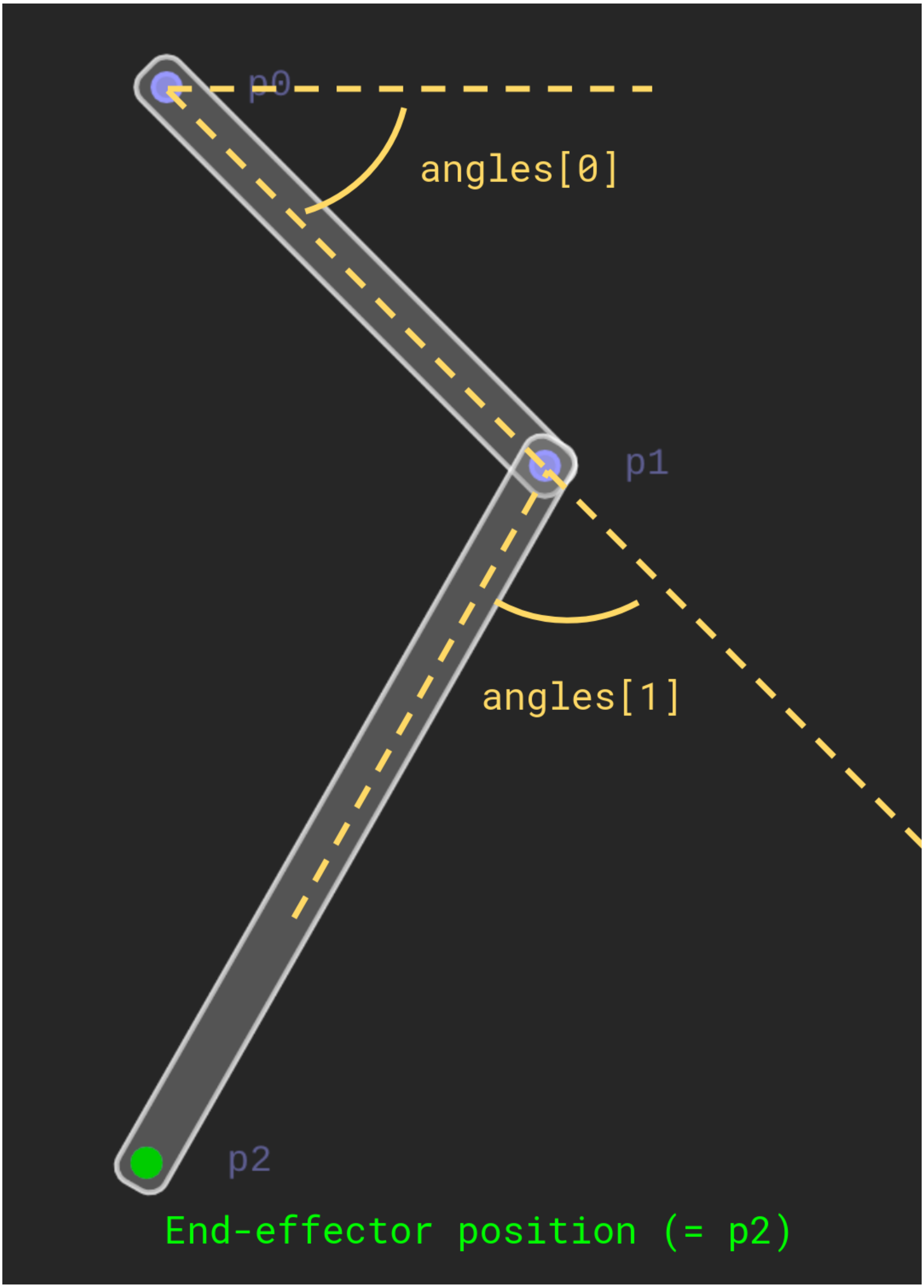
Assignment 1

Inverse Kinematics

For a more readable version of this document, see Readme.pdf.

In this assignment you will implement forward and inverse kinematics of a two-bar linkage.



1 - Forward Kinematics

Code: File `src/kinematics/ForwardKinematics.h`, function `forwardKinematics(...)`.

Task: Implement forward kinematics for the linkage above by computing the coordinates of points `p0`, `p1` and `p2` given angles `angles`.

Details: The function `forwardKinematics` returns an array of size 3 containg the positions of the 3 points that define the state of the linkage, `p0`, `p1` and `p2`. The angles of the linkage are defined in the figure above.

Test: Compile and run `src/test-a1/test.cpp`. Test 1 should pass.

Compile and run `src/app/linkage-app.cpp`. You can change the angles with the sliders or by left-clicking in the left half of the app.

2 - Derivatives of Forward Kinematics

We want to solve the inverse kinematics using an optimization-based approach. For this, we will need derivatives of thee forward kinematics, or in other words, how the end-effector coordinates change when we change the angles.

Code: File `src/kinematics/ForwardKinematics.h`, functions `dendEffector_dangles(...)` and `ddendEffector_ddangles(...)`.

Task: Implement the 1st and 2nd derivative (Jacobian and derivative of Jacobian) of the end-effector position with respect to the angles.

Details: Implementing these functions will help us later setup inverse kinematics.

Test: Compile and run `src/test-a1/test.cpp`. Test 2 and 3 should pass.

3 - Inverse Kinematics

Now we are ready to solve inverse kinematics. We want to construct an objective function that describes our goal: Reach a target position (`target`) with the end-effector (`endEffectorPosition(...)`).

Code: File `src/kinematics/InverseKinematics.h`.

Tasks:

1. Implement the objective function

$$f(x) = \frac{1}{2}(e(x) - x_t)^T(e(x) - x_t)$$

   where x are the angles and $e(x)$ returns the position of the end-effector.
   Code: `InverseKinematics::evaluate(x)`. `x` are the angles.
   Test: Compile and run `src/test-a1/test.cpp`. Test 4 should pass.

2. Implement the *analytic* gradient

$$\frac{\partial f}{\partial x} = \nabla_x f.$$

   Code: `InverseKinematics::gradient(x)`.
   Hint: Leverage the functions you implemented in part 2!
   Test: Compile and run `src/test-a1/test.cpp`. Test 5 should pass.

3. Implement the *analytic* Hessian

$$\frac{\partial^2 f}{\partial x^2} = \nabla_x^2 f.$$

   Code: `InverseKinematics::hessian(x)`.
   Hint: Leverage the functions you implemented in part 2!
   Test: Compile and run `src/test-a1/test.cpp`. Test 6 should pass.

Now that you have completed all of part 3, you can try out the app. Change the optimization strategy, and change the energy landscape by changing the target.