.	L	Lq	rou	W	Wq
utilization	1	0.111235	0.011004	0.100231	1.10983	0.109793
0.1	2	0.111497	0.011246	0.100251	1.1111	0.112069
	3	0.111147	0.011256	0.099892	1.1115	0.112558
	4	0.111139	0.011088	0.100051	1.11566	0.111304
	5	0.111898	0.011182	0.100716	1.1145	0.111368
	6	0.111522	0.011242	0.100281	1.11503	0.112398
	7	0.111025	0.011181	0.099844	1.10483	0.111264
	8	0.111194	0.011217	0.099977	1.11328	0.112303
	9	0.111146	0.01118	0.099966	1.10749	0.111401
	10	0.110984	0.010935	0.10005	1.10844	0.109207
	AVG	0.111279	0.011153	0.100126	1.111166	0.111367
	STD	0.00028	0.000109	0.000257	0.003554	0.001106
	CILOW	0.111079	0.011075	0.099942	1.108626	0.110576
	CLHIGH	0.111479	0.011231	0.100309	1.113706	0.112157
	Error	0.0002	7.81E-05	0.000183	0.00254	0.00079

utilization	11	0.247369	0.048941	0.198427	1.2399	0.245312
0.2	12	0.251624	0.051646	0.199978	1.25809	0.258223
	13	0.250866	0.050416	0.20045	1.24962	0.251132
	14	0.247307	0.048808	0.198499	1.24349	0.245412
	15	0.251202	0.050634	0.200569	1.25462	0.252889
	16	0.250495	0.050625	0.19987	1.2501	0.252647
	17	0.247168	0.048276	0.198892	1.24041	0.242271
	18	0.249193	0.049657	0.199537	1.24518	0.248126
	19	0.248326	0.049072	0.199254	1.24165	0.245364
	20	0.248394	0.04935	0.199045	1.24633	0.247614
	AVG	0.249194	0.049742	0.199452	1.246939	0.248899
	STD	0.001726	0.00105	0.000759	0.00611	0.004779
	CILOW	0.247961	0.048992	0.19891	1.242572	0.245483
	CLHIGH	0.250428	0.050493	0.199994	1.251306	0.252315
	Error	0.001234	0.000751	0.000542	0.004367	0.003416
utilization	21	0.666995	0.268103	0.398891	1.67518	0.673352
0.4	22	0.671605	0.270862	0.400743	1.67981	0.677477
	23	0.666159	0.26441	0.40175	1.66561	0.661109
	24	0.680122	0.27887	0.401252	1.69317	0.694248
	25	0.6776	0.274028	0.403572	1.68662	0.682086
	26	0.675348	0.272782	0.402566	1.683	0.679788
	27	0.673657	0.271283	0.402374	1.68721	0.679443
	28	0.6718	0.269707	0.402093	1.67688	0.673216
	29	0.668334	0.269383	0.39895	1.67076	0.673429
	30	0.671492	0.270353	0.40114	1.6839	0.677964
	AVG	0.672311	0.270978	0.401333	1.680214	0.677211
	STD	0.004514	0.003814	0.001506	0.00829	0.008371
	CILOW	0.669085	0.268253	0.400257	1.67429	0.671228
	CLHIGH	0.675537	0.273704	0.402409	1.686138	0.683194
	Error	0.003226	0.002726	0.001076	0.005924	0.005983
utilization	31	1.46821	0.871966	0.596248	2.44879	1.45433
0.6	32	1.49315	0.891049	0.602098	2.48769	1.48455
	33	1.49505	0.893969	0.60108	2.47768	1.48153
	34	1.50354	0.903335	0.600202	2.50468	1.50483
	35	1.48276	0.883233	0.599523	2.47241	1.47274
	36	1.46782	0.874641	0.593175	2.46537	1.46906
	37	1.46305	0.867153	0.595892	2.44291	1.44793
	38	1.4821	0.884677	0.59742	2.4653	1.47156
	39	1.49727	0.898101	0.59917	2.50434	1.50217
	40	1.54911	0.943736	0.605372	2.58356	1.57394
	AVG	1.490206	0.891186	0.599018	2.485273	1.486264
	STD	0.024884	0.021835	0.003495	0.040208	0.035737
	CILOW	1.472422	0.875581	0.59652	2.456537	1.460723

	CLHIGH	1.50799	0.906791	0.601516	2.514009	1.511805
	Error	0.017784	0.015605	0.002498	0.028736	0.025541
utilization	41	3.99316	3.19214	0.801016	4.97971	3.98079
0.8	42	4.01649	3.21501	0.801481	5.01558	4.01474
	43	4.0587	3.25535	0.803352	5.06033	4.05872
	44	3.91491	3.11546	0.799447	4.89674	3.8968
	45	3.88909	3.09172	0.797374	4.87774	3.87767
	46	4.03206	3.22821	0.803846	5.04855	4.04206
	47	4.10668	3.30681	0.799873	5.1233	4.12542
	48	3.89929	3.10254	0.796746	4.88734	3.8887
	49	4.05531	3.2584	0.796907	5.09011	4.08986
	50	3.80061	3.00356	0.797049	4.75809	3.76024
	AVG	3.97663	3.17692	0.799709	4.973749	3.9735
	STD	0.096154	0.094632	0.002677	0.115425	0.114539
	CILOW	3.907911	3.109289	0.797796	4.891258	3.891642
	CLHIGH	4.045349	3.244551	0.801622	5.05624	4.055358
	Error	0.068719	0.067631	0.001913	0.082491	0.081858

NO.	Service mean	Arrival mean	rou
Rou=0.1			
1	1.00004	9.977352316	0.100231
2	0.999032	9.965307079	0.100251
3	0.998945	10.00027029	0.099892
4	1.00435	10.03838043	0.100051
5	1.00313	9.959986497	0.100716
6	1.00263	9.998205044	0.100281
7	0.993568	9.951193911	0.099844
8	1.00098	10.01209277	0.099977
9	0.996088	9.96431769	0.099966
10	0.999233	9.987336332	0.10005
AVG	0.9997996	9.985444236	0.100126
Rou=0.2			
11	0.994592	5.012382387	0.198427
12	0.999869	4.999894988	0.199978
13	0.99849	4.981242205	0.20045
14	0.998081	5.0281412	0.198499
15	1.00173	4.994440816	0.200569
16	0.997454	4.990513834	0.19987
17	0.998135	5.018477365	0.198892
18	0.997057	4.996852714	0.199537
19	0.996282	5.000060225	0.199254
20	0.99872	5.017558843	0.199045

AVG	0.998041	5.003956458	0.199452
Rou=0.4			
21	1.00183	2.51153824	0.398891
22	1.00233	2.50117906	0.400743
23	1.0045	2.500311139	0.40175
24	0.998919	2.489505348	0.401252
25	1.00454	2.489122139	0.403572
26	1.00322	2.492063413	0.402566
27	1.00777	2.504560434	0.402374
28	1.00366	2.496089213	0.402093
29	0.997333	2.499894724	0.39895
30	1.00594	2.507703046	0.40114
AVG	1.0030042	2.499196676	0.401333
Rou=0.6			
31	0.994464	1.667869745	0.596248
32	1.00314	1.666074294	0.602098
33	0.996142	1.65725361	0.60108
34	0.999852	1.665859161	0.600202
35	0.999668	1.667438947	0.599523
36	0.996308	1.679618999	0.593175
37	0.994989	1.669747203	0.595892
38	0.993741	1.663387567	0.59742
39	1.00217	1.672597093	0.59917
40	1.00962	1.667767918	0.605372
AVG	0.9990094	1.667761454	0.599018
Rou=0.8			
41	0.998914	1.247058735	0.801016
42	1.00085	1.24875075	0.801481
43	1.00161	1.246788456	0.803352
44	0.999942	1.25079211	0.799447
45	1.00008	1.254216967	0.797374
46	1.0065	1.252105503	0.803846
47	0.997883	1.247551799	0.799873
48	0.998637	1.253394432	0.796746
49	1.00025	1.255165283	0.796907
50	0.997847	1.251926795	0.797049
AVG	1.0002513	1.250775083	0.799709

- Theoretical result:

The steady-state parameters of M/M/1:

$$\rho = \lambda / \mu, \quad P_0 = (1-\rho), \quad P_n = (1-\rho)\rho^n$$

$$L = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1-\rho}, \quad L_Q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho}$$

$$w = \frac{1}{\mu - \lambda} = \frac{1}{\mu(1-\rho)}, \quad w_Q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\rho}{\mu(1-\rho)}$$

Thus, the theoretical result is :

Actual:					
rou	0.1	0.2	0.4	0.6	0.8
L	0.111111	0.25	0.666667	1.5	4
Lq	0.011111	0.05	0.266667	0.9	3.2
W	1.111111	1.25	1.666667	2.5	5
Wq	0.111111	0.25	0.666667	1.5	4

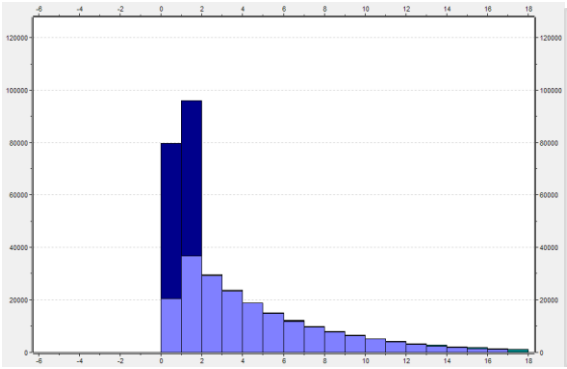
the simulation result is :

Simulate:					
Rou	0.100126	0.199452	0.401333	0.599018	0.799709
L	0.111279	0.249194	0.672311	1.490206	3.97663
Lq	0.011153	0.049742	0.270978	0.891186	3.17692
W	1.111166	1.246939	1.680214	2.485273	4.973749
Wq	0.111367	0.248899	0.677211	1.486264	3.9735

Compared data from simulation, the theory data is close enough according to confidence intervals.

- Histogram result:

  Actual histogram: Theoretical histogram:



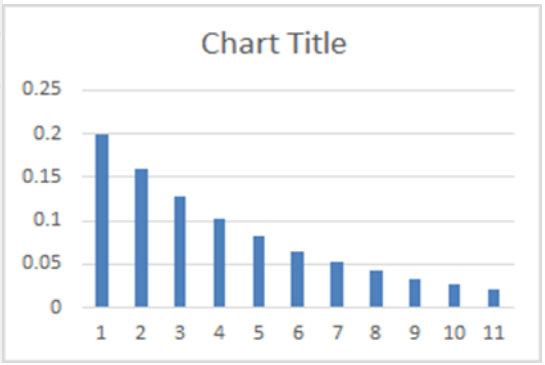


Chart Title
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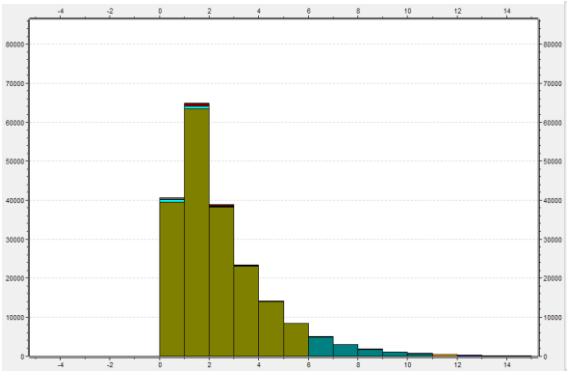
0.2
0.15
0.1
0.05
0
1 2 3 4 5 6 7 8 9 10 11



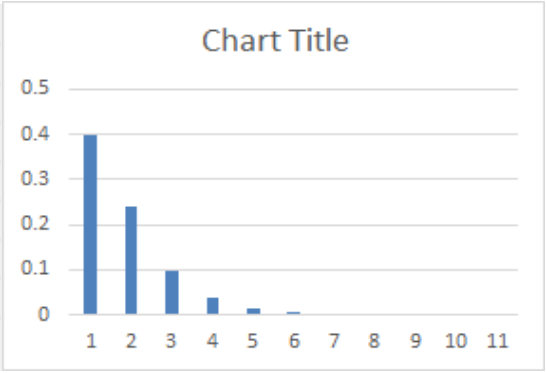


Chart Title
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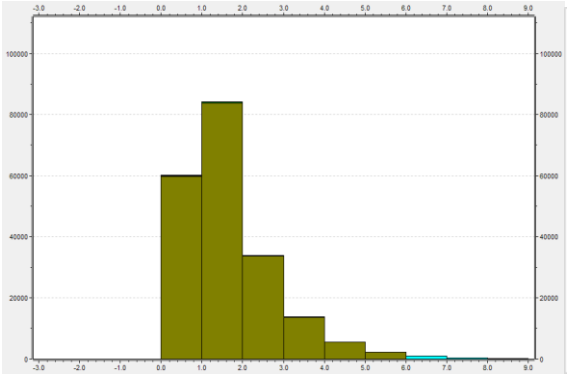
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11



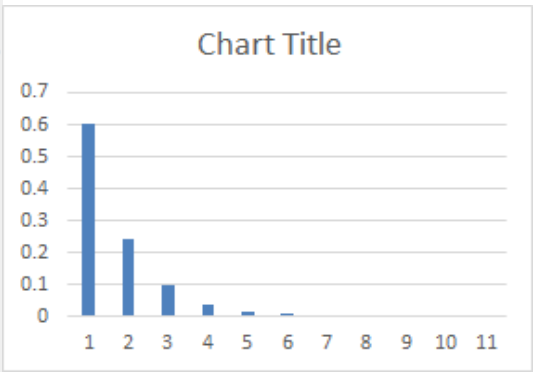


Chart Title
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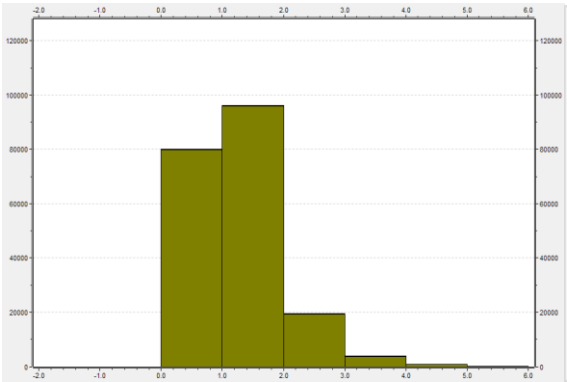
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11



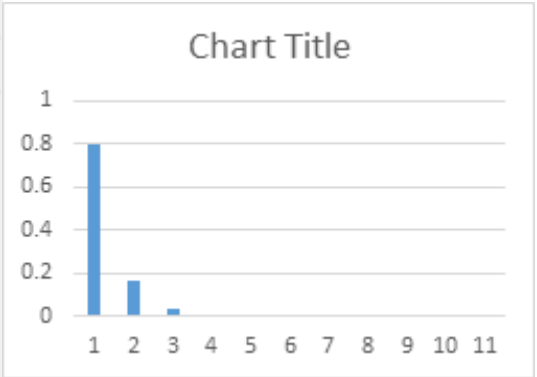
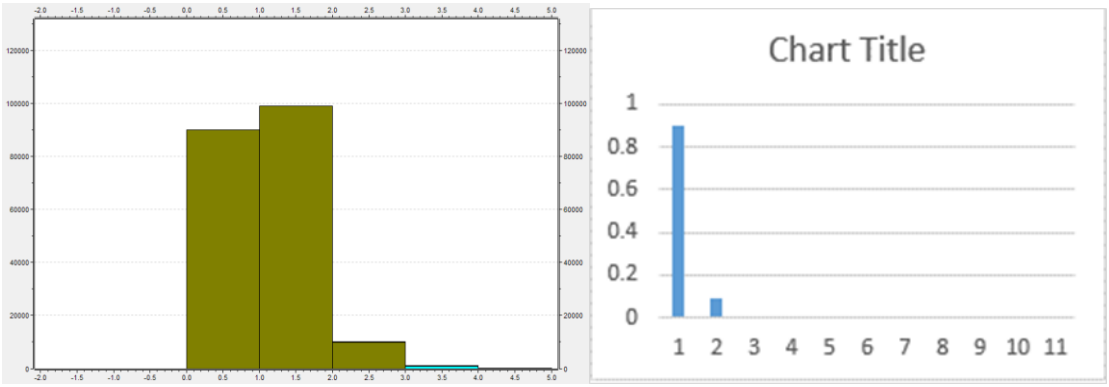


Chart Title
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10 11



Utilization: from top to bottom is 0.8, 0.6, 0.4, 0.2, 0.1.

Comparing these histogram, they are similar.

➢ Analysis of M/G/1:

- Statistic of 50 runs and 5 utilizations :

This table contains: result data of L, Lq, W, Wq, rou , the confidence intervals , the actual service mean , the actual interarrivals mean and average stats across ten different runs using different initial seeds.

	0	L	Lq	rou	W	Wq
utilization	1	0.107613	0.007463	0.10015	1.07363	0.074454
0.1	2	0.107626	0.007463	0.100162	1.0728	0.074393
	3	0.107235	0.007441	0.099794	1.07279	0.074438
	4	0.107356	0.007493	0.099862	1.07725	0.075193
	5	0.107862	0.007462	0.100401	1.0743	0.074317
	6	0.107885	0.007468	0.100417	1.07775	0.074602
	7	0.107776	0.007574	0.100203	1.07203	0.075334
	8	0.107306	0.007472	0.099834	1.07478	0.07484
	9	0.107528	0.007504	0.100025	1.07187	0.074799
	10	0.107555	0.007407	0.100148	1.07399	0.073962
	AVG	0.107574	0.007475	0.1001	1.074119	0.074633
	STD	0.000226	4.37E-05	0.00022	0.002018	0.000415
	CILOW	0.107412	0.007443	0.099942	1.072677	0.074336
	CLHIGH	0.107736	0.007506	0.100257	1.075561	0.07493
	Error	0.000162	3.13E-05	0.000157	0.001442	0.000297

utilization	11	0.231919	0.032912	0.199008	1.16205	0.164907
0.2	12	0.234022	0.033958	0.200064	1.16955	0.169709
	13	0.234164	0.033739	0.200425	1.16699	0.168145
	14	0.231867	0.033039	0.198828	1.16621	0.166174
	15	0.23391	0.033708	0.200201	1.1686	0.168404
	16	0.233606	0.033427	0.200179	1.16575	0.166808
	17	0.232674	0.033102	0.199572	1.16595	0.165878
	18	0.233259	0.033431	0.199829	1.16454	0.166901
	19	0.231804	0.032285	0.199519	1.15982	0.161535
	20	0.232397	0.033112	0.199285	1.16558	0.166075
	AVG	0.232962	0.033271	0.199691	1.165504	0.166454
	STD	0.000943	0.000488	0.000539	0.002869	0.002229
	CILOW	0.232288	0.032923	0.199306	1.163453	0.16486
	CLHIGH	0.233636	0.03362	0.200076	1.167555	0.168047
	Error	0.000674	0.000349	0.000385	0.002051	0.001593
utilization	21	0.571702	0.174094	0.397609	1.43788	0.437862
0.4	22	0.575644	0.176217	0.399427	1.44198	0.441422
	23	0.579796	0.178119	0.401677	1.4485	0.444995
	24	0.583428	0.182693	0.400735	1.45457	0.45548
	25	0.582968	0.181047	0.40192	1.45206	0.450953
	26	0.576844	0.175984	0.400861	1.43979	0.439252
	27	0.577797	0.177471	0.400326	1.44761	0.444636
	28	0.579953	0.17848	0.401473	1.44653	0.445169
	29	0.576614	0.17803	0.398583	1.44235	0.445326
	30	0.581399	0.180467	0.400932	1.45492	0.451611
	AVG	0.578615	0.17826	0.400354	1.446619	0.445671
	STD	0.003613	0.002576	0.001403	0.006046	0.005593
	CILOW	0.576032	0.176419	0.399351	1.442298	0.441673
	CLHIGH	0.581197	0.180102	0.401357	1.45094	0.449668
	Error	0.002582	0.001841	0.001003	0.004321	0.003997
utilization	31	1.19845	0.599107	0.599342	1.99703	0.998318
0.6	32	1.20322	0.601455	0.601764	2.00612	1.0028
	33	1.22408	0.619804	0.604273	2.02761	1.02667
	34	1.2071	0.604216	0.60288	2.00705	1.00463
	35	1.19191	0.591404	0.600509	1.98597	0.985399
	36	1.1937	0.597476	0.596224	2.00114	1.00162
	37	1.20105	0.600114	0.60094	2.0009	0.99976
	38	1.22268	0.621807	0.600873	2.03027	1.03251
	39	1.20144	0.603627	0.597808	2.01112	1.01043
	40	1.19356	0.594296	0.599268	1.99867	0.995175
	AVG	1.203719	0.603331	0.600388	2.006588	1.005731
	STD	0.011396	0.010019	0.002359	0.013601	0.01422
	CILOW	1.195574	0.59617	0.598702	1.996868	0.995569

	CLHIGH	1.211864	0.610491	0.602074	2.016308	1.015894
	Error	0.008145	0.007161	0.001686	0.00972	0.010162
utilization	41	2.91968	2.11845	0.801229	3.63802	2.63966
0.8	42	2.87273	2.0759	0.796834	3.60376	2.60416
	43	3.03715	2.23429	0.802858	3.78708	2.78598
	44	2.93822	2.13545	0.802773	3.67016	2.66741
	45	2.91092	2.11	0.800915	3.63863	2.63749
	46	2.93833	2.13873	0.799602	3.68631	2.68316
	47	2.92476	2.11969	0.805075	3.63589	2.63507
	48	2.86342	2.06393	0.799485	3.58143	2.58147
	49	2.87401	2.07856	0.795449	3.61199	2.61229
	50	2.9962	2.19217	0.804025	3.73401	2.73199
	AVG	2.927542	2.126717	0.800825	3.658728	2.657868
	STD	0.055061	0.053038	0.003065	0.062865	0.062095
	CILOW	2.888191	2.088812	0.798634	3.6138	2.61349
	CLHIGH	2.966893	2.164622	0.803015	3.703656	2.702246
	Error	0.039351	0.037905	0.00219	0.044928	0.044378

NO.	Service mean	Arrival mean	rou
rou=0.1	Second	second	
1	0.999173	9.976764853	0.10015
2	0.998409	9.967941934	0.100162
3	0.998352	10.00409843	0.099794
4	1.00206	10.03441739	0.099862
5	0.999985	9.959910758	0.100401
6	1.00315	9.989842357	0.100417
7	0.9967	9.94680798	0.100203
8	0.999937	10.01603669	0.099834
9	0.997072	9.968227943	0.100025
10	1.00003	9.985521428	0.100148
AVG	0.9994868	9.984956976	0.1001
rou=0.2			
11	0.997148	5.010592539	0.199008
12	0.999842	4.997610765	0.200064
13	0.998849	4.983654734	0.200425
14	1.00003	5.029623594	0.198828
15	1.00019	4.995929091	0.200201
16	0.998946	4.990263714	0.200179
17	1.00007	5.011073698	0.199572
18	0.997643	4.992483573	0.199829
19	0.998282	5.003443281	0.199519

20	0.999508	5.015470306	0.199285
AVG	0.9990508	5.00301453	0.199691
rou=0.4			
22	1.00056	2.504988396	0.399427
23	1.00351	2.498300874	0.401677
24	0.999092	2.493148839	0.400735
25	1.0011	2.490794188	0.40192
26	1.00054	2.495977409	0.400861
27	1.00298	2.505408092	0.400326
28	1.00136	2.494215053	0.401473
29	0.99702	2.501411249	0.398583
30	1.00331	2.502444305	0.400932
AVG	1.001052444	2.498520934	0.400659
rou=0.6			
32	1.00332	1.667298143	0.601764
33	1.00094	1.656436743	0.604273
34	1.00241	1.662702362	0.60288
35	1.00057	1.666203171	0.600509
36	0.99952	1.676416917	0.596224
37	1.00114	1.665956668	0.60094
38	0.997753	1.660505631	0.600873
39	1.00069	1.673932099	0.597808
40	1.0035	1.674542942	0.599268
AVG	1.001093667	1.66711052	0.600504
rou=0.8			
42	0.999606	1.254472073	0.796834
43	1.0011	1.246920377	0.802858
44	1.00275	1.24910778	0.802773
45	1.00114	1.249995318	0.800915
46	1.00315	1.254561644	0.799602
47	1.00082	1.243138838	0.805075
48	0.999959	1.250753923	0.799485
49	0.999703	1.256778247	0.795449
50	1.00202	1.246254781	0.804025
AVG	1.001138667	1.250220331	0.80078

- Theoretical result:

The steady-state parameters of M/G/1:

$\rho = \lambda / \mu, \quad P_0 = (1-\rho)$ is the probability server is idle (no customers)

$$L = \rho + \frac{\lambda^2(1/\mu^2 + \sigma^2)}{2(1-\rho)} = \rho + \frac{\rho^2(1+\sigma^2\mu^2)}{2(1-\rho)}$$

$$L_Q = \frac{\rho^2(1+\sigma^2\mu^2)}{2(1-\rho)}$$

$$w = \frac{1}{\mu} + \frac{\lambda(1/\mu^2 + \sigma^2)}{2(1-\rho)}$$

$$w_Q = \frac{\lambda(1/\mu^2 + \sigma^2)}{2(1-\rho)}$$

Thus, the theoretical result is :

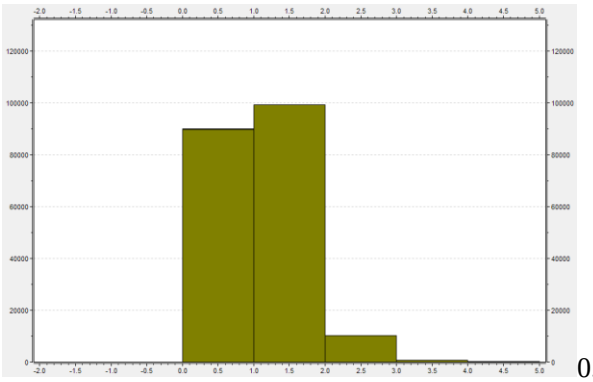
Actual:					
rou	0.1	0.2	0.4	0.6	0.8
L	0.107407	0.233333	0.577777	1.199999	2.933328
Lq	0.007407	0.033333	0.177777	0.599999	2.133328
W	1.074074	1.166666	1.444443	1.999998	3.66666
Wq	0.074074	0.166666	0.444443	0.999998	2.66666

the simulation result is :

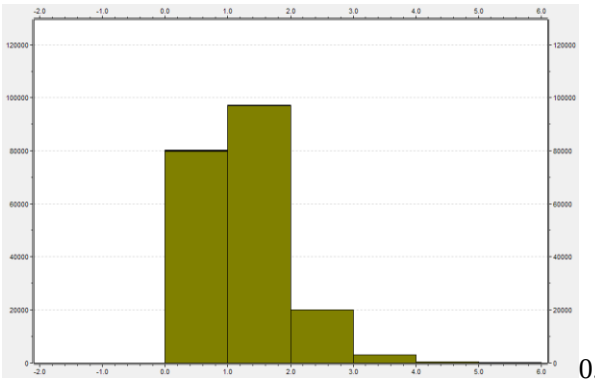
Simulate:					
rou	0.1001	0.199691	0.400354	0.600388	0.800825
L	0.107574	0.232962	0.578615	1.203719	2.927542
Lq	0.007475	0.033271	0.17826	0.603331	2.126717
W	1.074119	1.165504	1.446619	2.006588	3.658728
Wq	0.074633	0.166454	0.445671	1.005731	2.657868

Compared data from simulation, the theory data is close according to confidence intervals.

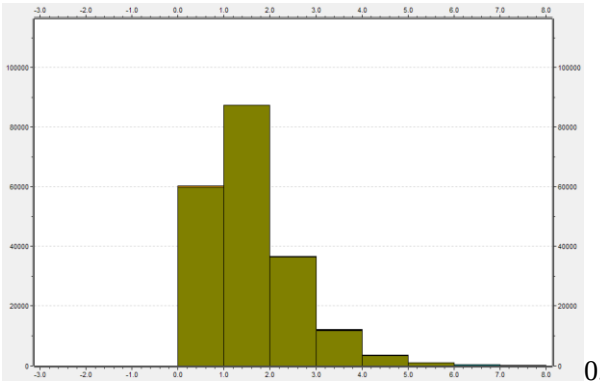
- Histogram result of M/G/1 simulation:



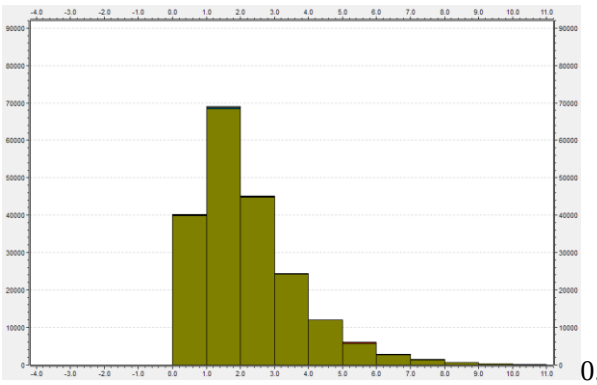
0.1



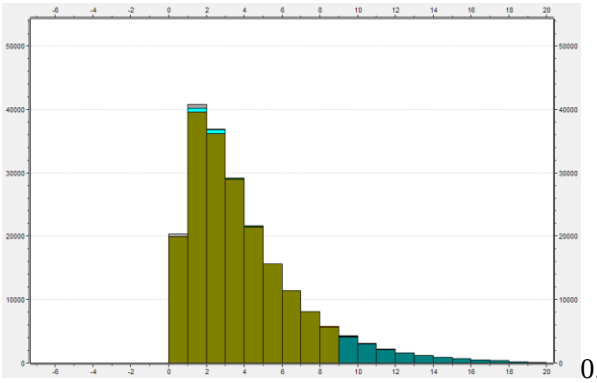
0.2



0.4



0.6


0.8

Utilization: from top to bottom is 0.1, 0.2, 0.4, 0.6, 0.8.

➢ Analysis of M/M/3/20:

- Statistic of 50 runs and 5 utilizations :

This table contains: result data of L, Lq, W, Wq, rou , the confidence intervals , the actual service mean , the actual interarrivals mean, probability of loss and average stats across ten different runs using different initial seeds.

	0	L	Lq	rou	W	Wq
utilization	1	0.301	0.000442	0.300558	1.00295	0.001474
0.1	2	0.300746	0.000404	0.300341	1.00206	0.001348
	3	0.299169	0.000421	0.298748	0.998739	0.001407
	4	0.302436	0.00048	0.301956	1.00755	0.001599
	5	0.301544	0.000434	0.30111	1.00086	0.00144
	6	0.301506	0.000439	0.301067	1.00489	0.001465
	7	0.299702	0.00039	0.299312	0.999824	0.001302
	8	0.29907	0.000427	0.298644	0.999113	0.001425
	9	0.301986	0.000474	0.301512	0.999506	0.00157
	10	0.303383	0.000394	0.302989	1.00474	0.001303
	AVG	0.301054	0.000431	0.300624	1.002023	0.001433
	STD	0.001417	3.05E-05	0.001408	0.002959	0.000101
	CILOW	0.300042	0.000409	0.299617	0.999909	0.001361
	CLHIGH	0.302067	0.000452	0.30163	1.004138	0.001505
	Error	0.001012	2.18E-05	0.001006	0.002115	7.19E-05
utilization	11	0.611797	0.00605	0.605747	1.0159	0.010047
0.2	12	0.602216	0.005634	0.596582	1.00982	0.009447

	13	0.605225	0.006218	0.599007	1.00903	0.010367
	14	0.599996	0.005609	0.594387	1.00617	0.009406
	15	0.606567	0.006207	0.60036	1.0104	0.01034
	16	0.601885	0.005983	0.595902	1.006	0.010001
	17	0.605967	0.006433	0.599534	1.0096	0.010718
	18	0.603215	0.006574	0.596641	1.00708	0.010975
	19	0.610466	0.006004	0.604462	1.01388	0.009971
	20	0.607848	0.006526	0.601322	1.01216	0.010867
	AVG	0.605518	0.006124	0.599394	1.010004	0.010214
	STD	0.00381	0.000337	0.003706	0.003249	0.000545
	CILOW	0.602796	0.005883	0.596746	1.007682	0.009824
	CLHIGH	0.608241	0.006365	0.602043	1.012326	0.010603
	Error	0.002723	0.000241	0.002649	0.002322	0.00039
utilization	21	1.29417	0.095636	1.198534	1.08403	0.080108
0.4	22	1.300047	0.099221	1.200826	1.08486	0.082798
	23	1.294258	0.09447	1.199788	1.08016	0.078843
	24	1.302642	0.097848	1.204794	1.08218	0.081289
	25	1.307899	0.099029	1.20887	1.08596	0.082225
	26	1.289834	0.090202	1.199632	1.07522	0.075194
	27	1.296176	0.096519	1.199657	1.07972	0.080401
	28	1.280789	0.09162	1.189169	1.07594	0.076967
	29	1.289654	0.093688	1.195966	1.07741	0.07827
	30	1.283884	0.091683	1.192201	1.07617	0.07685
	AVG	1.293935	0.094992	1.198944	1.080165	0.079294
	STD	0.008316	0.003203	0.005657	0.003954	0.002499
	CILOW	1.287992	0.092703	1.194901	1.077339	0.077508
	CLHIGH	1.299878	0.097281	1.202987	1.082991	0.08108
	Error	0.005943	0.002289	0.004043	0.002826	0.001786
utilization	31	2.307173	0.509698	1.797475	1.27867	0.282482
0.6	32	2.302916	0.514668	1.788248	1.28443	0.287052
	33	2.330759	0.535109	1.79565	1.29516	0.297351
	34	2.295576	0.505452	1.790124	1.28018	0.281877
	35	2.34233	0.541244	1.801086	1.29933	0.300237
	36	2.314006	0.524999	1.789007	1.29032	0.292746
	37	2.325245	0.529641	1.795604	1.2949	0.29495
	38	2.371357	0.557517	1.81384	1.30857	0.307651
	39	2.311803	0.517754	1.794049	1.28844	0.288562
	40	2.363337	0.555019	1.808318	1.31178	0.308065
	AVG	2.32645	0.52911	1.79734	1.293178	0.294097
	STD	0.025598	0.018113	0.008351	0.011127	0.009394
	CILOW	2.308156	0.516165	1.791372	1.285226	0.287384
	CLHIGH	2.344745	0.542055	1.803309	1.30113	0.300811
	Error	0.018295	0.012945	0.005968	0.007952	0.006713

utilization	41	4.610057	2.22753	2.382527	1.92984	0.932478
0.8	42	4.732164	2.33722	2.394944	1.97259	0.974266
	43	4.868513	2.45761	2.410903	2.03305	1.02628
	44	4.6186	2.23863	2.37997	1.93158	0.936232
	45	4.626423	2.24789	2.378533	1.94059	0.942894
	46	4.842703	2.4414	2.401303	2.03005	1.02343
	47	4.66179	2.27454	2.38725	1.95379	0.953278
	48	4.784442	2.37643	2.408012	1.99749	0.99215
	49	4.718567	2.33011	2.388457	1.97127	0.973446
	50	4.772438	2.38463	2.387808	1.99521	0.996939
	AVG	4.72357	2.331599	2.391971	1.975546	0.975139
	STD	0.093424	0.083352	0.011438	0.037845	0.034265
	CILOW	4.656802	2.27203	2.383796	1.948499	0.950651
	CLHIGH	4.790337	2.391168	2.400145	2.002593	0.999628
	Error	0.066767	0.059569	0.008174	0.027047	0.024489

NO.	Service mean	Arrival mean
rou=0.1	second	second
1	1.00148	3.332069018
2	1.00071	3.331910507
3	0.997332	3.33837661
4	1.00595	3.331445641
5	0.999424	3.319132543
6	1.00342	3.332879392
7	0.998523	3.336064043
8	0.997687	3.340726759
9	0.997936	3.309772082
10	1.00344	3.311803399
AVG	1.0005902	3.328417999
rou=0.2		
11	1.00585	1.660511732
12	1.00038	1.67685247
13	0.998663	1.667197545
14	0.996768	1.676968036
15	1.00006	1.665767206
16	0.996002	1.671419126
17	0.99888	1.666094
18	0.996101	1.669514834
19	1.0039	1.660815734
20	1.00129	1.665147791
AVG	0.9997894	1.668028847
rou=0.4		

21	1.00392	0.837623296
22	1.00207	0.83448393
23	1.00132	0.834580776
24	1.0009	0.830764429
25	1.00374	0.830312606
26	1.00003	0.833613975
27	0.999323	0.833007268
28	0.998972	0.840058898
29	0.999142	0.83542676
30	0.999322	0.838216039
AVG	1.0008739	0.834808798
rou=0.6		
31	0.996187	0.554214662
32	0.997381	0.557741991
33	0.997811	0.555682343
34	0.998306	0.557674217
35	0.999093	0.554716987
36	0.997572	0.557612128
37	0.999948	0.556886708
38	1.00092	0.551823755
39	0.999882	0.557332604
40	1.00371	0.555051711
AVG	0.999081	0.555873711
rou=0.8		
41	0.99736	0.418614354
42	0.998328	0.41684816
43	1.00678	0.417594569
44	0.995345	0.418217457
45	0.997694	0.419457708
46	1.00662	0.419197411
47	1.00051	0.419105666
48	1.00534	0.417497919
49	0.99782	0.417767622
50	0.998266	0.418067952
AVG	1.0004063	0.418236882

- Theoretical result:

The steady-state parameters of M/M/C/N:

$$\rho = \lambda / c\mu; \quad a = \lambda / \mu$$

$$P_0 = \left[1 + \sum_{n=1}^{c} \frac{a^n}{n!} + \frac{a^c}{c!} \sum_{n=c+1}^{N} \rho^{n-c} \right]^{-1}$$

$$P_N = \frac{a^N P_0}{c! c^{N-c}}$$

$$L_Q = \frac{\rho a^c P_0}{c!(1-\rho)^2} \left[1 - \rho^{N-c} - (N-c)\rho^{N-c}(1-\rho) \right]$$

$$\lambda_e = \lambda(1 - P_N)$$

$$w_Q = \frac{L_Q}{\lambda_e}$$

$$w = w_Q + \frac{1}{\mu}$$

$$L = \lambda_e . w$$

Thus, the theoretical result is :

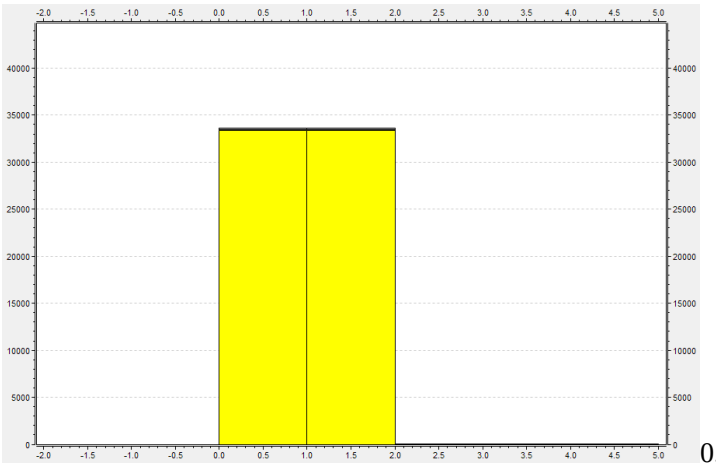
Actual:					
rou	0.1	0.2	0.4	0.6	0.8
a	0.3	0.6	1.2	1.8	2.4
P0	0.740741	0.547945	0.294118	0.145991	0.056842
PN	3.33E-20	2.59E-14	1.46E-08	2.4E-05	0.002949
lamda	0.3	0.6	1.2	1.799957	2.392922
L	0.300412	0.606164	1.294117	2.33139	4.752706
Lq	0.000412	0.006164	0.094117	0.531433	2.359784
W	1.001372	1.010274	1.078431	1.295248	1.986151
Wq	0.001372	0.010274	0.078431	0.295248	0.986151

the simulation result is :

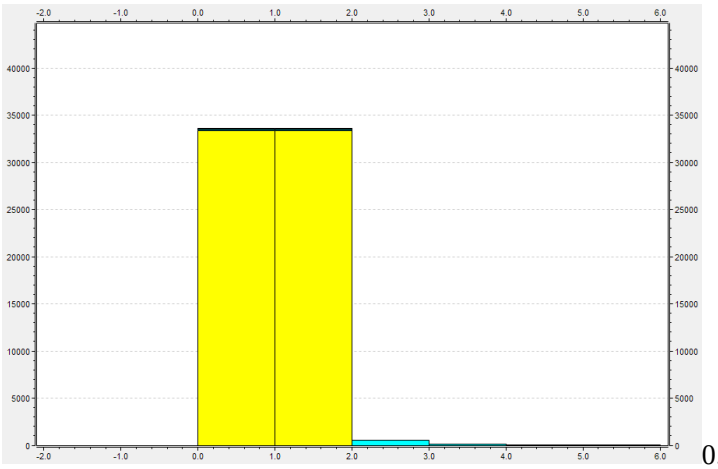
Simulate:					
Rou(total)	0.300624	0.599394	1.198944	1.79734	2.391971
L	0.301054	0.605518	1.293935	2.32645	4.72357
Lq	0.000431	0.006124	0.094992	0.52911	2.331599
W	1.002023	1.010004	1.080165	1.293178	1.975546
Wq	0.001433	0.010214	0.079294	0.294097	0.975139

Compared data from simulation, the theory data is close.

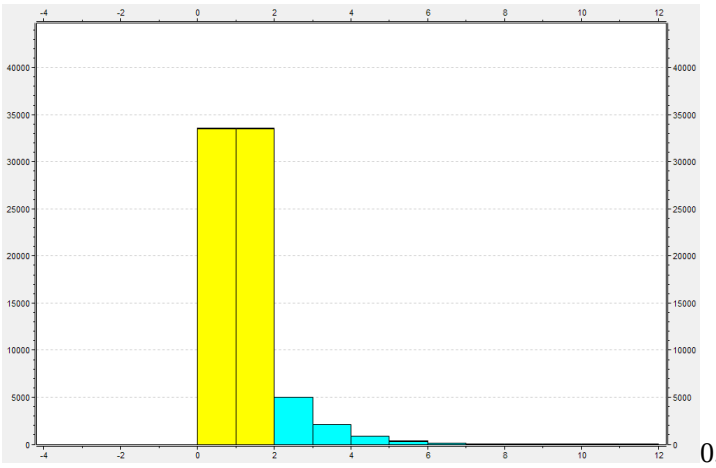
- Histogram result:



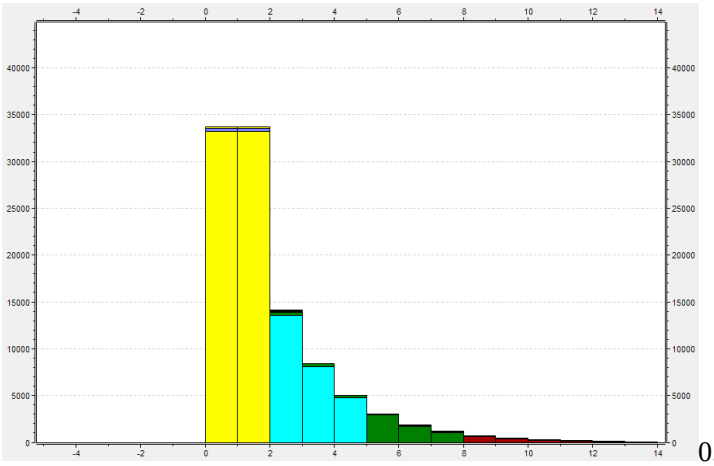
0.1



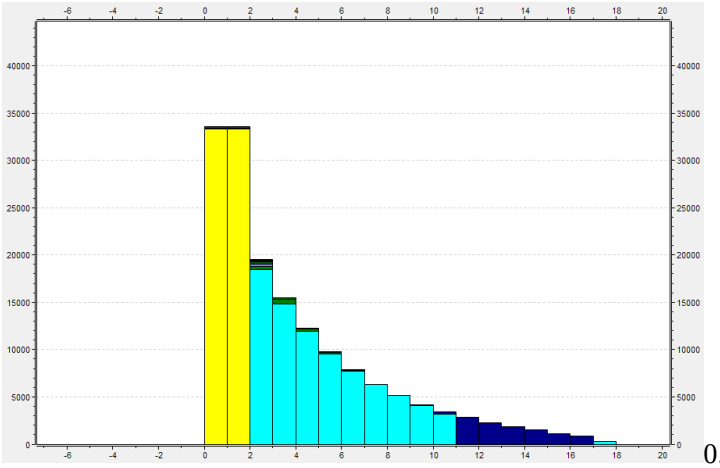
0.2



0.4



0.6



0.8

Utilization: from top to bottom is 0.1, 0.2, 0.4, 0.6, 0.8.

- **Conclusion:**

For M/M/1 models in the supermarket, the simulation results are consistent with theoretical results according to table in analysis of M/M/1.

For M/G/1 models in the supermarket, the simulation results are consistent with theoretical results according to table in analysis of M/G/1.

For M/M/3/20 models in the supermarket, the simulation results are consistent with theoretical results according to table in analysis of M/M/3/20.