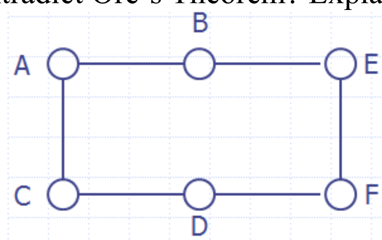
Lab 14

1. Answer the following questions about the graph G having n = 6 vertices, below.
   a. Is G Hamiltonian?
   b. Can you find two non-adjacent vertices the sum of whose degrees is less than 6?
   c. Do these facts contradict Ore's Theorem? Explain.



2. Show that TSP is NP-complete. (Hint: use the relationship between TSP and HamiltonianCycle discussed in the slides. You may assume that the HamiltonianCycle problem is NP-complete.)

3. Below is another variation of the Knapsack problem.
   Given a set S = $\{s_0, s_1, \ldots, s_{n-1}\}$ of items, weights $\{w_0, w_1, \ldots, w_{n-1}\}$, values $\{v_0, v_1, \ldots, v_{n-1}\}$, a max weight W, and a min value V, find a subset T of S whose total value is no less than V and total weight is at most W.

   Show that the SubsetSum problem is polynomial reducible to this Knapsack problem.

4. Show that the worst case for VertexCoverApprox can happen by giving an example of a graph G which has these properties:
   a. G has a smallest vertex cover of size *s*
   b. VertexCoverApprox outputs size 2*s as its approximation to optimal size.

5. Find an O(n) algorithm that does the following: Given a size n input array of integers, output the first numbers in the array (from left to right) whose sum is exactly 10 (or indicate that no such numbers can be found).

6. The decision problem formulation of the Vertex Cover problem is this: Given a positive integer k, and a graph G, is there a vertex cover for G having size $\leq k$? Show that this decision problem belongs to NP.