International summer camp “Formulo de Integreco”
Losevo, 18 July – 1 August 2015

# IO

Materials of mathematical circle “IO”

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# GAMES and GRAPHS

## LET'S PLAY MATHEMATICS! 20 July 2015

1) Using exactly 24 matches each time make:
   А) 1 square; Б) 2 equal squares; В) 3 equal squares; Г) 9 equal squares; Д) 50 equal squares.

2) Scrooge McDuck is selling his diamonds. The first client bought half of all his diamonds and also half of a diamond more. The second client bought the half of the rest and half of a diamond more. The third client bought the last diamond. How many diamonds had Scrooge?

3) Nils with his son and Karl with his son were fishing. Nils caught as many fishes as his son, Karl caught three times more than his son. It is known that only 25 fishes were caught. What is the name of the son of Karl?

4) Harry and Hermione are eating sweets and play such a game: from a box of "Raffaello" one takes 1 or 2 sweets turn by turn. Who takes the last sweet wins. Hermione starts the game. Who will win and what is the winner's strategy? If the box contains:
   А) 15 sweets; Б) 333 sweets; В) 1001 sweets.

5) Erika and Diego put knights (horse) on a chessboard one by one in such a way that the knights do not kill each other. The one who can not put the knight loses. Prove that Erika can always win, if she starts.

6) Two persons put 1-euro-coins on a round table without overlapping. A person lose if he can not put a coin on the table. Who is the winner and how he should play?

7) How many diagonals has a (convex) polygon with 20 angles?

8) John was at Disneyland and was telling about a mysterious lake that he saw there: this is a lake with 7 islands. 1, 3 or 5 bridges starts at each island. Is it possible that no bridge leads from the coast (all the bridges connect just the islands)?

9) A student told to his friend: there are 35 persons in our class and each of them is friend of 11 classmates exactly. Viktor Ivanov, the winner of the mathematical Olympiad exclaimed: "it is impossible!" Explain why he was so sure.

## WHAT IS A GRAPH? 21 July 2015

**Definition:** A *graph* is a set of points connected by some links. The points are called the *vertices* of the graph. The links are called the *edges* of the graph.
The number of edges coming to the vertex is called the *degree* of a vertex. The vertices that have odd (even) degree we will call odd (even) vertices correspondingly.
**Theorem:**
The sum of degrees of all verices is equal to the number of all edges of the graph multiplied by 2.
*Proof:*
Let's think that at any vertex of the graph there is a person holding ends of a rope. Each rope corresponds to the edge of the graph. Let's assume that the length of each rope is one meter. The total length of all ropes is equal to the number of edges of the graph.

Let's cut each rope in the middle. Each person will hold $1/2 \cdot$ (vertex degree) meters of the rope. Let's sum up all the vertices. As a result we have

$$\frac{1}{2}\sum_{\substack{\text{all vertices}\\ \text{of the graph}}} \text{vertex degree} = \#\text{ edges of the graph}\ , \qquad \text{Q.E.D.}$$

**Corollary 1:** The sum of all vertex's degrees is an even number.
**Corollary 2:** The number of odd vertices in any graph is even.

**PROBLEMS:**

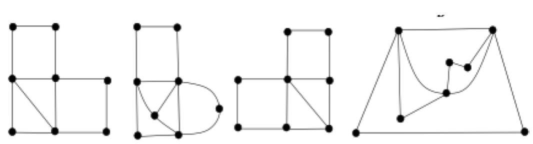
1) On IO (what is IO?) live 27 IO-people (boys and girls). Every IO-boy is a friend of 4 IO-girls and every IO-girl is a friend of 5 IO-boys. How many IO-boys and IO-girls live on IO?

2) On IO every town is connected with 3 other towns. Can be there exactly 100 roads?

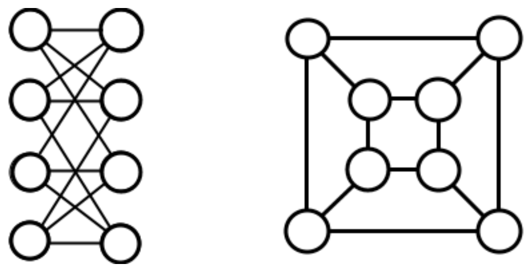
3) Masha wants to connect 25 telephones with each other by cables of 24 different colors. Every telephone should be connected with exactly 24 other telephones and all cables (in each telephone) should be of different colors. Is it possible?

4) Every participant of FdI camp calculates the number of friends in the camp. Prove at least two people have the same number.

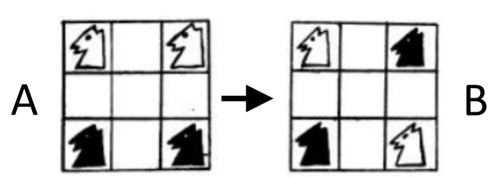
5) Are these graphs equivalent?



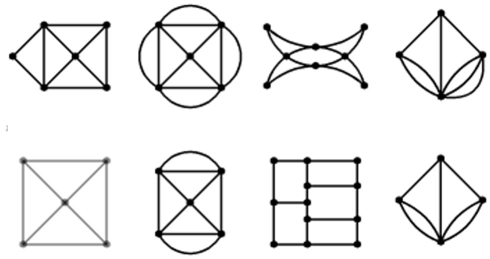
6) And what about these graphs?



7) Is it possible to move hourses on a board $3 \times 3$ from position A to position B?




8) Can you «travel» in these graphs going through each side one time?



9) Can you draw 9 segments on a plane such that every segment intersects with 3 other segments?

10) A «cross» is a table $4 \times 4$ without corners. Can a horse visit all cells of the «cross» and return to a starting point if it stands on each cell only one time?

**23 July 2015**

1) Draw a graph which contains:
А) 5 vertices and 10 edges; Б) 6 vertices and 15 edges;
В) 7 vertices and 21 edges; Г) 8 vertices and 30 edges;
Д) What do А), Б) and В) have in common? Formulate your hypothesis.
Е) Let's call such graph the *full* one. How many edges are there in a full graph with 2015 vertices?

2) On the moon IO there are 27 towns, each one is connected (at least) with 13 another ones. Prove that if one day you visit IO you can reach any town from any other town (may be not directly, but through others).

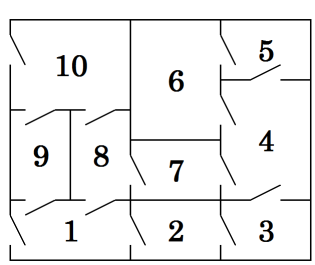
3) The capital of the moon GANYMEDE is connected with 21 cities, the village CALLISTO is connected only with 1 city, all other cities are connected exactly with 20 cities. Prove that you can go from capital to village CALLISTO (may be through the other cities).

4) On the moon EUROPA 20 girls live. On July, 1, one of them learned something very interesting and immediatly told to all her friends. On July, 2, those girls (friends of the first girl) told the news to all their friends. And so on. Can it be that:

   А) on July, 15, not all girls know the news, but on July, 18, everyone knows?
   Б) on July, 25, not all girls know the news, but on July, 28, everyone knows?

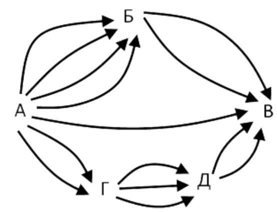
5) There was a generation of russian heroes which was founded by 3 sons of Ilya Muromets. All men in this generation had 3 children, except 7 men who didn't have children at all. There were 2015 women in the generation (wives of men are not counted). How many people were in the generation of Ilya Muromets? A person belongs to the generation if his father belongs to the generation.

6) (TOP SECRET) Here you can see the plan of the house of russian president Putin. Can you go through all doors just one time? From which room should you start?



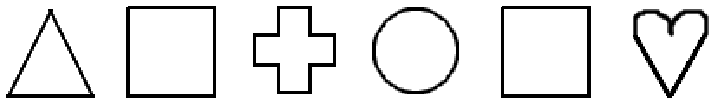
# COMBINATORICS and PROBABILITY

**In how many different ways you can solve these problems?**
**25 July 2015**




1) On the picture you can see the map of Ukhta (city in Russia). On every street you can go only by one direction. How any ways are to go from A to B?

2) Mihai draw 6 figures in a row. In how many ways can you put these figures in the row so that the neighbours of each figure don't change?



3) How many words (not necessarily existing!) can you make using russian letters:

   А) Ю, Я; Б) Ю, Ю, Я; В) Ю, Л, Я; Г) Ю, Л, Л, Я;
   Д) Ю, Л, Е, Н, Ь, К, А? Е) Ю, Л, И, А, Н, Н, А

4) There are 15 people in the first group.

   А) In how many ways can we choose the helper of the leader for today and for tomorrow? They should be 2 different people.
   Б) In how many ways can we choose 3 people to help serving the lunch?

5) Let's call the number «pretty» if all digits are odd. How many 6-digit numbers are «pretty»?

6) Let's call the number «excellent» if at least one digit is 5. How many 4-digit numbers are «excellent»?

### What is BICOMRITONACS and BATYPROBILI? 28 July 2015

What is a probability? Let's toss a coin. There could be two possible results of such experiment:

1) the head with probability 1/2;
2) the tail also with probability 1/2.

*Example 1:*
Let us toss a coin twice. Now we will have 4 possible variants: HT (Head, Tail), TH, TT, HH.
What is the probability to have 2 heads one after each other? Answer: 1/4.
What is the probability to have one head and one tail? Answer: $2/4 = 1/2$.

### General rule for computation of probability of an event A:

1) Calculate the total number of variants in the experiment.
2) Calculate the number of successive variants for the event A (a variant is called a success for the event A, if this variant guaranties that the event A is true).
3) Then compute:

$$P(A) = \frac{\#\text{successes for A}}{\#\text{all possible variants}}.$$

*Example 2:*
Let us toss a dice. What is the probability to get 6? Answer: $1/6$.
Let us toss it twice. What is the probability to get 6 and 6? Answer: $1/36$.
What is the probability to get 1 and 5? The answer: $2/36 = 1/18$.

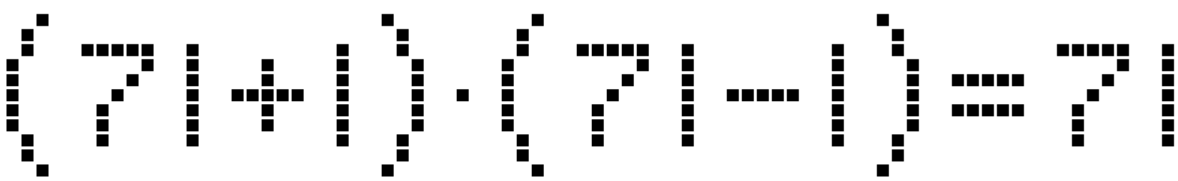
## PROBLEMS:

1) *"Problem of test checking".* Professor gave to 4 his students a test and proposed them to check it themselves (a student checks a test of one of his colleges, but he can not check his own test). How many different variants of such checking exist?

2) *"Problem of letters-1".* There are 4 letters each one in its own envelope. A group of leaders mixed all the letters and envelopes. How many possible variants leave at least one letter in its own envelope?

3) *"Problem of coincidence".* From an alphabet of 31 letters construct a word from 7 letters. A) How many words are there? B) How many words have all letters different?

4) CORRECT THE EQUATION BY DELETING ONLY ONE PIXEL



5) *"Wild monkey problem".* A wild monkey types words using keyboard with only 7 letters (Ь, Л, Е, А, Ю, К, Н). What is the probability that it types the word "ЮЛЕНЬКА", if it typed exactly 7 letters?

6) *"Card problem".* A deck of 52 cards consists of 4 suits and within each suit there are 13 cards of different ranks: Ace, 2, 3, ..., 10, Jack, Queen, King.

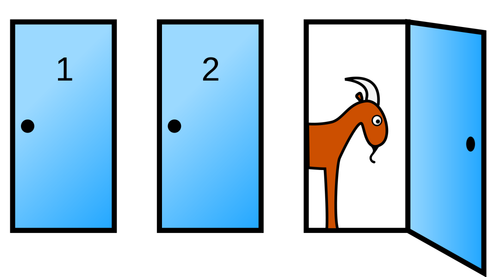
   А) What is the probability to choose all 4 Aces at once?
   Б) What is the probability to choose 4 cards all of different suits and different ranks?

7) *"Problem of letters-2"* (in conditions of problem №2). What is the probability that at least one letter will be its own envelope?

8) *"Birthday paradox"* . There is a group of 23 persons (chosen at random from our population). Find the probability that two or more persons have their birthday at one day (the day and the month coincide, the year could be different). Hint: use the problem of coincidence.

## PROBABILITY and GEOMETRY, 30 July 2015

1) *"Monty Hall paradox".* Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No.1, and the host, who knows what's behind the doors, opens another door, say No.3, which has a goat. He then says to you, "Do you want to pick door No.2?". Is it to your advantage to switch your choice?



2) Ioana writes by random a 2-digits number. Find the probability that the sum of the digits is equal to 5.

3) There are 25 children. To serve the lunch a teacher choose two children by random. The probability that all of them are boys is equal to $3/25$. How many girls are in the class?

4) Braulio and 49 boys and 50 girls are standing in a circle. Braulio is happy when his right and left neighbors are girls. What is the probability that Braulio is happy?

5) Hugo cuts a stripe of length 1m at random place. What is the probability that the length of the biggest part is more than 80cm?

6) In a circle of radio 10cm there is a right-angle triangle with catheters 12 and 7cm. Inside the circle Andrei puts a point by random. What is the probability that the point is not in a triangle?

7) Participants of camp FdI randomly come to the canteen between 14:30 and 15:30. Every child eats for 20 minutes. Find the probability that:

   А) Mai Do and Sergi meet in the canteen?
   Б) Mai Do and Sergi won't meet in the canteen?
   В) What is more probable: that they meet or not?

8) On an interval OA of length 1 there are 2 points B and C marked at random place. Find the probability that:

   А) BC is twice less than the distance from O to the closest point.
   Б) BC is less than $1/3$.
   В) BC is 3 times more than the distance from O to the closest point.

## A LITTLE BIT OF THEORY...

**Definition:** The events A and B are independent, if the fact that one of the events happens have no influence to the second event. Mathematically this means the following:

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

Examples:

1) Let us have a pack of 52 cards. Denote A the event that the first card we took was an Ace. We know: $P(A) = 4/52$. Denote B the event that the second card we took was also an Ace. It is clear that $P(A \text{ and } B) = 4 \times 3/(52 \times 51)$. These events are dependent. The computations of the second probability depends on the first event.

   *Exercise:* how changes the probability to get two Aces, if we put the first Ace back to the pack?

2) We toss a coin 7 times. It is known that each time it was the head, so we have a series of 7 heads. What is the probability to have the tail if we toss a coin one more time?

   The answer: $1/2$ (because the result of every toss is independent of any other toss).

3) *"Second child's paradox".*
   Mr. Smith says that he has two children and AT LEAST ONE of them is a boy. What is the probability that another child is also a boy?

Answer: 1/3 (we know an additional information that one of the kids is a boy, thus the total number of variants is 3, and only one of them is “boy and boy”).

How this probability will change, if Mr. Smith says that the FIRST his child is a boy?
Answer: 1/2 (the total number of variants reduces to 2, and only one of them is “boy and boy”).

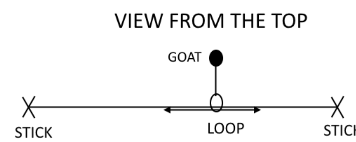
*Note:* that if we have no additional information about the kids of Mr. Smith the probability that he has two boys is $1/2 \times 1/2 = 1/4$. This is because the sex of the first child is independent of the sex of the other child.

## GOATS & GRASS, 26 July 2015

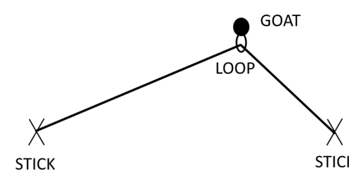
Goats like to eat very much and eat all the grass that they can touch. That's why clever men leash them to a stick.

1) Leash a goat so that it can eat a circle.

2) Between two sticks there is a stretched rope (can not be extended!) and a goat is leashed with another rope to this one by a loop (see picture). The loop can easily move on the first rope. What figure will the goat eat?

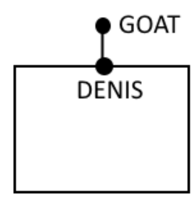



3) Between two sticks there is a rope (can not be extended but it's length is bigger than the distance between sticks). The goat is leashed to it by a loop (see picture). The loop can easily move on the rope. What figure will the goat eat?



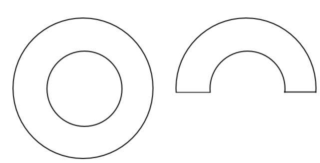

4) Now the goat is leashed to 2 sticks with 2 ropes of different lengths. What figure will it eat?

5) Every morning the mathematician Denis is walking with his goat on the road. The road has a form of rectangular $3 \times 5$ $m^2$. The length of the rope which connects Denis and the goat is 1 $m$. What figure will the goat eat?




6) How can you leash the goat so that it eats exactly:
   - A) semicircle;
   - Б) square;
   - В) rectangle (there are at least 2 different solutions!).

7) On a farm FdI live not only goats but also dogs. The goat is clever and don't go to the territory of the dog (even if the goat is extremely hungry and the dog is supernice). Leash the goat and one dog so that the goat eats exactly:
   - A) a ring of width 1;
   - Б) a semiring of width 1.



# COMPETITIONS

## STARTING OLYMPIAD, 19 July 2015

*First stage:* to pass into the second stage, you should solve 3 problems from these 5. You have three tries for each problem.

1) Is it possible to write numbers from 1 to 16 in a row in such a way that sum of any two adjacent numbers is a square? Each number should be used once.

2) Cheese is made from milk. The fatness of milk is 5%, the fatness of cheese is 15.5% and the fatness of waste is 0.5%. How many cheese can be made from 1 ton of milk? (All the milk is divided into two parts: cheese and waste.)

3) Computer designer Maria wants to make a list of all possible patterns. A pattern is an $8 \times 8$ square with each cell painted in black or white. How many patterns are there?

4) $AE$ is a bisector of a triangle $ABC$ (that is, point E lies on BC and $\angle BAE = \angle CAE$). It is known that $AE = EC$ and $AC = 2AB$. Find the angles of the triangle $ABC$.

5) Is it possible to find integers $a$ and $b$ such that $a^2 + b^2 = 2012201320142015$?

*Second stage:* you can also solve problems from the first stage which you haven't solved yet.

6) Some dates are printed in a book. Each date is written using 6 digits (for example, 19.07.15). Peter count the amount of 0s, 1s, ..., 9s in these dates. Is it possible that all these 10 amounts are equal?

7) Computer designer Maria now considers two patterns different only if they don't coincide after any rotation. How many different patterns she can find now?

8) Does such an infinite set of integers exist, that each two of them have a common divisor greater than 1, but each three of them are coprime (=have not such a common divisor)?

## MATHEMATICAL FIGHT, 23 July 2015

*Rules:* It is a team game. Each problem has a cost. In the beginning the cost equals to a number of teams. Each time when the team gives a right answer to the problem the team gets points and the cost decreases by 1, BUT if the answer is wrong the team gets minus 1 and the price increases by 1. Everybody sees the costs of problems and that makes the game dynamic. Normally the problems are not difficult, but you need to make them as quick as possible.

1) How will the price change if firstly it increases by 100% and then it decreases by 50%?

2) Erika and Stalyn want to buy a book. Erika needs 6 kopeikas to buy a book, Stalyn needs 1 kopeika to buy a book. If they share the money, they still don't have enough money to buy a book. How many kopeikas costs a book?

3) There are 2 numbers on the blackboard: 30 and 51. You can write the difference of any two numbers which are already on the blackboard (from the bigger you substract the smaller). What is the biggest number that can be written on the blackboard (except 51)?

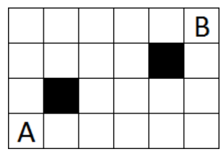
4) King Gvidon had 3 children. In 1200 Gvidon and his wife were 30 years old, and the sum of ages of all his children was 30 years too. It is known that they lived for a long time. In what year the sum of years of Gwindon and his wife is the same as the sum of years of all their children?

5) What is the biggest natural $n$ that number $1 \times 11 \times 111 \times \ldots \times \underbrace{11\ldots11}_{\textbf{24 times}}$ is divisible by $3^n$?

6) What is the smallest number, two last digits of which are 15, is divisible by 15 and and the sum of it's digits is 15.

7) Find the sum $(x + y + z)$, where $x$, $y$ and $z$ are natural numbers, and $28x + 30y + 31z = 365$.

8) How many ways are to go from A to B? You can go only to the right or up and stand only on white cells.




9) There are two cups: one with milk and another with coffee. Alex takes 1/4 part of milk and pours out into a cup of coffee and mixes it. Then Alex takes 1/4 part of the mixed coffee with milk and pours it out into the cup of milk. What is more: the part of milk in coffee or the part of coffee in milk?

10) In one copybook there were 40 statements:

— In this copybook exactly 1 wrong statement

— In this copybook exactly 2 wrong statements

.............................................

— In this copybook exactly 40 wrong statements

How many wrong statements are?

11) In the equality $101 - 102 = 1$ move ONE digit (you can move it to any place) so that the equality becomes correct.

12) Is it possible to choose signs in expression

$$1 \pm 2 \pm 3 \pm 4 \pm 5 \pm 6 \pm 7 \pm 8 \pm 9 \pm 10 \pm 11 \pm 12 \pm 13 \pm 14 \pm 15 \pm 16 \pm 17 \pm 18 \pm 19 \pm 20$$

and make it equal a 20?

13) 4 girls are standing in a circle: Teresa, Eugenia, Miruna and Maria. The girl in green dress (not Teresa and not Eugenia) is standing between the girl in blue dress and Maria. The girl in white dress is standing between the girl in pink dress and Eugenia. What dress wears each girl?

**MATHEMATICAL BATTLE, 27 July 2015**

1) The calculator "Cheburashka" can add numbers, subtract them, find the inverse number and remember all the results. New numbers can appear only as a result of the mentioned above operations. Can we get the number 1 starting from the number 19/99?

2) Let's call a mushroom a bad one if there are more than 11 worms inside it. Also let's call a worm thin if it had eaten not more that 20% of the mushroom which it lives in. It is known, that a quarter of all mushrooms in the forest are bad. Prove that not less than a third part of all worms are thin.

3) The table $4 \times 4$ is filled by signs minus except one adjacent to a corner cell where there is a plus. In one move you can swap all signs in some diagonal into opposite, including one-celled diagonals. Can you make the table filled only by pluses?

4) There is a point $E$ on the diagonal $BD$ of the convex quadrangle $ABCD$ such that $AB = CE$ and the angle $ABD$ is equal to the angle $BCD$ and to the angle $ECD$. Also the angle $DAB$ is equal to the angle $ABC$. Prove that the triangle $BCD$ is isosceles (with 2 equal sides).

5) Santa brought 13 packages of sweets to a classroom. Using one question kids can know the summary weight of 2 packages. What minimal amount of questions can kids use to know total weight of all 13 packages?

6) There are three different prime numbers $a$, $b$ and $c$. Prove that $ab + bc + ca + 29 \leq 2abc$.

## MATHEMATICAL DOMINO, 20 July 2015

**0–0:** There is a rook in each cell of a board $10 \times 10$. You can take away the rook which beats odd number of rooks. What is the greatest amount of rooks you can take away? (Two rooks beat each other if they are standing in the same row or column and there is no other rook between them)

**0–1:** We have some sheep and chickens. They have 36 heads and 100 legs together. How many sheep are there?

**0–2:** How many are there two-digit numbers of the form $ab$ , where $a < b$?

**0–3:** 24 trains go along the ring subway line. They go in the same direction with the same speed and the same intervals between trains. How many trains with the same speed should be added for reduce the interval on 20%?

**0–4:** Find all the prime numbers that can not be expressed as a sum of two composite numbers.

**0–5:** The sum of the squares of 2006 some integer positive numbers is a perfect square. Find the smallest possible value of the sum of these numbers.

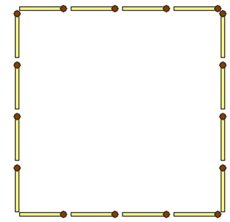
**0–6:** Find a 100-digit positive integer number without 0-digit such that it is divided by the sum of its digits.

**1–1:** The sum of four consecutive integer numbers is equal 2010. Find the numbers.

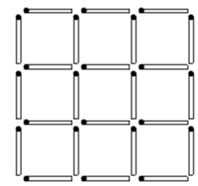
**1–2:** The nuts are in three boxes. In the first box the amount of nuts is 6 less than in the other two together, and in the second - 10 less than the first and third together. How many nuts are there third box?

**1–3:** There were 43 students on the X-mas party. It is known that first girl danced with 8 boys, second one – with 9 boys, . . ., the last one – with all the boys. How many girls came to the party?

**1–4:** Add the odd amount of matches to cut this square into 4 pieces of area 4. You can't break, flex matches and they can't intersect one another.

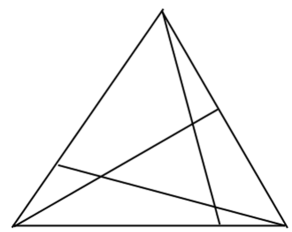


**1–5:** 24 matches are put in the way you see on the picture. Move away 6 matches so that the other matches form 5 equal squares.



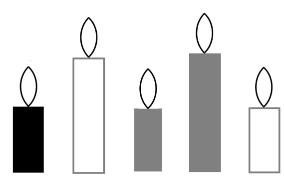
**1–6:** In the four-digit number each digit was increased by 1 or 5 after that it increased in 4 time. What could be the original number? Find all the options.

**2–2:** How many triangles are there on the picture?



**2–3:** Claudio has 43 mathematical problems. He want to write them on some sheets of paper in such a way on each two of them were the different amount of problems. What is the greatest amount of sheets for he could do it.

**2–4:** The candles of Alba and Benito have the same size. The candles of Benito and Clara have the same colours. The candles of Clara and Paco have different size. And finally the candles of Paco and Alba have different colours.And which of these candles belongs to Katarina? Find all the possible variants.



**2–5:** Find the largest positive integer number of different digits for which in each pair of adjacent digit one digit is divisible by another.

**2–6:** Milk and cream are sold in the similar bottles In the shop for 5 empty bottles you can get 1 bottle of milk and for 10 empty bottles – a bottle of cream. Dor found in the forest 60 empty bottles and decided to bring them to the shop. When he got a bottle of milk or cream he drank it and then used new empty bottle for next exchanges. After all this drinking activity he had only 1 empty bottle. How many exchanges did he make?

**3–3:** Find the smallest ten-digit number for which in each pair of adjacent digits one digit is divisible by another.

**3–4:** Two cars simultaneously went from cities A and B towards each other. After 7 hours of moving the distance between them was 136 km. Find the distance between A and B if all the distance one car can go in 10 hours and the one in 12 hours.

**3–5:** If we subtract 2 from a 2-digit number, the result is divisible by 3, and if we subtract 3, the result is divisible by 2. If we add 4 to this number, the result is divisible by 5, but if we subtract 5, the result is divisible by 4. Moreover if we subtract 5, the result is divisible by 6. In addition if we add to this number 7, the result is divisible by 8, and if we add 8, the result s divisible by 7. Find this magic number.

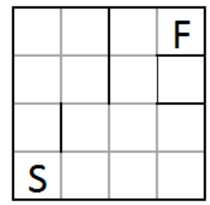
**3–6:** 14 different positive integer numbers are written in a row one by one so appears a one big number consisting of all of them. Find the smallest such a number.

**4–4:** Draw any decagon which you can cut into 5 triangles.

**4–5:** There are 2010 shareholders in the joint-stock company. It is known that every 1005 of them have not lesser than 50 percents of all shares together. What is the greatest possible percent of shares one shareholder can have?

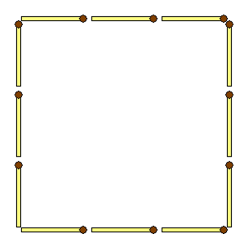
**4–6:** Enrique wants to obtain the key from the labyrinth. He can't enter to the labyrinth but he can put a robot there. The robot knows following commands:

- Up (go to one cell up)
- Down (go to one cell down)
- Left (go to one left cell)
- Right (go to one right cell)




The command which the robot can't do is omitted and then it does the next command. The robot has a limited memory, that's why Enrique can write a program which consists only of 4 commands. When the robot finishes doing the program, it starts from the beginning. When the robot is in the cell with the key, it stops doing the program. How should Enrique make a program to move the robot from the cell S to the cell F?

**5–5:** On the picture there is a square made of 12 matches. Its area is equal to 9. Make from these 12 matches a figure with area 4 using all the matches. You can't break, flex or make intersections between the matches. There should be no hanging matches.



**5–6:** Pasha cut the cheese hat into 10 pieces and ate the smallest one. Then he cut one of the pieces into 2 and ate the smallest of 10 ones. After that one more time he cut one of the pieces into 2 and ate the smallest one of 10 ones. What is the greatest part of cheese hat that Pasha could eat?

**6–6:** Recover the way of a horse going through all the cells of the table $6 \times 6$ for one time by the numbers you see (so fill in the table with numbers from 1 to 36 so that numbers with difference 1 stand in such cells that a horse can move from one of them to another)

| | | | | | |
|---|---|---|---|---|---|
| 17 | | | | 11 | |
| 2 | | | 25 | | |
| 23 | 16 | 1 | | | |
| 30 | | | 19 | | |
| 15 | | | | 13 | |
| 8 | | | | | 35 |

# ABACUS, 29 July 2015

| | **10** | **20** | **30** | **40** | **50** | **60** |
|---|---|---|---|---|---|---|
| *NUMBERS* | From the numbers 1, 2, …, 19, all even numbers (=divisible by 2) are removed. After that, all such $x$ that $19-x$ is divisible by 3 are removed. How many numbers are rest? | How many three-digit numbers exist such that the last digit is the product of first two digits? (A 3-digit number cannot start with 0). | A positive integer $a$ is divided to a positive integer $m$ with remainder $q$: $a=mb+q$. Number $a$ ends with 1, and $b$ and $q$ both end with 9. List all possible last digits of $m$. | From all positive integer numbers whose squares are divisible by 24, the smallest is chosen. What is the sum of digits of the chosen number? | A square of a positive integer number is combined from digits 0, 2, 3, 5. Find this number. | There are two lines on a coordinate plane: $y=2+x$ and $y=1-x$. They divide the plane into 4 parts. Let us enumerate these parts counterclockwise, starting with the part containing (0, 0). In which of the parts the point A(–2003; 2003) is located? |
| *TEXT PROBLEMS* | A weight of an iron cub is 10 g. Find the weight of a cube whose edge is twice longer. | A cow eats a portion of dry grass in 4 days, and a goat eats the same portion in 12 days. In how many days they eat that portion together? | A car has driven 300 km. The first half of this distance it drove with the speed 100 km/h, and the second half with the speed 60 km/h. Find the average speed of the car. | A bath is filled by cold water in 6 min 40 sec, and it is filled by hot water in 8 minutes. If we pull the plug out of the full bath, it becomes empty in 13 min 20 sec. How much time is needed to fill the bath if both the taps (cold and hot) are opened but the plug is pulled out? | Each of 10 consecutive positive integers was decreased by 1. As a result, the product became three times smaller. Find the smallest of the initial 10 numbers. | A cyclist rode 96 km, and spent 2 hours less than he expected. During each hour he covered 1 km more than he expected to cover during 1 hour 15 minutes. At what speed did he ride? |
| *COMBINATORICS* | How many ways are there to choose a vowel (A, E, …) and a consonant (B, C, …) from the word CIRCLE? | Bogdan paints five New Year trees of different size, each tree in one color. He has three paints (green, blue and silver). How many ways are there to paint them? | 25 people participate in a contest. How many ways are there to award one first, one second and one third prize? | Find the sum of all 3-digit numbers which are constructed from 3 different nonzero digits. | There are six members in an organization: A,B,C,D,E and F. How many ways exist to create a committee of three persons if A don't want to be in one committee with B, C with D, and E with F? | How many 4-digit numbers don't contain three identical digits one after another? (e.g. 1131 is allowed, but 1113 is not) |
| *LOGIC* | Basile has 20 balloons of different colors: yellow, green, blue and black. 17 of these balloons are not green, 5 are black, and 12 are not yellow. How many blue balloons Basile has? | The seven Dwarfs lined growth, and Snow White gave them 707 mushrooms. First he gave some mushrooms to the smallest Dwarf, and each of the next Dwarfs receives 1 mushroom more than the previous one. How many mushrooms the biggest Dwarf receives? | Five participants of an Olympiad won it: they got 15, 14 and 13 points and received first, second and third place respectively. How many people received each of the places if they totally got 69 points? | There are 2015 people on the Island of Gentlemen and Liars. Gentlemen always tell the truth, and liars always lie. One day, each of the islanders said: "Among all other islanders (except me), more than a half are liars". How many liars are on the island? | The age of Sister now is three times bigger, than the age of Brother in the day, when the age of Sister was so big as the age of Brother now. In the day when Brother will be as old as Sister now, they will together have 28 years. What is the age of Sister and the age of Brother now? | All the digits of a 4-digit number $x$ are different. Each of the numbers 5860, 1674, 9432, 3017 contains exactly two digits of $x$, but none of that digits is located at the same place as in $x$. Find $x$. |

# GAMES TO WAKE UP

These games are good at the beginning of your lesson to involve sleeping students in the process. They take from 15 to 50 minutes (including analysis and making conclusions).

1) *Bulls and cows:*
One player, the Chooser, thinks of a four-digit number and the other player, the Guesser, tries to guess it. At each turn the Guesser tries a four digit number, and the Chooser says how close it is to the answer by giving:
   - The number of Bulls — digits correct in the right position.
   - The number of Cows — digits correct but in the wrong position.

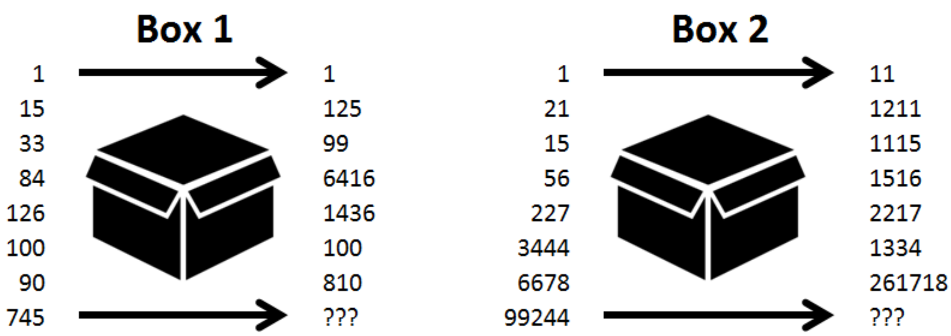
   The Guesser tries to guess the answer in the fewest number of turns.

   There are different types of these game: with numbers or with words, for 2 players or for many players.

   For more information see Wikipedia: `https://en.wikipedia.org/wiki/Bulls_and_Cows`.

   To play online:
   - with numbers: `http://www.papg.com/show?1`;
   - with words: `http://www.papg.com/show?1TLX`.

2) *Black box:*
The Leader invents some rule (operation with numbers) and the others should guess this rule. One by one they tell the number to the Leader and he tells the result. For example:



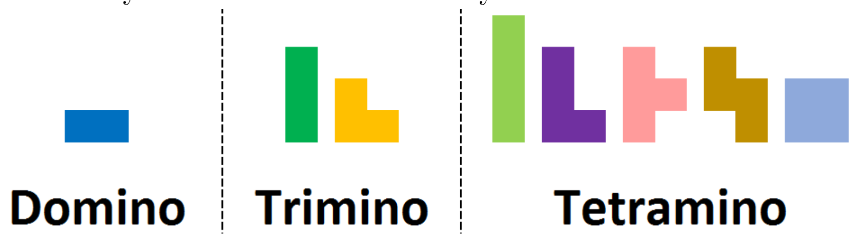

3) *Telephone (or T9):*
Some years ago many people used telephones with buttons (see the picture). To write a letter it was necessary to push the corresponding digit. So we can say that every word can be written as some number, for example, cow = 269, geometry=43663879, dance=32623. What word is written here: 53666? 2523786? 835374663? Note: sometimes there is more than 1 answer.

| Telephone | | |
|---|---|---|
| 1 | 2 ABC | 3DEF |
| 4 GHI | 5 JKL | 6 MNO |
| 7 PQRS | 8 TUV | 9 WXYZ |

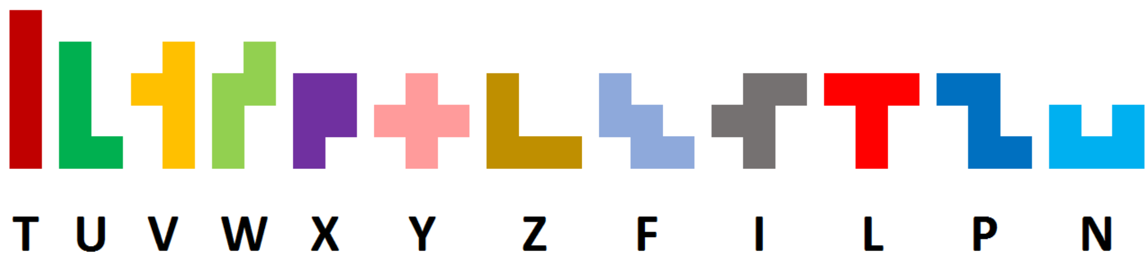
4) *Game of "7":*
Everybody stands in a circle and starts to count loudly one by one clockwise: one, two, three, etc. If a number has a digit 7 or is divisible by 7, then the person whose turn is to say this number, claps instead of telling the number and the the direction changes to the opposite one (clockwise to counterclockwise and vice versa). The person who makes an error (f.e. tells number instead of clapping or claps instead of telling number or does something not in his turn), goes away from a circle. A goal is to count up to 100 (or more as you wish). When this happens you win the game. Otherwise the game wins you.

5) *Pentaminos:*

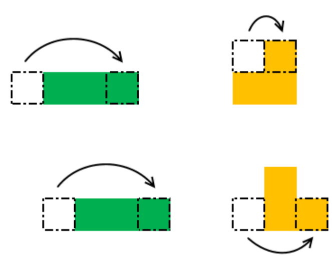
Everybody knows what is a domino — 2 squares that have one side in common (we don't write numbers on them). Let's agree that vertical and horizontal domino are the same for us, because we can rotate one of them and get another one. So there is only 1 domino. And how many trimino exist? That's easy to see that there are only 2 of them. And what about tetraminos?




Now we are interested in pentaminos. That's a good task for children to draw all possible variants (in fact there are 12 of them). There is an easy way to remember: write last 7 letters of english alphabet (T, U, V, W, X, Y, Z) and word FILIPINI. Each written letter has a pentamino which looks like it! Can you guess what letter corresponds to what pentamino?




Now we are ready for classification. We say that a figure moves when we can change the position of one square and obtain the same figure but in a different location. For example, trimino can move like this:

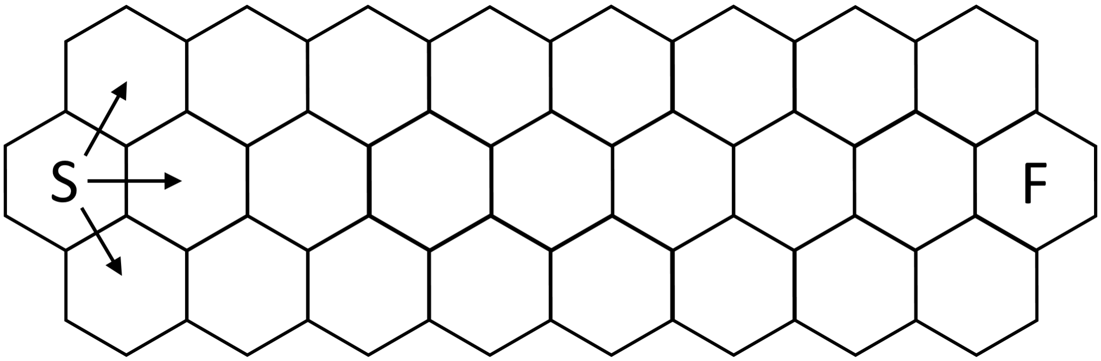


There are 3 different kinds of objects:

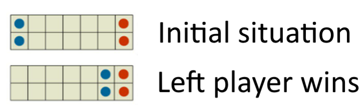
- an animal — a figure can make moves and go "far away" from the original position
- a plant — a figure can make a move but no matter how many moves it makes it can not go "far away" from the original position
- a stone — a figure can not make a move.

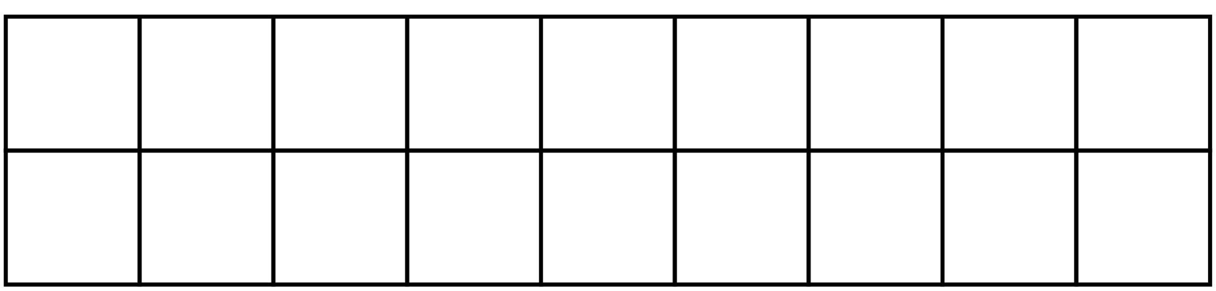
For example, letter T (bright red) is a stone, dark red letter is an animal, and the last letter in a row (of the color of sky) is a plant (it can move but it can't go "far away"). So the task is to classify all pentaminos. It may seem easy, but be careful!

6) *Bumblebee game:*
The game is played on a hexagonal board by 2 players, where S = Starting point, F = Finishing point. One by one you move a figure in one of 3 directions: straight, up or down (you can not go back!) Standing on the first or on the third row you have only 2 variants (straight or down; straight or up, respectively). Who comes to F wins. The global task is to understand who can always win and what is the best strategy to win.
You can experiment with different numbers of cells in each row or even with different numbers of rows. Also it is useful to change the winner's condition: who comes to F looses. Have FUN!




7) *Fencing:*
The game is played on a board $2 \times N$ ($N$ can be choosen, f.e. 9) by 2 players. Each of them has 2 figures of the same color (in the picture 2 blue and 2 red circles). Turn by turn players move one of their figures only horizontally by any number of cells. It is not allowed to stand on the opponent's cell and to jump the figure of your opponent. The looser is the person who can not make a move. The global task is to understand who can always win and what is the best strategy to win.
After trying with 2 figures for a player, you can make an experiment with 3 figures (the strategy is quite easy!) and with 4 figures (that is more complicated!).



8) *Numbers game:*
There are two cards with numbers "1", "2", "3", "4", "5", "6", "7", "8", "9", "10", and one card with numbers "25", "50", "75", "100". Firstly we take at random six cards of these ones. These are our basic numbers. Then we take at random some three digit number (may be using calculator), let's call it X. Our task is using basic numbers and standard mathematical operations (addition, substaction, multiplication, division and exponentiation=making powers) construct a number as closer as possible to X.

Good way to play this game is the foolowing:
we devide all children into several teams with 3-4 people in each team. First two minutes after the number X is known, everyone thinks by himself. Then for a half of a minute each team shares the results and they choose the best one and CHECK the calculations. Then the time is over each team tells what best number it can get. The best result of all teams (or best results if there are more than one) needs to be checked. The team shows all operations how it gets the result. It is possible to have 3 different situations:

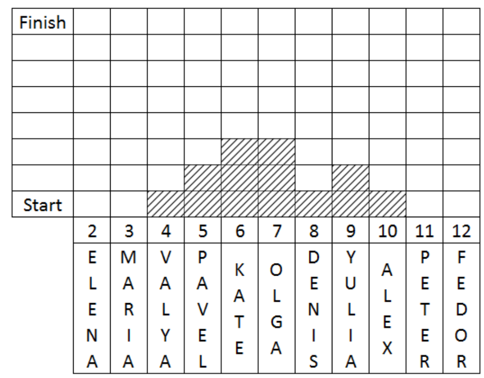
А) If the result is correct and is equal to number X — the team gets 2 points.

Б) If the result is correct but is not equal to X — the team gets 1 point.

В) If there is some error in calculations — the team gets minus 1 point. And we check the next closest result to number X.

Good luck!

9) *Horses:*
We are on a horserace! First of all everyone chooses one number from 2 to 12, this will be the number of your horse. Note: every participant should choose the number of his horse before he knows rules of the game. Then one by one they roll two dices and sum the numbers. The horse with this number goes one cell forward. And so on. Can you guess who will win?
For example, in this picture you can see that one time won the horse of Valya, two times — the horse of Pavel, three times — the horse of Kate, etc.
Here is the way how you can show it on the paper.




This game is not very fair, because the result almost doesn't depend on your moves. But it is a good game to show some probabilities and (may be) speak about normal distributions.
One of variations of the game is not to sum numbers on dices but substract them (always from the biggest number we substract the smallest one). Here you can have 0, 1, 2, 3, 4 and 5. Which horse will win in this horserace?


**MATHEMATICAL JOKES**

- Why the Math's book committed suicide? Because it has many problems.
- Why Math's book died? One operation went wrong.
- Why is 6 scared of 7? Because 7 ate 9.
- Why are numbers so friendly? Because you can always count on them.
- A Roman soldier went into the canteen and ordered "give me V beers"
- OK, Bart, what's 32 oranges minus 17 oranges?
  Sorry, my teacher tough me calculus with apples.
- If you have 7 cows and you kill 3 of them, how many cows do you have?
  7 cows, 4 alive and 3 died.
- If there are 7 flies on the table, and you kill 3 of them, how many flies do you have?
  Any, as the other will fly.

Formulo de Integreco
2015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Somera Tendaro
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Ferne Alexandru
Smileva Elena
Vasile Marian Daniel
Sosea Miruna
Sancho Mukhachev Sergi
Semen Valentin Ion
Ronquillo Diego
Petrova Yulia
Muchitsch Mai Do
Indacochea Rosado Maria Gratzia