1.3 a) $T(n) = \begin{cases} 1 & n \leq a, \ a > 0 \\ T(n-a) + 1, & n > a. \end{cases}$

$T(n) = T(n-a) + 1 = T(n-2a) + 2 = T(n-ka) + k = |n - ka \leq a| =$

$= k + 1 = [-1 + \frac{n}{a}] + 1 = \frac{n}{a}$

$n - ka \leq a \qquad ka \geq -a + n \qquad k \geq [-1 + \frac{n}{a}]$

b) $T(n) = \begin{cases} 1 & k = 0 \\ T(n-1) + 2^n, & n \geq 1 \end{cases}$

$T(n) = T(n-1) + 2^n = T(n-2) + 2^{n-1} + 2^n = T(n-3) + 2^{n-2} + 2^{n-1} + 2^n =$

$= T(n-n) + \sum_{k=0}^{n} 2^k = \sum_{k=0}^{n} 2^k = 2^{n+1} - 1.$

c) $T(n) = \begin{cases} 1 & n = 1 \\ 2T([\frac{n}{2}]) + 1, & n \geq 2 \end{cases}$

Нехай $n = 2^m \Rightarrow m = \log_2 n.$

$$T(n) = 2T\left(\left[\frac{n}{2}\right]\right) + 1 = 2T\left(2^{m-1}\right) + 1 = 2\left(2T\left(2^{m-2}\right) + 1\right) + 1 =$$

$$= 4T\left(2^{m-2}\right) + 2 + 1 = 4\left(2T\left(2^{m-3}\right) + 1\right) + 2 + 1 =$$

$$= 8T\left(2^{m-3}\right) + 2^2 + 2 + 1 = 2^3 T\left(2^{m-3}\right) + 2^2 + 2 + 1 =$$

$$= 2^m T\left(2^{m-m}\right) + 2^{m-1} + \ldots + 2 + 1 = 2^m + \sum_{i=0}^{m-1} 2^i =$$

$$= 2^m + 2^m - 1 = 2^{m+1} - 1 = 2n - 1.$$

d) $T(n) = \begin{cases} 1 & n = 1 \\ aT\left(\left[\frac{n}{a}\right]\right) + n & n \geq 2, \ a \geq 2 \end{cases}$

flescau $n = a^m$, $m = \log_a n$

$$T(n) = aT\left(a^{m-1}\right) + n = a\left(aT\left(a^{m-2}\right) + a^{m-1}\right) + a^m =$$

$$= a^2 T\left(a^{m-2}\right) + a^m + a^m = a^2\left(aT\left(a^{m-3}\right) + a^{m-2}\right) + a^m + a^m =$$

$$= a^3 T\left(a^{m-3}\right) + a^m + a^m + a^m = a^m T\left(a^{m-m}\right) + \underbrace{a^m + \ldots + a^m}_{m} =$$

$$= a^m + m a^m = n + \log_a(n) \cdot n = n\left(1 + \log_a(n)\right)$$

d)

```
1    i=0
2    while i<n:
3        j=0
4        while j+1*i:
5            k+=1
6            j+=1
7        i+=1
```

2

$3 \cdot (n+1)$

$2 \cdot n$

$5 \left(n + \dfrac{n(n-1)(2n-1)}{6} \right)$

$4 \cdot \left(\dfrac{n(n-1)(2n-1)}{6} \right)$

$4 \cdot \dfrac{n(n-1)(2n-1)}{6}$

$4 \cdot n$

$4: \quad 5 \cdot \displaystyle\sum_{i=0}^{n-1} i^2 + 1 = 5\left(\sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} 1 \right) = 5\left(0 + 1^2 + 2^2 + \dots + (n-1)^2 + n \right) =$

$= 5\left(n + \dfrac{n(n-1)(2n-1)}{6} \right)$

$5 \; \text{i} \; 6: \; 4 \cdot \displaystyle\sum_{i=0}^{n-1} i^2 = 4 \cdot \dfrac{n(n-1)(2n-1)}{6}$

$T(n) = 2 + 3n + 3 + 2n + 5n + \dfrac{5n(n-1)(2n-1)}{6} + \dfrac{4n(n-1)(2n-1)}{6} + \dfrac{4n(n-1)(2n-1)}{6} +$

$+ 4n = 5 + 14n + \dfrac{26n^3 - 39n^2 + 13n}{6}$

e)

```
1    i=1
2    while i<n:
3        j=1
4        while j<n:
5            k+=1
6            j*=2
7        i*=2
```

2

$3 \cdot (m+1)$

$2 \cdot m$

$3 \cdot m \cdot (m+1)$

$4 \cdot m \cdot m$

$4 \cdot m \cdot m$

$4 \cdot m$

$n = 2^m \qquad m = \log_2(n)$

$T(n) = 2 + 3m + 3 + 2m + 3m^2 + 3m + 4m^2 + 4m^2 + 4m = 11m^2 + 12m + 5 =$

$= 11 \log_2^2(n) + 12\log_2(n) + 5$