№2.13 a) $T(n) = \begin{cases} O(1) & , n=0 \\ T(n-1) + O(1) & , n \geq 1 \end{cases}$

$T(n) <= T(n-1) + C <=$

$<= T(n-2) + C + C <=$

$<= T(n-3) + C + C + C =$

$= T(n-3) + C(1+1+1) <= \ldots <=$

$<= T(n-n) + C(n) <= O(1) + C(n) <= C + C(n) = O(n)$

d) $T(n) = \begin{cases} O(1) & , n \leq a,\ a > 1 \\ a\,T(n-a) + O(1) & , n > a \end{cases}$

$T(n) <= aT(n-a) + C <=$

$<= a(aT(n-2a) + C) + C =$

$= a^2 \cdot T(n-2a) + a \cdot C + C = a^2 T(n-2a) + C(a+1) <=$

$<= a^2(a\,T(n-3a) + C) + C(a+1) =$

$= a^3 T(n-3a) + C(a^2 + a + 1) <=$ $\qquad n - ka \leq a$

$\qquad k \geq [-1 + \frac{n}{a}]$

$<= a^k T(n-ka) + C(a^{k-1} + a^k + \ldots + 1) =$

$= a^k \cdot C + C\left(\sum_{i=0}^{k-1} a^i\right) = a^k \cdot C + C\left(\frac{a^{k-1}}{a-1}\right) =$

$= C(a^{[\frac{n}{a}]}) = O(a^n)$

g) $T(n) = \begin{cases} O(1) & , n = 1 \\ aT([\frac{n}{a}]) + O(1) & , n \geq 2, a \geq 2 \end{cases}$

$n = a^m \qquad m = \log_a n$

$T(n) <= aT(a^{m-1}) + C <=$

$<= a(aT(a^{m-2}) + C) + C =$

$= a^2T(a^{m-2}) + C(a+1) <=$

$<= a^3T(a^{m-3}) + C(a^2+a+1) <= \_\_ <=$

$<= a^mT(a^{m-m}) + C(a^{m-1} + \dots + 1) =$

$= a^m \cdot C + C\left(\frac{a^{m-1}-1}{a-1}\right) = C\left(\frac{a^{\log_a n}-1}{a-1}\right) = C(a^{\log_a n}) =$

$= O(n)$

h) $T(n) = \begin{cases} O(1) & , n = 1 \\ aT([\frac{n}{a}]) + O(n) & , n \geq 2, a \geq 2 \end{cases}$

$T(n) <= aT([\frac{n}{a}]) + C \cdot n = \qquad n = a^m \quad m = \log_a n$

$= aT(a^{m-1}) + C \cdot a^m <=$

$<= a(aT(a^{m-2}) + Ca^{m-1}) + Ca^m =$

$= a^2T(a^{m-2}) + C(a^m + a^m) <=$

$<= a^2(aT(a^{m-3}) + C(a^{m-2})) + C(a^m + a^m) =$

$= a^3T(a^{m-3}) + C(a^m + a^m + a^m) <=$

$<= a^mT(a^{m-m}) + C(ma^m) = C(\log_a n \cdot n) =$

$= O(n \log n)$