Questions for exam. Last update: 18/04/2023.

Part 1: Around wave equation.

1. Wave equation, “physical” derivation (balls and springs).
2. Wave equation, derivation from general principles.
3. D'Alambert's formula for 1D wave equation, and well-posedness of Cauchy problem on real line.
4. Inhomogeneous wave equation. Duhamel principle.
5. Mixed initial-boundary value problem for wave equation: existence and uniqueness of solution.
6. Mixed initial-boundary value problem for wave equation: solution by a Fourier series.

Part 2: Conservation and balance laws.

7. Fluid flow: Eulerian vs. Lagrangian point of view; flow map; incompressibility condition.
8. Fluid flow: scalar transport equation, conservation of mass.
9. Scalar conservation law. Weak form of solution. Rankine-Hugoniot condition.
10. Burgers equation: blow-up in finite time, explicit solutions to different Riemann problems, multiplicity of solutions, definition of entropy solution, irreversibility.
11. Scalar conservation law with convex flux function: equivalent definitions of entropy conditions.
12. Scalar conservation law with convex flux function: theorem on existence of entropy solution. Lemmas 1 and 2 describing properties for discrete approximation (boundedness, entropy condition).
13. Scalar conservation law with convex flux function: theorem on existence of entropy solution. Lemmas 3, 4 and 5 describing properties for discrete approximation (space and time estimates, stability).
14. Scalar conservation law with convex flux function: theorem on existence of entropy solution. Lemma 6 on convergence and properties of the limiting solution.
15. Scalar conservation law with convex flux function: theorem on existence of entropy solution. Lemmas 7 and 8 on properties of the limiting solution.