**13** a)
```
1   k += 1                    4
2   i = n                     2
3   while i > 0:              3 · (n+1)
4       i -= 1                4 · n
```

$T(n) = 7n + 9$

b)
```
1   i = n                     2
2   while i > 1:              3 · (m+1)
3       k += 1                4 · m
4       i //= 2               4 · m
```

$n = 2^m \qquad m = \log_2(n)$

$n = 1 \qquad m = 0 \quad \rightarrow \quad$ loop block: 0
$n = 2 \qquad m = 1 \quad \rightarrow \quad$ loop block: 1
$n = 4 \qquad m = 2 \quad \rightarrow \quad$ loop block: 2
$n \qquad\quad m \qquad \rightarrow \quad$ loop block: m

$T(n) = 11m + 5 = 11 \log_2(n) + 5$

c)
```
1   i = 0                     2
2   while i < n:              3 (n/2 + 1)
3       j = 0                 2 · n/2
4       while j < n:          3 (n/2 + 1) · n/2
5           k += 1            4 · n/2 · n/2
6           j += 2            4 · n/2 · n/2
7       i += 2                4 · n/2
```

$T(n) = 2 + 3 + \dfrac{3n}{2} + \dfrac{2n}{2} + \dfrac{3n^2}{4} + \dfrac{3n}{2} + \dfrac{4n^2}{4} + \dfrac{4n^2}{4} + \dfrac{4n}{2} = \dfrac{11n^2}{4} + 6n + 5$

**d)**

```
1      i=0
2      while i<n:
3          j=0
4          while j+1*i:
5              k+=1
6              j+=1
7          i+=1
```

2

3·(n+1)

2·n

$5\left(n + \dfrac{n(n-1)(2n-1)}{6}\right)$

$4\cdot\left(\dfrac{n(n-1)(2n-1)}{6}\right)$

$4\cdot \dfrac{n(n-1)(2n-1)}{6}$

4·n

4. $5\cdot\displaystyle\sum_{i=0}^{n-1} i^2+1 = 5\left(\sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} 1\right) = 5\left(0+1^2+2^2+ \ldots + (n-1)^2 + n\right) =$

$= 5\left(n + \dfrac{n(n-1)(2n-1)}{6}\right)$

5 ; 6 : $4\cdot\displaystyle\sum_{i=0}^{n-1} i^2 = 4\cdot \dfrac{n(n-1)(2n-1)}{6}$

$T(n) = 2+3n+3+2n+5n+ \dfrac{5n(n-1)(2n-1)}{6} + \dfrac{4n(n-1)(2n-1)}{6} + \dfrac{4n(n-1)(2n-1)}{6} +$

$+ 4n = 5+14n+ \dfrac{26n^3-39n^2+13n}{6}$

**e)**

```
1      i=1
2      while i<n:
3          j=1
4          while j<n:
5              k+=1
6              j*=2
7          i*=2
```

2

3·(m+1)

2·m

3·m·(m+1)

4·m·m

4·m·m

4·m

$n=2^m \qquad m=\log_2(n)$

$T(n) = 2+3m+3+2m+3m^2+3m+4m^2+4m^2+4m = 11m^2 +12m+5 =$

$= 11\log_2^2(n)+12\log_2(n)+5$

f)

| | | |
|---|---|---|
| 1 | $i = 1$ | 2 |
| 2 | while $i < n$: | $3(m+1)$ |
| 3 | $j = i$ | $3m$ |
| 4 | while $j < n$: | $3 \cdot (m + 1 + 2 + \cdots + (m-1) + m) = \dfrac{3(m+1)m}{2} + 3m$ |
| 5 | $k += 2$ | $4 \quad \dfrac{m(m+1)}{2}$ |
| 6 | $j *= 2$ | $4 \cdot \dfrac{m(m+1)}{2}$ |
| 7 | $i *= 2$ | $4m$ |

$n = 2^m \qquad m = \log_2(n)$

$n = 1 \qquad m = 0 \quad \rightarrow \quad 0$

$n = 2 \qquad m = 1 \quad \rightarrow \quad 1$

$n = 4 \qquad m = 2 \quad \rightarrow \quad 2$

$T(n) = 2 + 3m + 3 + 3m + 3m + \dfrac{3(m+1)m}{2} + \dfrac{4m(m+1)}{2} + \dfrac{4m(m+1)}{2} + 4m =$

$= \dfrac{11m^2 + 11m}{2} + 5 + 13m = \dfrac{11m^2 + 37m}{2} + 5$