

Graded viscosity banks on the rear end of the polymer slug

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1 June 2021

Joint work with F. Bakharev, A. Enin,
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InterPore2021
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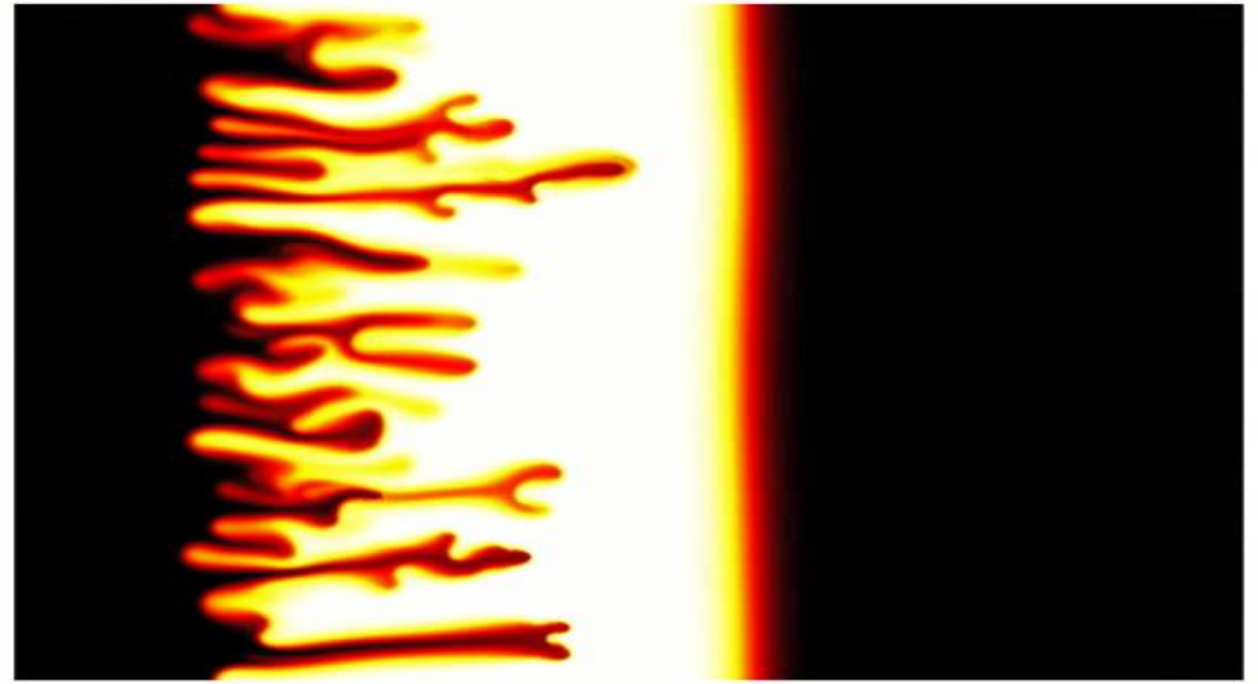
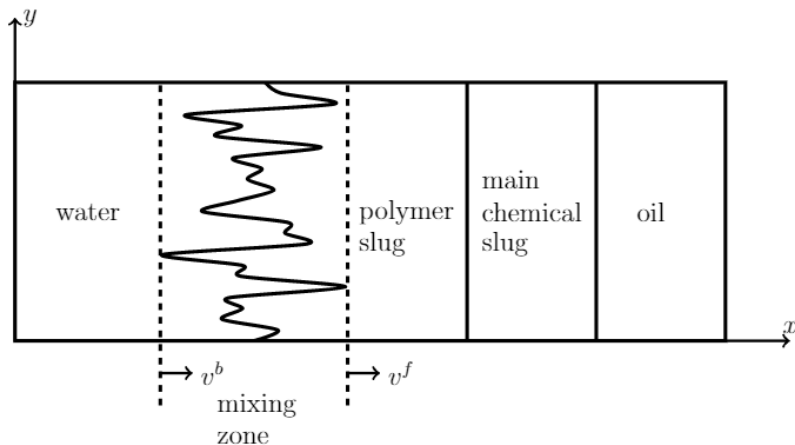


Supported by



Breakthrough of polymer slug

- Homogeneous porous media
- Instability occur due to different viscosities (viscous fingering effect)
- After the breakthrough of the polymer slug the positive effect decreases
- Chemical EOR:
 - Polymer flooding
 - Surfactant flooding
 - ASP-flooding



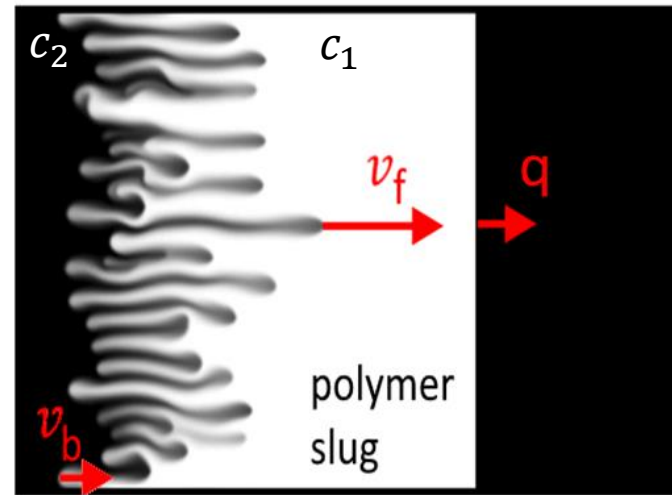
Question: what size of polymer slug?

Mathematical models

- One-phase miscible displacement (Peaceman model)

$$\begin{aligned} c_t + u \cdot \nabla c &= \varepsilon \Delta c \\ \operatorname{div} u &= 0 \\ u &= -\frac{k}{\mu(c)} \nabla p \end{aligned}$$

c – polymer concentration
 $\mu(c)$ – water viscosity



q – Velocity of the stable front.
 Take $q = 1$

v_f – Velocity of the front end of the mixing zone **is constant**

v_b – Velocity of the rear end of the mixing zone **is constant**

Empirical models of velocities

- Averaging - “effective viscosity” M_e :
 - Koval (1963)
 - Todd-Longstaff (1972)
- Transverse Flow Equilibrium (TFE)
 Otto-Menon, Yortsos-Salin (2006)
 - $p(x, y) = p(x)$
 - $M = \frac{\mu(c_1)}{\mu(c_2)} > 1$ – ratio of viscosities

Koval	$v_f = M_e \quad v_b = \frac{1}{M_e} \quad M_e = \left(\alpha \cdot M^{\frac{1}{4}} + (1 - \alpha) \right)^4$
Todd-Longstaff	$v_f = M_e \quad v_b = \frac{1}{M_e} \quad M_e = M^\omega$
TFE	$v_f \leq \frac{\bar{m}(c_1, c_2)}{m(c_2)} \quad v_b \geq \frac{v_f}{M} \quad m(c) = 1/\mu(c)$

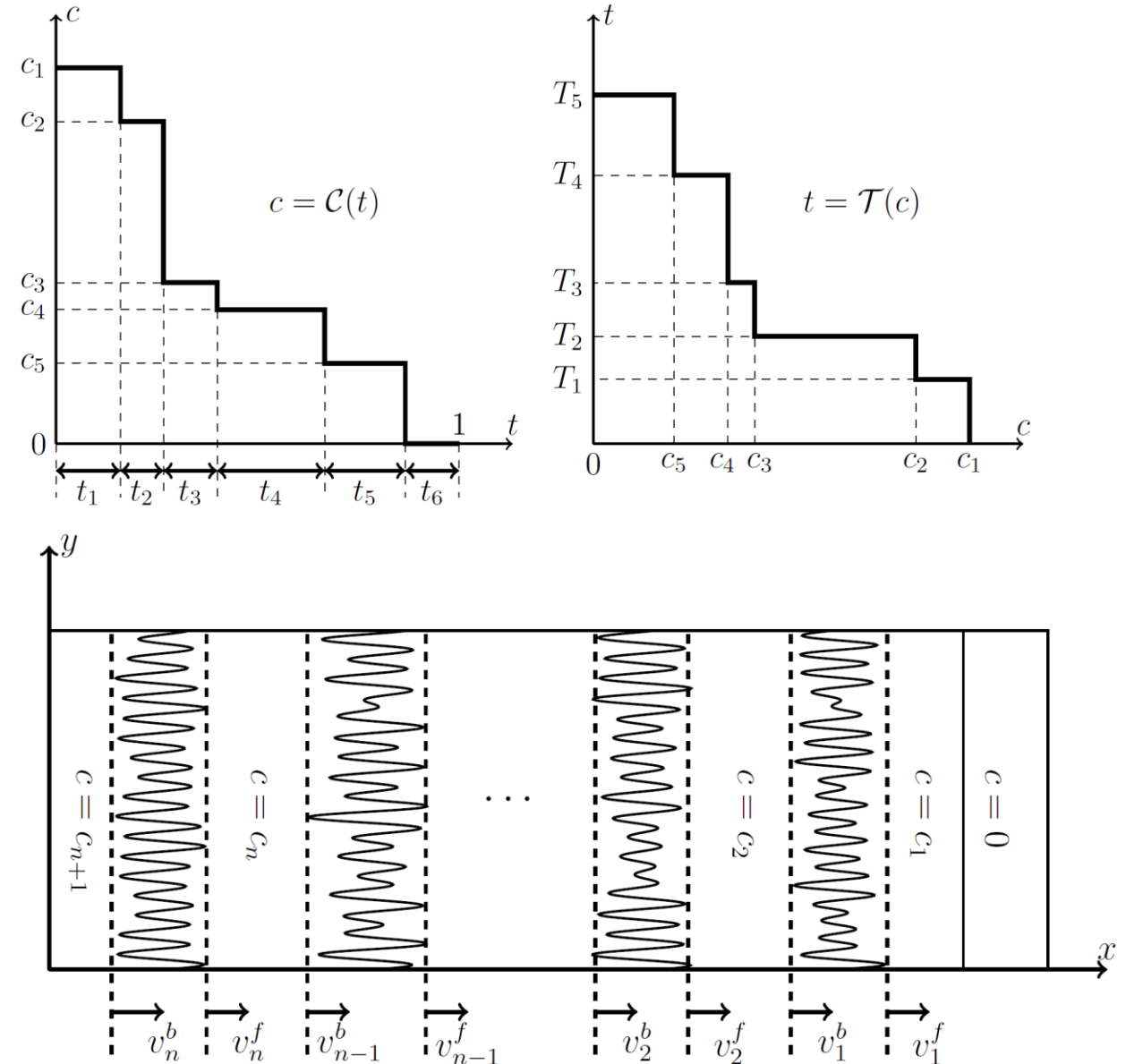
Problem Statement

- Goal: reduce amount of polymer
- Strategy: graded viscosity banks (GVB, tapering) Claridge (1978)
- We want no breakthrough in any slug
- Given concentrations c_n and v_b, v_f we can find sizes of slugs t_n without breakthrough

- Choose concentrations c_n to minimize amount of polymer

$$V_n = \sum_{i=1}^n c_i t_i \rightarrow \min$$

- Questions:
 - n – small ($n = 2, 3, 5$)
 - $n \rightarrow \infty$



Results for small n

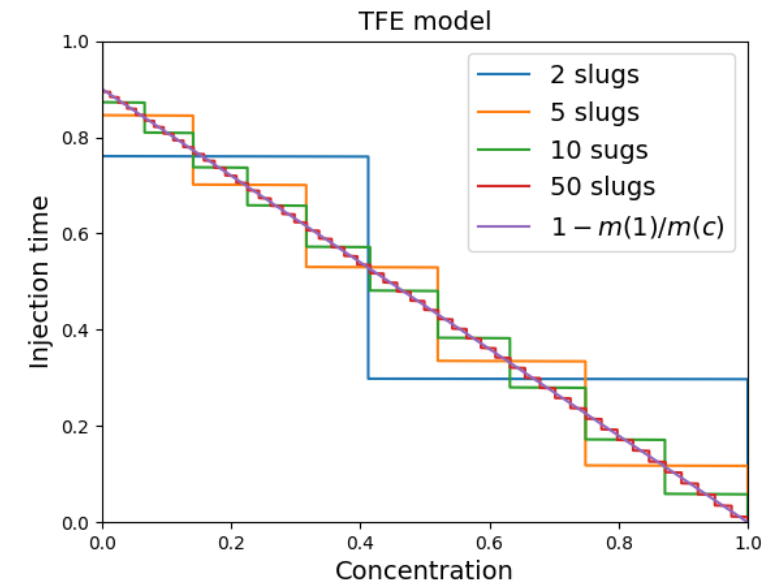
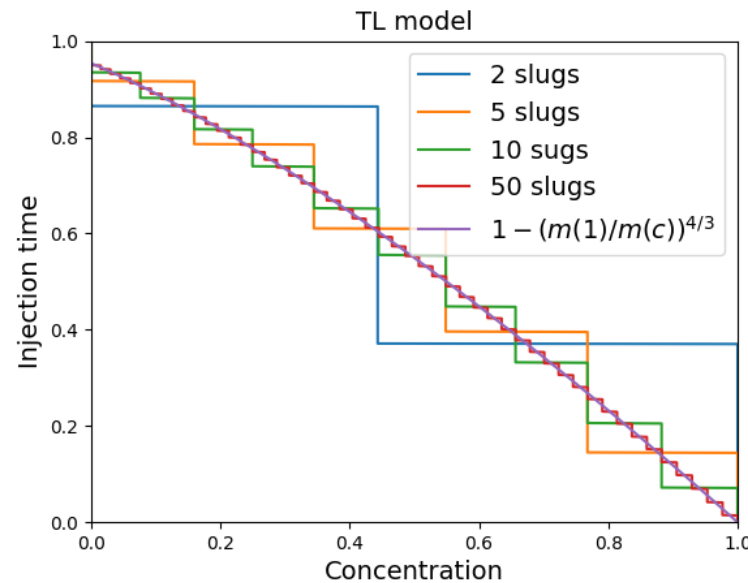
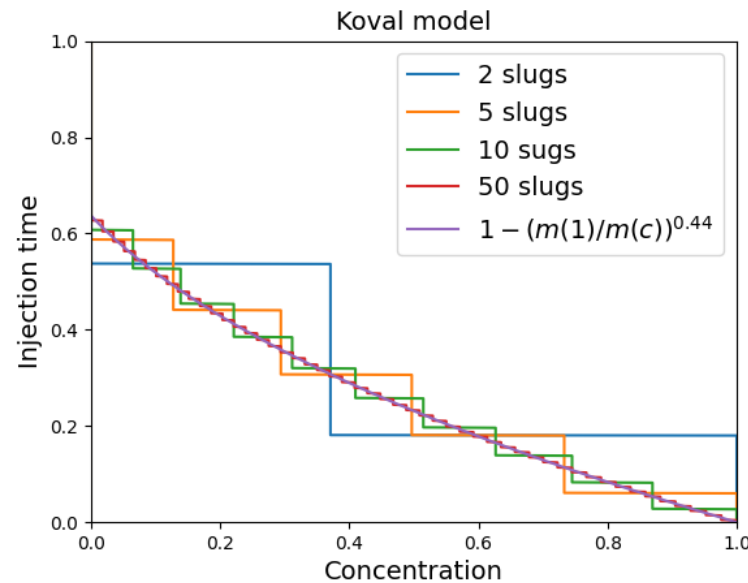
V_n - polymer mass for n slugs

$\eta = \frac{V_1 - V_n}{V_1}$ - percentage

of gain in polymer mass

	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 10$	Limit
TFE	19,83%	23,35%	24,57%	25,13%	25,88%	26,12%
Todd-Longstaff	24,84%	29,36%	30,93%	31,66%	32,63%	32,95%
Koval	33,21%	39,24%	41,46%	42,55%	44,24%	45,28%

- Conclusion: in practice it is enough to use 2-3 slugs
- For details see [arxiv:2012.03114](https://arxiv.org/abs/2012.03114)



Graded viscosity banks: $n \rightarrow \infty$

Theorem [Bakharev, Enin, Kalinin, P., Rastegaev, Tikhomirov, 2021]

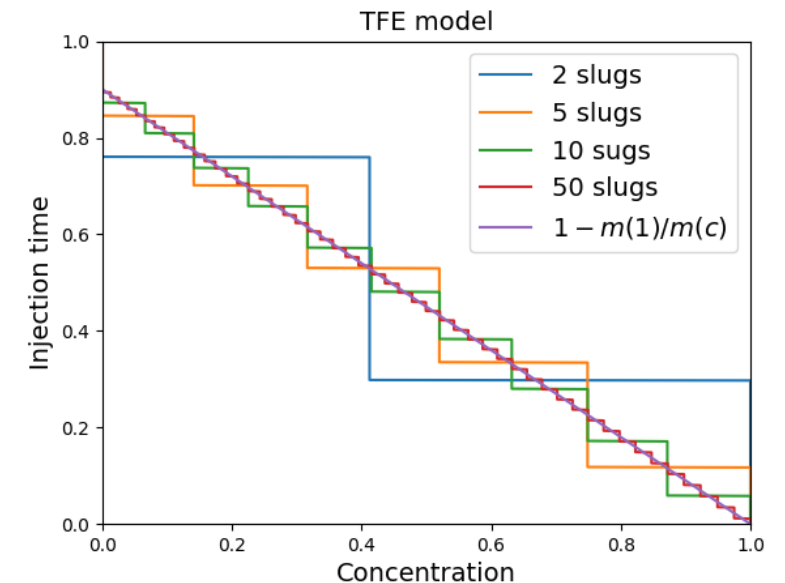
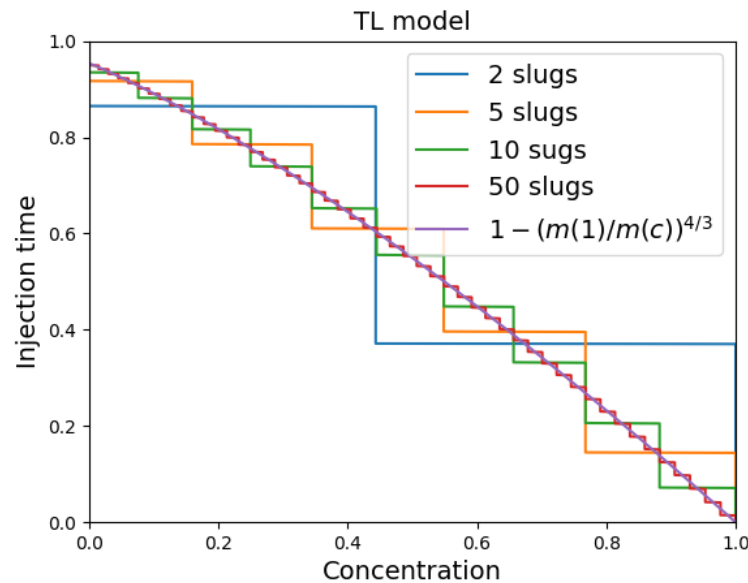
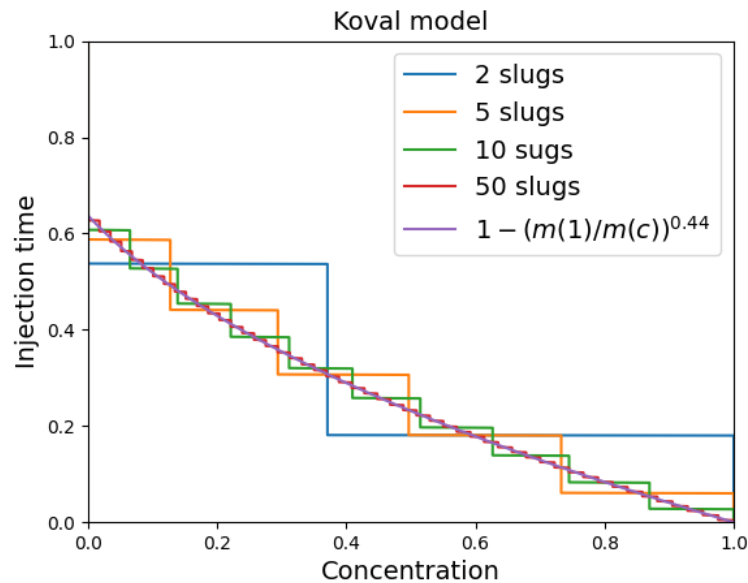
As $n \rightarrow \infty$ the optimal limiting injection profile

$$T^\infty(c) = 1 - \left(\frac{\mu(c)}{\mu(c_1)} \right)^\beta$$

Koval: $\beta = 2\alpha$

Todd-Longstaff: $\beta = 2\omega$

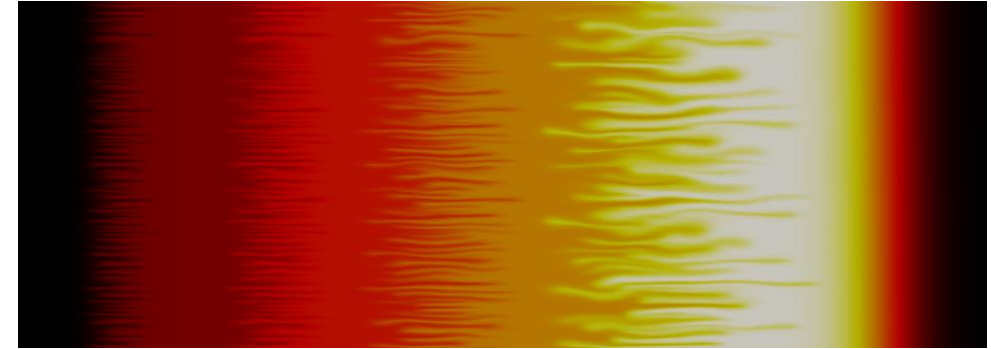
TFE: $\beta = 1$



For details see [arxiv:2012.03114](https://arxiv.org/abs/2012.03114)

Conclusions

1. Graded viscosity banks helps to reduce polymer mass with the same efficiency
2. In practice it is enough to inject 2-3 slugs
3. The choice of model for “finger velocities ” is an open problem – no rigorous results, only empirical



Simulation of GVB in DuMuX by our group in SPSU

The talk is based on:

- <https://arxiv.org/abs/2012.03114> - “Optimal polymer slugs injection profiles”
F. Bakharev, A. Enin, K. Kalinin, Yu. Petrova, N. Rastegaev, S. Tikhomirov
- <https://arxiv.org/abs/2012.02849> - “Velocity of viscous fingers in miscible displacement”
F. Bakharev, A. Enin, A. Groman, A. Kalyuzhnuk, S. Matveenko, Yu. Petrova, I. Starkov, S. Tikhomirov

If you have any questions, please, ask me:

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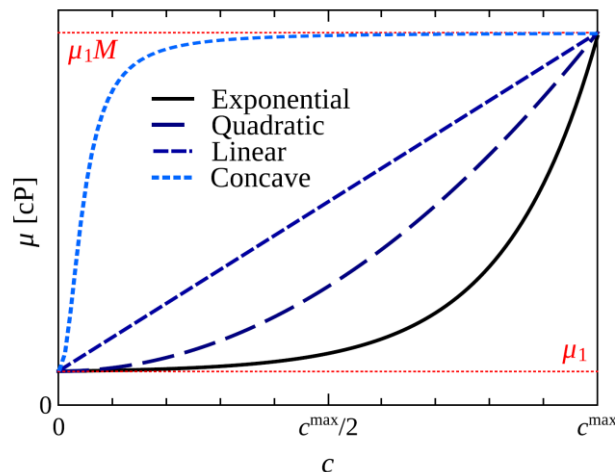
Yulia Petrova

Acknowledgements:

- the work was supported by the Russian Science Foundation grant 19-71-30002 and PJC “GazpromNeft”

Koval & TFE model. Numerical validation

- Numerical validation of TFE (Otto-Yortsos) model: for different viscosity curves: always gives a pessimistic estimate
- Koval model not always gives a pessimistic estimate
- Examples when TFE model is exact:
Exponential viscosity – at the rear end
Concave viscosity – at the front end

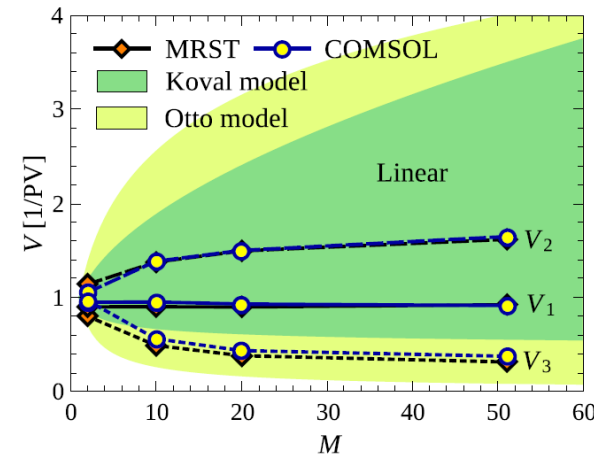


Exponential viscosity

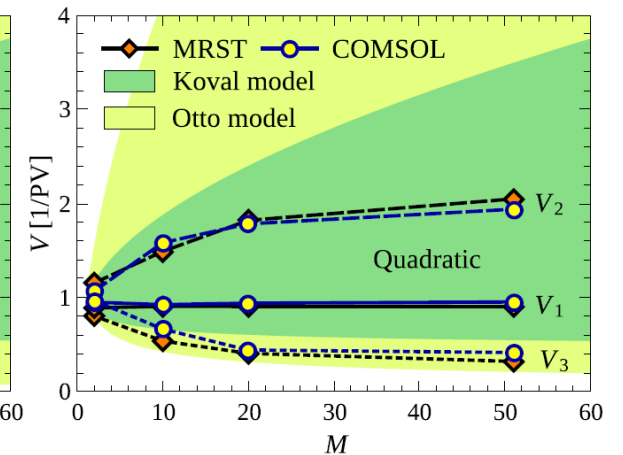
$$u^x = \frac{m(c)}{m(c)} u \approx u \cdot \frac{m(c)}{m(0)}$$

Concave viscosity

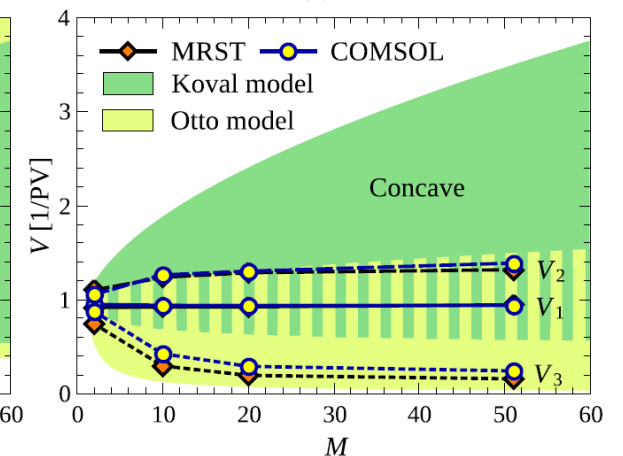
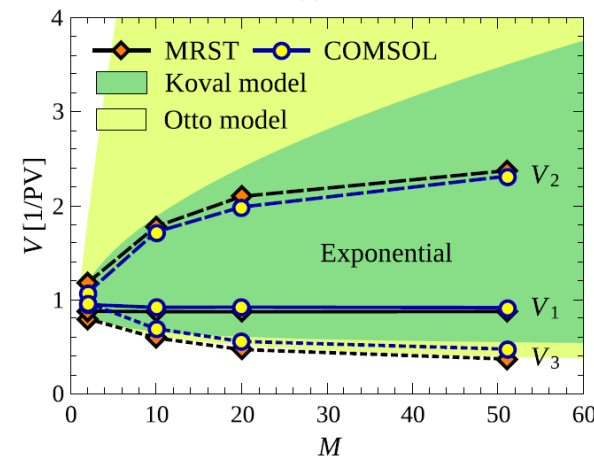
$$u^x = \frac{m(c)}{m(c)} u \approx u \cdot \frac{m(c)}{m(1)}$$



(a)



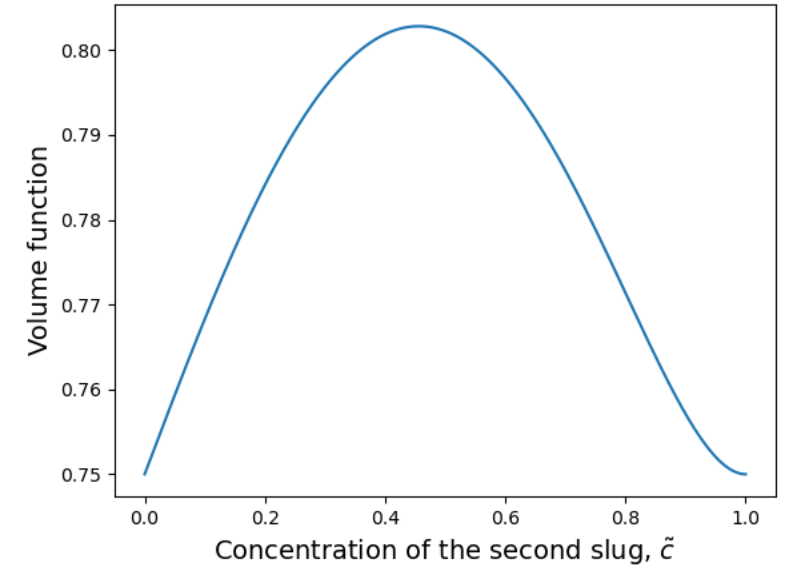
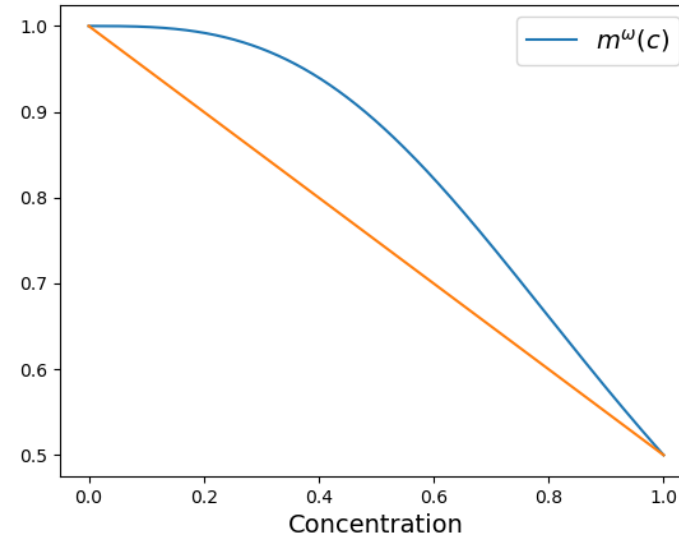
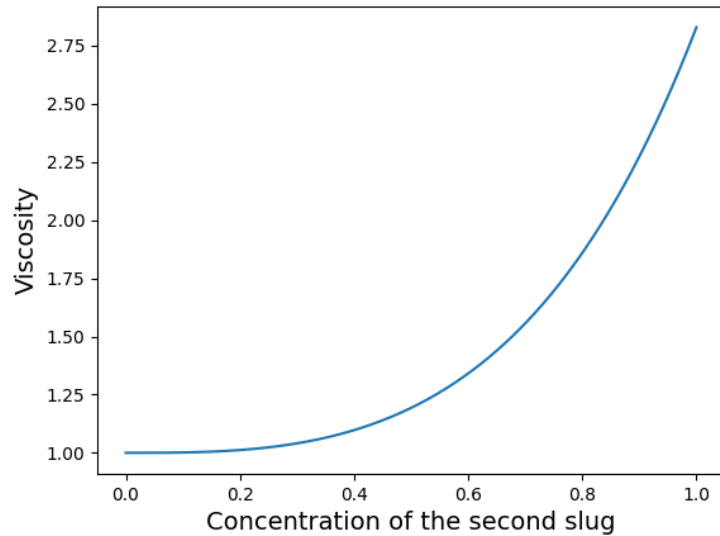
(b)



For details see [arxiv:2012.02849](https://arxiv.org/abs/2012.02849)

GVB gives gain not for all viscosities

- Counterexample – Todd-Longstaff model and $\mu = (1 + c^3)^{\frac{3}{2}}$



- There is no gain in polymer mass even if you change 1 slug for 2 slugs!
- In the Theorem we have extra assumptions