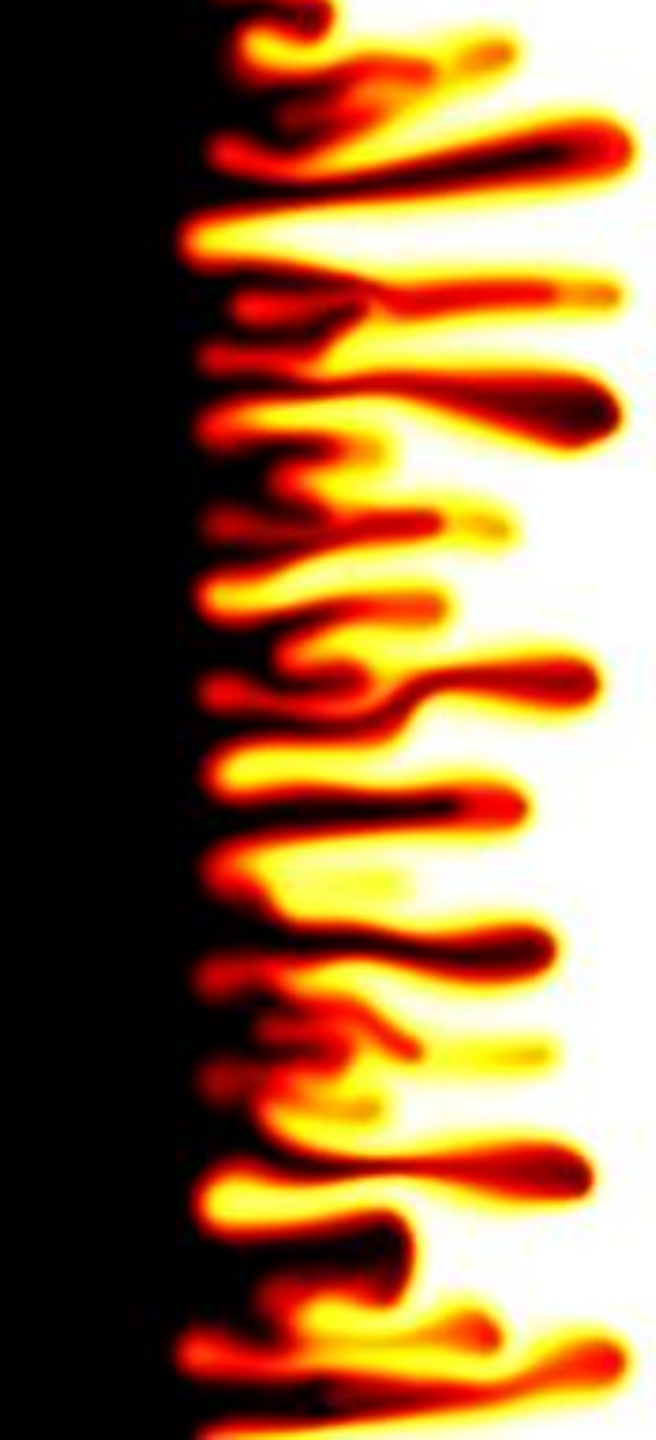

Graded viscosity banks on the rear end of the polymer slug

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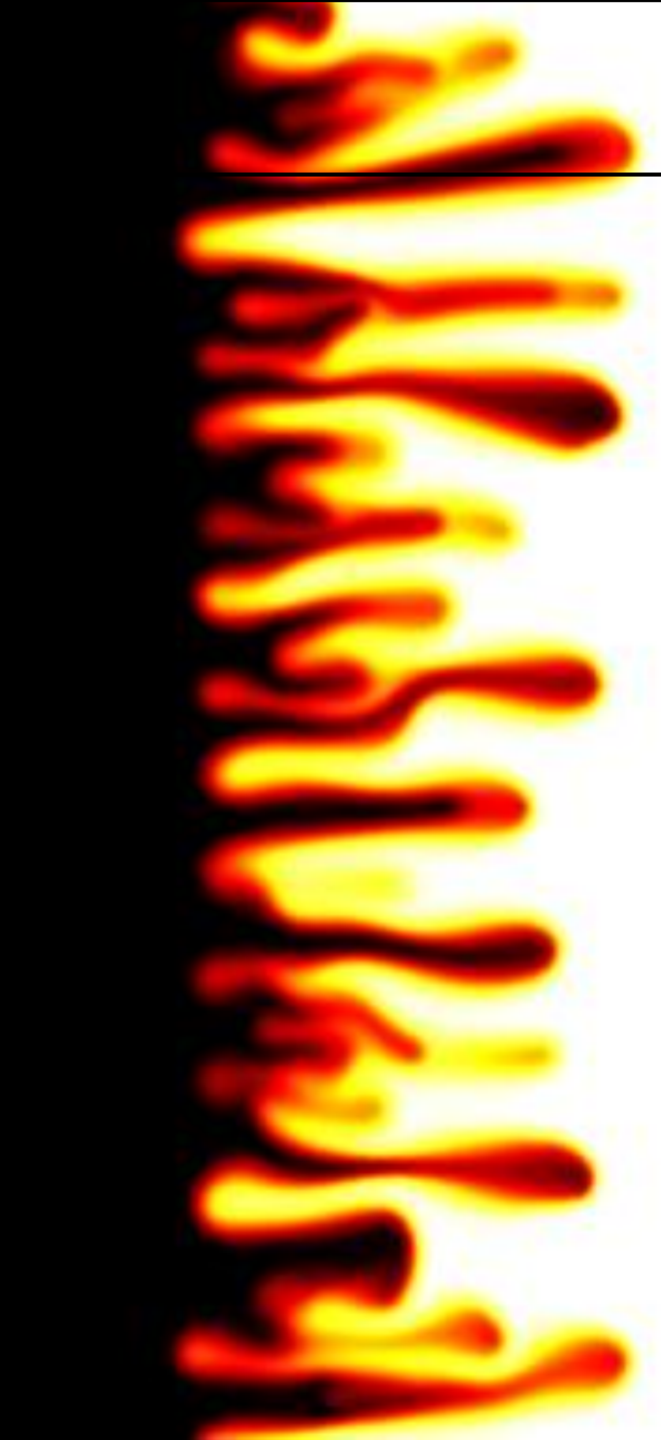


Outline of the talk

1. General context
   a. Chemical EOR, polymer slugs
   b. Viscous fingering phenomenon
   c. Velocities of the mixing zones

2. Graded Viscosity banks = GVB (tapering)
   a. Main idea of Claridge
   b. Problem statement
   c. Results for small number of slugs
   d. Results for $n \to \infty$
   e. Numerical validation of GVB
   f. Generalizations and discussions

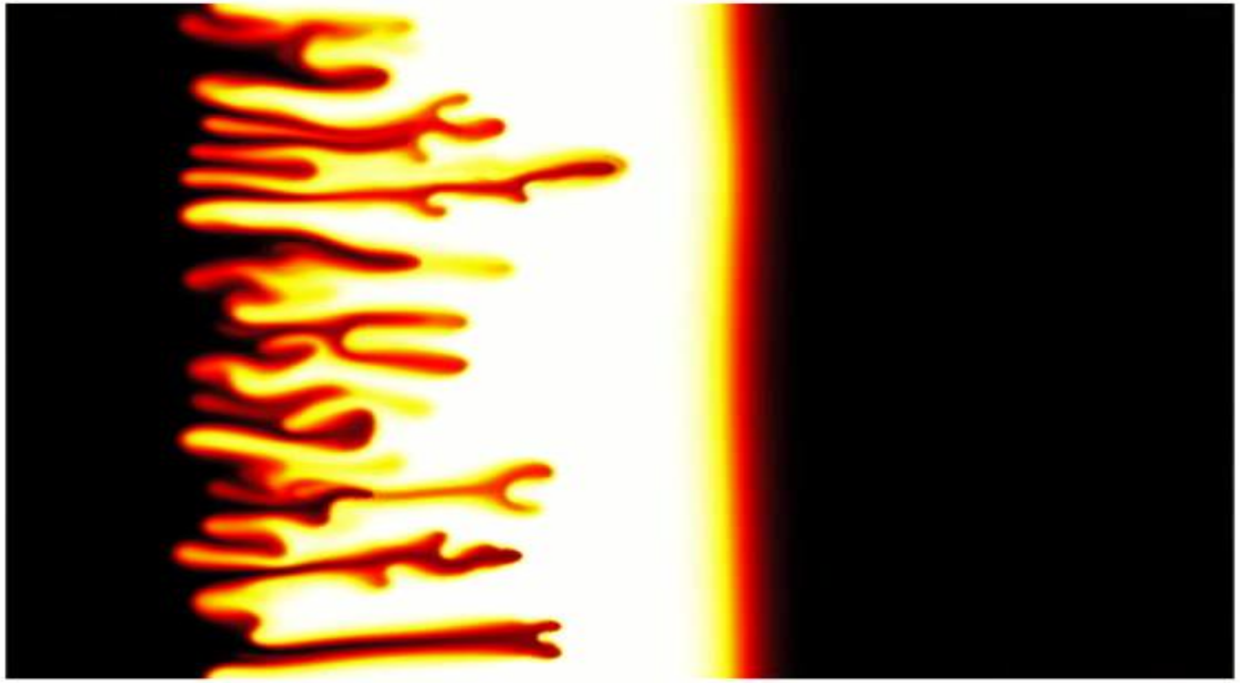
3. Transverse Flow Equilibrium model
   a. Does it give “pessimistic” estimates?
   b. How to take into account adsorption?

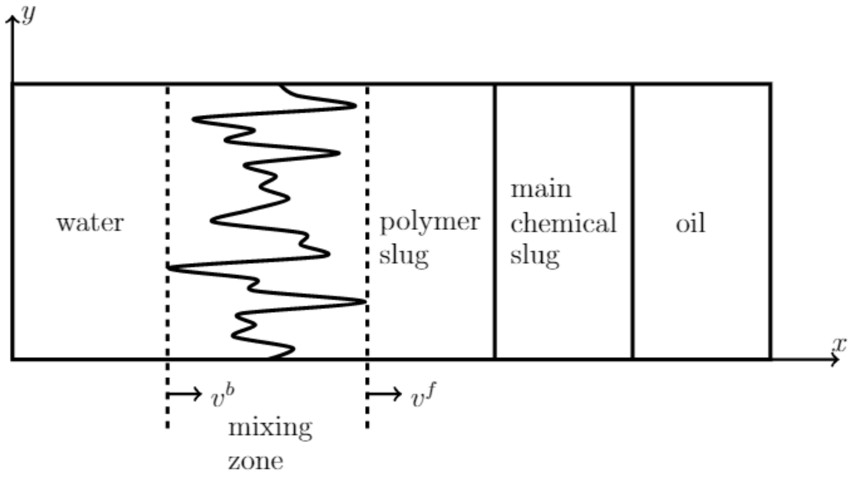
4. Conclusions, acknowledgements
   a. Industrial collaboration with IMPA via Centro Pi https://centropi.impa.br/

Breakthrough of polymer slug

- Homogeneous porous media
- Instability occur due to different viscosities (viscous fingering effect)
- After the breakthrough of the polymer slug the positive effect decreases

- Chemical EOR:
  - Polymer flooding
  - Surfactant flooding
  - ASP-flooding





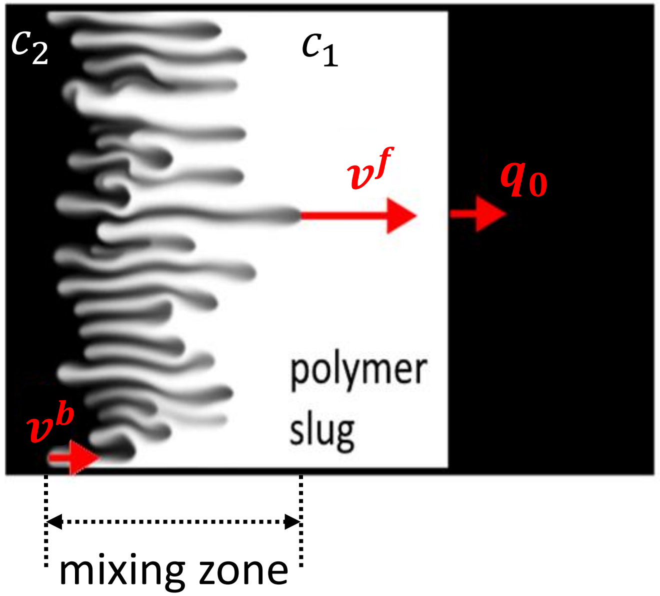
Question: what size of polymer slug?

Velocities of the mixing zone

- One-phase miscible displacement (Peaceman model)

$$c_t + u \cdot \nabla c = \varepsilon \, \Delta c$$
$$div\, u = 0$$
$$u = -\frac{k}{\mu(c)} \nabla p$$

c – polymer concentration
$\mu(\mathrm{c})$ – water viscosity



q_0 – Velocity of the stable front. Take $q = 1$

v^f – Velocity of the front end of the mixing zone is constant [1]

v^b – Velocity of the rear end of the mixing zone is constant [1]

How to determine the velocity of the mixing zone?

- Development and implementation of an oil-field experiment
- Laboratory tests
- Numerical simulation
- Analytical expressions

c_1 – concentration of injected polymer
c_2 – decreased concentration of injected polymer (for one slug $c_2 = 0$ – injection of water)

- often v_f is considered to be a function of $M = \frac{\mu(c_1)}{\mu(c_2)}$

[1] *Nijjer, J.S., Hewitt, D.R. and Neufeld, J.A., 2018. The dynamics of miscible viscous fingering from onset to shutdown*

Velocities of the mixing zone

- One-phase miscible displacement (Peaceman model)

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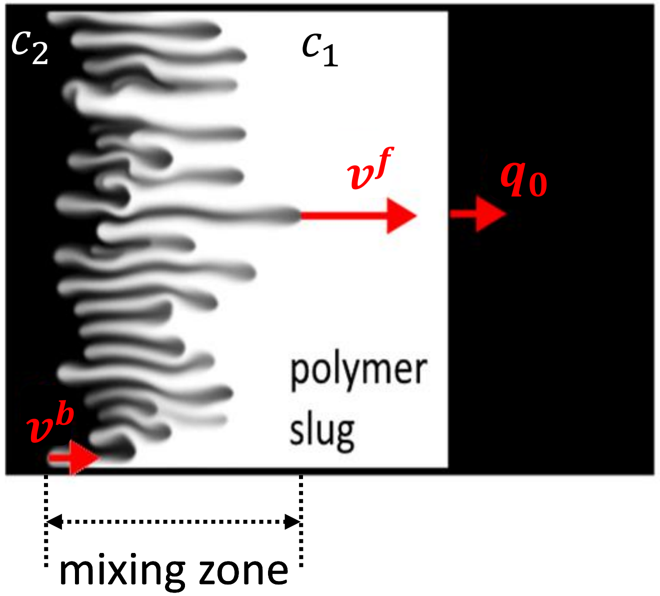

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v^f – Velocity of the front end of the mixing zone is constant

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Empirical models of velocities

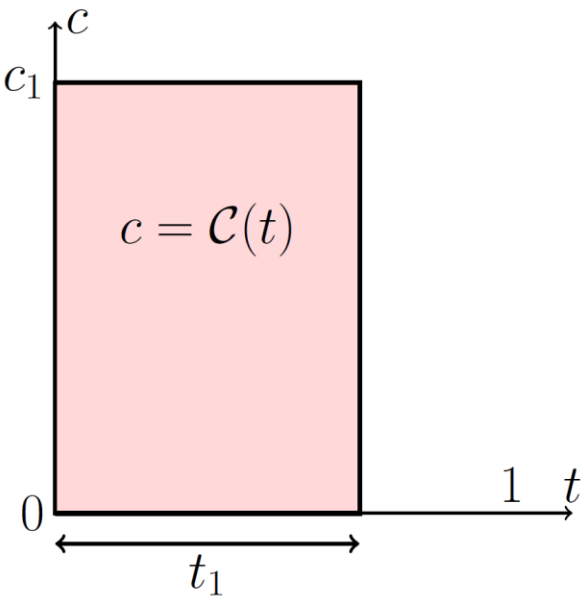
- Averaging - “effective viscosity” M_e:
  - Koval (1963)
  - Todd-Longstaff (1972)
- Transverse Flow Equilibrium (TFE) Otto-Menon, Yortsos-Salin (2006)
  - $p(x, y) = p(x)$

$M = \frac{\mu(c_1)}{\mu(c_2)} > 1$ – ratio of viscosities

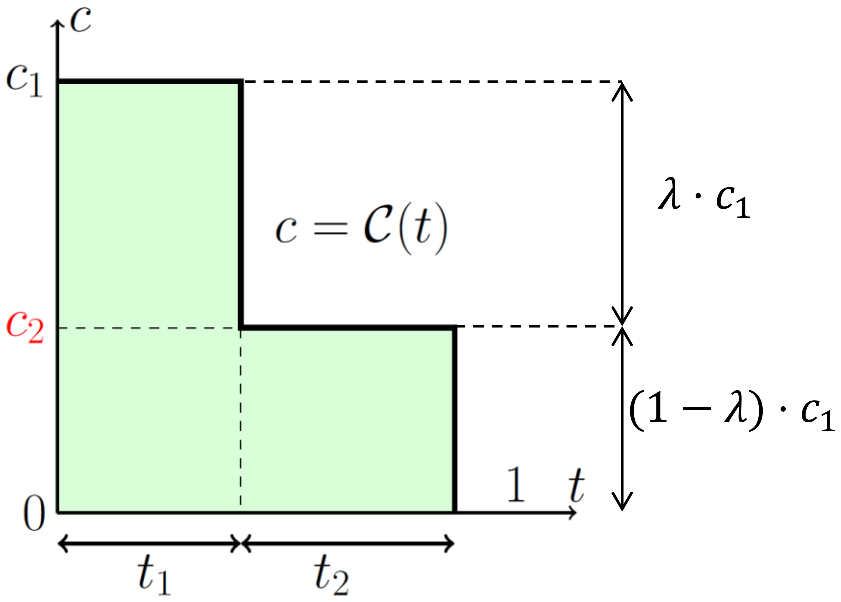
Koval	$v^f = M_e \quad v^b = \frac{1}{M_e} \quad M_e = \left(\alpha \cdot M^{\frac{1}{4}} + (1-\alpha)\right)^4$
Todd-Longstaff	$v^f = M_e \quad v^b = \frac{1}{M_e} \quad M_e = M^{\omega}$
TFE	$v^f \leq \frac{\overline{m}(c_1, c_2)}{m(c_2)} \quad v^b \geq \frac{v^b}{M} \quad m(c) = 1/\mu(c)$

Main idea

Injecting two slugs may give gain in polymer mass (Claridge, 1978)



Add concentration c_2



“Convexity” argument

$$\begin{cases} \lambda \dfrac{1}{m(c_1, c_2)} + (1-\lambda) \dfrac{1}{m(c_2, 0)} > \dfrac{1}{m(c_1, 0)} & (TFE) \\[2em] \lambda\, m^{\omega}(0) + (1-\lambda)\, m^{\omega}(c_1) > m^{\omega}(c_2) & (TL) \end{cases} \iff V > V \quad \forall c_2 \in (0, c_1)$$

Problem Statement

- Goal: reduce amount of polymer

- Strategy: graded viscosity banks (GVB, tapering) Claridge (1978)

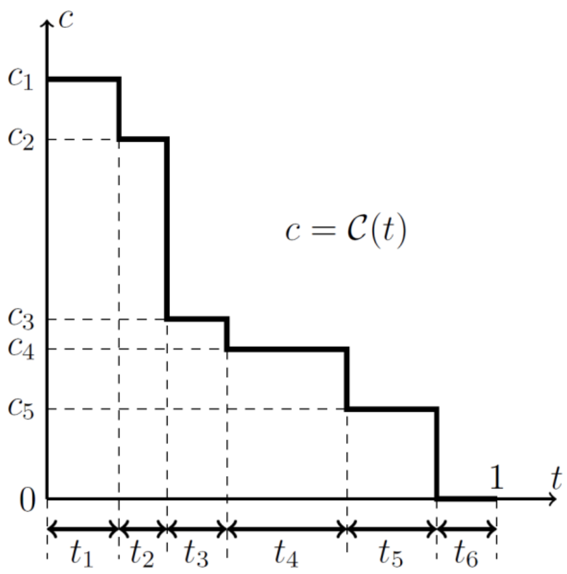
- We want no breakthrough in any slug

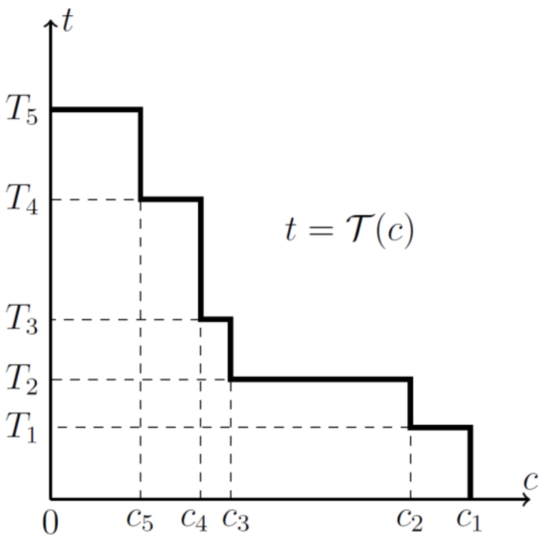
- Given concentrations c_n and v_b, v_f we can find sizes of slugs t_n without breakthrough

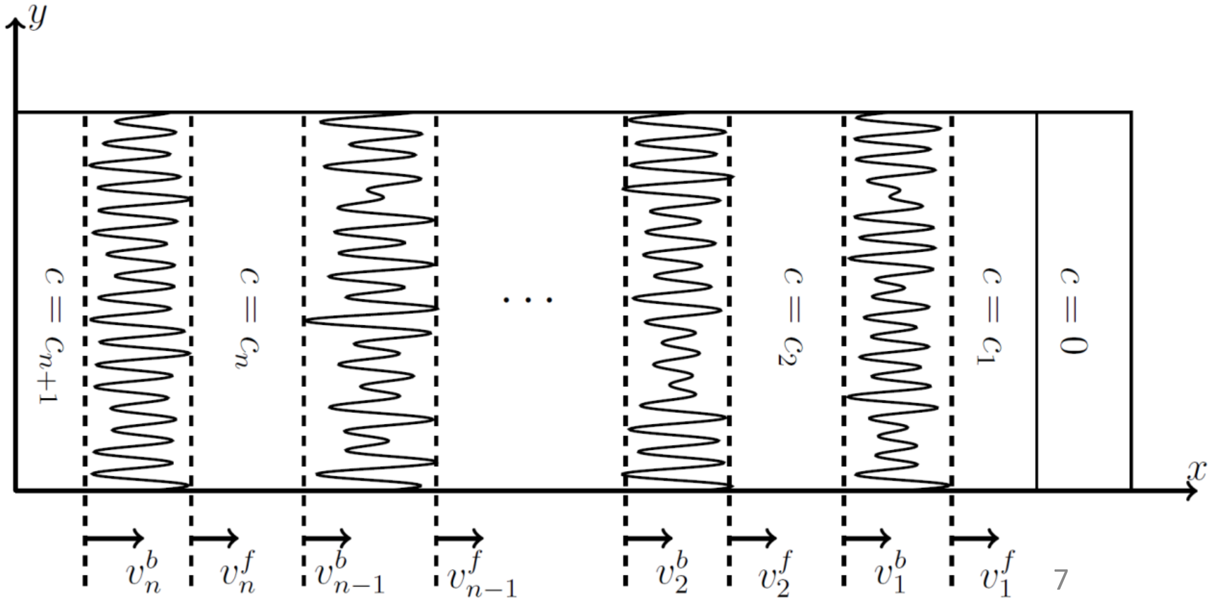
- Choose concentrations c_n to minimize amount of polymer

$$\mathrm{V_n} = \sum_{i=1}^{n} c_i t_i \rightarrow \ \min$$

- Questions:
  - n – small ($n = 2, 3, 5$)
  - $n \rightarrow \infty$







Problem Statement

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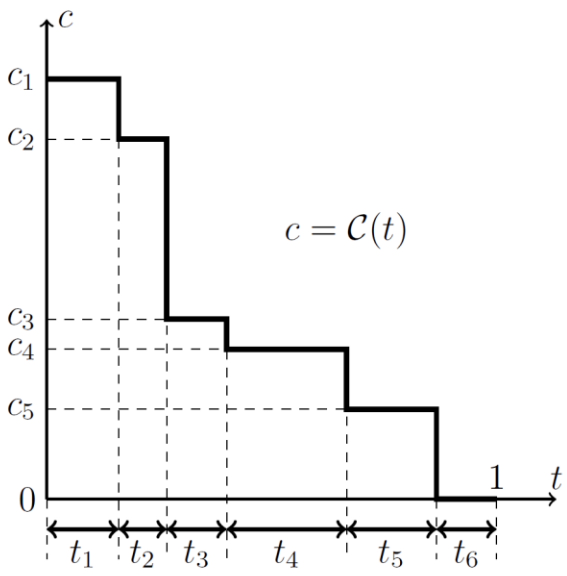
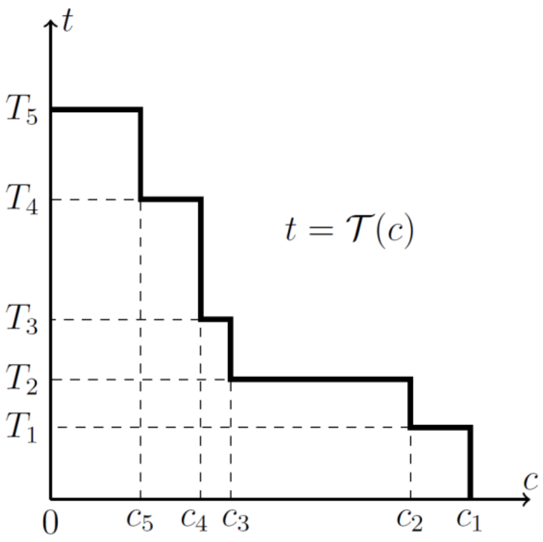
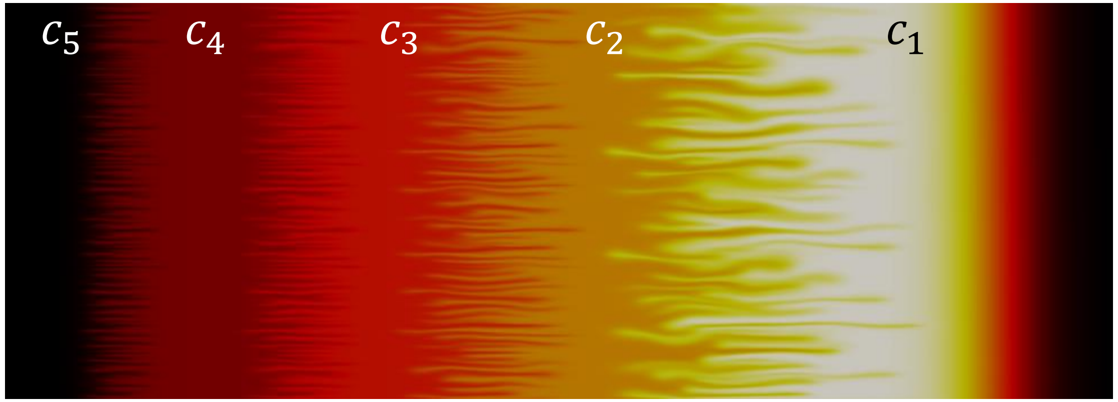






Simulation of GVB in DuMuX by a member of our group in SPSU Dmitry Pavlov

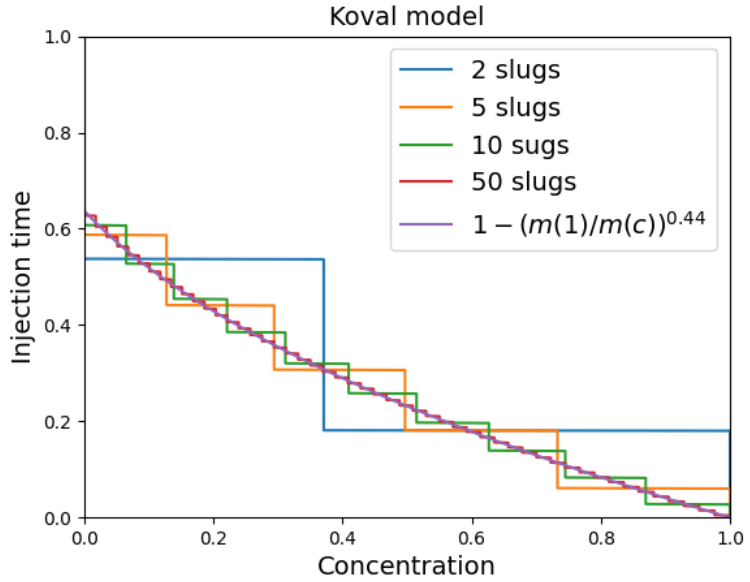
Results for small n

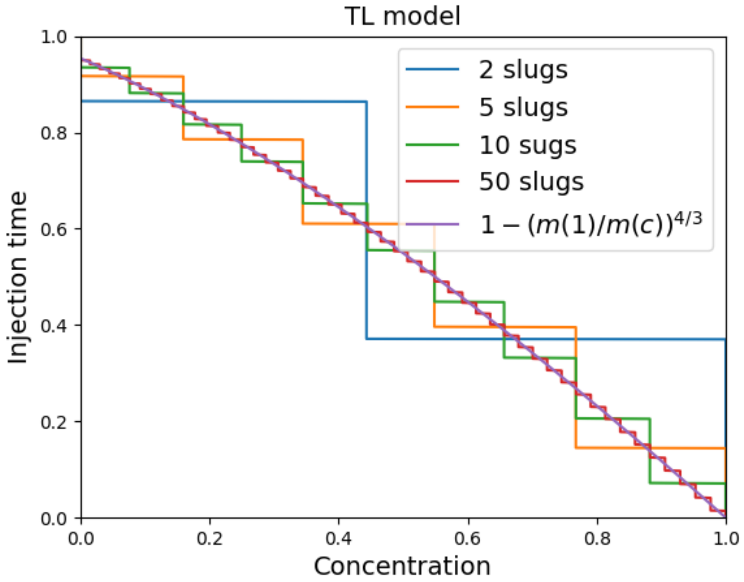
V_n - polymer mass for n slugs

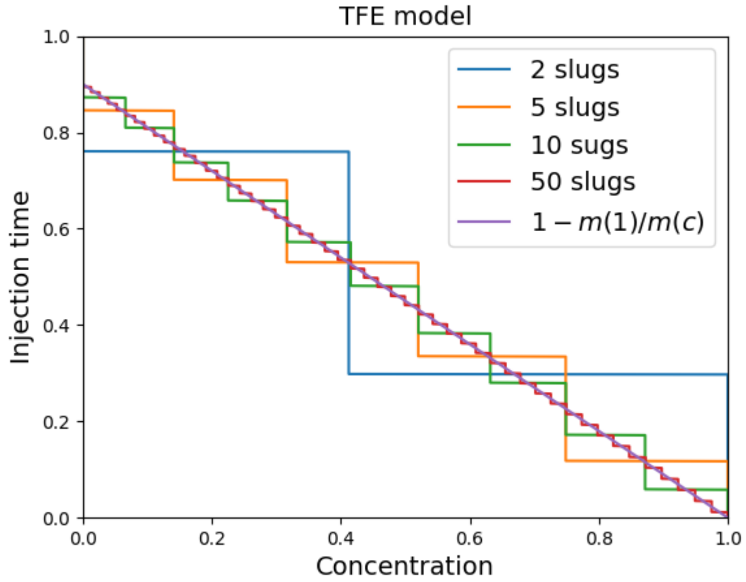
$\eta = \frac{V_1 - V_n}{V_1}$ - percentage of gain in polymer mass

	n = 2	n = 3	n = 4	n = 5	n = 10	Limit
TFE	19,83%	23,35%	24,57%	25,13%	25,88%	26,12%
Todd-Longstaff	24,84%	29,36%	30,93%	31,66%	32,63%	32,95%
Koval	33,21%	39,24%	41,46%	42,55%	44,24%	45,28%

- Conclusion: in practice it is enough to use 2-3 slugs
- For detailes see arxiv:2012.03114







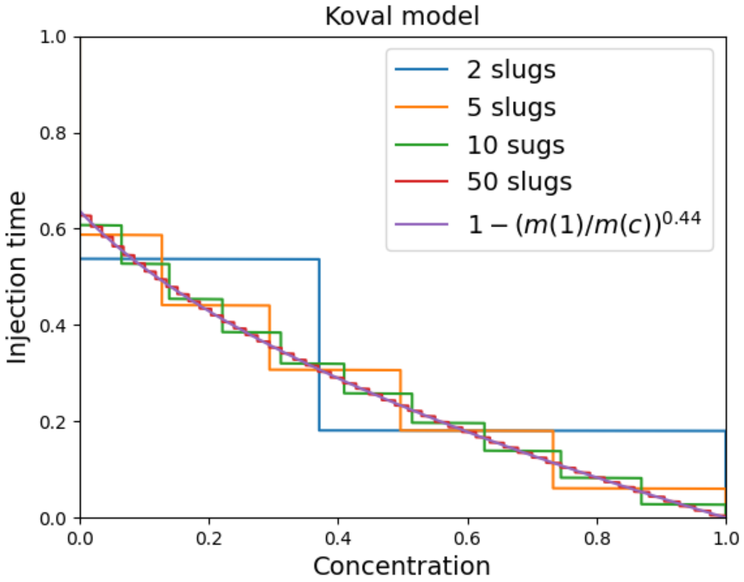
Graded viscosity banks: $n \to \infty$

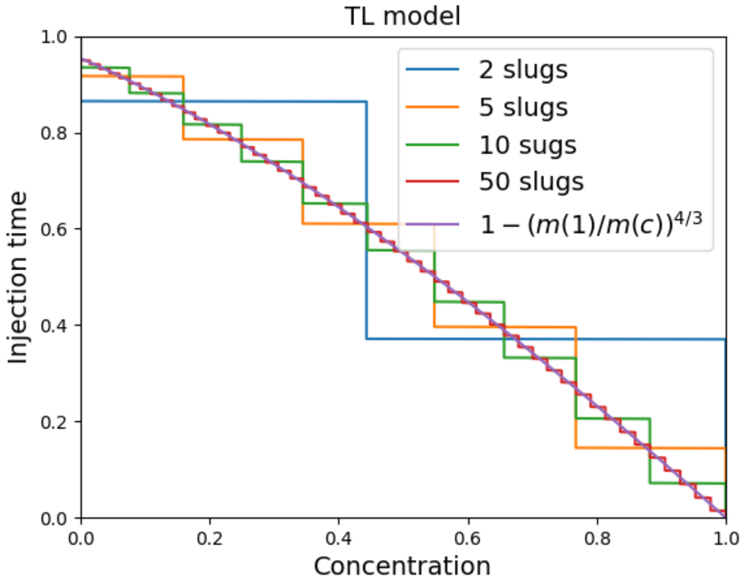
Theorem [Bakharev, Enin, Kalinin, P., Rastegaev, Tikhomirov, 2021]

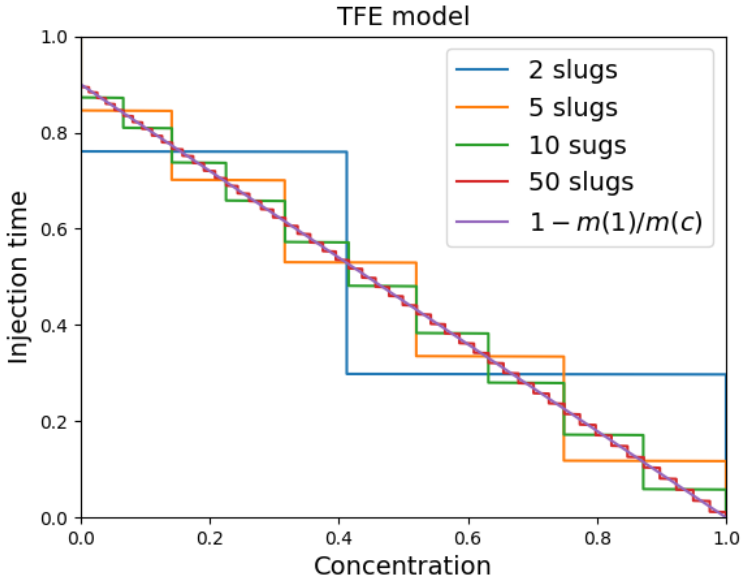
As $n \to \infty$ the optimal limiting injection profile

$$T^{\infty}(c) = 1 - \left(\frac{\mu(c)}{\mu(c_1)}\right)^{\beta}$$

Koval: $\beta = 2\alpha$	Todd-Longstaff: $\beta = 2\omega$	TFE: $\beta = 1$





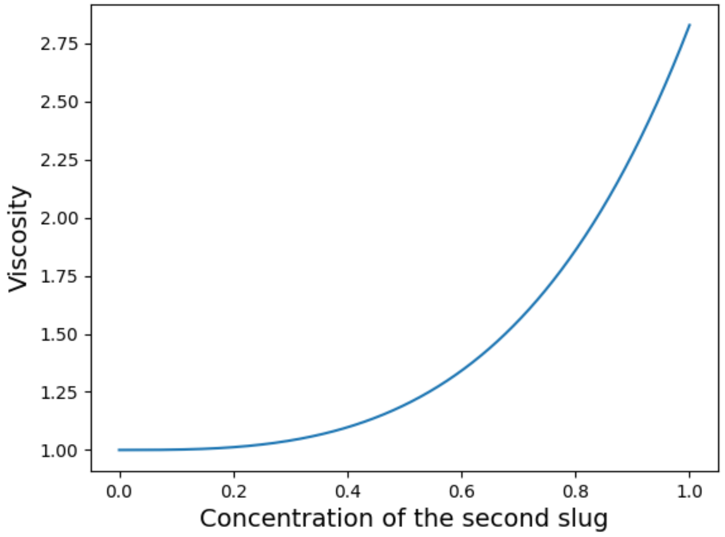


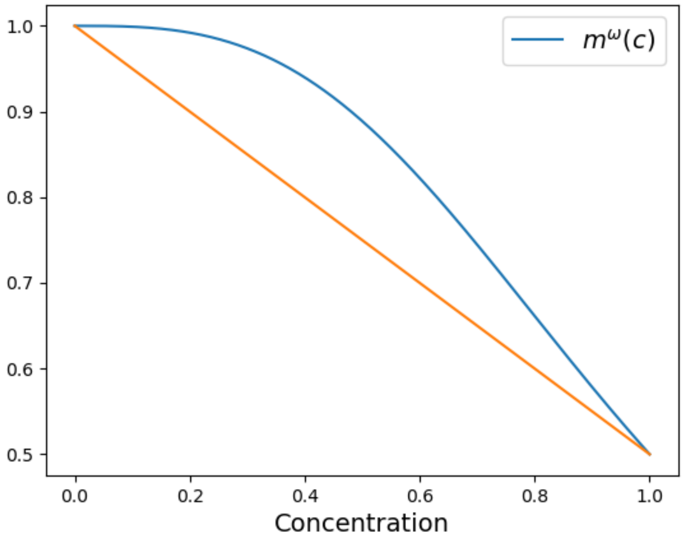
For detailes see arxiv:2012.03114

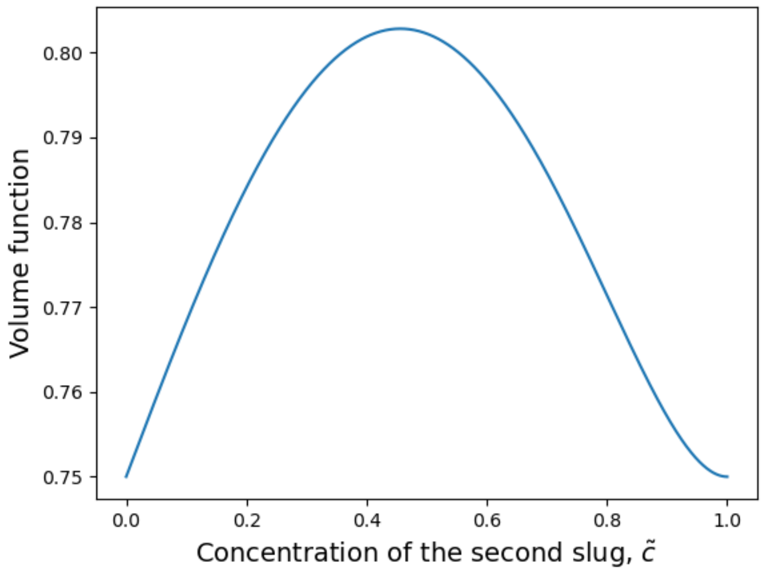
Discussions

- For TL model GVB gives gain not for all viscosities

Counterexample – $\mu = (1 + c^3)^{\frac{3}{2}}$ – there is no gain in polymer mass even if you change 1 slug for 2 slugs!







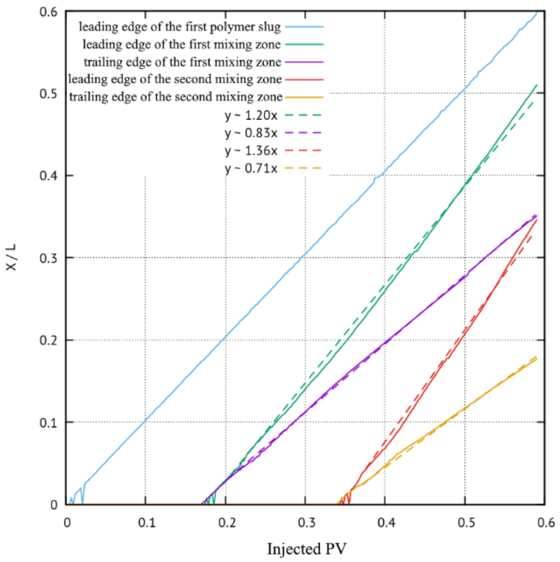
- In the Theorem we have extra assumptions for Koval and Todd Logstaff models. The main two are:
  - $f\left(\frac{m(c')}{m(c)}\right)$ is convex function of c for any $c' < c$, where $v_j^f = \frac{1}{f\left(\frac{m(c_{j+1})}{m(c_j)}\right)}$ and $v_j^b = f\left(\frac{m(c_{j+1})}{m(c_j)}\right)$
  - $f(ab) \leq f(a)f(b)$
- For TFE model – no extra assumptions
- So the effectiveness of GVB is *model dependent*.

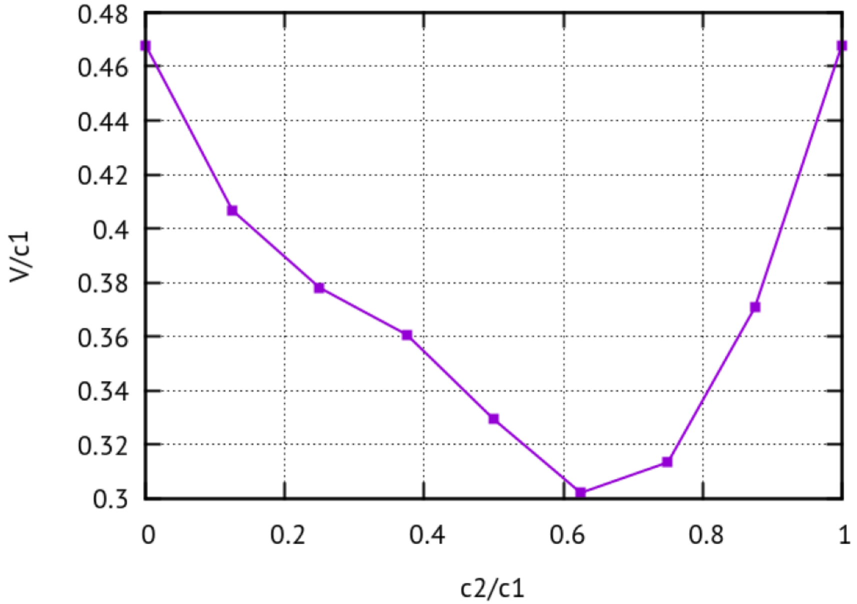
Numerical validation of graded viscosity banks

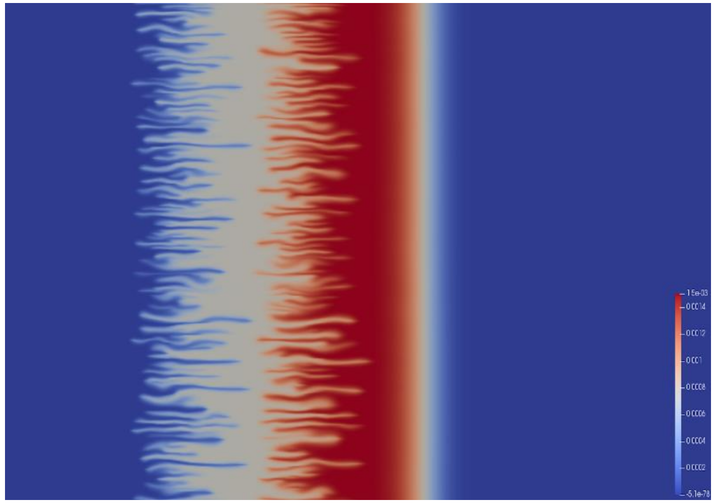
- Simulations in DuMuX, 2 slugs
- Velocities are taken from simulations

Theoretical assumptions that need to be validated:

- the velocities of the mixing zones edges remain constant even in the case of the presence of several slugs;
- the velocities of the mixing zones edges do not depend on the presence of additional slugs, on the sizes of the slugs and on the size of the modeling area;







Possible generalizations

- **Adsorption**

$a(c)$ – adsorption of polymer – concave function (e.g. Langmuir isotherm)
Easy to take into account in TFE model, and not clear for Koval or Todd-Longstaff models
Consider irreversible adsorption: only v_0 changes

$$\partial_t\big(c + a(c)\big) + q_0 \partial_x c = \varepsilon \partial_{xx} c$$

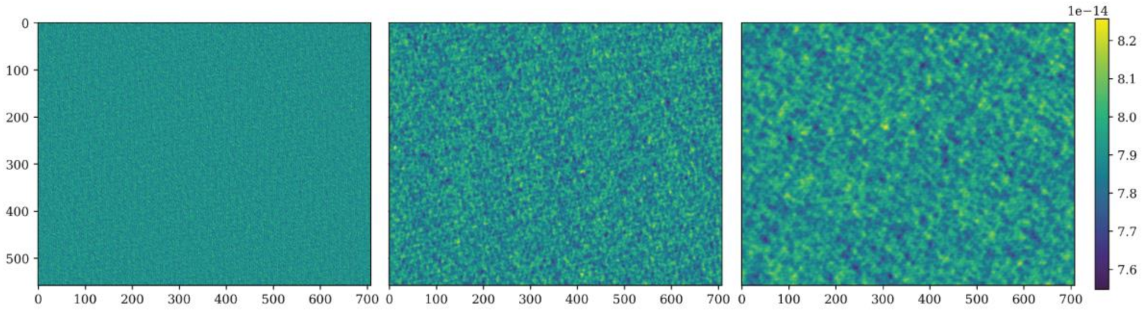
$$\boxed{v_0 = \frac{1}{1 + \frac{a(c_1)}{c_1}} q_0.}$$

Consider fully reversible adsorption:

$$\boxed{v_j^f \le \frac{1}{1 + a'(c_j)} \frac{\overline{m(c_j, c_{j+1})}}{m(c_j)} q_0, \qquad v_j^b \ge \frac{1}{1 + a'(c_{j+1})} \frac{\overline{m(c_j, c_{j+1})}}{m(c_{j+1})} q_0}$$

- **Permeability**

Considering permeability k to be random (e.g. lognormal).
How spatial correlation influence the velocity of the mixing zone?



- **Two-phase flow**

Are velocities constants? Probably, not. Can we write pessimistic estimates?
More advanced questions: can we take into account hysteresis of relative permeabilities? Non-Newtonian effects?

Transverse Flow Equilibrium model. Intro.

- Let us consider Peaceman model with an extra assumption

$$p(x,y) \sim p(x), \qquad p_x(x,y) \sim p_x(x)$$

Peaceman model		TFE model
$c_t + u \cdot \nabla c = \varepsilon \, \Delta c$ $div\, u = 0$ $u = -\dfrac{1}{\mu(c)} \nabla p = -m(c) \nabla p$	$\longleftrightarrow$	$c_t + u \cdot \nabla c = \varepsilon \, \Delta c$ $div\, u = 0$ $u = (u^x, u^y)$ $u^x = \dfrac{m(c)}{\overline{\overline{m(c)}}} q_0$

- TFE model has no pressure
- In 2d it is closed system of equations
  - Is it well-posed?
- TFE model contradicts to assumptions which deduces it

- Hypothesis:

TFE model always gives pessimistic estimates for velocities of the mixing zones for original Peaceman model

TFE model. Sub and super solutions

TFE model

$$
\begin{aligned}
c_t + u \cdot \nabla c &= \varepsilon \, \Delta c \\
div\, u &= 0 \\
u &= (u^x, u^y) \\
u^x &= \frac{m(c)}{\overline{\overline{m(c)}}} q_0
\end{aligned}
$$

Making rough estimates:

$$q_0 \cdot \frac{m(c)}{m(0)} \le u^x = \frac{m(c)}{\overline{\overline{m(c)}}} q_0 \le q_0 \cdot \frac{m(c)}{m(1)}$$

Consider 1-dim equations

$$c_t^{max} + q_0 \cdot \frac{m(c^{max})}{m(0)} \cdot c_x^{max} = \varepsilon \, (c^{max})_{xx}$$

$$c_t^{min} + q_0 \cdot \frac{m(c^{min})}{m(1)} \cdot c_x^{min} = \varepsilon \, (c^{min})_{xx}$$

Theorem [Otto-Menon for gravity driven fingers with extra assumptions, Yortsos heuristics for Peaceman model]

- If $c(0,x,y) < c^{max}(0,x)$ then $c(t,x,y) \le c^{max}(t,x)$
- If c$(0,x,y) > c^{min}(0,x)$ then $c(t,x,y) \ge c^{min}(t,x)$

- Doesn't say anything about $\bar{c}$, but give estimates on velocities

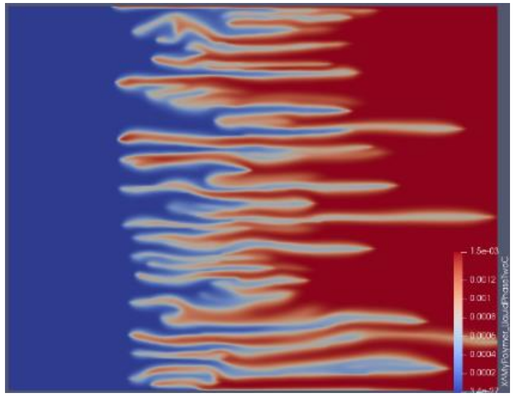
Consequence

- Speed of the tip of the fastest finger is less than

$$q_0 \cdot \frac{\int_0^1 m(c)\, dc}{m(1)}$$

- Speed of the rear front is bigger than

$$q_0 \cdot \frac{\int_0^1 m(c)\, dc}{m(0)}$$



TFE model. Taking into account adsorption

TFE model with adsorption

$$(c + a(c))_t + u \cdot \nabla c = \varepsilon\, \Delta c$$
$$div\, u = 0$$
$$u = (u^x, u^y)$$
$$u^x = \frac{m(c)}{\overline{m(c)}} q_0$$

Consider 1-dim equations

$$(c^{max} + a(c^{max}))_t + q_0 \cdot \frac{m(c^{max})}{m(0)} \cdot c_x^{max} = \varepsilon\, (c^{max})_{xx}$$

$$(c^{min} + a(c^{min}))_t + q_0 \cdot \frac{m(c^{min})}{m(1)} \cdot c_x^{min} = \varepsilon\, (c^{min})_{xx}$$

Theorem

- If $c(0, x, y) < c^{max}(0, x)$ then $c(t, x, y) \leq c^{max}(t, x)$
- If $c(0, x, y) > c^{min}(0, x)$ then $c(t, x, y) \geq c^{min}(t, x)$

Consequence

- Speed of the tip of the fastest finger is less than
$$q_0 \cdot \frac{\int_0^1 m(c)\, dc}{m(1)} \cdot \frac{1}{1 + a'(1)}$$
- Speed of the rear front is bigger than
$$q_0 \cdot \frac{\int_0^1 m(c)\, dc}{m(0)} \cdot \frac{1}{1 + a'(0)}$$

- The argument heavily relies on monotonicity of $c^{max}(x, t)$ and $c^{min}(x, t)$ in variable x

- What if the initial condition is non-monotone? This is the case for polymer slug

- Does the theorem stays valid for two-phase flow?

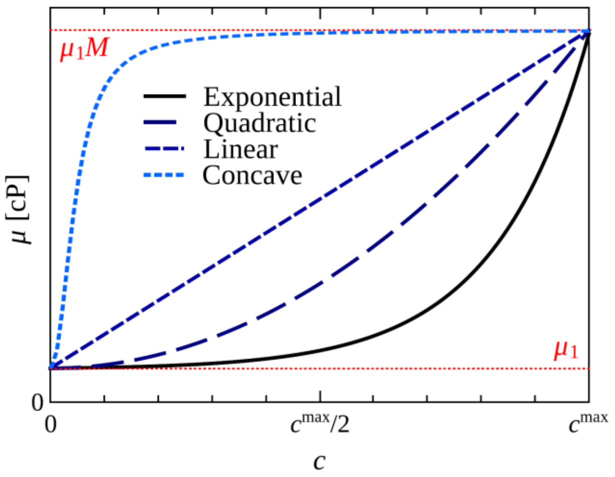
TFE model – always pessimistic?

- Numerical validation of TFE model:
  for different viscosity curves:
  always gives a pessimistic estimate

- Koval model not always gives a pessimistic estimate

- Examples when TFE model is exact:

Exponential viscosity – at the rear end
Concave viscosity – at the front end

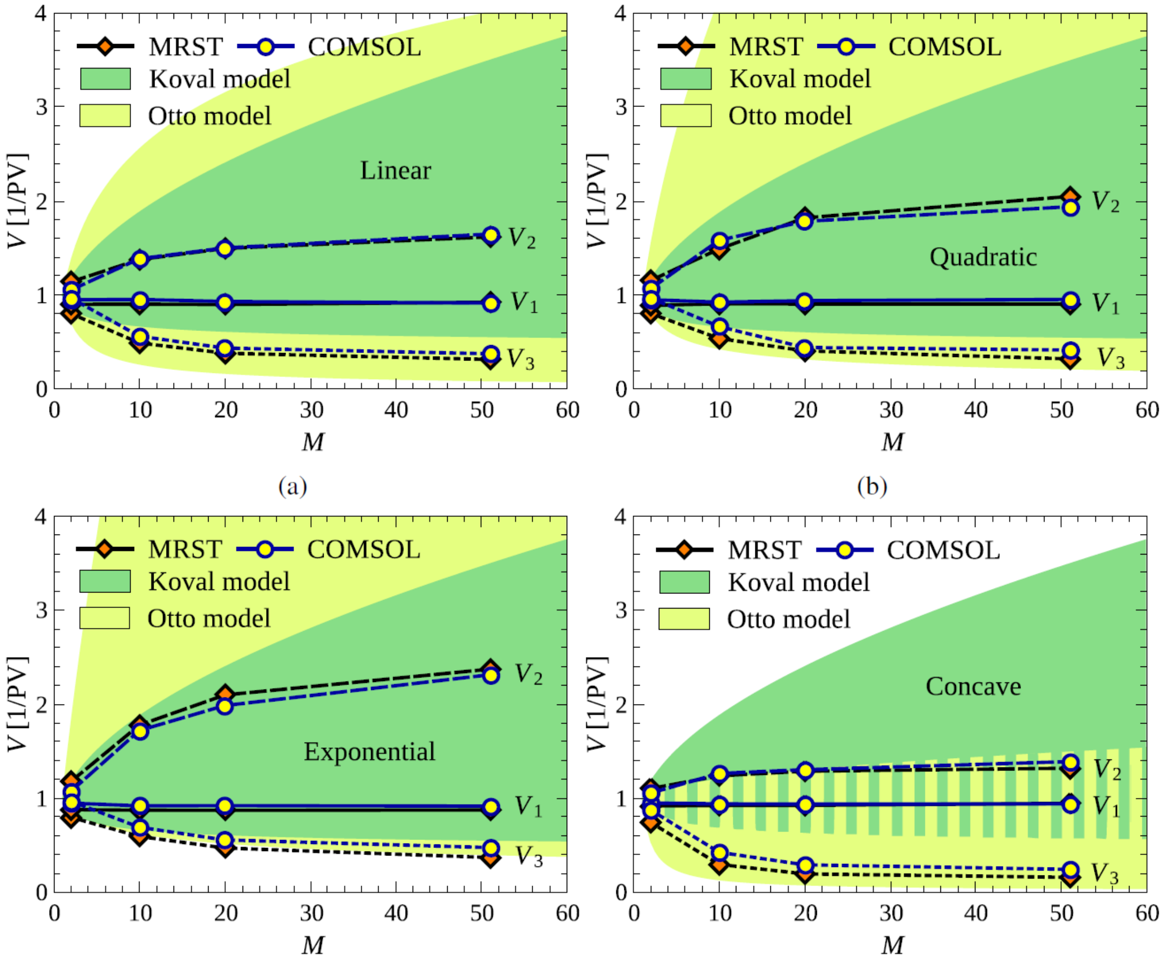


Exponential viscosity

$$u^x = \frac{m(c)}{\overline{m(c)}} u \approx u \cdot \frac{m(c)}{m(0)}$$

Concave viscosity

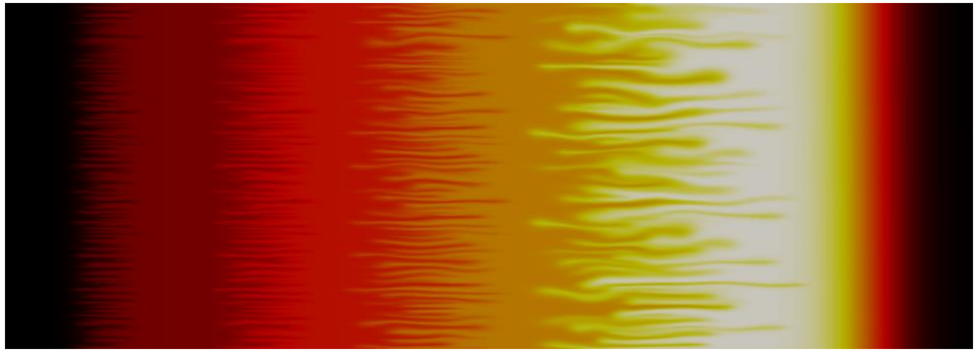
$$u^x = \frac{m(c)}{\overline{m(c)}} u \approx u \cdot \frac{m(c)}{m(1)}$$



For detailes see arxiv:2012.02849

Conclusions

1. Graded viscosity banks helps to reduce polymer mass with the same efficiency
2. In practice it is enough to inject 2-3 slugs
3. The choice of model for “finger velocities ” is an open problem – no rigorous results – TFE model looks promising



Simulation of GVB in DuMuX by our group in SPSU

The talk is based on:

- https://arxiv.org/abs/2012.03114 - “Optimal polymer slugs injection profiles”
  F. Bakharev, A. Enin, K. Kalinin, Yu. Petrova, N. Rastegaev, S. Tikhomirov

- https://arxiv.org/abs/2012.02849 - “Velocity of viscous fingers in miscible displacement”
  F. Bakharev, A. Enin, A. Groman, A. Kalyuzhnuk, S. Matveenko, Yu. Petrova, I.Starkov, S. Tikhomirov

If you have any questions, please, ask me: yulia.petrova@impa.br Yulia Petrova

Acknowledgements:

- the work was supported by the Russian Science Foundation grant 19-71-30002 and PJC “GazpromNeft”
- Very BIG thanks to all members of our group in Saint-Petersburg and colleagues from “GazpromNeft”!

Collaboration with IMPA

- Laboratory of Fluid Dynamics

http://fluid.impa.br

- Center PI – Centro de Projetos e Inovação IMPA

https://centropi.impa.br/

6 – 10 September – 7º Workshop de Soluções Matemáticas para Problemas Industriais
https://centropi.impa.br/7wsmpi/

Thank you for your attention!

If you have any questions, please, ask me:
yulia.petrova@impa.br Yulia Petrova

What mathematical theorem is hidden here?