Defensa de Memorial
Concurso IM-UFRJ MC-012

Yulia Petrova [1,2]

[1] IMPA, Instituto de Matematica Pura e Aplicada, Rio de Janeiro, Brazil

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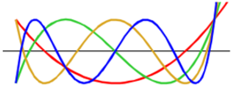
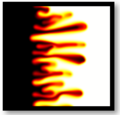
https://yulia-petrova.github.io/





30 November 2022
IM-UFRJ

A story of my academic path

About me

- Name: Petrova Yulia (Iuliia)
- Date of birth: 29 June 1991
- Place of birth: Ukhta, Russia
- Marital status: married, no kids
- Languages:
  - Russian (native)
  - English (fluent)
  - Portuguese (upper intermediate)
  - Spanish (intermediate)
- Personal web-site: https://yulia-petrova.github.io/

About me

2008–2013 | MSc in Mathematics, SPbSU, St. Petersburg, Russia. **Department of PDEs**

2013–2018 | PhD, SPbSU, Russia. "Exact L_2-small ball asymptotics for finite-dimensional perturbations of Gaussian processes". **Probability theory & spectral theory of operators**. Scient. advisor: Alexander I. Nazarov[a]

2017–2021 | Researcher at Chebyshev Laboratory, SPbSU, Russia. Participant of industrial projects with "GazpromNeft". **Fluid dynamics**

2021–2023 | Postdoc of excellence at IMPA, Brazil. Researcher at Centro Pi. **Fluid dynamics**. Mentor: Dan Marchesin.

Yulia Petrova

yulia-petrova.github.io

[a]ICM 2022 speaker

Teaching experience

Problem solving classes

09.2018 – 06.2021 Assistant at Department of Mathematics and Computer Science St. Petersburg State University, Russia

- probability theory
- calculus of variations
- complex analysis
- calculus

09.2014 – 06.2018 Assistant at Department of Mathematics and Information Technology, St. Petersburg Academic University, Russia

- calculus

09.2012 – 06.2015 Assistant at Institute of Physics, Nanotechnology and Telecommunications, St. Petersburg Polytecnic University, Russia

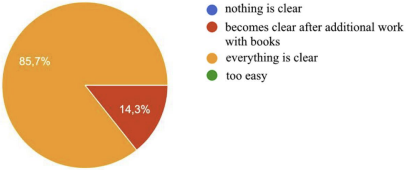
- partial differential equations

Olympiad mathematics

06.2012 – 06.2017 Teaching *Olympiad Mathematics* in "Formulo de Integreco", International educational center for gifted high-school students.

Teaching: students reviews

- usual evaluation by students $\approx 4.9 - 5$ out of 5
- use "gamification" (e.g. math battles, Abaka —— traditional math circles games)
- try to be as clear as possible



Quotes from anonymous student reviews at St. Petersburg (calculus, 2020):

- The best practice in the observable universe (possibly in the unobservable) Game Abaka is super
- Everything is wonderful
- Everything is perfect
- Yulia explains the material very well and clearly!
- A very good teacher, at each practice sheets with problems and reminders of the material, active work in the classroom, a very fast check of homeworks

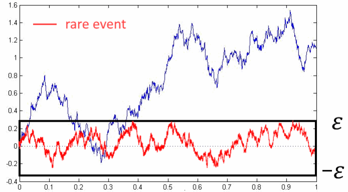
Research activity

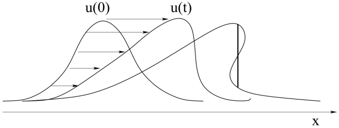
- **Publications:** 7 published papers, 3 preprints (arxiv), 1 conference paper (SPE).
- **Active participation at conferences and schools: 24**.
  Among them: Russia (11), Brazil (4), Germany (2), Italy (3), Spain (1), Canada (1), China (1), India (1).
- **Invited speaker** at *13th Americas Conference on Diff. Equations and Nonlinear Analysis and ICMC Summer Meeting on Differential Equations.* Sao Carlos, Brazil. Section "Conservation Laws and Transport Equations". Will be in Jan-Feb 2023.
- **Active participation at seminars: 25 talks**.
  Among them: Oberseminar "Nonlinear dynamics" at Freie Universitat Berlin, Applied Math/PDE seminar UC Davis, Seminário Luiz Adauto de Análise/EDP and Seminario de Probabilidade at IM-UFRJ
- **Organization of seminars:** seminar at St. Petersburg State University, seminar at IMPA, small reading groups (reading Dafermos book on hyperbolic laws)

Why me at IM-UFRJ?

- **Research**: active research agenda in
  - Analysis
  - Nonlinear PDEs
  - Probability
  - Dynamical Systems

  I will be happy to start collaboration with Prof. Wladimir Neves on the Buckley-Leverett model.

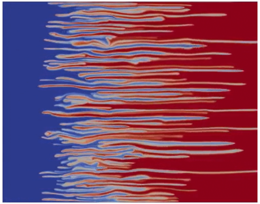




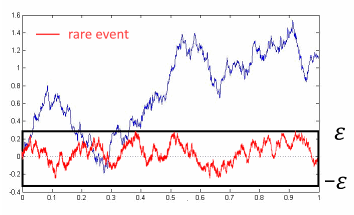
- **Teaching**:
  I like teaching and have 10 years of experience.

- **Brazil**: after 24 of February 2022, when the war between Russia and Ukraine started, me and my husband (also mathematician) decided to stay in Brazil. I have learnt Portuguese in one year (see my results on the preparation to Celpe-Bras in October 2022).

I like Brazil!



PhD Thesis: small ball probabilities for Gaussian processes



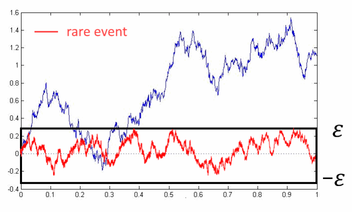
Let $(\mathcal{X}, \|\cdot\|)$ be a Banach space.
Let X be a random element in $\mathcal{X}$.

Small deviations problem

Find asymptotics as $\varepsilon \to 0$ of

$$\mathbb{P}\left(\|X\| < \varepsilon\right)$$

PhD Thesis: small ball probabilities for Gaussian processes

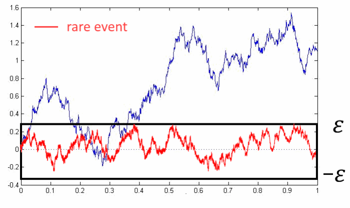


Example:

$W(t)$ — Wiener process in $C[0,1]$

$$\mathbb{P}\left(\|W\| < \varepsilon\right) \underset{\varepsilon\to 0}{\sim} \frac{4}{\pi}\, \exp\left(-\frac{\pi^2}{8}\,\varepsilon^{-2}\right)$$

PhD Thesis: small ball probabilities for Gaussian processes



Stochastic processes

$X(t)$, $t \in (0,1)$, —

- Gaussian process
- $\mathbb{E}X(t) \equiv 0$
- $G(s,t) = \mathbb{E}X(s)X(t)$.

Small ball asymptotics

$$\mathbb{P}\left(\|X\|_2 < \varepsilon\right), \quad \varepsilon \to 0$$

Example:

$W(t)$ — Wiener process in $C[0,1]$

$$\mathbb{P}\left(\|W\| < \varepsilon\right) \underset{\varepsilon\to 0}{\sim} \frac{4}{\pi} \exp\left(-\frac{\pi^2}{8}\varepsilon^{-2}\right)$$

Spectral theory

$\mathbb{G} : L_2[0,1] \to \mathrm{Im}(\mathbb{G})$

- integral operator of trace class
$$(\mathbb{G}u)(s) = \int_0^1 G(s,t)u(t)\,dt$$
- eigenvalues: $\sum \mu_k < \infty$

Asymptotics of eigenvalues μ_k

Find "good" approximation to μ_k

PhD Thesis — an example of the theorem

Theorem (P. '2017)

For a process $X(t)$, corresponding to Gumbel distribution, $G_0(s,t) = \min(s,t) - st$

$$G_X(s,t) = G_0(s,t) - \psi(t)\psi(s), \quad \psi(t) = C\, t\ln(t)\cdot\ln(-\ln(t))$$

asymptotics of eigenvalues is

$$\mu_k^{-1/2} = \pi k + \frac{\pi}{2} + (-1)^k \cdot 2\mathrm{arctg}\Big(\frac{1}{\ln(\ln(k))+1}\Big) - \frac{1}{\ln(k)\ln(\ln(k))} + O\Big(\frac{\ln(k)^{-1}}{(\ln(\ln(k)))^2}\Big)$$

L_2-small ball probability asymptotics is

$$\mathbb{P}\Big\{\|X\| < \varepsilon\Big\} \sim C\cdot \ln^{-1}(\ln(\varepsilon^{-1}))\cdot\varepsilon^{-1}\cdot\exp\Big(-\frac{1}{8\varepsilon^2}\Big)$$

PhD Thesis — an example of the theorem

Theorem (P. '2017)

For a process $X(t)$, *corresponding to Gumbel distribution,* $G_0(s,t) = \min(s,t) - st$

$$G_X(s,t) = G_0(s,t) - \psi(t)\psi(s), \quad \psi(t) = C\, t\ln(t) \cdot \ln(-\ln(t))$$

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L_2*-small ball probability asymptotics is*

$$\mathbb{P}\Big\{\|X\| < \varepsilon\Big\} \sim C \cdot \ln^{-1}(\ln(\varepsilon^{-1})) \cdot \varepsilon^{-1} \cdot \exp\Big(-\frac{1}{8\varepsilon^2}\Big)$$

Key ingredient: asymptotic expansion for oscillation integrals with amplitude SVF

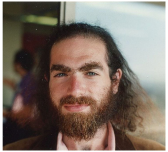
Let $F(t) =: F_0(t)$ be SVF at 0, and $F_{n+1}(t) = tF_n'(t)$ also be SVF at 0, then:

$$\int_0^{\frac{1}{2}} F(t)\cos(\omega t)\, dt = \frac{1}{\omega}\sum_{k=1}^{N} c_k F_k\left(\frac{1}{\omega}\right) + R_N, \qquad \omega \to \infty$$

Publications and awards

2019 Laureat of the "Young Mathematician" prize of the St. Petersburg Mathematical Society

2018 Winner of 22nd Möbius Contest in nomination "Undergraduates and graduates", Moscow, Russia

5. *L_2-small ball asymptotics for a family of finite-dimensional perturbations of Gaussian functions.* Zapiski Nauchnykh Seminarov POMI, vol. 501. Nikitin's memorial volume, pp. 236–258, 2021. (In Russian). English version: arXiv:1905.07804.
4. *On spectral asymptotics for a family of finite-dimensional perturbations of operators of trace class.* Doklady Math., 2018, vol. 98, №1, pp. 367–369;
3. *Exact L_2-small ball asymptotics for some Durbin processes.* Zap. nauchn. sem. POMI, 2017, vol. 466, pp. 211–233. (In Russian) Translated: Journal of Mathematical Sciences (USA), 2020, 244(5), pp. 842-857;
2. *Spectral asymptotics for problems with integral constraints.* Mat. Zametki, 2017, vol. 102(3), pp. 405–414 (In Russian). Translated: Mathematical Notes, 2017, 102(3-4), pp. 369-377;
1. (with A. I. Nazarov) *The small ball asymptotics in Hilbertian norm for the Kac–Kiefer–Wolfowitz processes.* Teor. Veroyatnost. i Primenen., 2015, Volume 60, Issue 3, Pages 482–505. Translated: Theory of Probability and its Applications, 2016, 60(3), pp. 460-480.

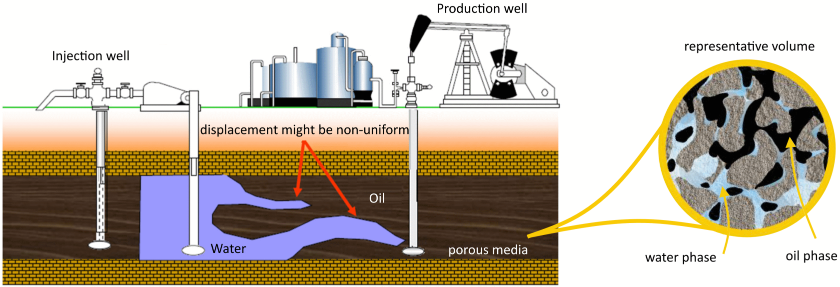
Experience with applied research

2018-2019 "Gazprom-Neft" Scholarship

2019-2021 Participant of Industrial projects with "Gazprom-Neft" at Chebyshev Laboratory, St. Petersburg

- mathematics of enhanced oil recovery (EOR)

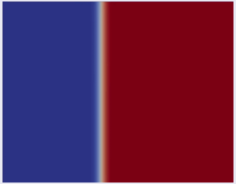
2022 **Patent** No. 2772808 C1 Russian Federation, IPC E21B 43/16, C09K 8/58.



Fundamental research: two main directions

1-dim in spatial variable

- Stable displacement



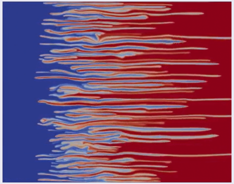
- main questions: find an exact solution to a Riemann problem; analyze stability
- hyperbolic conservation laws

$$
\begin{aligned}
s_t + f(s,c)_x &= 0,\\
(cs + a(c))_t + (cf(s,c))_x &= 0.
\end{aligned}
$$

Example: chemical flooding model

2-dim (or 3-dim) in spatial variable

- Unstable displacement



- source of instability: water and oil/polymer have different viscosities
- viscous fingering phenomenon

$$
\begin{aligned}
c_t + u \cdot \nabla c &= \varepsilon \Delta c,\\
\mathrm{div}(u) &= 0,\\
u &= -\nabla p / \mu(c).
\end{aligned}
$$

Example: Peaceman model

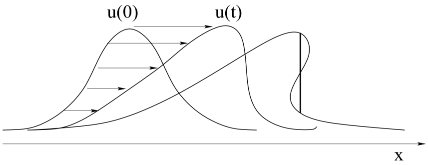
Research topic I: hyperbolic conservation laws

2021-2023 Postdoc of excellence at IMPA.
Working with Profs. Dan Marchesin and Bradley Plohr

$$\begin{aligned} s_t + f(s,c)_x &= 0, && \text{(conservation of water)} \\ (cs + a(c))_t + (cf(s,c))_x &= 0. && \text{(conservation of chemical)} \end{aligned} \tag{1}$$

Generalization of the Buckley-Leverett equation.

- $s = s(x,t)$ — water phase saturation;
- $f(s,c)$ — fractional flow function;
- $c = c(x,t)$ — concentration of a chemical in water;
- $a(c)$ — adsorption of a chemical on a rock.



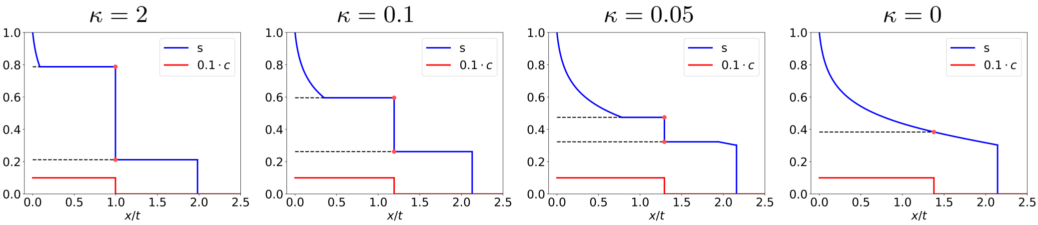
7. (with B. Plohr, D. Marchesin) *Vanishing adsorption admissibility criterion for contact discontinuities in the polymer model*, 2022. arXiv:2211.10326
6. (with F. Bakharev, A. Enin, N. Rastegaev) *Impact of dissipation ratio on vanishing viscosity solutions of the Riemann problem for chemical flooding model*, 2021. arXiv:2111.15001. Accepted to Journal of Hyperbolic Differential Equations.

Research topic I: results

- use vanishing viscosity criterion — add small diffusion/capillary terms

$$\boxed{\begin{aligned} s_t + f(s,c)_x &= \varepsilon_c (A(s,c)s_x)_x, \\ (cs + a(c))_t + (cf(s,c))_x &= \varepsilon_c (cA(s,c)s_x)_x + \varepsilon_d c_{xx}. \end{aligned}}$$

- $f(s,c)$ monotone in $c \Rightarrow$ uniqueness of vanishing viscosity solution (1988)
- Main result: $f(s,c)$ non-monotone in $c \Rightarrow$ exist multiple vanishing viscosity solutions, depending on ratio $\kappa = \varepsilon_d / \varepsilon_c$



Such shocks are known as transitional (undercompressive)

Research topic II: fingering instability

1. Transport of species ($\varepsilon \geq 0$)

$$c_t + u \cdot \nabla c = \varepsilon \Delta c$$

2. Incompressibility condition

$$\mathrm{div}(u) = 0,$$

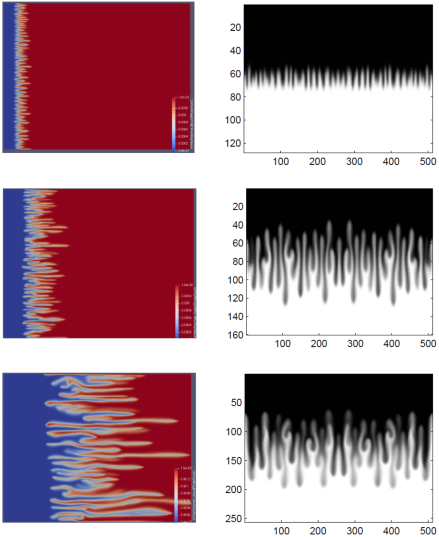
3a. Darcy's law (viscosity-driven)

$$u = -\nabla p / \mu(c).$$

3b. Darcy's law (gravity-driven)

$$u = -\nabla p + (0, c).$$

viscosity-driven fingers gravity-driven fingers



NB: 1 + 2+ 3b for $\varepsilon = 0$ is known as IPM (incompressible porous media equation)

Research topic II: questions of interest & publications

1. **Well-posedness:**
   - existence of a global solution vs finite-time blow-up:
     active scalar: $u = A(c)$ — singular integral operator (like in SQG)
2. **Mixing zone:**
   - many laboratory and numerical experiments show linear growth of the mixing zone
   - I want to find mathematically rigorously estimates of speed of the linear growth

11. (with F. Bakharev, A. Enin, K. Kalinin, N. Rastegaev, S. Tikhomirov) *Optimal polymer slugs injection profiles*. arXiv:2012.03114. Submitted.
10. (with F. Bakharev, A. Enin, A. Groman, A. Kalyuzhnuk, S. Matveenko, I.Starkov, S. Tikhomirov) *Velocity of viscous fingers in miscible displacement: Comparison with analytical models*. Journal of Computational and Applied Mathematics, March 2022
9. (with S. Tikhomirov, F. Bakharev, A. Groman, A. Kalyuzhnyuk, A. Enin, K. Kalinin, N. Rastegaev) *Calculation of graded viscosity banks profile on the rear end of the polymer slug.* Paper SPE-206426-MS, SPE Russian Petroleum Technology Conference, 2021.
8. (with F. Bakharev, L. Campoli, A. Enin, S. Matveenko, S. Tikhomirov, A. Yakovlev) *Numerical investigation of viscous fingering phenomenon for raw field data.* Transport in Porous Media, 2020, pp. 1–22;

Thank you for your attention!

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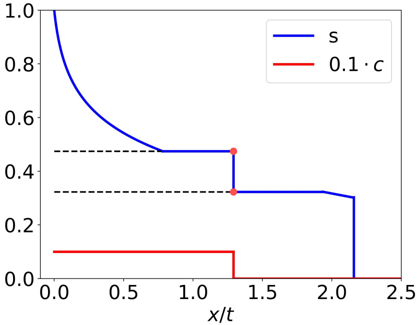
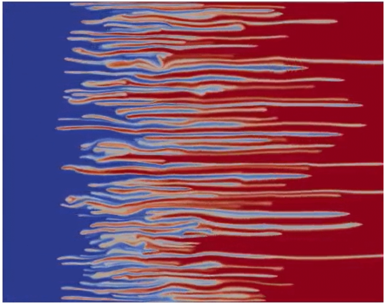
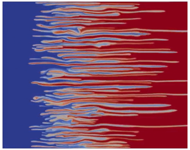




Toy model of viscous fingering (work in progress)

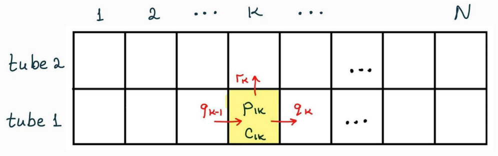


Discrete case

- system of $2N$ ODEs and N algebraic equations

Unknowns:

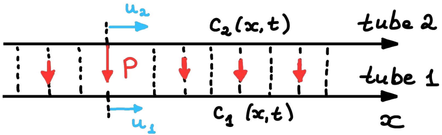
- $c_{1k}(t), c_{2k}(t)$ — concentrations
- p_{1k}, p_{2k} — pressures
- $q_k(t), r_k(t)$ — velocities



Continuous case

- two coupled advection-diffusion eqs

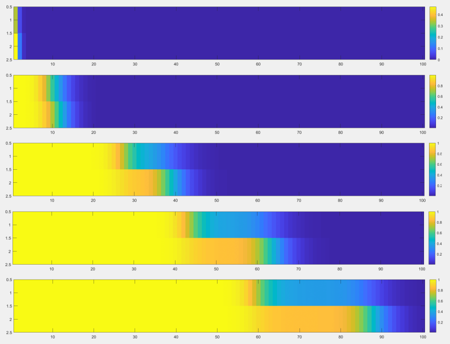
$$\partial_t c_1 = -\partial_x(u_1 c_1) + p + \varepsilon \partial_{xx} c_1,$$
$$\partial_t c_2 = -\partial_x(u_2 c_2) - p + \varepsilon \partial_{xx} c_2.$$



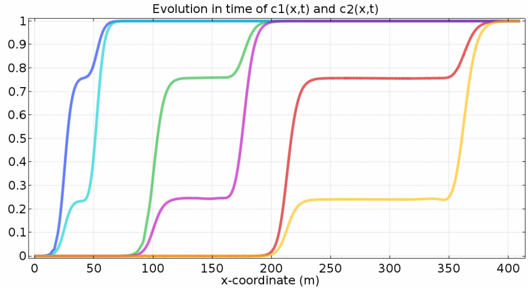
Flow between tubes: $p = (-1)^{1,2}\partial_x u_{1,2} c_{1,2}$. Work in progress with S. Tikhomirov, Y. Efendiev.

Toy model of viscous fingering: numerical experiments

Discrete setting



Continuous setting



Result of experiments: cascade of two travelling waves (TW)

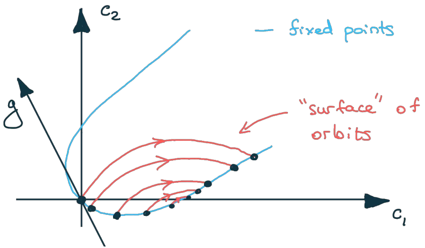
$$(0,0) \xrightarrow{\mathrm{TW}_1} (c_1^*, c_2^*) \xrightarrow{\mathrm{TW}_2} (1,1)$$

Toy model of viscous fingering: theoretical approach

For a travelling wave $c_1 = c_1(x - vt)$ and $c_2 = c_2(x - vt)$ we have a dynamical system.

Travelling wave dynamical system

$$\begin{aligned} c_1' &= g, \\ g' &= (u - v)g, \\ c_2' &= (u - v)c_1 + (2 - u - v)c_2 - g. \end{aligned}$$



Here $u = u(c_1, c_2)$, depends on $\mu(c_1)$ and $\mu(c_2)$.