ON ADMISSIBILITY CRITERIA FOR CONTACT DISCONTINUITIES IN GLIMM-ISAACSON MODEL ARISING IN CHEMICAL FLOODING

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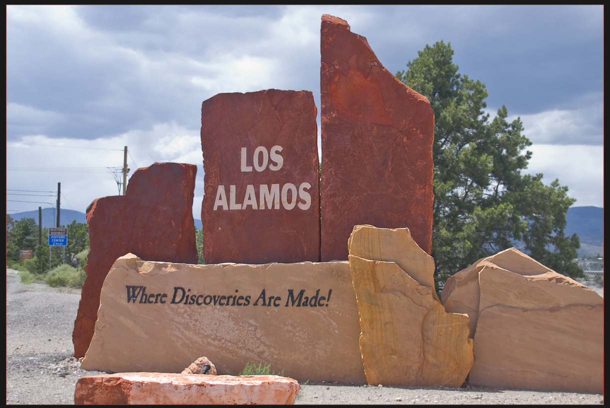

HYP conference

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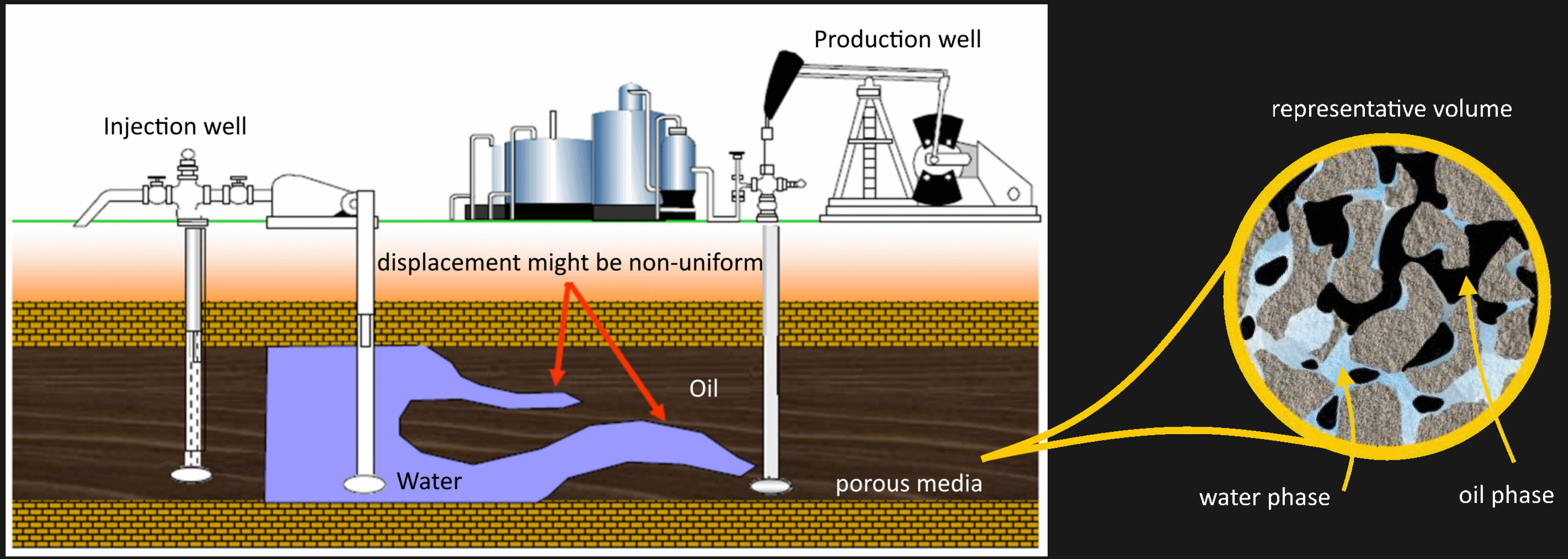
MOTIVATION: ENHANCED OIL RECOVERY (EOR)

We are interested in the mathematical model of oil recovery. Some features:

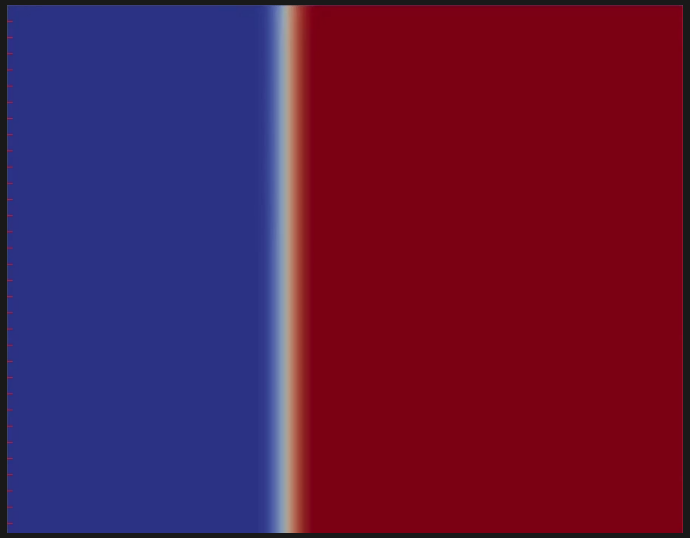
- *porous media* (averaged models of flow)
- unknown variables: $s \in [0, 1]$ - water saturation, $1 - s$ - oil concentration
- relatively *small speeds* ($\approx$ 1 meter per day): Navier-Stokes $\rightarrow$ Darcy's law
- multiphase flow: oil, water, gas.
- applications to EOR methods: thermal, gas, *chemical flooding*



TWO MAIN DIRECTIONS OF INVESTIGATION

Stable dislacement

- 1-dim in spatial variable



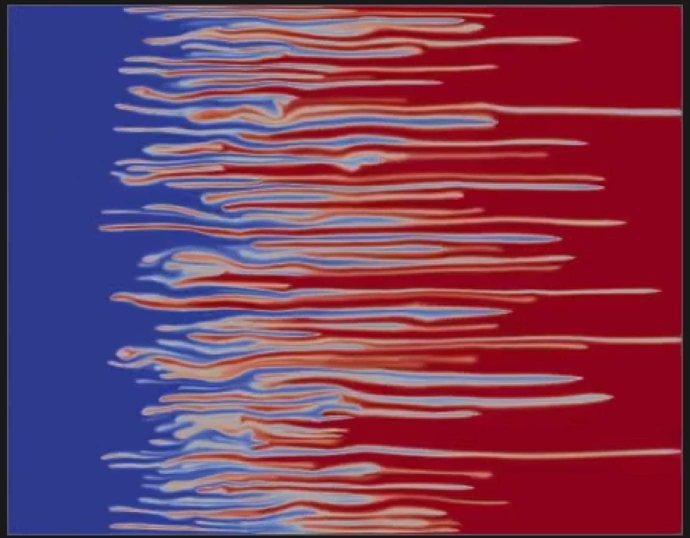
- main question: find an exact solution for a Riemann problem
- hyperbolic conservation laws

$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ (cs)_t + (cf(s,c))_x &= 0. \end{aligned}$$

Example: chemical flooding model

Unstable displacement

- 2-dim (or 3-dim)



- source of instability: water and oil/polymer have different viscosities
- viscous fingering phenomenon

$$c_t + u \cdot \nabla c = \varepsilon \Delta c,$$
$$\mathrm{div}(u) = 0, \quad u = -\frac{1}{\mu(c)} \nabla p.$$

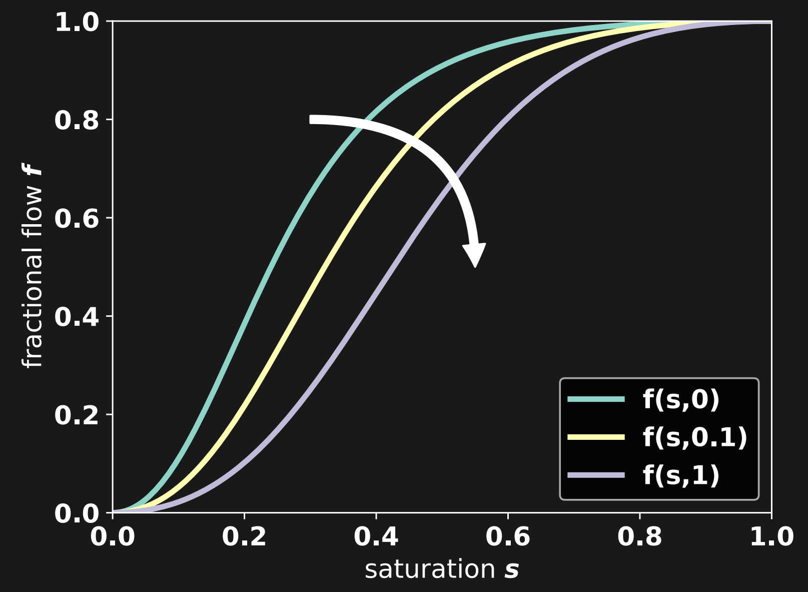
Example: Peaceman model

GLIMM-ISAACSON MODEL (KKIT MODEL)

Two-phase oil-water flow with *polymer* in the water

$$\begin{aligned} s_t + f(s,c)_x &= 0 \\ (cs)_t + (cf(s,c))_x &= 0 \end{aligned}$$

- $s \in [0,1]$ - water saturation
- $c \in [0,1]$ - polymer concentration in water
- f - fractional flow function: affected by polymer
  - S-shaped in s
  - *f is monotone in c*



Initial data: $$(s,c)\big|_{t=0} = \begin{cases} (s_L, c_L), & \text{if } x \leqslant 0, \\ (s_R, c_R), & \text{if } x \geqslant 0. \end{cases}$$

Question: find an exact solution $s(x,t)$ and $c(x,t)$ to this Riemann problem

- 1980 — Isaacson, Glimm (polymer flooding): $\exists!$ under Isaacson-Glimm admissibility criterion
- 1980 — Keyfitz, Kranzer (elastisity theory)

Our goals: justify the Isaacson-Glimm admissibility criterion from physical principles

find an exact solution to any Riemann problem for non-monotone f in c (surfactants, salty water)

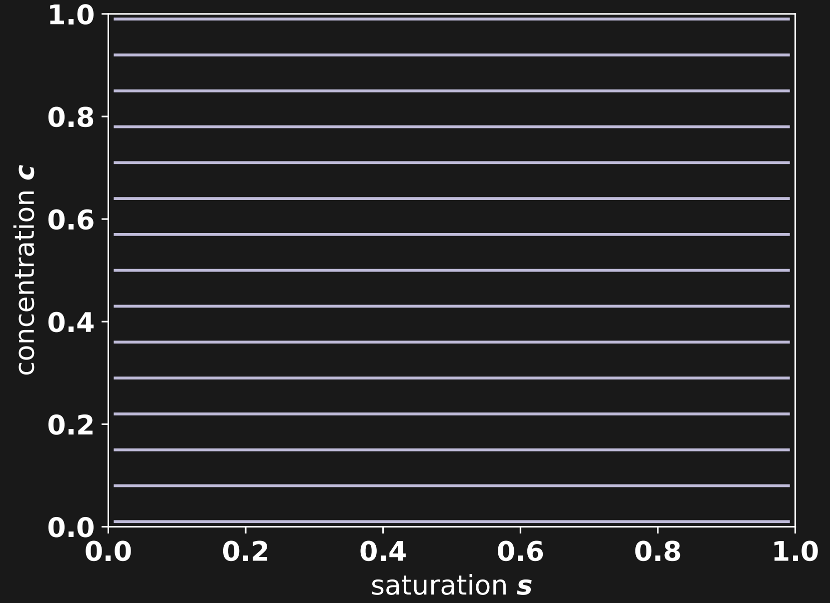
CHARACTERISTIC FAMILIES: S AND C-WAVES

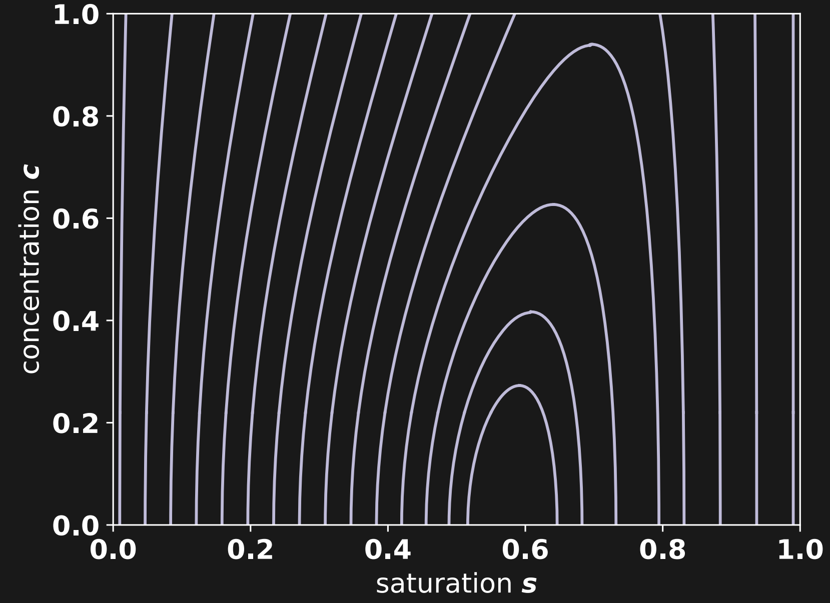
s-waves

- $\lambda^{(s)} = f_s$
- solve the Buckley-Leverett equation $c = const$
- Riemann invariant $c = const$
- *line family*

c-waves

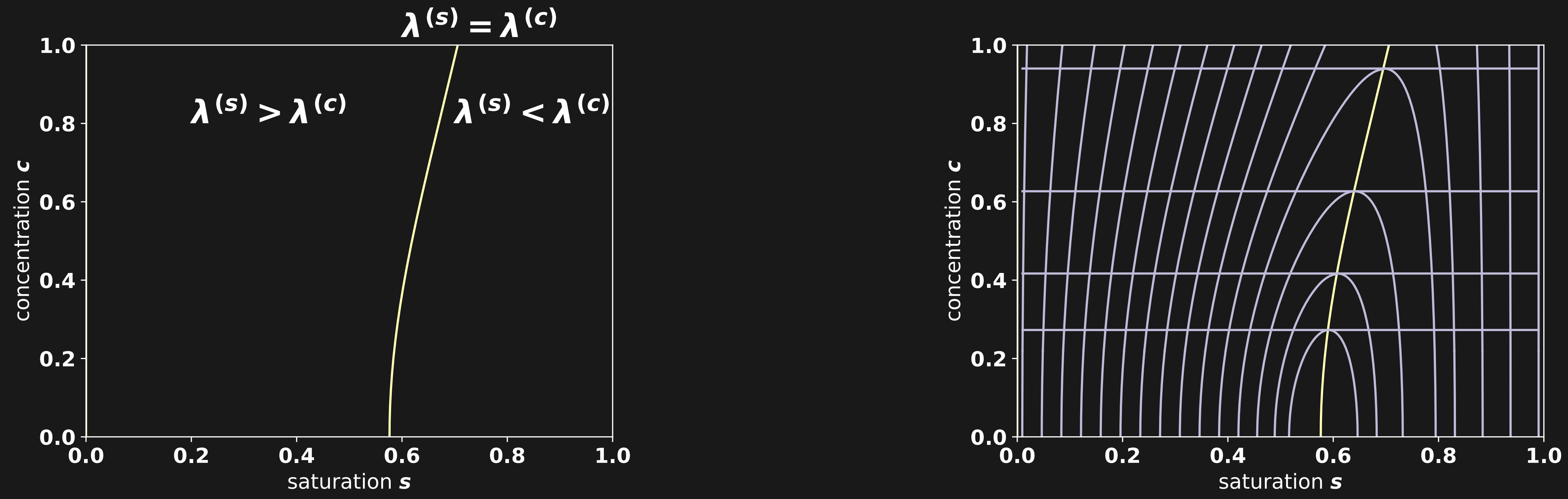
- $\lambda^{(c)} = f/s$
- are contact discontinuities (linearly degenerate)
- Riemann invariant $f/s = const$
- *contact family*





- For both families the *rarefaction and shock curves coincide*! But in a different way (Temple'1983)
- Any solution to a Riemann problem is a combination of s and c-waves

NON-STRICTLY HYPERBOLIC SYSTEM

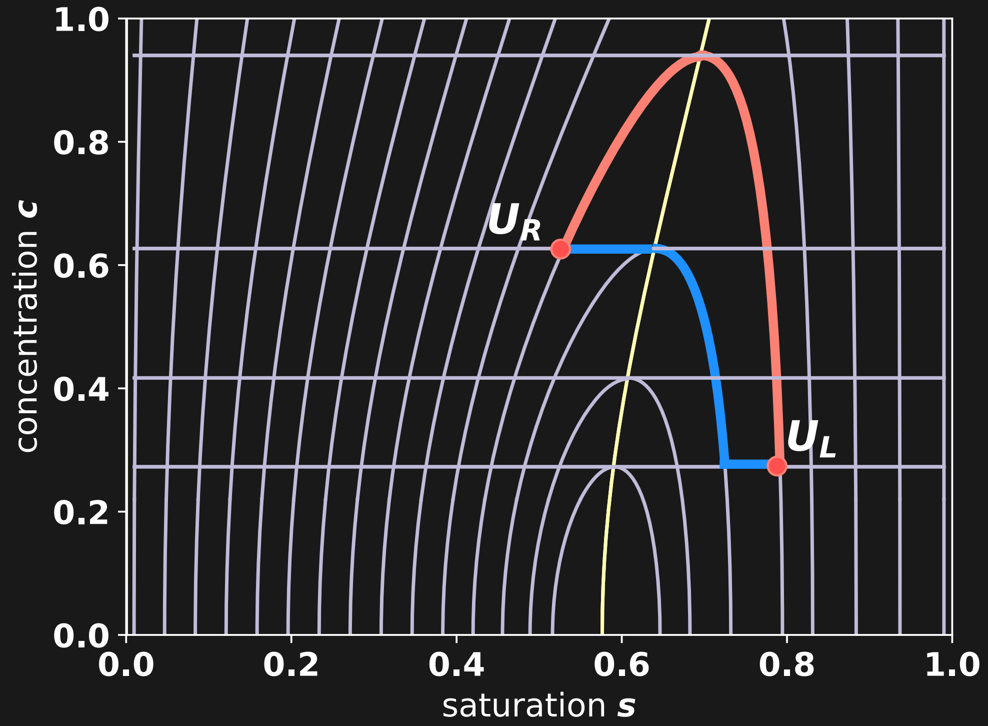


The coordinate system of wave curves is singular and wave speeds coincide on a co-dimension one curve (*coincidence locus*)

$$\lambda^{(s)} = f_s = \frac{f}{s} = \lambda^{(c)}$$

s and c-waves are tangent on coincidence curve.

NON-UNIQUENESS OF SOLUTIONS



- vanishing viscosity criterion doesn't help (no nonlinear forcing to balance the diffusion terms)
- *admissibility criterion of E. Isaacson and J. Glimm*: a contact is admissible if and only if c is continuous and monotone along the sequence of contact curves, connecting U_- and U_+.
- *consequence*: existence and uniqueness of solutions for all Riemann problems

What is the (physical) motivation of this criterion?

GENERAL ADMISSIBILITY CRITERION

- a model M_0
- a parameterized family of models M_α with its own admissibility criterion

Definiton: a solution for M_0 is admissible provided it is the L^1_{loc} limit of a family of admissible solutions of M_α as $\alpha \to 0$.

For instance, a solution of M_α could be any wave group: a shock, rarefaction, or composite, or more general wave group that is admissible for M_α.

Example: consider chemical flooding model M_α with adsorption (T. Johansen, R. Winther' 1988)

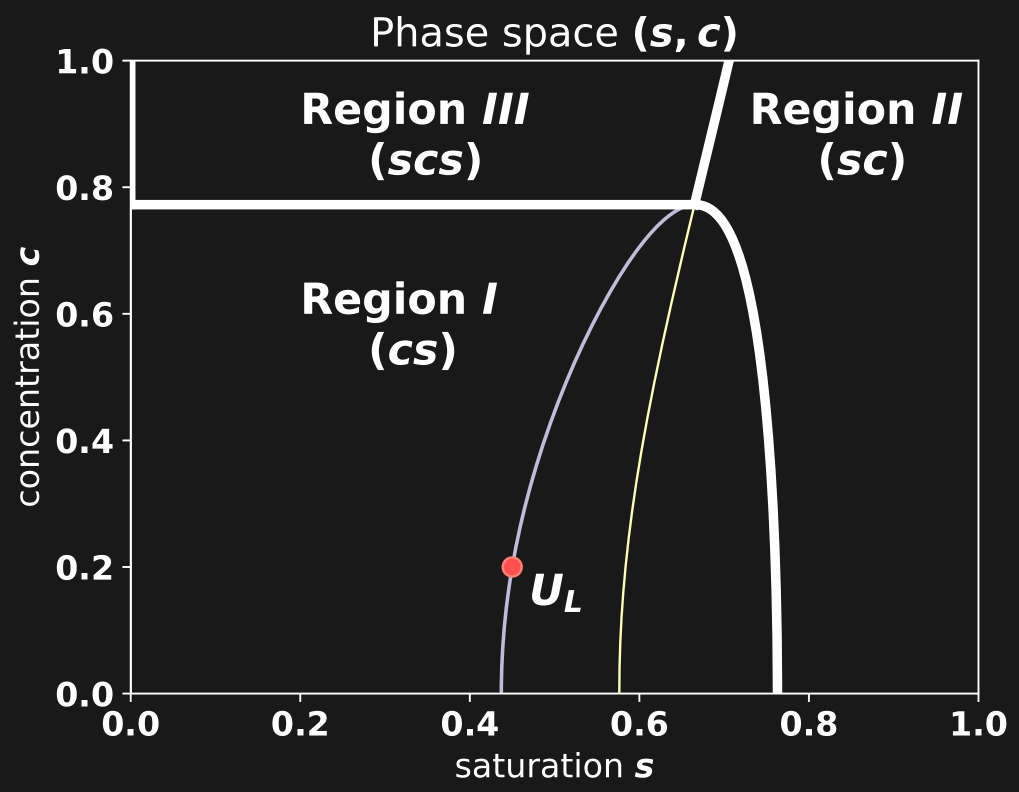
$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ (cs + \alpha a(c))_t + (cf(s,c))_x &= 0. \end{aligned} \qquad (M_\alpha)$$

- *vanishing adsorption criterion*
- if adsorption depends nonlinearly on the concentration, then *contact discontinuities become rarefactions and shock waves*
- For moderate values of c we have $a''(c) \leq 0$. When $a'' \equiv 0$, c-waves are contacts.

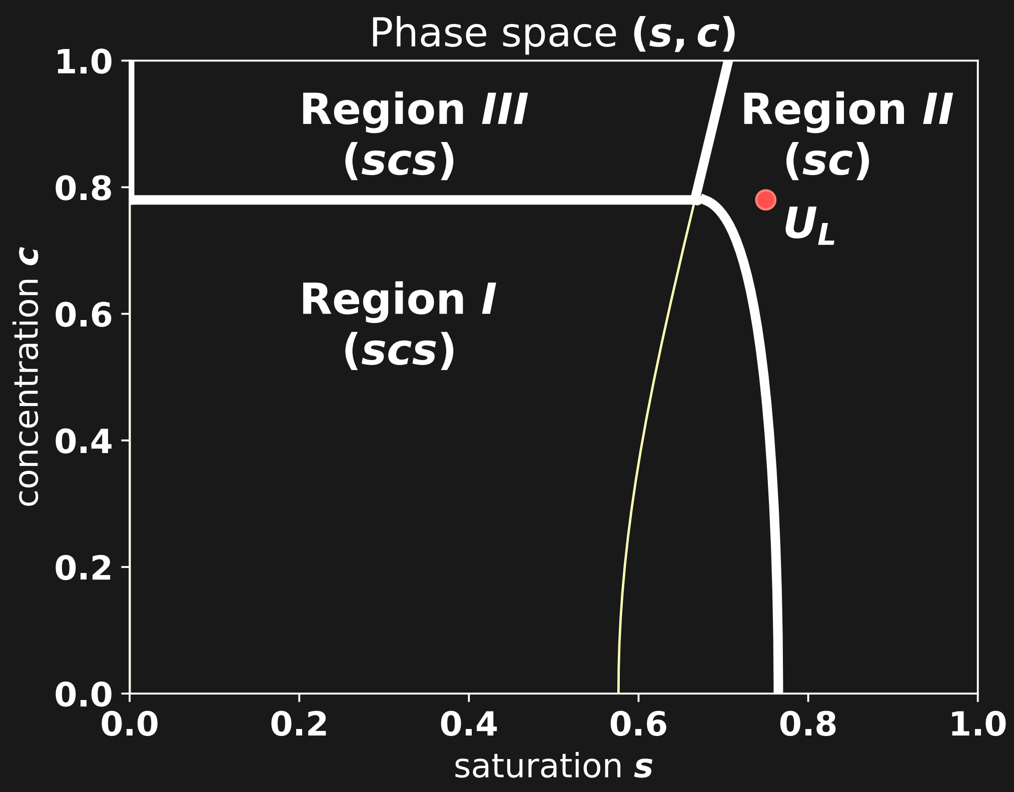
MAIN RESULT

Theorem (P., Marchesin, Plohr ‘2022). The set of admissible Riemann solutions is the same for the Isaacson-Glimm and vanishing adsorption admissibility criteria.

The solutions may be presented by two diagrams (E. Isaacson ‘1980):



U_L to the left of coincidence



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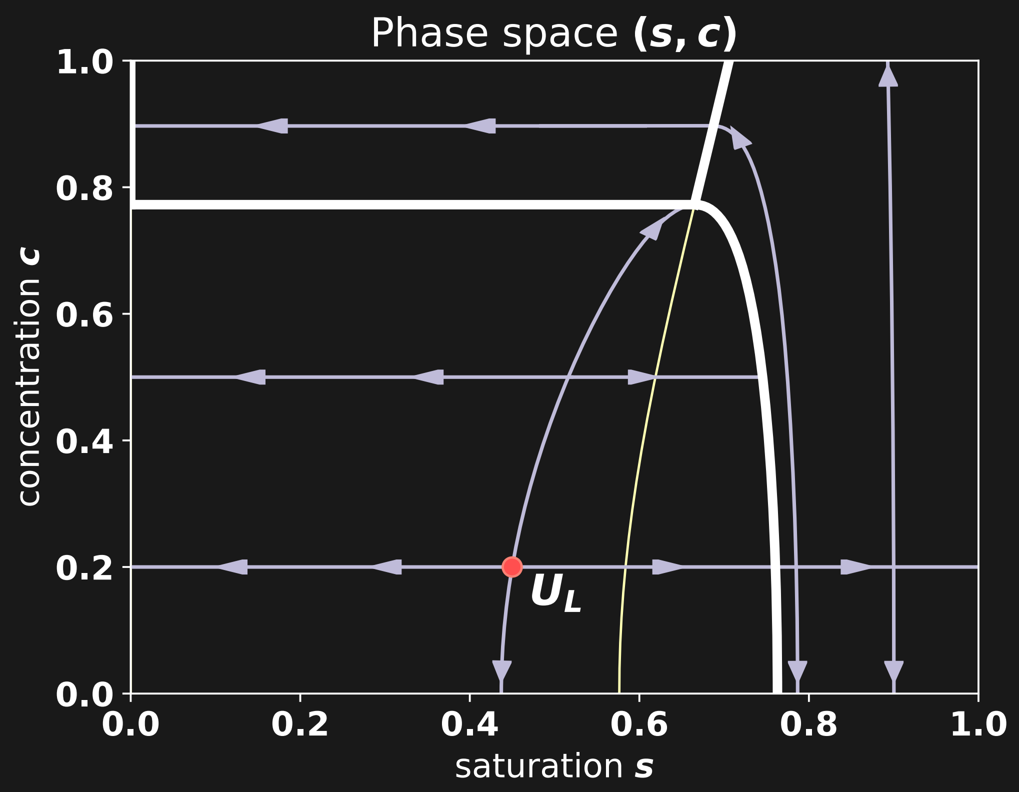
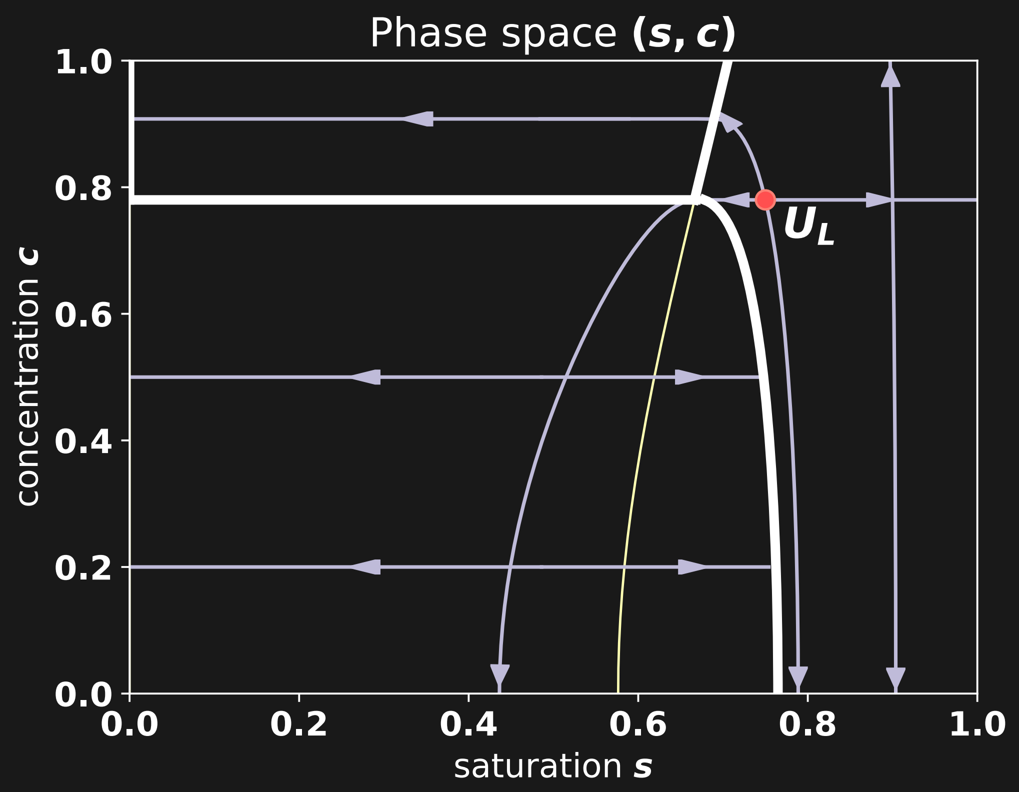
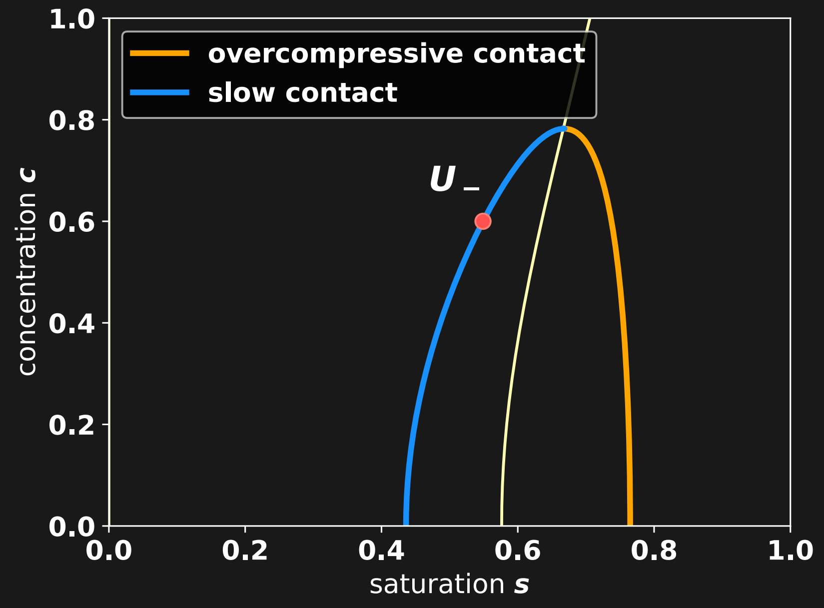


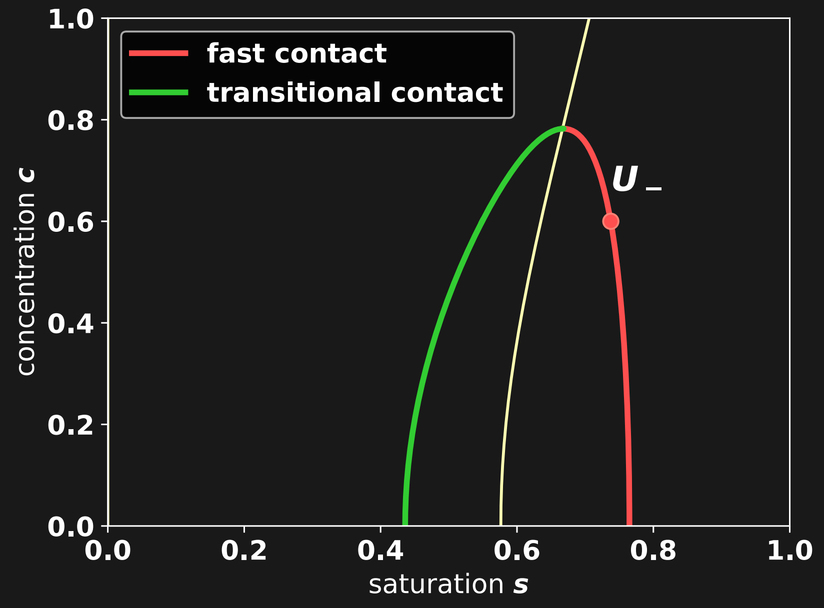
U_L to the left of coincidence



U_L to the right of coincidence

MAIN STEP: ADMISSIBLE CONTACTS





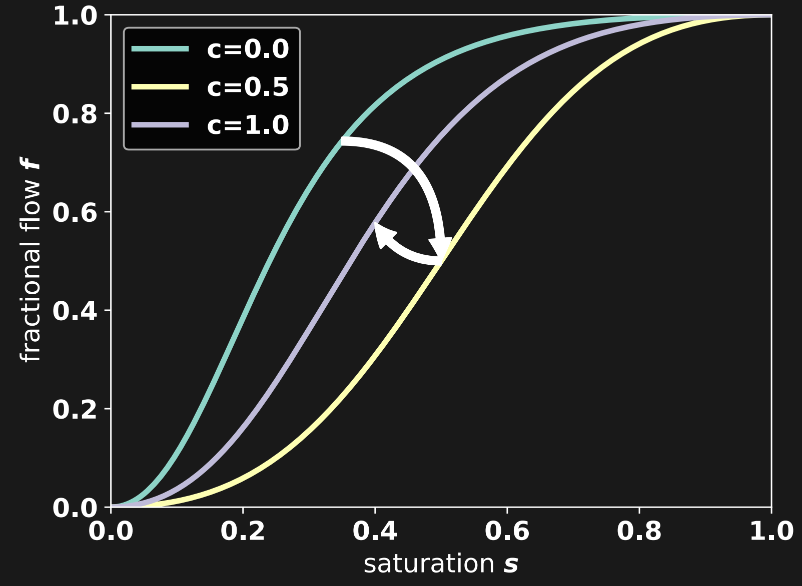
- *Isaacson-Glimm criterion*: slow and fast (Lax) contact discontinuities are admissible, overcompressive and undercompressive are not.
- *Vanishing adsorption criterion*: slow, fast and overcompressive contact discontinuities are admissible, undercompressive are not.

P.S. Overcompressive contacts can be represented as a sequence of two waves (c and s). Whether or not they are regarded as admissible does not affect the Riemann problem solution.

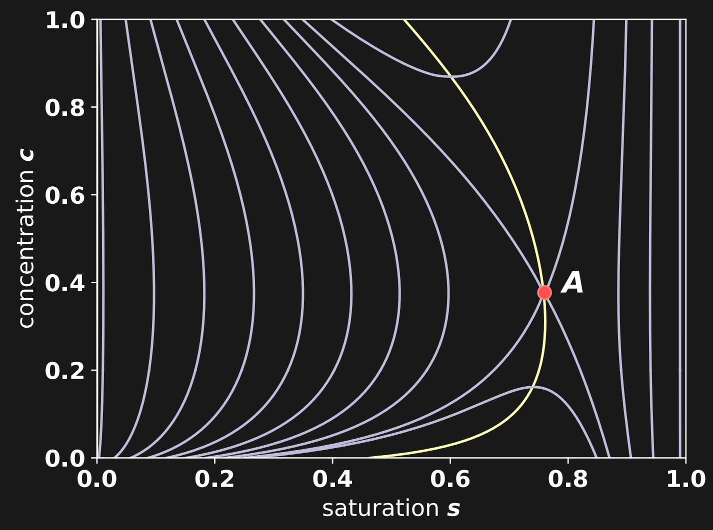
NON-MONOTONE CHEMICAL FLOODING MODEL

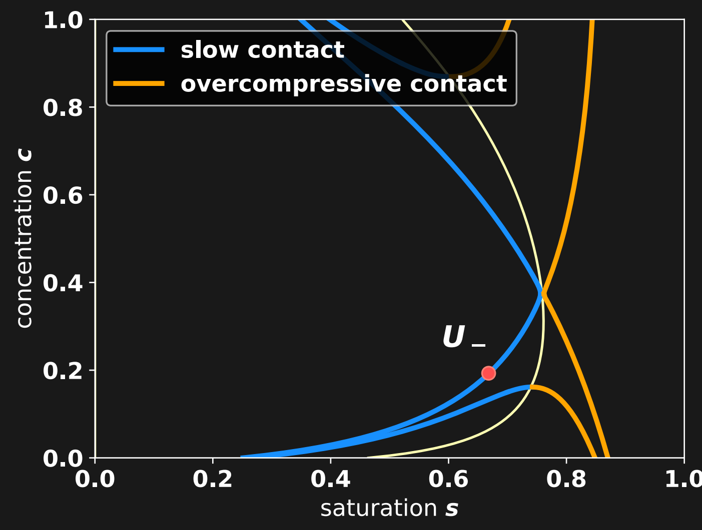
Two-phase oil-water flow with *surfactant* in the water

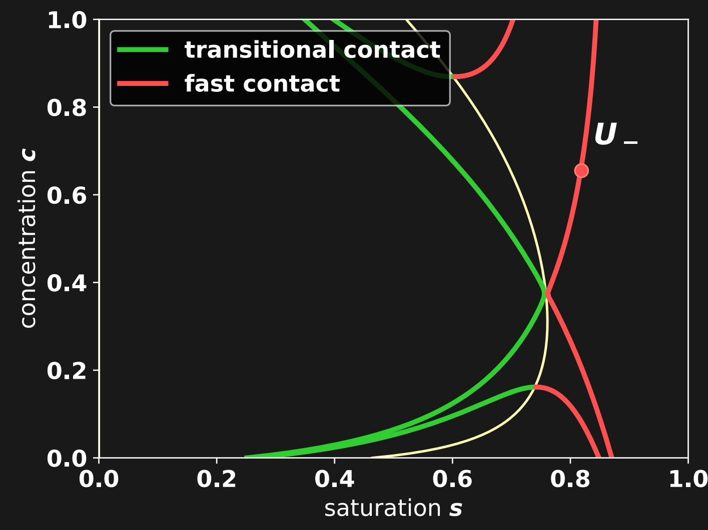
$$s_t + f(s,c)_x = 0,$$
$$(cs + \alpha a(c))_t + (cf(s,c))_x = 0.$$



- *f is non-monotone in c*: $\exists c^* \forall s \in [0,1]$
  - $f_c < 0$ for $c < c^*$
  - $f_c > 0$ for $c > c^*$
- work in progress
- umbilic point A is present
- curves $\lambda^{(s)} = \lambda^{(c)}$ and $f_c = 0$ intersect transversally

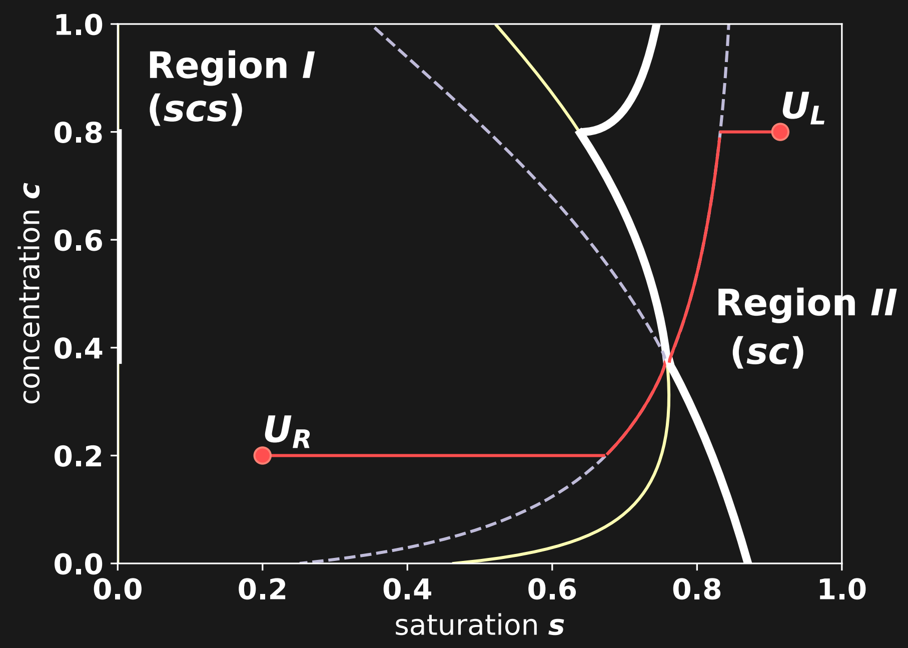






NON-MONOTONE CHEMICAL FLOODING MODEL

Theorem (P., Marchesin, Plohr '2022). The set of admissible Riemann solutions is the same for the Isaacson-Glimm and vanishing adsorption admissibility criteria for non-monotone chemical flooding model.



For model with adsorption M_α:

- there exist *transitional shocks and rarefactions*
- See: Entov, Kerimov (1986), Shen (2017), Bakharev-Enin-Rastegaev-P. (2021)
- *Consequence*: there exist admissible transitional contacts as a limit of transitional shocks

TRANSITIONAL SHOCKS FOR NON-MONOTONE CASE

Vanishing viscosity admissibility criterion for M_α (capillarity and diffusion):

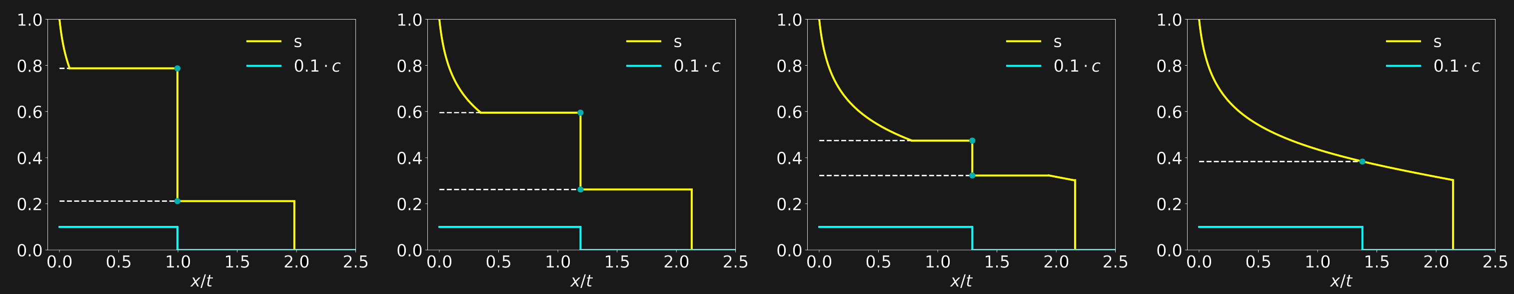
$$\begin{aligned} s_t + f(s,c)_x &= \varepsilon_c (A(s,c)s_x)_x\,, \\ (cs + a(c))_t + (cf(s,c))_x &= \varepsilon_c (cA(s,c)s_x)_x + \varepsilon_d (c_x)_x\,. \end{aligned}$$

Theorem (Bakharev, Enin, P., Rastegaev '2021).

Fix $c^- > c^+$. There exist $0 < v_{\min} < v_{\max} < \infty$, such that $\forall \kappa = \varepsilon_d/\varepsilon_c \in (0,+\infty)$, there exists a unique

- points $s^\pm(\kappa) \in [0,1]$
- velocity $v(\kappa) \in [v_{\min}, v_{\max}]$

such that there exists a travelling wave, connecting two saddle points $u^-(\kappa) = (s^-(\kappa), c^-)$ and $u^+(\kappa) = (s^+(\kappa), c^+)$ with velocity $v(\kappa)$. Moreover, $v(\kappa)$ is monotone and continuous; $v(\kappa) \to v_{\min}$ as $\kappa \to \infty$ and $v(\kappa) \to v_{\max}$ as $\kappa \to 0$.



Thank you!

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FLUID

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Own works:

- **Yu. Petrova**, D. Marchesin, B. Plohr. Work in progress.
- F. Bakharev, A. Enin, **Yu. Petrova**, N. Rastegaev, 2021. Impact of dissipation ratio on vanishing viscosity solutions of the Riemann problem for chemical flooding model. arXiv:2111.15001

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- B. Temple. Systems of conservation laws with invariant submanifolds. Transactions of the American Mathematical Society, 280(2), pp.781-795, 1983.
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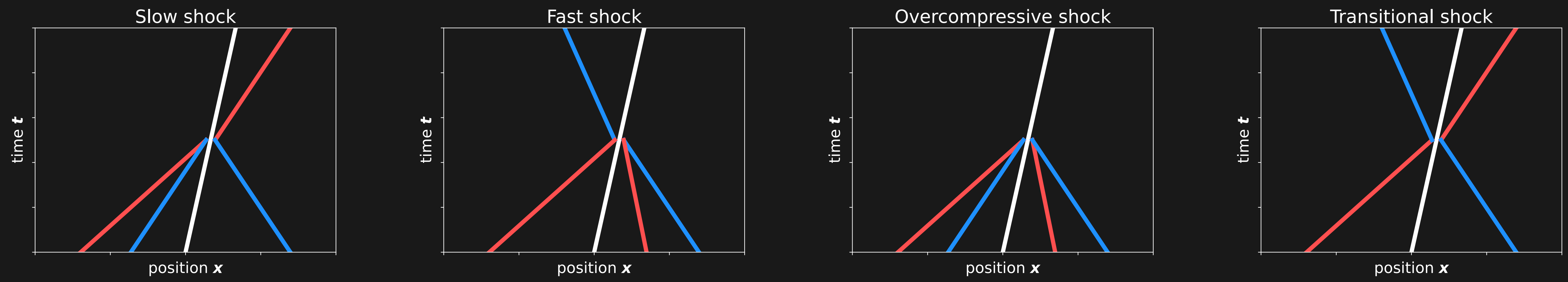
- T. Johansen and R. Winther. The solution of the Riemann problem for a hyperbolic system of conservation laws modeling polymer flooding. SIAM Journal on Mathematical Analysis, 19(3), pp.541-566, 1988.
- Keyfitz, B.L. and Kranzer, H.C., 1980. A system of non-strictly hyperbolic conservation laws arising in elasticity theory. Archive for Rational Mechanics and Analysis, 72(3), pp.219-241.

SHOCKS TYPES



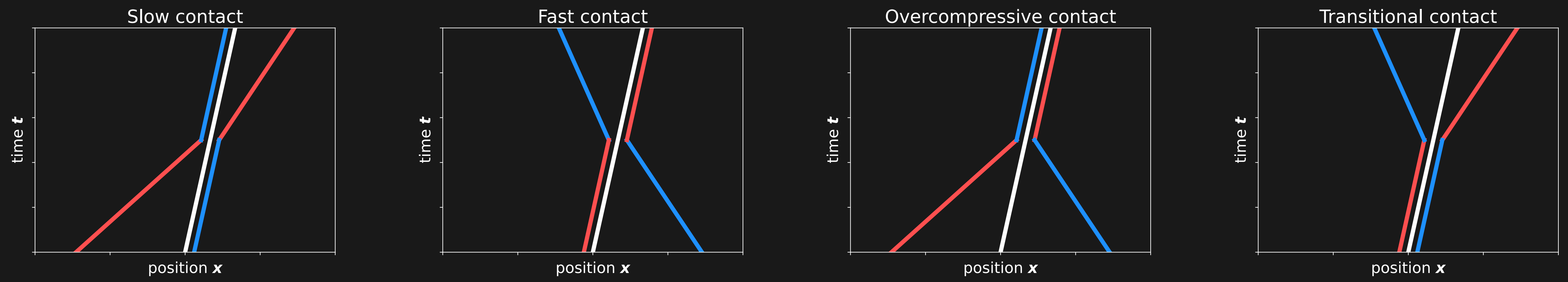
A shock from U_- to U_+ with speed σ is called

- slow if $\lambda_1(U_-) > \sigma > \lambda_1(U_+)$ and $\sigma < \lambda_2(U_-), \lambda_2(U_+)$;
- fast if $\lambda_2(U_-) > \sigma > \lambda_2(U_+)$ and $\sigma > \lambda_1(U_-), \lambda_1(U_+)$;
- overcompressive if $\sigma > \lambda_2(U_+), \lambda_1(U_+)$ and $\sigma < \lambda_1(U_-), \lambda_2(U_-)$;
- undercompressive if $\sigma < \lambda_1(U_-), \lambda_1(U_+)$ and $\sigma > \lambda_2(U_-), \lambda_2(U_+)$.

Undercompressive $\equiv$ transitional.

Fast and slow are also called Lax shocks (classical).

TYPES OF CONTACT DISCONTINUITIES



We will call a contact discontinuity from U_- to U_+ with speed σ

- slow if $\lambda_2(U_-) > \lambda_1(U_-) = \sigma = \lambda_1(U_+) < \lambda_2(U_+)$;
- fast if $\lambda_1(U_-) < \lambda_2(U_-) = \sigma = \lambda_2(U_+) > \lambda_1(U_+)$;
- overcompressive if $\lambda_2(U_-) > \lambda_1(U_-) = \sigma = \lambda_2(U_+) > \lambda_1(U_+)$;
- undercompressive if $\lambda_1(U_-) < \lambda_2(U_-) = \sigma = \lambda_1(U_+) < \lambda_2(U_+)$.

Undercompressive $\equiv$ transitional.