Lab 1 Math problems Answer

Math Review Problem 1

(1) Increasing Function $f(x) = x^3 + x$

(2) Eventually non-decreasing function: $f(x) = x^2 + 2x + 1$

Math Review Problem 2

(1) $\lim_{n\to\infty} \frac{(2n^2+3n)}{(n^3-4)} = \lim_{n\to\infty} \frac{4n+3}{3n^2} = \lim_{n\to\infty} \frac{4}{6n} = 0$

(2) $\lim_{n\to\infty} \frac{n^2}{2^n} = \lim_{n\to\infty} \frac{2n}{2^n \ln 2} = \lim_{n\to\infty} \frac{2}{2^n \cdot \ln 2 \cdot \ln 2} = 0$

Math Review Problem 3:

for all $n > 4$, $2^n < n!$

Using Induction.

(1) $n = 5$, $2^5 = 32$, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \Rightarrow 2^5 < 5!$

(2) Assume $2^n < n!$, $2^{(n+1)} = 2 \cdot 2^n = 2^n + 2^n$

$(n+1)! = (n+1)n! = n \cdot n! + n! = (n-1)n! + n! + n$

$\because n - 1 > 3 \Rightarrow (n-1)n! > 0$,

$\because 2^n < n! \Rightarrow 2^n + 2^n < (n-1)n! + n! + n!$

$\Rightarrow 2^{(n+1)} < (n+1)!$

$\Rightarrow$ For all $n > 4$, $2^n < n!$

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