On admissibility criteria for contact discontinuities in Glimm-Isaacson model arising in chemical flooding

Monotone chemical flooding model

Problem statement

Consider two-phase oil-water flow with dissolved chemical in water in porous media:

$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ [c\,s]_t + [c\,f(s,c)]_x &= 0, \end{aligned} \tag{1}$$

- $s \in [0,1]$ — water saturation
- $c \in [0,1]$ — concentration of chemical in water. $U = (s,c)$
- $f(s,c)$ — fractional flow function (usually S-shaped, see Fig.): f is monotone in c ($f_c < 0$)

Find exact solution $s(x,t)$ and $c(x,t)$ to any Riemann problem:

$$(s,c)\Big|_{t=0} = \begin{cases} (s_L, c_L), & x < 0, \\ (s_R, c_R), & x \geq 0. \end{cases} \tag{2}$$

and physically motivate the Isaacson-Glimm admissibility criterion.

Isaacson-Glimm admissibility criterion (KKIT condition)

A contact discontinuity between U_+ and U_- is admissible if and only if c is continuous and monotone along the sequence of contact curves, connecting U_- and U_+ (see Fig. 2b on the right).

- Correspond to Lax admissible contacts
- Monotone case: U_- and U_+ are on the same side of the coincidence

Theorem 1 (on existence and uniqueness [1])

Under KKIT entropy condition the solution to any Riemann problem (1), (2) exists and unique (see Fig. 1).

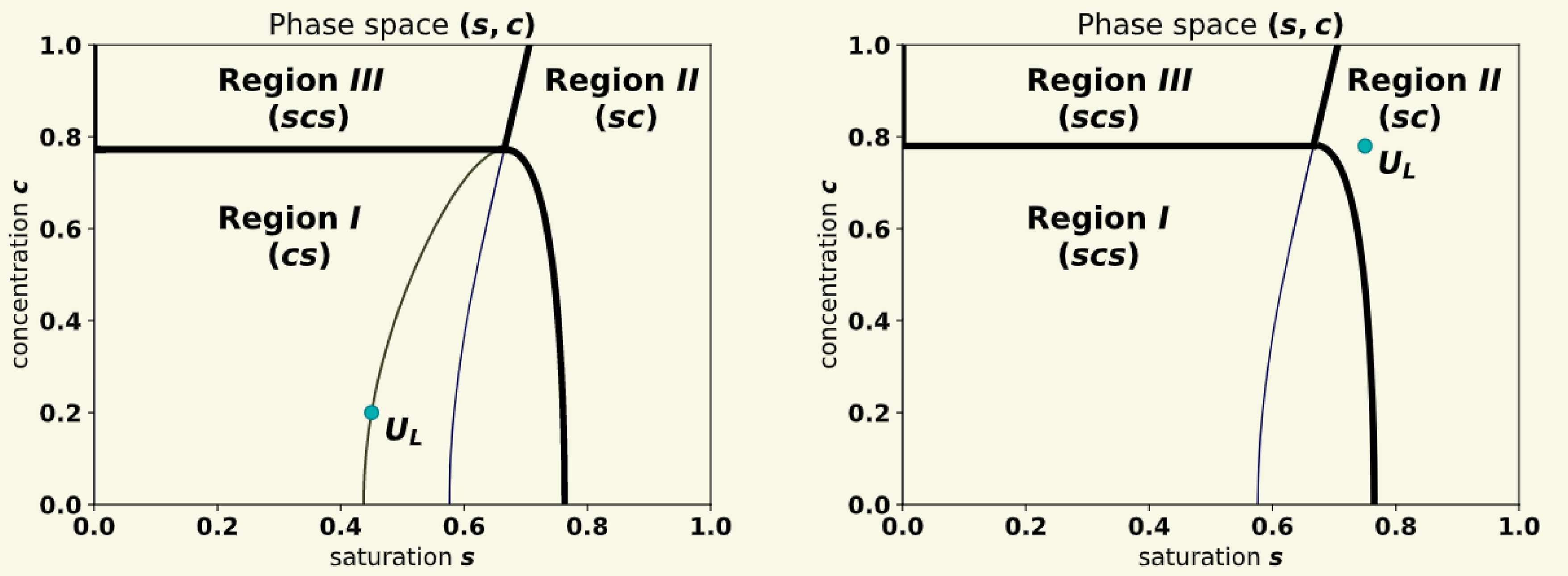


Fig. 1: Solution diagrams (U_R-regions for U_L to the left and right of coincidence)

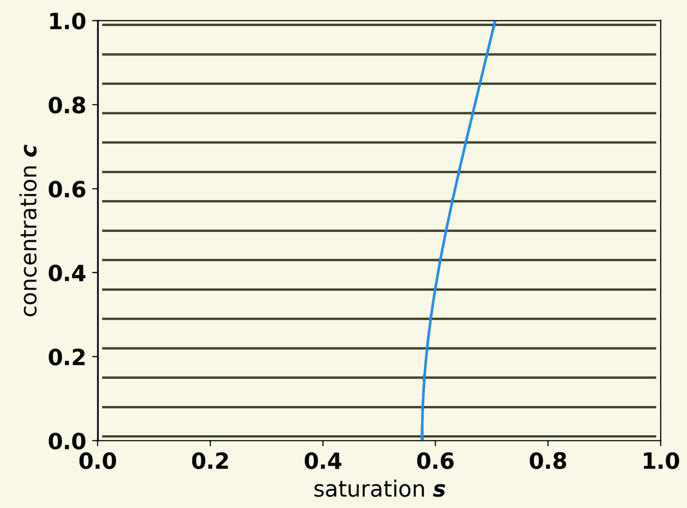
Characteristic families

s-wave

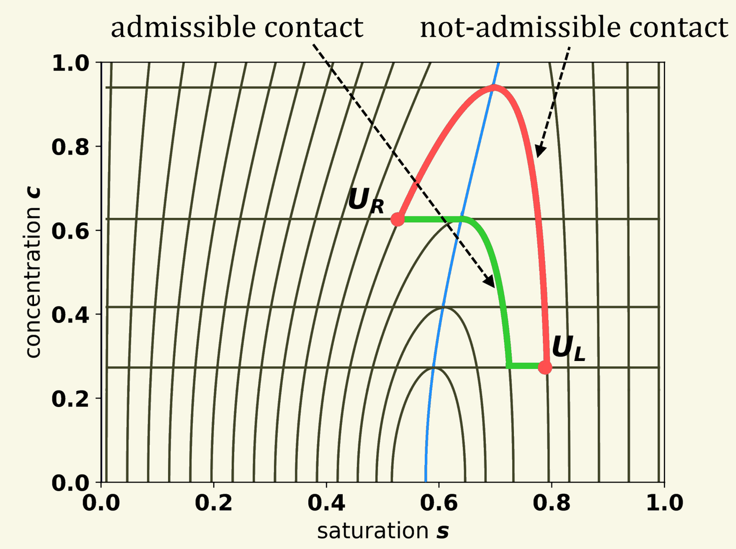
- $\lambda^{(s)} = f_s$
- solve the Buckley-Leverett equation $c = const$
- Riemann invariant $c = const$

c-wave

- $\lambda^{(c)} = f/s$
- are contact discontinuities (linearly degenerate field)
- Riemann invariant $f/s = const$



2a:



2b:

– coincidence locus $\{\lambda^{(s)} = \lambda^{(c)}\} \Rightarrow$ non-strictly hyperbolic system

- non-uniqueness of solutions $\Rightarrow$ need for admis. criteria for c-waves

Vanishing adsorption admissibility criterion

A contact discontinuity between U_+ and U_- is **admissible** provided it is the L^1_{loc} limit of a family of admissible solutions of the system (3), as $\alpha \to 0$

$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ [c\,s + \alpha a(c)]_t + [c\,f(s,c)]_x &= 0, \end{aligned} \tag{3}$$

Here $a(c)$ — adsorption function (bdd, positive, convex, $a(0) = 0$).

Theorem 2 (P., Marchesin, Plohr [2])

The set of admissible Riemann solutions for the model (1),(2) is the same for the KKIT entropy condition and vanishing adsorption admissibility criterion.

Corollary

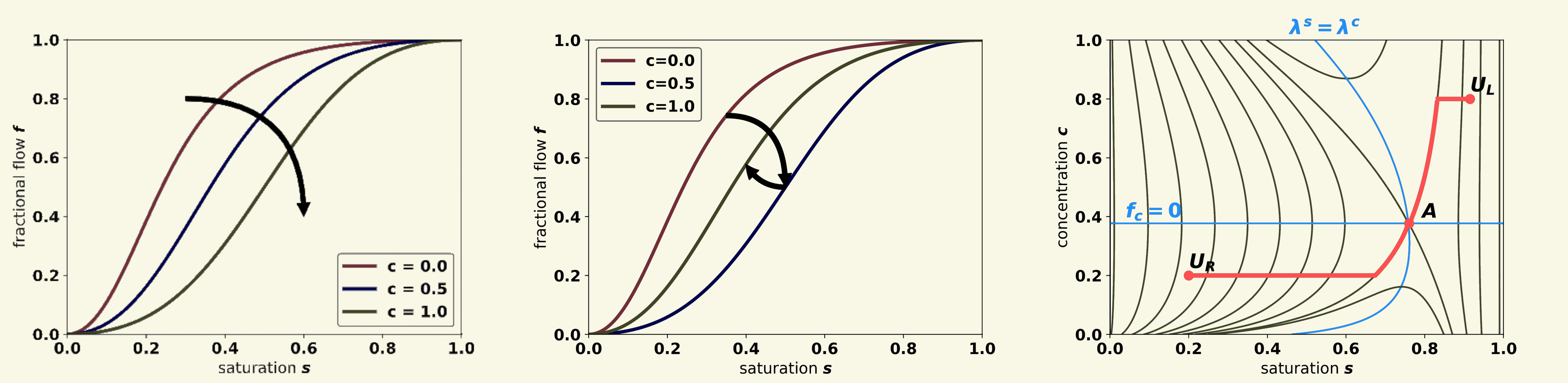
Under vanishing adsorption admissibility criterion the solution to a Riemann problem (1), (2) exists and unique.

History

- 1980 — Isaacson, Glimm (polymer flooding): ∃! for model (1)
- 1980 — Keyfitz, Kranzer (elastisity theory)
- 1988 — Johansen, Winther (polymer flooding): ∃! for model (3)

Non-monotone chemical flooding model

Fig. 3: left — fractional flow functions $f(s,c)$; right — c-waves; red — solution with non-Lax c-contact



Consider simple non-monotone chemical flooding model:

A1 f is non-monotone in c, that is $\forall s \quad \exists c^* = c^*(s)$ such that

$$f_c < 0 \quad \text{for } c < c^* \qquad \text{and} \qquad f_c > 0 \quad \text{for } c > c^*.$$

A2 Curves $\{\lambda^{(s)} = \lambda^{(c)}\}$ and $\{f_c = 0\}$ intersect at point $A = (s^*, c^*)$ transversally in phase space $(s,c) \in [0,1]^2$.

- Theorems 1 and 2 are valid under assumptions A1, A2.
- In contrast to monotone case, non-Lax contacts are admissible in non-monotone case (see Fig. 3). They can be seen as a limit of transitional rarefactions and shocks that are admissible for M_α.

For model M_α we call a c-shock **admissible** if it could be obtained as a limit of smooth travelling wave solutions of (4) as $\varepsilon_{c,d} \to 0$

$$\begin{aligned} s_t + f(s,c)_x &= \varepsilon_c (s_x)_x, \\ (cs + \alpha a(c))_t + (cf(s,c))_x &= \varepsilon_c (cs_x)_x + \varepsilon_d (c_x)_x. \end{aligned} \tag{4}$$

Here $A(s,c)$ bdd from zero and infinity function. Denote $\kappa := \varepsilon_d/\varepsilon_c$.

Theorem 3 (Bakharev, Enin, P., Rastegaev [3] for $c^- = 1, c^+ = 0$)

Fix $c^- > c^* > c^+$. There exist $0 < v_{\min} < v_{\max} < \infty$, such that for all $\kappa = \varepsilon_d/\varepsilon_c \in (0, +\infty)$, there exists a unique

- points $s^-(\kappa) \in [0,1]$ and $s^+(\kappa) \in [0,1]$;
- velocity $v(\kappa) \in [v_{\min}, v_{\max}]$,

such that there exists a travelling wave, connecting two saddle points $u^-(\kappa) = (s^-(\kappa), c^-)$ and $u^+(\kappa) = (s^+(\kappa), c^+)$ with velocity $v(\kappa)$. Moreover, $v(\kappa)$ is monotone and continuous; $v(\kappa) \to v_{\min}$ as $\kappa \to \infty$; $v(\kappa) \to v_{\max}$ as $\kappa \to 0$. Thus, transitional shock depends on ratio of small diffusion/capillary terms, but limiting transitional contact don't

References

[1] Isaacson, E.L., 1980. Global solution of a Riemann problem for a non-strictly hyperbolic system of conservation laws arising in enhanced oil recovery. Rockefeller University, New York, NY, preprint

[2] Yu. Petrova, D. Marchesin, B. Plohr. Work in progress

[3] F. Bakharev, A. Enin, Yu. Petrova, N. Rastegaev. Impact of dissipation ratio on vanishing viscosity solutions of the Riemann problem for chemical flooding model. arxiv:2111.15001, 2021.

[4] Johansen, T. and Winther, R., 1988. The solution of the Riemann problem for a hyperbolic system of conservation laws modeling polymer flooding. SIAM journal on mathematical analysis, 19(3), p.541-566.

[5] Keyfitz, B.L. and Kranzer, H.C., 1980. A system of non-strictly hyperbolic conservation laws arising in elasticity theory. Archive for Rational Mechanics and Analysis, 72(3), pp.219-241.

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