ON ADMISSIBILITY CRITERIA FOR CONTACT DISCONTINUITIES IN GLIMM-ISAACSON MODEL ARISING IN CHEMICAL FLOODING

Yulia P. Petrova

Rio de Janeiro, Brazil

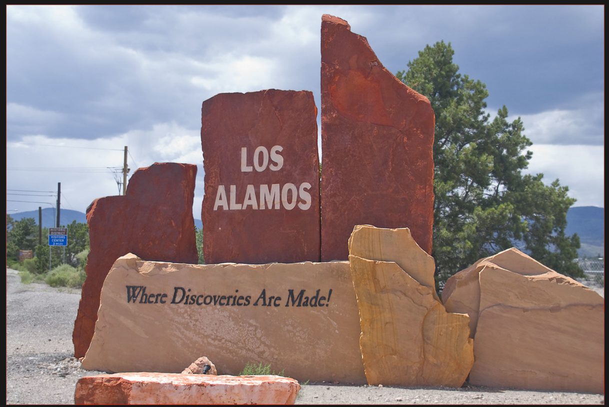
Dan Marchesin

Rio de Janeiro, Brazil

Bradley J. Plohr

Los Alamos, USA

75 anos impa
Instituto de Matemática Pura e Aplicada



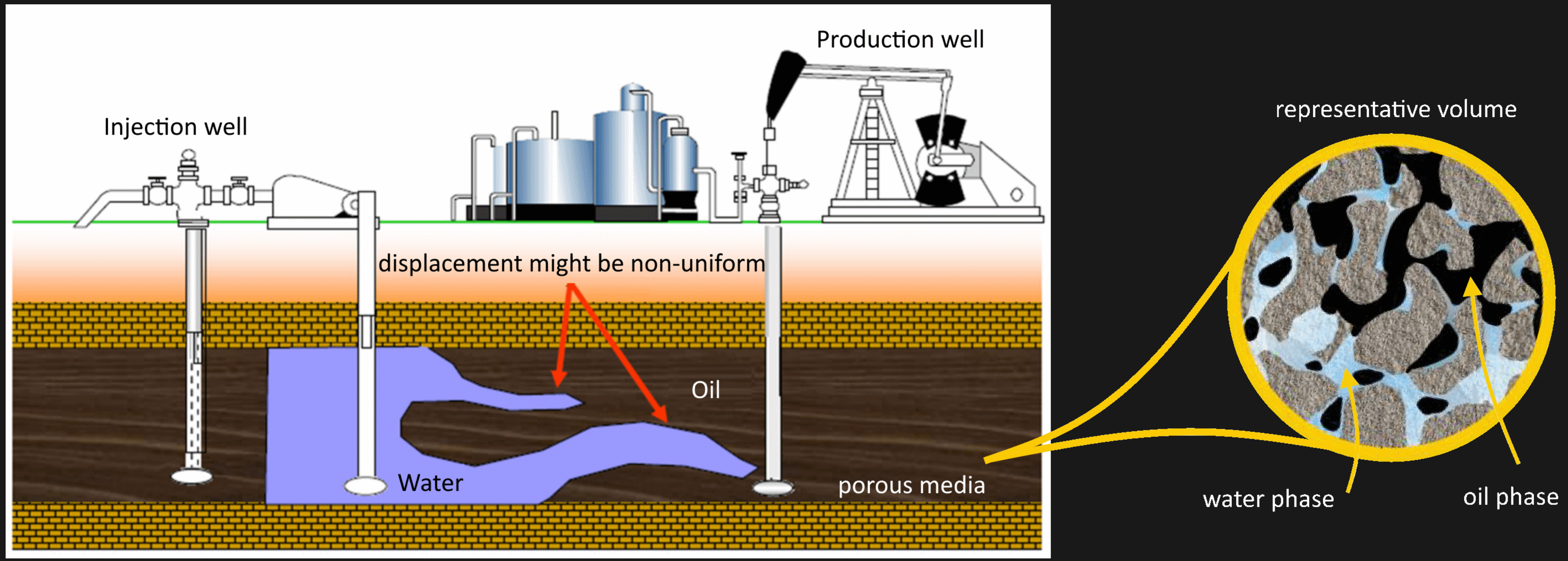
HYP conference

20 June 2022

MOTIVATION: ENHANCED OIL RECOVERY (EOR)

We are interested in the mathematical model of oil recovery. Some features:

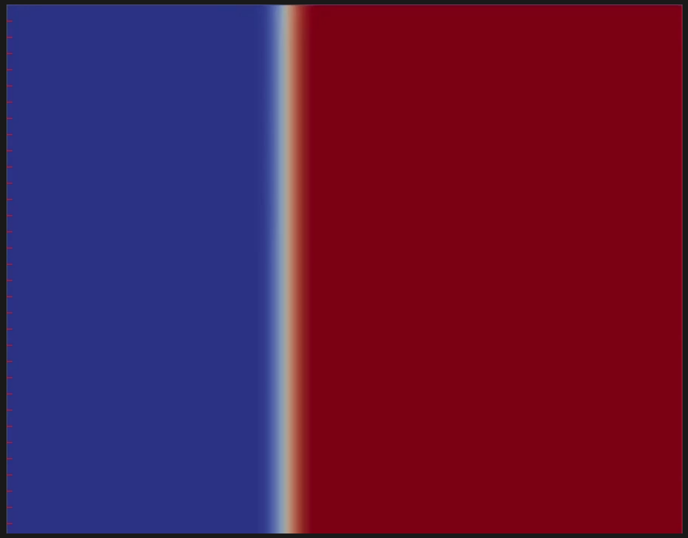
- *porous media* (averaged models of flow)
- unknown variables: $s \in [0,1]$ - water saturation, $1 - s$ - oil concentration
- relatively *small speeds* (≈ 1 meter per day): Navier-Stokes $\to$ Darcy's law
- multiphase flow: oil, water, gas.
- applications to EOR methods: thermal, gas, *chemical flooding*



TWO MAIN DIRECTIONS OF INVESTIGATION

Stable dislacement

- 1-dim in spatial variable



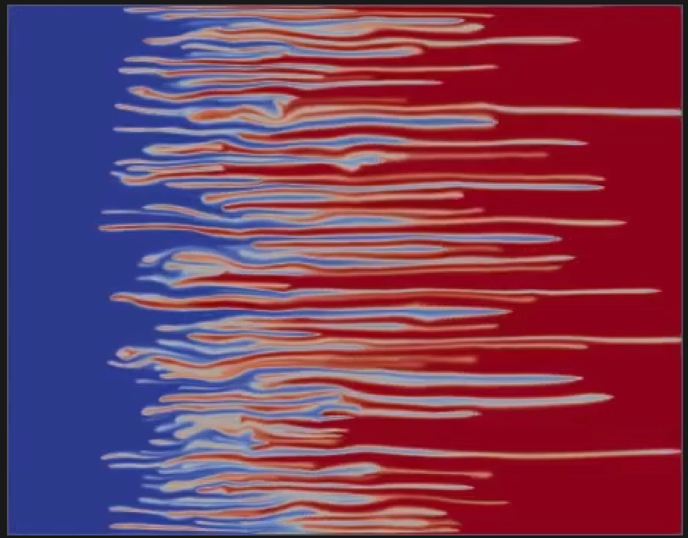
- main question: find an exact solution for a Riemann problem
- hyperbolic conservation laws

$$
\begin{aligned}
s_t + f(s,c)_x &= 0,\\
(cs)_t + (cf(s,c))_x &= 0.
\end{aligned}
$$

Example: chemical flooding model

Unstable displacement

- 2-dim (or 3-dim)



- source of instability: water and oil/polymer have different viscosities
- viscous fingering phenomenon

$$
c_t + u \cdot \nabla c = \varepsilon \Delta c,
$$
$$
\operatorname{div}(u) = 0, \quad u = -\frac{1}{\mu(c)} \nabla p.
$$

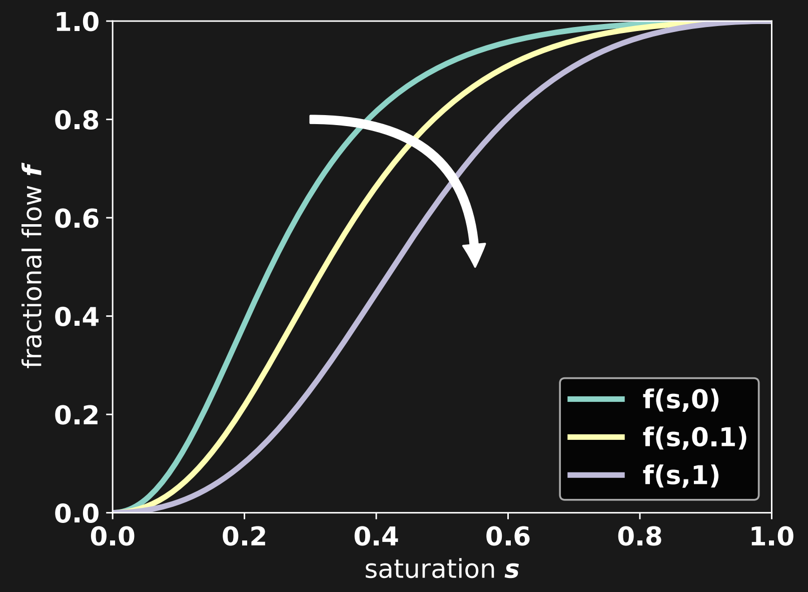
Example: Peaceman model

GLIMM-ISAACSON MODEL (KKIT MODEL)

Two-phase oil-water flow with *polymer* in the water

$$\begin{aligned} s_t + f(s,c)_x &= 0 \\ (cs)_t + (cf(s,c))_x &= 0 \end{aligned}$$

- $s \in [0,1]$ - water saturation
- $c \in [0,1]$ - polymer concentration in water
- f - fractional flow function: affected by polymer
  - S-shaped in s
  - *f is monotone in c*



Initial data: $\quad (s,c)\big|_{t=0} = \begin{cases} (s_L, c_L), & \text{if } x \leqslant 0, \\ (s_R, c_R), & \text{if } x \geqslant 0. \end{cases}$

Question: find an exact solution $s(x,t)$ and $c(x,t)$ to this Riemann problem

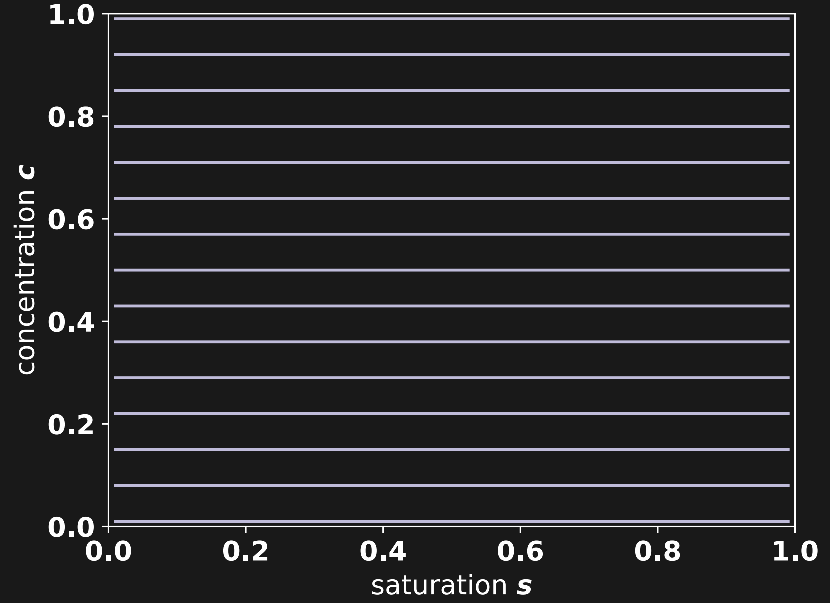
- 1980 — Isaacson, Glimm (polymer flooding): $\exists!$ under Isaacson-Glimm admissibility criterion
- 1980 — Keyfitz, Kranzer (elastisity theory)

Our goal: justify the Isaacson-Glimm admissibility criterion from physical principles

CHARACTERISTIC FAMILIES: S AND C-WAVES

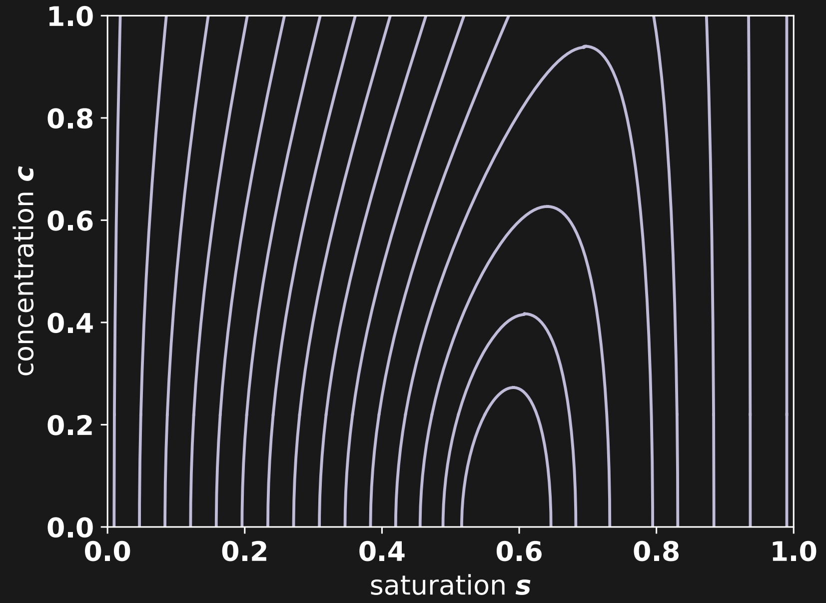
s-waves

- $\lambda^{(s)} = f_s$
- solve the Buckley-Leverett equation $c = const$
- Riemann invariant $c = const$
- *line family*



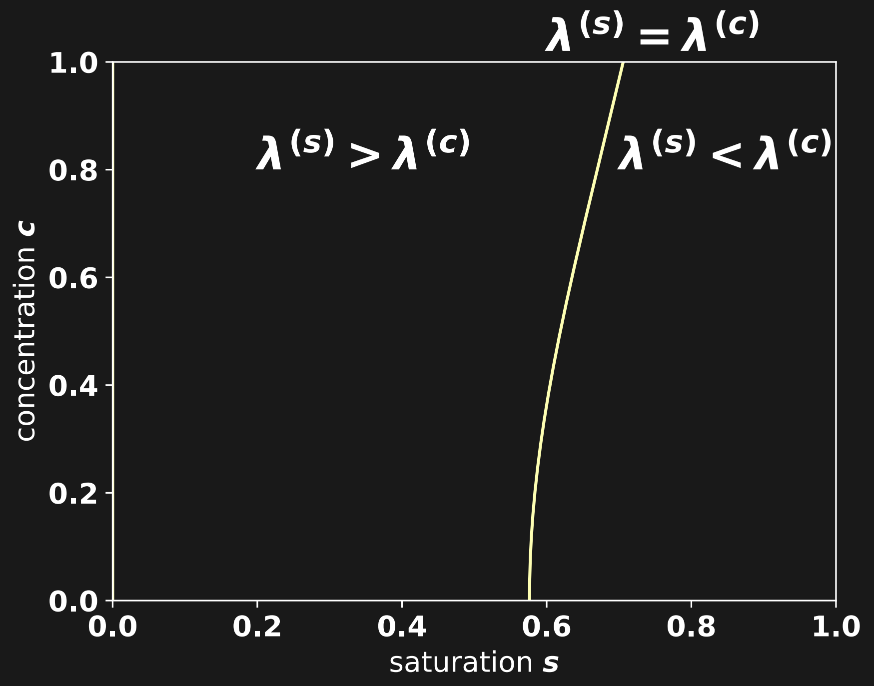
c-waves

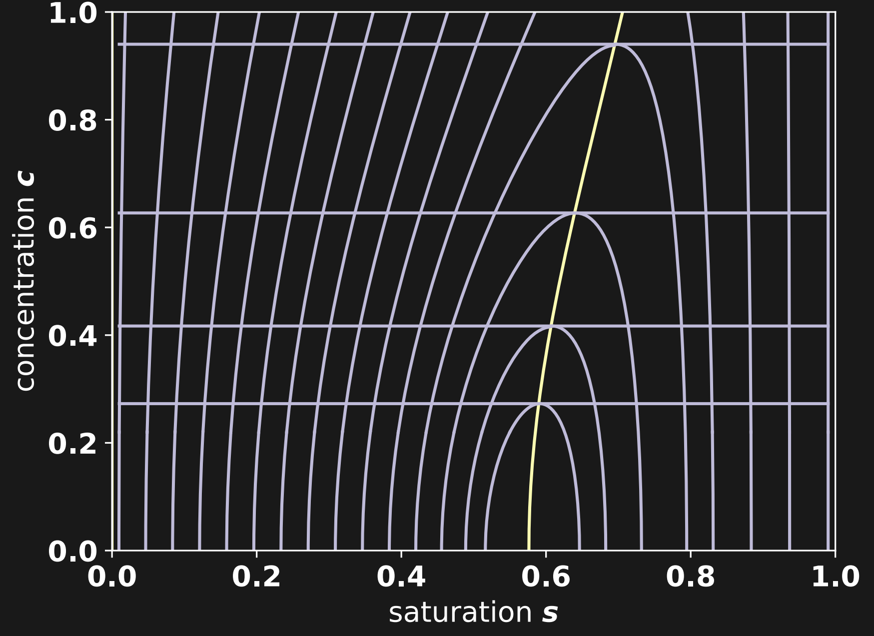
- $\lambda^{(c)} = f/s$
- are contact discontinuities (linearly degenerate)
- Riemann invariant $f/s = const$
- *contact family*



For both families the *rarefaction and shock curves coincide*! But in a different way (Temple'1983)

NON-STRICTLY HYPERBOLIC SYSTEM



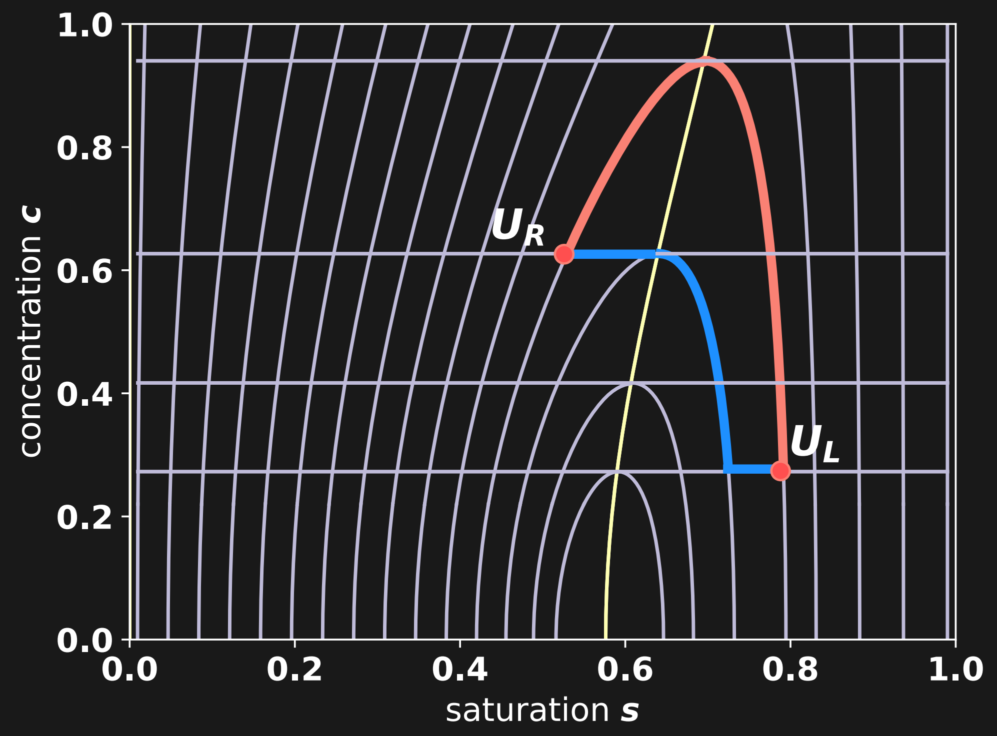


The coordinate system of wave curves is singular and wave speeds coincide on a co-dimension one curve (*coincidence locus*)

$$\lambda^{(s)} = f_s = \frac{f}{s} = \lambda^{(c)}$$

s and c-waves are tangent on coincidence curve.

NON-UNIQUENESS OF SOLUTIONS



- vanishing viscosity criterion doesn't help (no nonlinear forcing to balance the diffusion terms)
- *admissibility criterion of E. Isaacson and J. Glimm*: a contact is admissible if and only if c is continuous and monotone along the sequence of contact curves, connecting U_- and U_+.
- *consequence*: existence and uniqueness of solutions for all Riemann problems

What is the (physical) motivation of this criterion?

GENERAL ADMISSIBILITY CRITERION

- a model M_0
- a parameterized family of models M_α with its own admissibility criterion

Definiton: a solution for M_0 is admissible provided it is the L^1_{loc} limit of a family of admissible solutions of M_α as $\alpha \to 0$.

For instance, a solution of M_α could be any wave group: a shock, rarefaction, or composite, or more general wave group that is admissible for M_α.

Example: consider chemical flooding model M_α with adsorption (T. Johansen, R. Winther' 1988)

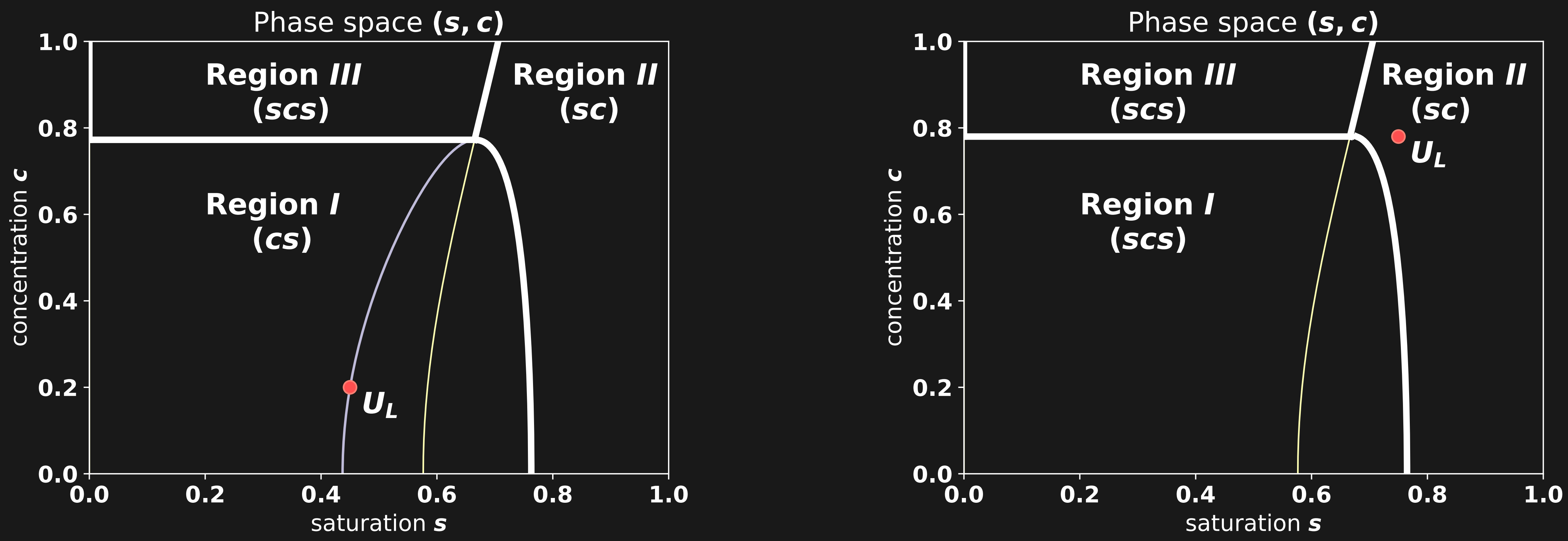
$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ (cs + \alpha a(c))_t + (cf(s,c))_x &= 0. \end{aligned} \qquad (M_\alpha)$$

- *vanishing adsorption criterion*
- if adsorption depends nonlinearly on the concentration, then *contact discontinuities become rarefactions and shock waves*
- For moderate values of c we have $a''(c) \leq 0$. When $a'' \equiv 0$, c-waves are contacts.

MAIN RESULT

Theorem (P., Marchesin, Plohr '2022). The set of admissible Riemann solutions is the same for the Isaacson-Glimm and vanishing adsorption admissibility criteria.

The solutions may be presented by two diagrams (E. Isaacson '1980):



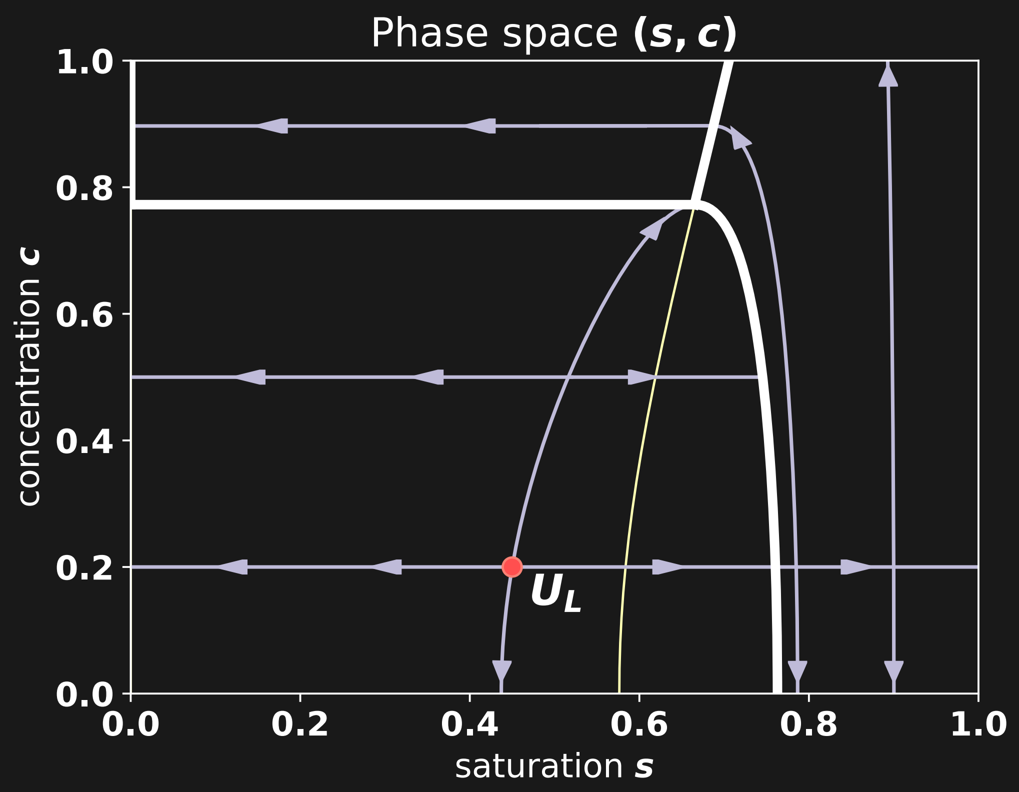
U_L to the left of coincidence

U_L to the right of coincidence

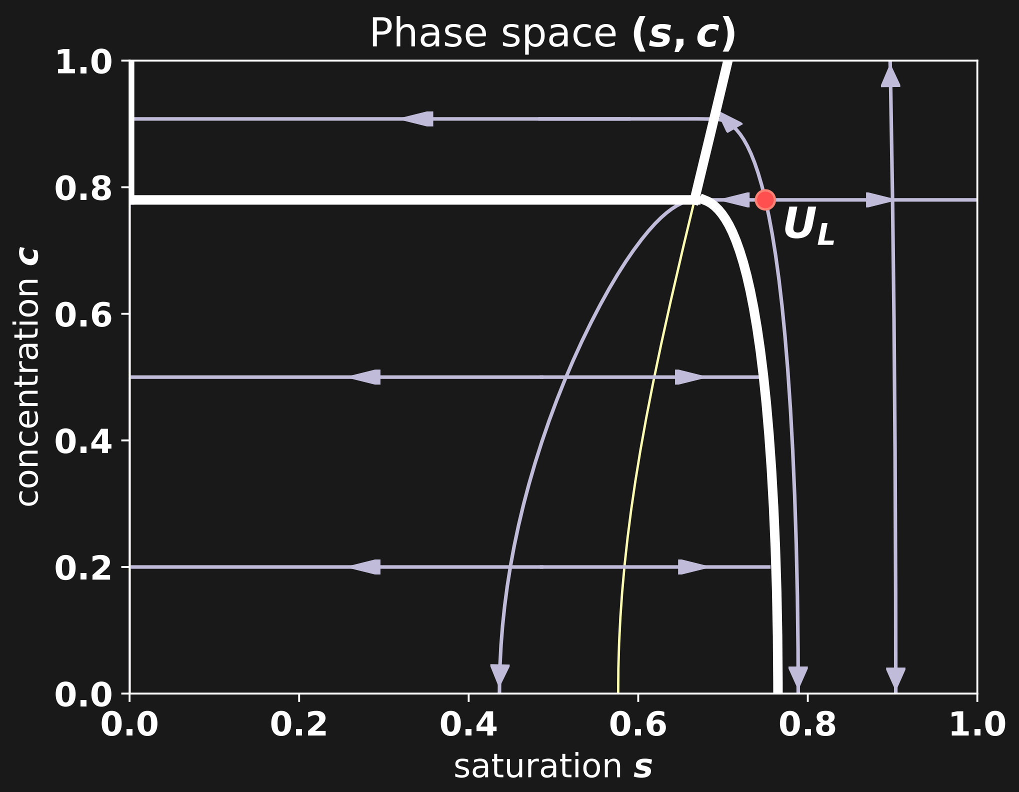
MAIN RESULT

Theorem (P., Marchesin, Plohr '2022). The set of admissible Riemann solutions is the same for the Isaacson-Glimm and vanishing adsorption admissibility criteria.

The solutions may be presented by two diagrams (E. Isaacson '1980):

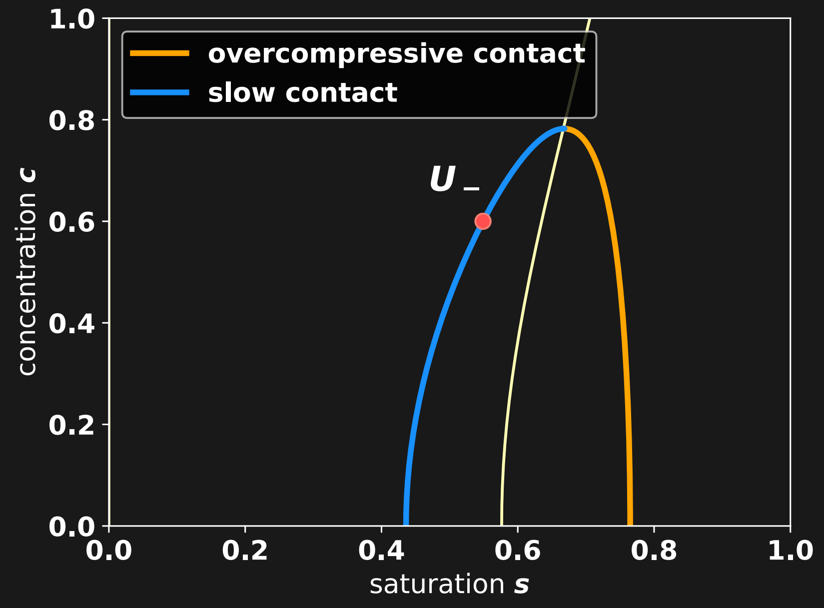


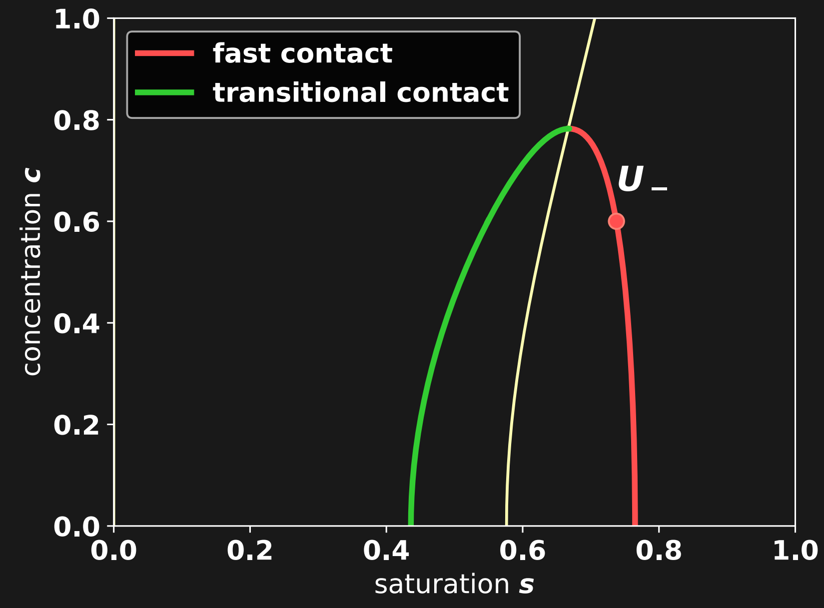
U_L to the left of coincidence



U_L to the right of coincidence

MAIN STEP: ADMISSIBLE CONTACTS





- *Isaacson-Glimm criterion*: slow and fast (Lax) contact discontinuities are admissible, overcompressive and undercompressive are not.
- *Vanishing adsorption criterion*: slow, fast and overcompressive contact discontinuities are admissible, undercompressive are not.

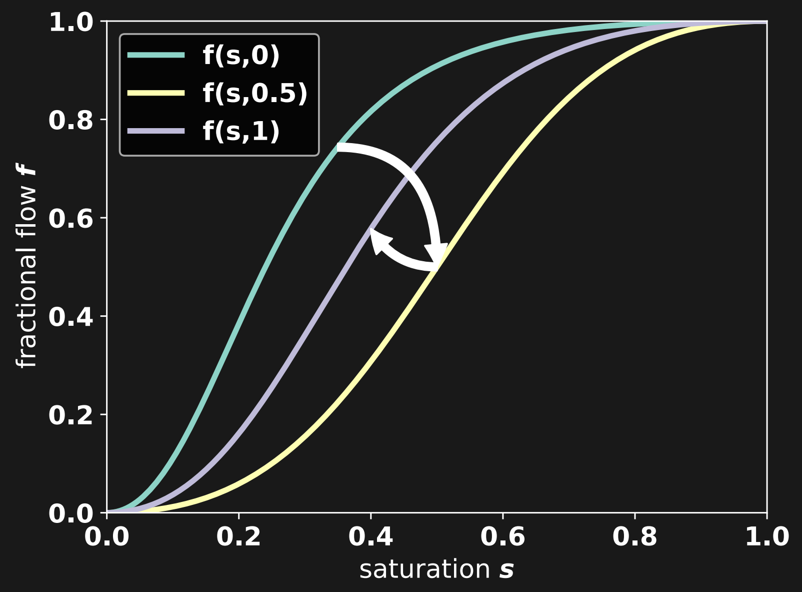
P.S. Overcompressive contacts can be represented as a sequence of two waves (c and s). Whether or not they are regarded as admissible does not affect the Riemann problem solution.

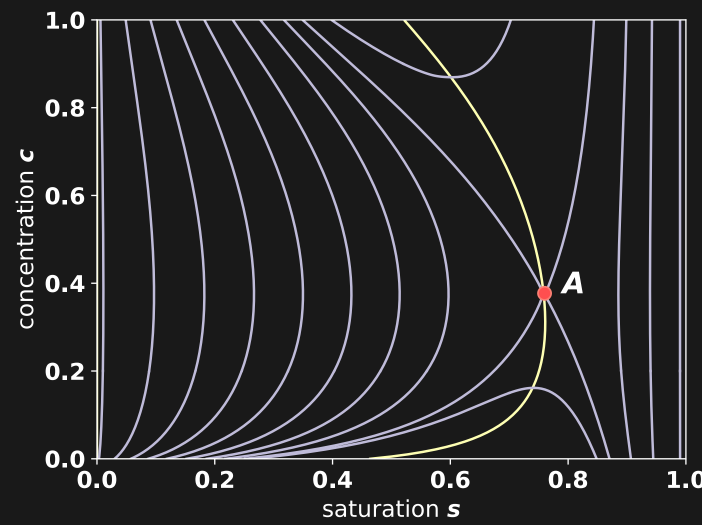
NON-MONOTONE CHEMICAL FLOODING MODEL

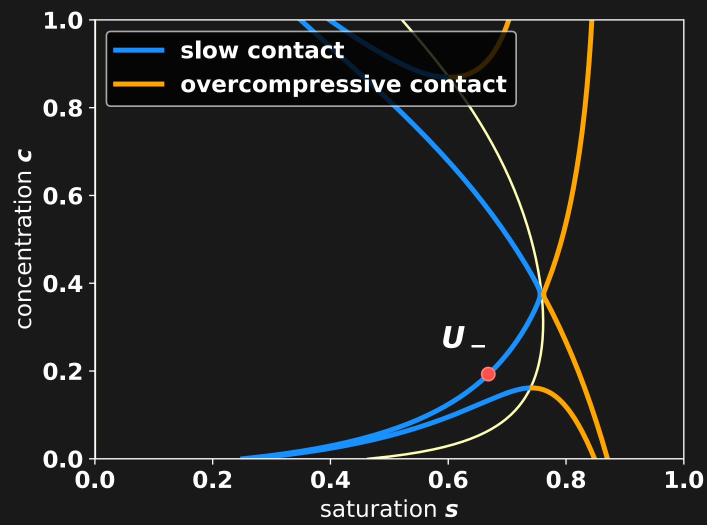
Two-phase oil-water flow with *surfactant* in the water

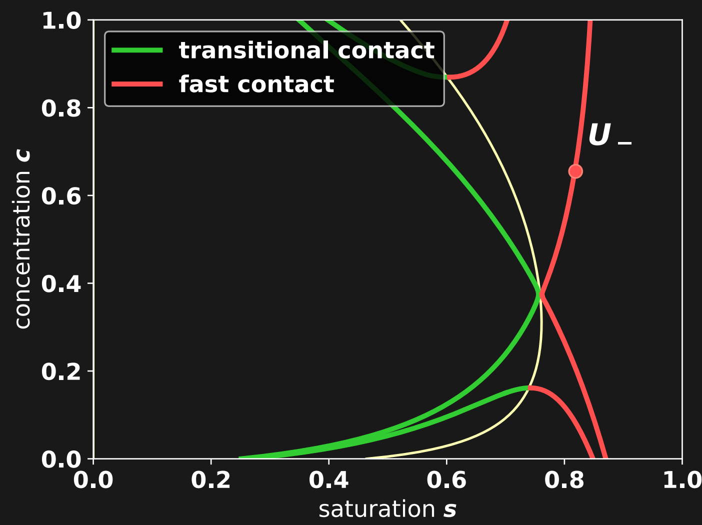
$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ (cs + \alpha a(c))_t + (cf(s,c))_x &= 0. \end{aligned}$$

- *f is non-monotone in c*: $\exists c^* \forall s \in [0,1]$
  - $f_c < 0$ for $c < c^*$
  - $f_c > 0$ for $c > c^*$
- work in progress
- umbilic point A is present



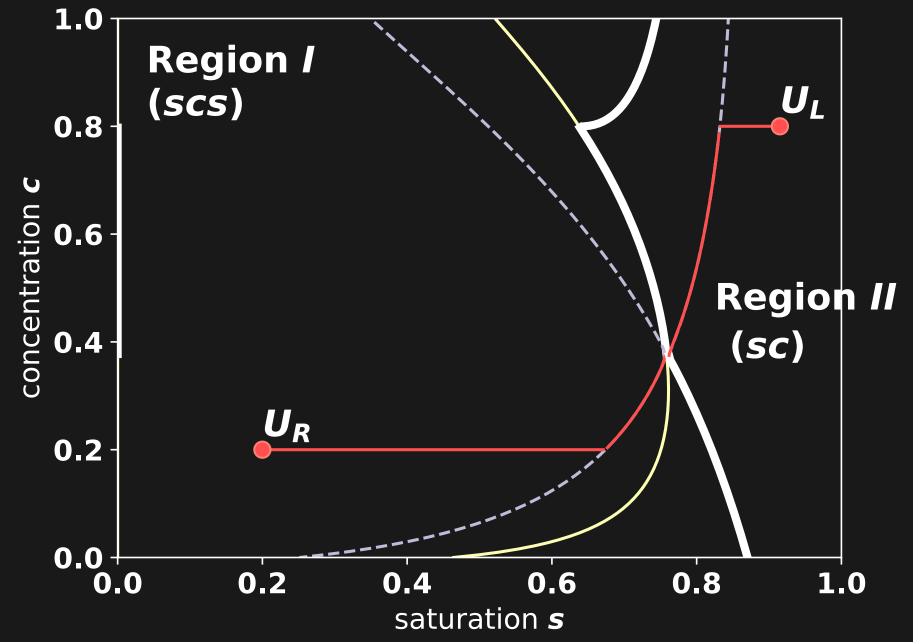






NON-MONOTONE CHEMICAL FLOODING MODEL

Theorem (P., Marchesin, Plohr '2022). The set of admissible Riemann solutions is the same for the Isaacson-Glimm and vanishing adsorption admissibility criteria for non-monotone chemical flooding model.



For model with adsorption M_α:

- there exist *transitional shocks and rarefactions*
- See: Entov, Kerimov (1986), Shen (2017), Bakharev-Enin-Rastegaev-P. (2021)
- *Consequence*: there exist admissible transitional contacts as a limit of transitional shocks

TRANSITIONAL SHOCKS FOR NON-MONOTONE CASE

Vanishing viscosity admissibility criterion for M_α (capillarity and diffusion):

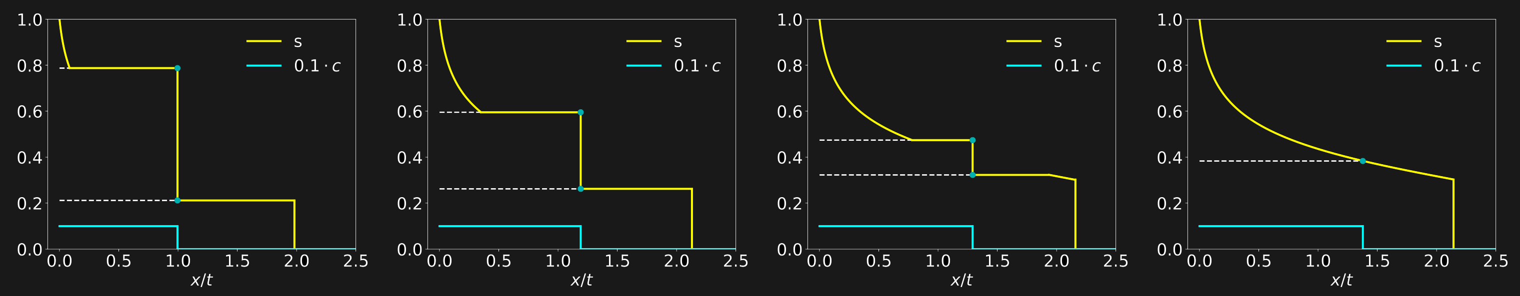
$$s_t + f(s,c)_x = \varepsilon_c (s_x)_x,$$
$$(cs + a(c))_t + (cf(s,c))_x = \varepsilon_c (cs_x)_x + \varepsilon_d (c_x)_x.$$

Theorem (Bakharev, Enin, P., Rastegaev '2021).

Fix $c^- > c^+$. There exist $0 < v_{\min} < v_{\max} < \infty$, such that $\forall \kappa = \varepsilon_d / \varepsilon_c \in (0, +\infty)$, there exists a unique

- points $s^\pm(\kappa) \in [0,1]$
- velocity $v(\kappa) \in [v_{\min}, v_{\max}]$

such that there exists a travelling wave, connecting two saddle points $u^-(\kappa) = (s^-(\kappa), c^-)$ and $u^+(\kappa) = (s^+(\kappa), c^+)$ with velocity $v(\kappa)$. Moreover, $v(\kappa)$ is monotone and continuous; $v(\kappa) \to v_{\min}$ as $\kappa \to \infty$ and $v(\kappa) \to v_{\max}$ as $\kappa \to 0$.



Thank you!

yulia.petrova@impa.br

https://yulia-petrova.github.io

75 anos impa
Instituto de Matemática Pura e Aplicada

FLUID
Grupo de Dinâmica dos Fluidos - IMPA

References:

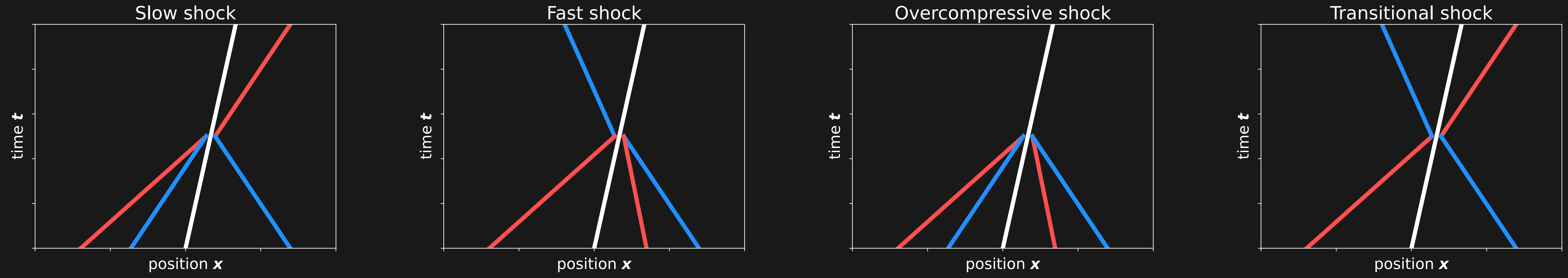
Own works:

- **Yu. Petrova**, D. Marchesin, B. Plohr. Work in progress.
- F. Bakharev, A. Enin, **Yu. Petrova**, N. Rastegaev, 2021. Impact of dissipation ratio on vanishing viscosity solutions of the Riemann problem for chemical flooding model. arXiv:2111.15001

Other works:

- E. Isaacson. Global solution of a Riemann problem for a non-strictly hyperbolic system of conservation laws arising in enhanced oil recovery, Rockefeller University, New York, NY, preprint, 1980.
- B. Temple. Systems of conservation laws with invariant submanifolds. Transactions of the American Mathematical Society, 280(2), pp.781-795, 1983.
- V. Entov and Z. Kerimov. Displacement of oil by an active solution with a nonmonotonic effect on the flow distribution function. Fluid Dynamics, 21(1), pp.64–70, 1986.
- W. Shen. On the uniqueness of vanishing viscosity solutions for Riemann problems for polymer flooding. Nonlinear Differential Equations and Applications NoDEA, 24(4), pp.1-25, 2017.
- T. Johansen and R. Winther. The solution of the Riemann problem for a hyperbolic system of conservation laws modeling polymer flooding. SIAM Journal on Mathematical Analysis, 19(3), pp.541-566, 1988.
- Keyfitz, B.L. and Kranzer, H.C., 1980. A system of non-strictly hyperbolic conservation laws arising in elasticity theory. Archive for Rational Mechanics and Analysis, 72(3), pp.219-241.

SHOCKS TYPES



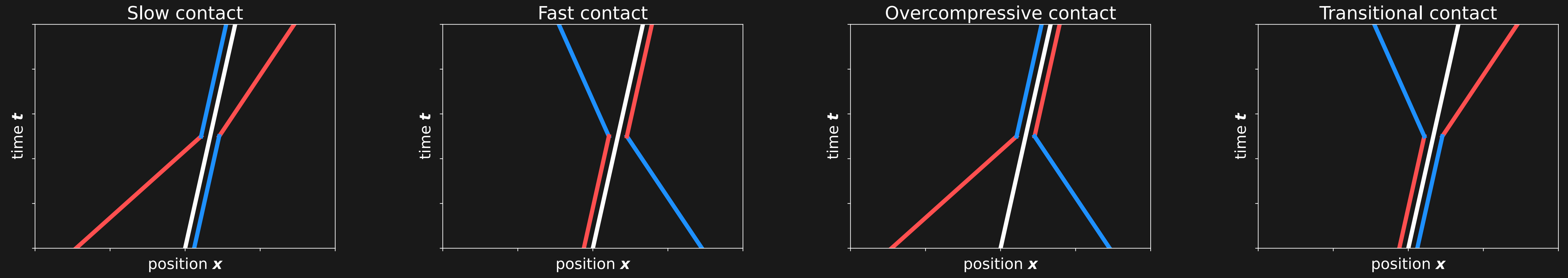
A shock from U_- to U_+ with speed σ is called

- slow if $\lambda_1(U_-) > \sigma > \lambda_1(U_+)$ and $\sigma < \lambda_2(U_-), \lambda_2(U_+)$;
- fast if $\lambda_2(U_-) > \sigma > \lambda_2(U_+)$ and $\sigma > \lambda_1(U_-), \lambda_1(U_+)$;
- overcompressive if $\sigma > \lambda_2(U_+), \lambda_1(U_+)$ and $\sigma < \lambda_1(U_-), \lambda_2(U_-)$;
- undercompressive if $\sigma < \lambda_1(U_-), \lambda_1(U_+)$ and $\sigma > \lambda_2(U_-), \lambda_2(U_+)$.

Undercompressive $\equiv$ transitional.

Fast and slow are also called Lax shocks (classical).

TYPES OF CONTACT DISCONTINUITIES



We will call a contact discontinuity from U_- to U_+ with speed σ

- slow if $\lambda_2(U_-) > \lambda_1(U_-) = \sigma = \lambda_1(U_+) < \lambda_2(U_+)$;
- fast if $\lambda_1(U_-) < \lambda_2(U_-) = \sigma = \lambda_2(U_+) > \lambda_1(U_+)$;
- overcompressive if $\lambda_2(U_-) > \lambda_1(U_-) = \sigma = \lambda_2(U_+) > \lambda_1(U_+)$;
- undercompressive if $\lambda_1(U_-) < \lambda_2(U_-) = \sigma = \lambda_1(U_+) < \lambda_2(U_+)$.

Undercompressive $\equiv$ transitional.