Oil Recovery: Fundamental research and Industrial applications

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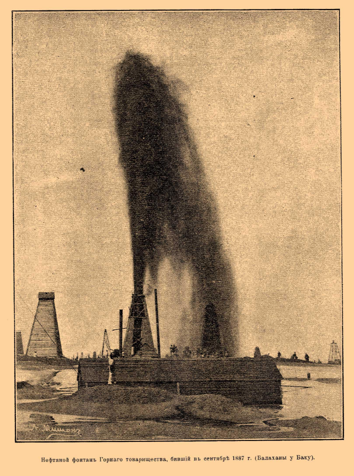
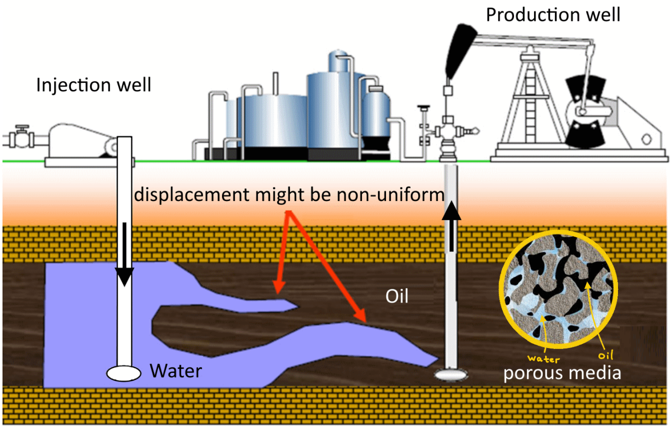
Seminar of "Centro PI"

Oil Recovery

How oil was recovered in the beginning? (Baku, 1857)

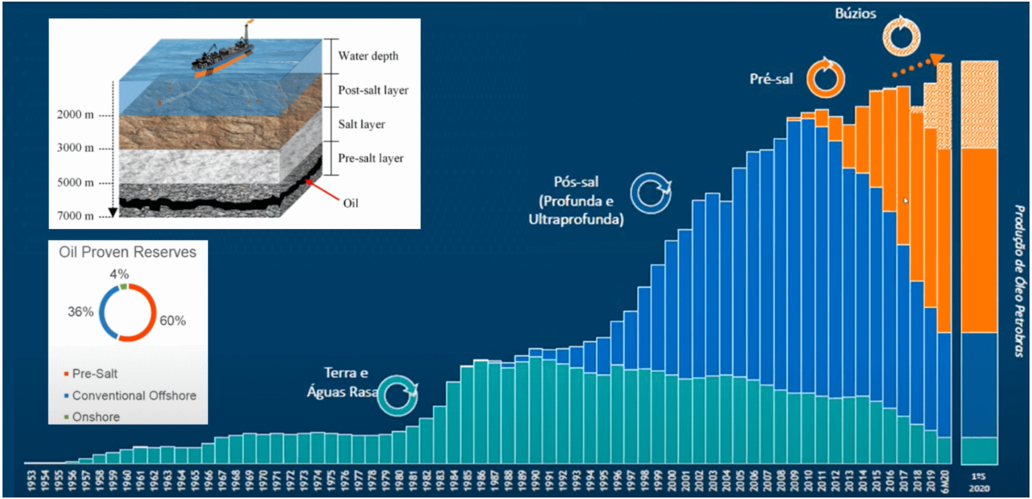
How oil is recovered now?





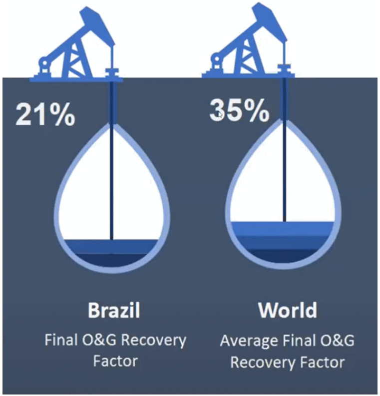
Oil recovery in Brazil

- has great potential: 13.2 billions of barrels ($\approx 1\%$ of the world's oil)



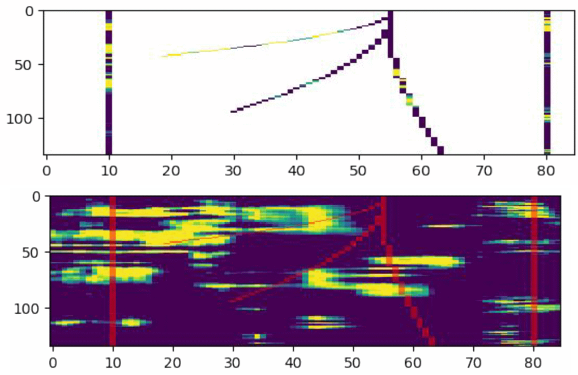
Fact: recovery factor is very low

Vivian Azor de Freitas "Brazilian exploration scenarios and opportunities". In: Deepwater, 2020



How to increase the recovery factor?

Problem: uncertainty in data

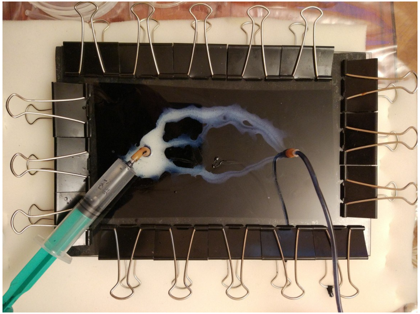


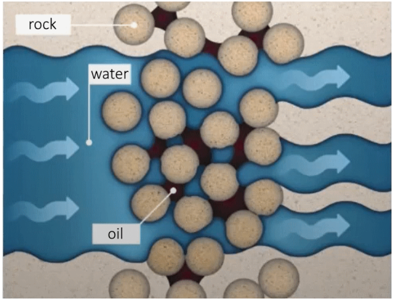
- unknown variables: permeability, porosity
- key words: geostatistics (ask e.g. Júlio Hoffimann and see Geostats.jl package)

Possible solution

Stochastic modelling (e.g. Gaussian processes, Markov chains, Bayesian approach)

Problems: macroscopic and microscopic sweep efficiency





- happens due to very viscous oil or inhomogeneous media
- local entrapment of oil in pores due to high capillary pressure

Possible solution

- Inject gas (CO_2, natural) to decrease the oil viscosity
- Add chemicals (polymer) to increase the water viscosity
- Add chemicals (surfactant) that reduce the surface tension etc

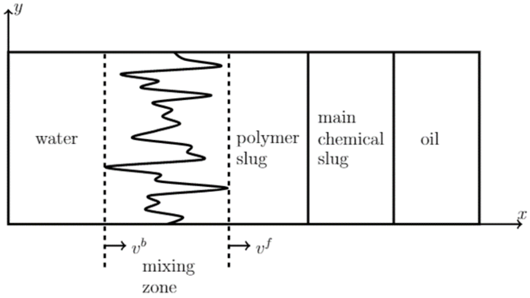
Collaboration with petroleum company GazpromNeft

2018–2022: St Petersburg (Russia), project on Enhanced Oil Recovery (EOR) methods:

- chemical flooding (polymer, surfactant, ASP etc)
- thermal flooding (hot water)
- gas flooding (natural gases)

Aims of the project:

1. for a given reservoir understand which EOR methods give better results
2. optimize the parameters for chosen EOR method



chemical flooding

- design of injection schemes (graded viscosity banks)
- how to save the amount of injected polymer?

Mathematical model of oil recovery: water flooding

- $s \in [0, 1]$ — water saturation
- $1 - s \in [0, 1]$ — oil concentration
- q_1, q_2 — velocity of the water and oil phases
- p_1, p_2 — pressure of the water and oil phases

Equations from first principles

$$
\begin{aligned}
(\phi s)_t + \mathrm{div}(q_1) &= \varepsilon \Delta s \\
(\phi(1-s))_t + \mathrm{div}(q_2) &= \varepsilon \Delta (1-s) \\
q_1 &= -\lambda_1(s) \nabla p_1 \\
q_2 &= -\lambda_2(s) \nabla p_2 \\
p_2 - p_1 &= p_c(s)
\end{aligned}
$$

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Equations from first principles

$$\begin{aligned}(\phi s)_t + \mathrm{div}(q_1) &= \varepsilon \Delta s \\ (\phi(1-s))_t + \mathrm{div}(q_2) &= \varepsilon \Delta (1-s) \\ q_1 &= -\lambda_1(s)\nabla p_1 \\ q_2 &= -\lambda_2(s)\nabla p_2 \\ p_2 - p_1 &= p_c(s)\end{aligned}$$

$\xrightarrow{\text{Mathematician}}$

Simplification (in 1d)

$$s_t + f(s)_x = \varepsilon s_{xx}$$

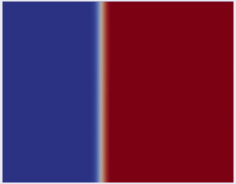
$$f(s) = \frac{\lambda_1(s)}{\lambda_1(s)+\lambda_2(s)}$$

Large-scale approximation ($\varepsilon \to 0$): $\quad s_t + f(s)_x = 0$

Fundamental research: two main directions

1-dim in spatial variable

- Stable displacement



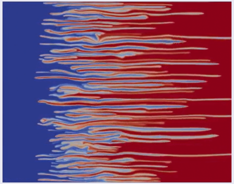
- main question: find an exact solution to a Riemann problem
- hyperbolic conservation laws

$$s_t + f(s,c)_x = 0,$$
$$(cs)_t + (cf(s,c))_x = 0.$$

Example: chemical flooding model

2-dim (or 3-dim) in spatial variable

- Unstable displacement



- source of instability: water and oil/polymer have different viscosities
- viscous fingering phenomenon

$$c_t + u \cdot \nabla c = \varepsilon \Delta c,$$
$$\mathrm{div}(u) = 0,$$
$$u = -\nabla p / \mu(c).$$

Example: Peaceman model

1-dim problems: exact formulas

- water flooding (Buckley-Leverett equation'1942)

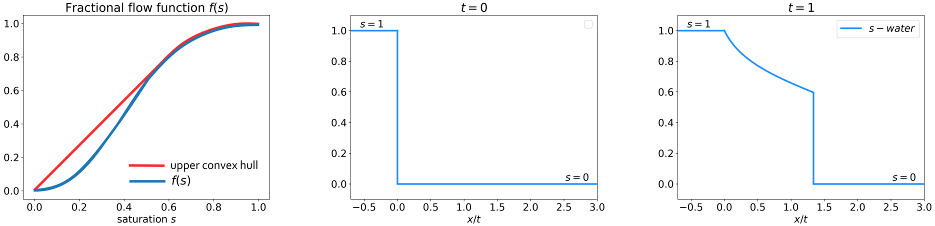
$$s_t + (f(s))_x = 0 \qquad \text{(conservation of water)}$$

- exact solution:

$$s(x,t) = \begin{cases} 0, & \frac{x}{t} < g(0), \\ s, & \frac{x}{t} = g(s), \\ 1, & \frac{x}{t} > g(1). \end{cases}$$

Here g is the derivative of the upper convex hull of f

- calculation time much smaller than than simulation



1d problems: chemical flooding

$$s_t + f(s,c)_x = \varepsilon_c s_{xx},$$
$$(cs + a(c))_t + (cf(s,c))_x = \varepsilon_c (cs_x)_x + \varepsilon_d c_{xx}.$$

s, $1-s$ and c — water, oil and chemical concentrations, $f(s,c)$ — flux function, $a(c)$ — adsorption, $\varepsilon_c, \varepsilon_d$ — capillary/diffusion coefficients (small)

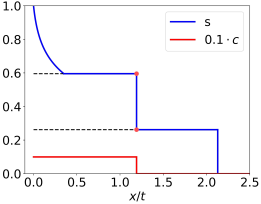
1d problems: chemical flooding

$$s_t + f(s,c)_x = \varepsilon_c s_{xx},$$
$$(cs + a(c))_t + (cf(s,c))_x = \varepsilon_c (c s_x)_x + \varepsilon_d c_{xx}.$$

s, $1-s$ and c — water, oil and chemical concentrations, $f(s,c)$ — flux function, $a(c)$ — adsorption, $\varepsilon_c, \varepsilon_d$ — capillary/diffusion coefficients (small)

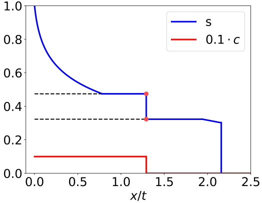
Polymer flooding [Johansen'88]

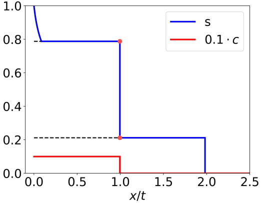
- monotone dependence of $f(s,c)$ on c
- unique solution as $\varepsilon_c, \varepsilon_d \to 0$



Surfactant flooding [Bakharev, P. et al'21]

- non-monotone dependence of $f(s,c)$ on c
- the solution depends on the ratio of $\varepsilon_d/\varepsilon_c$



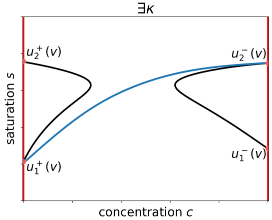


Such kind of research is one of the directions of the Fluid Dynamics Lab at IMPA, Prof. Dan Marchesin and his students: three-phase flows, foams, combustion etc.

1d problems: theorem for non-monotone chemical model

Consider a dynamical system for a travelling wave solution $s = s(x - vt)$ and $c = c(x - vt)$:

$$s_\xi = f(s, c) - v(s + d_1),$$
$$\kappa c_\xi = v(d_1 c - d_2 - a(c)).$$



Theorem (Bakharev, Enin, P.,Rastegaev'2021, arxiv: 2111.15001)

Under some technical assumptions there exist $0 < v_{\min} < v_{\max} < \infty$, such that for every $\kappa = \varepsilon_d/\varepsilon_c \in (0, +\infty)$, there exist unique

- *points $s^-(\kappa) \in [0, 1]$ and $s^+(\kappa) \in [0, 1]$;*
- *velocity $v(\kappa) \in [v_{\min}, v_{\max}]$,*

such that there exists a travelling wave, connecting two saddle points $u^-(\kappa) = (s^-(\kappa), 1)$ and $u^+(\kappa) = (s^+(\kappa), 0)$ with velocity $v(\kappa)$. Moreover, $v(\kappa)$ is monotone and continuous; $v(\kappa) \to v_{\min}$ as $\kappa \to \infty$; $v(\kappa) \to v_{\max}$ as $\kappa \to 0$.

1-dim problems: justification of Isaacson-Glimm criterion

Chemical flooding model without adsorption (Glimm-Isaacson model):

$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ (cs)_t + (cf(s,c))_x &= 0. \end{aligned}$$

- contact discontinuities $\Rightarrow$ non-uniqueness of solutions to a Riemann problem
- existing Isaacson-Glimm admissibility criterion needs to be justified

One can consider a chemical flooding model as a limit of models with vanishing adsorption ($\alpha \to 0$):

$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ (cs + \alpha a(c))_t + (cf(s,c))_x &= 0. \end{aligned}$$

Theorem (P., Marchesin, Plohr, work in progress)

For the polymer flooding model the set of admissible Riemann solutions is the same for the Isaacson-Glimm and vanishing adsorption admissibility criteria

- For more details see slides: https://yulia-petrova.github.io/uploads/talk-UCD-2022-02-15.pdf
- There will be a talk on this topic on PDE seminar at IMPA in April

2d problems

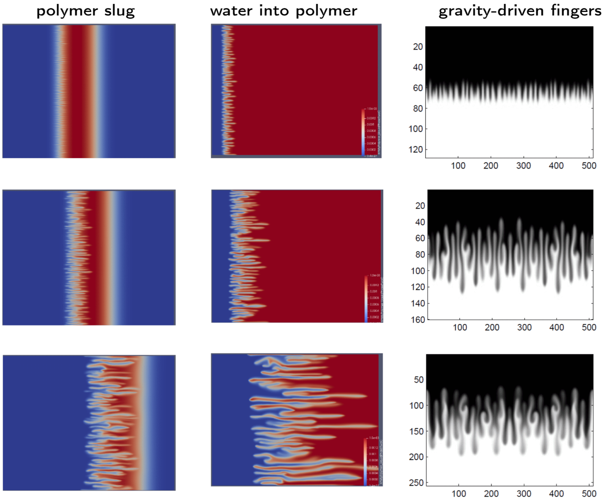
Show video

2d problems: motivation and main question

- water flooding;
- chemical flooding;
- cause problems for oil recovery.

Main questions of interest:

1. what are rigorous bounds on velocities of mixing zone propagation?
2. how to calculate the optimal size of the polymer slug?

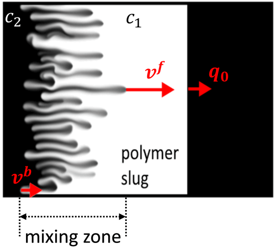


Velocities of the mixing zone

One-phase miscible displacement (Peaceman model)

$$c_t + u \cdot \nabla c = \varepsilon \Delta c,$$

$$\mathrm{div}(u) = 0, \quad u = -\frac{\nabla p}{\mu(c)}.$$



- q_0 — velocity of the stable front
- v_f — velocity of the front end of the mixing zone is constant
- v_b — velocity of the back end of the mixing zone is constant

How to determine the velocity of the mixing zone?

- Development and implementation of an oil-field experiment
- Laboratory tests
- Numerical simulation
- Analytical expressions

c_1 — concentration of injected polymer
c_2 — decreased concentration of injected polymer (for one slug $c_2 = 0$, water)

often v_f is considered to be a function of

$$M = \frac{\mu(c_1)}{\mu(c_2)}$$

$\mu(c)$ — polymer viscosity

Velocities of the mixing zone

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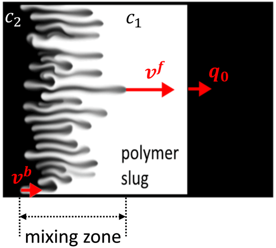

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- v_f — velocity of the front end of the mixing zone is constant
- v_b — velocity of the back end of the mixing zone is constant

Averaging — "effective viscosity" M_e:

- Koval (1963)
- Todd-Longstaff (TL, 1972)

Transverse Flow Equilibrium (TFE)

- F. Otto, G. Menon (2005)
- Yortsos, Salin (2006)

Koval	$v_f = M_e,\ v_b = \frac{1}{M_e}$,	$M_e = \left(0.22M^{1/4} + 0.78\right)^4$	
TL	$v_f = M_e$,	$v_b = \frac{1}{M_e}$,	$M_e = M^\omega$
TFE	$v_f \leqslant \frac{\overline{m}}{m(c_2)}$,	$v_b \geqslant \frac{v_f}{M}$,	$m(c) = \frac{1}{\mu(c)}$

TFE model

TFE model

$$c_t + u \cdot \nabla c = \varepsilon \Delta c,$$

$$\operatorname{div}(u) = 0, \qquad u = (u_1, u_2),\ u_1 = \frac{m(c)}{\overline{m}}$$

Making rough estimates

$$\frac{m(c)}{m(0)} \leqslant \frac{m(c)}{\overline{m}} \leqslant \frac{m(c)}{m(1)}$$

Consider the following 1-dim equations:

$$c_t^{\max} + \frac{m(c^{\max})}{m(0)} \cdot c_x^{\max} = \varepsilon (c^{\max})_{xx}$$

$$c_t^{\min} + \frac{m(c^{\min})}{m(1)} \cdot c_x^{\min} = \varepsilon (c^{\min})_{xx}$$

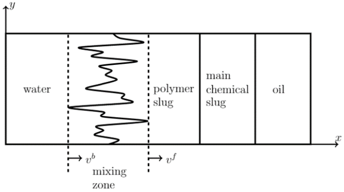
Theorem (Otto, Menon, Yortsos, Salin' 2006)

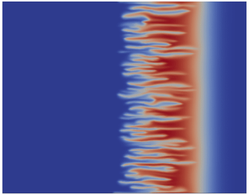
If $c(x,y,0) > c^{\min}(x,0)$, *then* $c(x,y,t) > c^{\min}(x,t)$
If $c(x,y,0) < c^{\max}(x,0)$, *then* $c(x,y,t) < c^{\max}(x,t)$

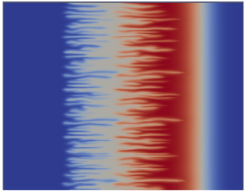
Corollary

$$v^f \leqslant \frac{1}{m(1)} \int_0^1 m(c)\, dc; \qquad v^b \geqslant \frac{1}{m(0)} \int_0^1 m(c)\, dc$$

Industrial application: graded viscosity banks technology

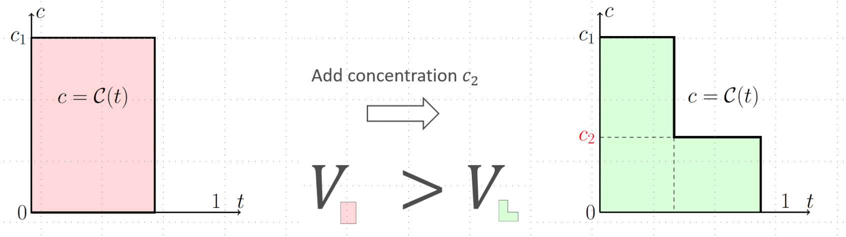






Main idea [Claridge, 1978]

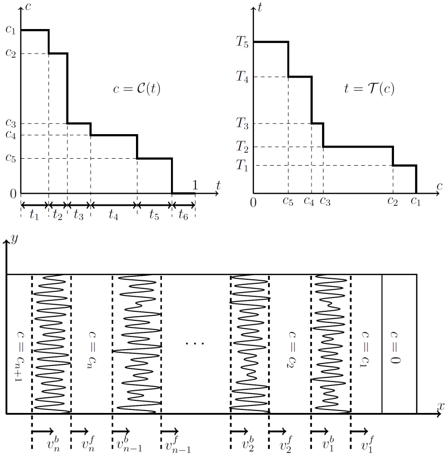
Injecting two slugs may give gain in polymer mass without loosing the effectiveness



Industrial application: graded viscosity banks technology

- Goal: reduce amount of polymer
- Strategy: graded viscosity banks (GVB, tapering) Claridge (1978)
- We want no breakthrough in any slug
- Given concentrations c_n, v^f and v^b find sizes of slugs t_n without breakthrough
- Choose concentrations c_n to minimize amount of polymer:

$$V_n = \sum_{i=1}^{n} c_i t_i \to \min$$



Current status: laboratory experiments show that GVB indeed helps (which was a surprise for petroleum engineers)!

Graded viscosity banks: n finite and $n \to \infty$

V_n — polymer mass for n slugs; $\eta = \frac{V_1 - V_n}{V_1}$ — percentage of gain in polymer mass

	n = 2	n = 3	n = 4	n = 5	n = 10	Limit
TFE	19,83%	23,35%	24,57%	25,13%	25,88%	26,12%
Todd-Longstaff	24,84%	29,36%	30,93%	31,66%	32,63%	32,95%
Koval	33,21%	39,24%	41,46%	42,55%	44,24%	45,28%

Conslusion: it is enough to use small amount of slugs (2–3)

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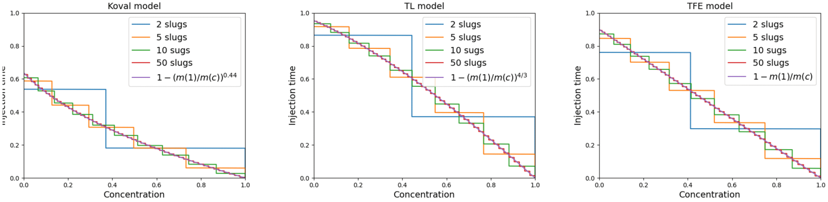
Conslusion: it is enough to use small amount of slugs (2–3)

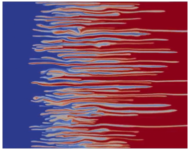
Theorem [Bakharev, Enin, Kalinin, P., Rastegaev, Tikhomirov, 2021]

As $n \to \infty$ the optimal limiting injection profile

$$T^{\infty}(c) = 1 - \left(\frac{\mu(c)}{\mu(c_1)}\right)^{\beta}$$



Toy model of viscous fingering (work in progress)

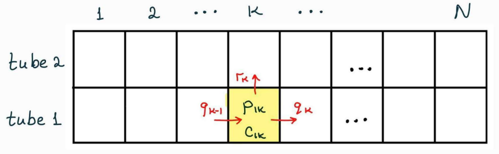


Discrete case

- system of $2N$ ODEs and N algebraic equations

Unknowns:

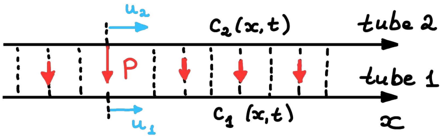
- $c_{1k}(t), c_{2k}(t)$ — concentrations
- p_{1k}, p_{2k} — pressures
- $q_k(t), r_k(t)$ — velocities



Continuous case

- two coupled advection-diffusion eqs

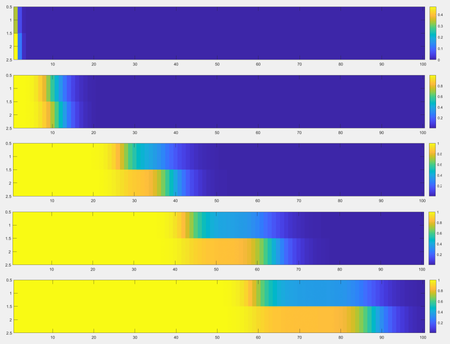
$$\partial_t c_1 = -\partial_x(u_1 c_1) + \partial_x u_1 \cdot c_{1,2} + \varepsilon \partial_{xx} c_1,$$
$$\partial_t c_2 = -\partial_x(u_2 c_2) - \partial_x u_1 \cdot c_{1,2} + \varepsilon \partial_{xx} c_2.$$

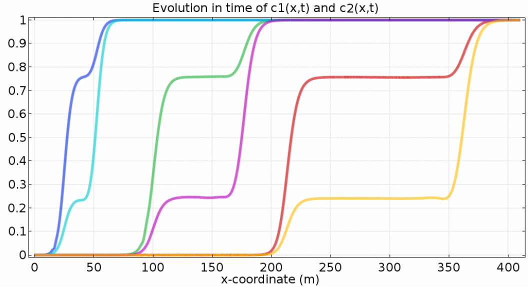


Work in progress with S. Tikhomirov, Y. Efendiev.

Toy model of viscous fingering: numerical experiments

Discrete setting	Continuous setting





Result of experiments: cascade of two travelling waves (TW)

$$(0,0) \xrightarrow{\mathsf{TW}_1} (c_1^*, c_2^*) \xrightarrow{\mathsf{TW}_2} (1,1)$$

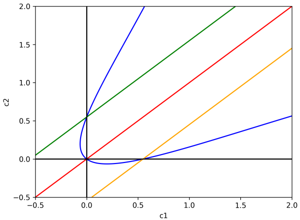
Toy model of viscous fingering: theoretical approach

For a travelling wave $c_1 = c_1(x - vt)$ and $c_2 = c_2(x - vt)$ we have a dynamical system.

Travelling wave dynamical system

$$c_1' = g,$$
$$g' = (u - v)g,$$
$$c_2' = (u - v)c_1 + (2 - u - v)c_2 - g.$$

Here $u = u(c_1, c_2)$, depends on $\mu(c_1)$ and $\mu(c_2)$.



Want to prove

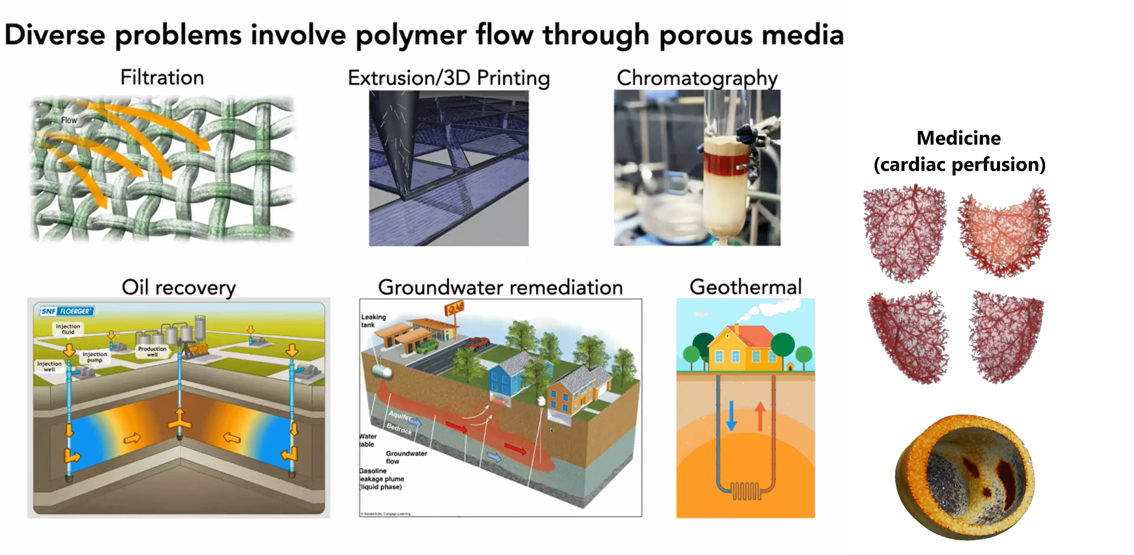
For all $v \in \mathbb{R}_+$ there exists a unique point (c_1^*, c_2^*): there exists a trajectory $U = U(\xi)$:

$$U(-\infty) = (0, 0, 0) \qquad U(+\infty) = (c_1^*, 0, c_2^*)$$

How to quantify the dependence $c_1^* = c_1^*(v)$ and $c_2^* = c_2^*(v)$?

Here $U = (c_1, g, c_2)$.

Another applications



Literature

Own works:

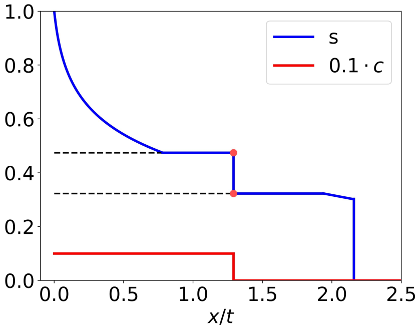
1. F. Bakharev, A. Enin, Yu. Petrova, N. Rastegaev, Impact of dissipation ratio on vanishing viscosity solutions of the Riemann problem for chemical flooding model. arXiv:2111.15001. Under consideration in a journal.
2. F. Bakharev, A. Enin, K. Kalinin, Yu. Petrova, N. Rastegaev, S. Tikhomirov, Optimal polymer slugs injection profiles. arXiv:2012.03114. Under consideration in a journal.
3. Yu. Petrova, D. Marchesin, B. Plohr, On admissibility criteria for contact discontinuities in Glimm-Isaacson model arising in chemical flooding. Work in progress. See slides: https://yulia-petrova.github.io/uploads/talk-UCD-2022-02-15.pdf
4. Y. Efendiev, S. Tikhomirov, Yu. Petrova, Toy model of viscous fingering. Work in progress.

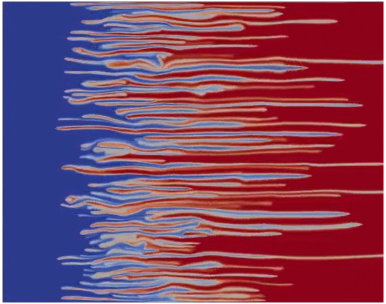
Other works:

1. Johansen, T. and Winther, R., 1988. The solution of the Riemann problem for a hyperbolic system of conservation laws modeling polymer flooding. SIAM journal on mathematical analysis, 19(3), pp.541-566.
2. Menon, G. and Otto, F., 2006. Diffusive slowdown in miscible viscous fingering. Communications in Mathematical Sciences, 4(1), pp.267-273.

Thank you for your attention!

yulia.petrova@impa.br https://yulia-petrova.github.io/





PEACE for Russia and Ukraine (and world)

These mathematicians will never prove a theorem because of the war...

Yuliia Zdanovska (Kiev)

Konstantin Olmezov (MIPT, Moscow)

Many Ukrainian mathematicians are under bomb attacks in Ukraine.
Many Russian mathematicians are under political pressure in Russia.

Example of validation of formulas

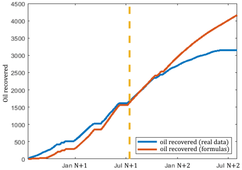
- real oilfield, "5-spot geometry"
- 1 production, 4 injection wells
- polymer-surfactant flooding

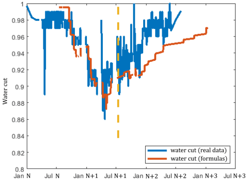
Aim

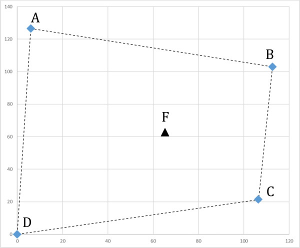
using formulas match the oil recovery rate

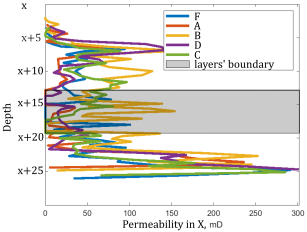
Result

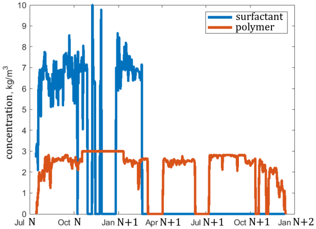
good agreement was achieved











2d problems: experiments in a Hele-Shaw cell

Beginning of the project with GazpromNeft

But...

- we investigate the displacement of fluids in porous medium
- "similar" instabilities occur
- show video