# C.1.1.

## Functions

In Exercises 1–6, find the domain and range of each function.

**1.** $f(x) = 1 + x^2$ **2.** $f(x) = 1 - \sqrt{x}$

**3.** $F(x) = \sqrt{5x + 10}$ **4.** $g(x) = \sqrt{x^2 - 3x}$

**5.** $f(t) = \dfrac{4}{3 - t}$ **6.** $G(t) = \dfrac{2}{t^2 - 16}$

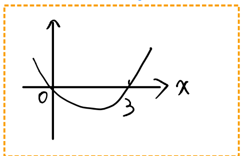
4. Domain : $x^2 - 3x \geq 0$

$\Rightarrow x(x-3) \geq 0$

$\Rightarrow x \leq 0 \cap x \geq 3$

$D: (-\infty, 0] \cup [3, +\infty)$

"Algebra + Figure"



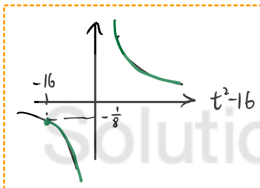

Range : when $x \leq 0$, $g(x) = \sqrt{x^2 - 3x} \geq 0$

when $x \geq 3$, $g(x) = \sqrt{x^2 - 3x} \geq 0$

Thus, $g(x) \geq 0$

$R: [0, +\infty)$

6. $G(t) = \dfrac{2}{t^2 - 16}$




Domain : $t^2 - 16 \neq 0$

$\Rightarrow t^2 \neq 16$

$\Rightarrow t \neq \pm 4$

Thus, $D: (-\infty, -4) \cup (-4, 4) \cup (4, +\infty)$

Range : $\begin{cases} t^2 - 16 \geq -16 \\ t^2 - 16 \neq 0 \end{cases}$

$\Rightarrow \dfrac{2}{t^2-16} \in (0, +\infty) \cup (-\infty, -\frac{1}{8}]$

$R: (0, +\infty) \cup (-\infty, -\frac{1}{8}]$

## Increasing and Decreasing Functions

Graph the functions in Exercises 37–46. What symmetries, if any, do the graphs have? Specify the intervals over which the function is increasing and the intervals where it is decreasing.

**37.** $y = -x^3$ **38.** $y = -\dfrac{1}{x^2}$

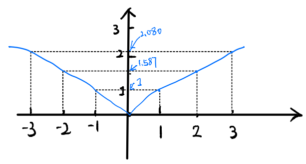
**39.** $y = -\dfrac{1}{x}$ **40.** $y = \dfrac{1}{|x|}$

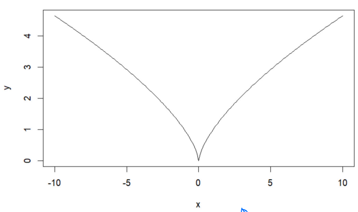
**41.** $y = \sqrt{|x|}$ **42.** $y = \sqrt{-x}$

**43.** $y = x^3/8$ **44.** $y = -4\sqrt{x}$

**45.** $y = -x^{3/2}$ **46.** $y = (-x)^{2/3}$

$y = (-x)^{\frac{2}{3}} = \sqrt[3]{(-x)^2}$




by software

Symmetric about the y-axis

Dec. : $x \in (-\infty, 0]$ or $-\infty < x \leq 0$

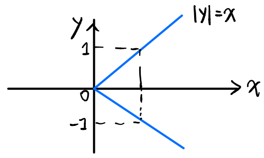
Inc. : $x \in [0, +\infty)$ or $0 \leq x < +\infty$

**23.** Graph the following equations and explain why they are not graphs of functions of $x$.

**a.** $|y| = x$ **b.** $y^2 = x^2$

a. $|y| = x$

$\Rightarrow x = |y| \ (x \geq 0)$



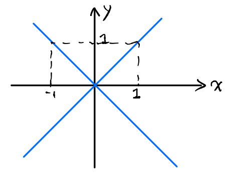

Reason : For each value of $x > 0$, there are two values of y.

b. $y^2 = x^2$

$\Rightarrow |y| = |x|$

$y \geq 0$

$y = |x|$



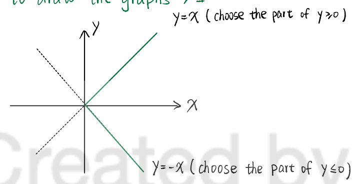

Reason : For each value of $x \neq 0$, there are two values of y.

[ Tip : ⟨About how to draw the graphs⟩ ]

$|y| = x$

$\Rightarrow \begin{cases} y = x, & y \geq 0 \\ -y = x, & y \leq 0 \end{cases}$

$\Rightarrow \begin{cases} y = x, & y \geq 0 \\ y = -x, & y \leq 0 \end{cases}$






## Even and Odd Functions

In Exercises 47–62, say whether the function is even, odd, or neither. Give reasons for your answer.

**47.** $f(x) = 3$ **48.** $f(x) = x^{-5}$

**49.** $f(x) = x^2 + 1$ **50.** $f(x) = x^2 + x$

**51.** $g(x) = x^3 + x$ **52.** $g(x) = x^4 + 3x^2 - 1$

**53.** $g(x) = \dfrac{1}{x^2 - 1}$ **54.** $g(x) = \dfrac{x}{x^2 - 1}$

**55.** $h(t) = \dfrac{1}{t - 1}$ **56.** $h(t) = |t^3|$

**57.** $h(t) = 2t + 1$ **58.** $h(t) = 2|t| + 1$

$h(t) = 2|t| + 1$ $D: t \in \mathbb{R}$

$h(-t) = 2|-t| + 1 = 2|t| + 1 = h(t)$

$\therefore h(-t) = h(t)$

Even

Recall:

Even function of $x$ if $f(-x) = f(x)$

Odd function of $x$ if $f(-x) = -f(x)$

T 72. a. Graph the functions $f(x) = 3/(x-1)$ and $g(x) = 2/(x+1)$ together to identify the values of $x$ for which

$$\frac{3}{x-1} < \frac{2}{x+1}.$$

b. Confirm your findings in part (a) algebraically.

a. $f(x) = \frac{3}{x-1}$ $(x \neq 1)$ $g(x) = \frac{2}{x+1}$ $(x \neq -1)$

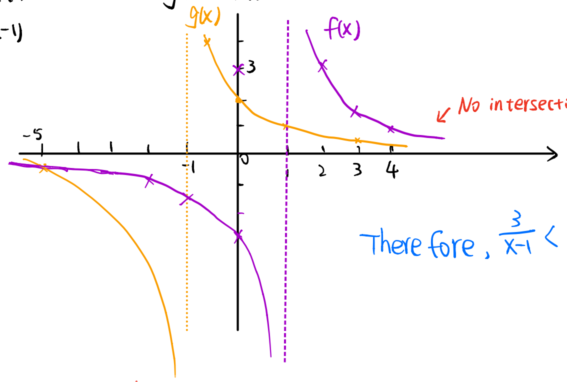
< Tip: How to draw by hand? >

S1. Find the intersection point.

$\frac{3}{x-1} = \frac{2}{x+1} \Rightarrow 3(x+1) = 2(x-1)$

$\Rightarrow x = -5$

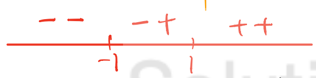
S2. Rough outline + Several Point




| g(x) | f(x) | intersection point |
|---|---|---|
| $(-\frac{1}{2}, 4)$ | $(0, 3)$ | $(-5, -\frac{1}{2})$ |
| $(0, 2)$ | $(2, 3)$ | |
| $(1, 1)$ | $(3, \frac{3}{2})$ | |
| $(3, \frac{1}{2})$ | | |
| | $(4, 1)$ | |

There fore, $\frac{3}{x-1} < \frac{2}{x+1}$ when $x \in (-\infty, -5) \cup (-1, 1)$

b. $\frac{3}{x-1} < \frac{2}{x+1}$




1) when $x \in (-1, 1)$

$\frac{2}{x+1} > 0$, $\frac{3}{x-1} < 0$

$\Rightarrow \frac{2}{x+1} > \frac{3}{x-1}$

$\therefore \frac{2}{x+1} > \frac{3}{x-1}$ $\forall x \in (-1, 1)$

in this condition.

2) when $x \in (-\infty, -1)$

$\Rightarrow x-1 < 0$, $x+1 < 0 \Rightarrow (x-1)(x+1) > 0$

$\Rightarrow \frac{3}{x-1}(x-1)(x+1) < \frac{2}{x+1}(x-1)(x+1)$

$\Rightarrow 3(x+1) < 2(x-1)$

$\Rightarrow 3x+3 < 2x-2$

$\Rightarrow x < -5$

$\therefore \frac{2}{x+1} > \frac{3}{x-1}$ when $x \in (-\infty, -5)$ in this condition

3) when $x > 1$

$\Rightarrow x-1, x+1 > 0$

$\Rightarrow \frac{3}{x-1}(x-1)(x+1) < \frac{2}{x+1}(x-1)(x+1)$

$3(x+1) < 2(x-1)$

$3x+3 < 2x-2$

$x < -5$

$\therefore \frac{2}{x+1} < \frac{3}{x-1}$, $\forall x \in (1, +\infty)$

Thus, $\frac{2}{x+1} > \frac{3}{x-1}$ when $x \in (-\infty, -5) \cup (-1, 1)$

# C 1.2

**16.** Evaluate each expression using the functions

$$f(x) = 2 - x, \quad g(x) = \begin{cases} -x, & -2 \le x < 0 \\ x - 1, & 0 \le x \le 2. \end{cases}$$ ✓

**a.** $f(g(0))$ **b.** $g(f(3))$ **c.** $g(g(-1))$
**d.** $f(f(2))$ **e.** $g(f(0))$ **f.** $f(g(1/2))$

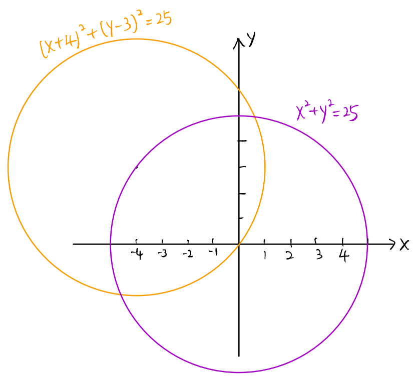
a. $g(0) = 0 - 1 = -1$ $f[g(0)] = f(-1) = 2 - (-1) = 3$
b. $f(3) = 2 - 3 = -1$ $g[f(3)] = g(-1) = -(-1) = 1$
c. $g(-1) = -(-1) = 1$ $g[g(-1)] = g(1) = 1 - 1 = 0$
d. $f(2) = 2 - 2 = 0$ $f[f(2)] = f(0) = 2 - 0 = 2$
e. $f(0) = 2 - 0 = 2$ $g[f(0)] = g(2) = 2 - 1 = 1$
f. $g(\frac{1}{2}) = \frac{1}{2} - 1 = -\frac{1}{2}$ $f[g(\frac{1}{2})] = f(-\frac{1}{2}) = 2 - (-\frac{1}{2}) = 2\frac{1}{2}$ or $\frac{5}{2}$

Exercises 27–36 tell how many units and in what directions the graphs of the given equations are to be shifted. Give an equation for the shifted graph. Then sketch the original and shifted graphs together, labeling each graph with its equation.

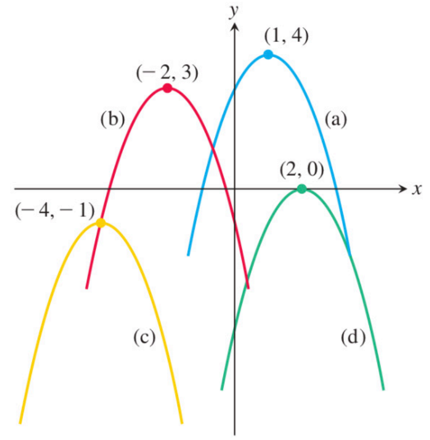
**27.** $x^2 + y^2 = 49$ Down 3, left 2

**28.** $x^2 + y^2 = 25$ Up 3, left 4

28. $x^2 + y^2 = 25$ up 3, left 4
center of the circle: Before $(0,0)$
After $(-4,3)$
Equation: $(x+4)^2 + (y-3)^2 = 25$




**26.** The accompanying figure shows the graph of $y = -x^2$ shifted to four new positions. Write an equation for each new graph.



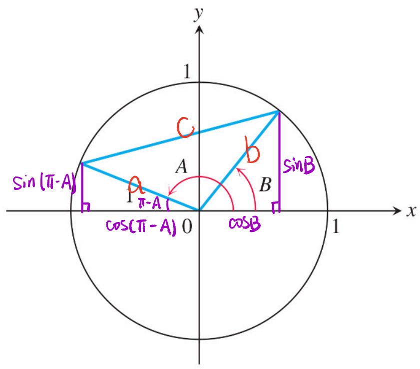

| Recall: | Right (a) | left (a) | Up (a) | Down (a) |
|---|---|---|---|---|
| | $x - a$ | $x + a$ | $+a$ | $-a$ |



(a) "Blue" $y = -(x-1)^2 + 4$
(b) "Red" $y = -(x+2)^2 + 3$
(c) "Yellow" $y = -(x+4)^2 - 1$
(d) "Green" $y = -(x-2)^2$

# C1.3

**57.** Apply the law of cosines to the triangle in the accompanying figure to derive the formula for $\cos(A - B)$.

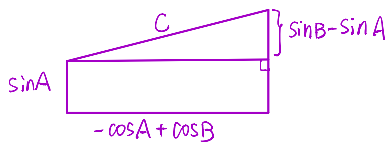



law of cosines of the triangle :

$$\cos(A-B) = \frac{a^2+b^2-c^2}{2ab} \qquad \cos\theta = \frac{a^2+b^2-c^2}{2ab}$$

$a = b = r = 1$

$\sin(\pi - A) = \sin A$ , $\cos(\pi - A) = -\cos A$




$$C^2 = (\cos B - \cos A)^2 + (\sin B - \sin A)^2$$
$$= \cos^2 B - 2\cos A \cdot \cos B + \cos^2 A + \sin^2 B - 2\sin A \sin B + \sin^2 A$$
$$= 2 - 2(\cos A \cos B + \sin A \cdot \sin B)$$

$$\cos(A-B) = \frac{1^2 + 1^2 - 2 + 2(\cos A \cos B + \sin A \sin B)}{2 \times 1 \times 1}$$
$$= \cos A \cos B + \sin A \sin B$$

**58.** **a.** Apply the formula for $\cos(A - B)$ to the identity $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ to obtain the addition formula for $\sin(A + B)$.

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$
$$\cos\left(\tfrac{\pi}{2} - \theta\right) = \cos\tfrac{\pi}{2} \cdot \cos\theta + \sin\tfrac{\pi}{2} \cdot \sin\theta$$
$$= 0 \times \cos\theta + 1 \times \sin\theta$$
$$= \sin\theta$$

$$\sin(\underbrace{A+B}_{\theta}) = \cos\left[\tfrac{\pi}{2} - \underbrace{(A+B)}_{\theta}\right]$$
$$= \cos\left[\left(\tfrac{\pi}{2} - A\right) - B\right]$$
$$= \cos\left(\tfrac{\pi}{2} - A\right) \cdot \cos B + \sin\left(\tfrac{\pi}{2} - A\right) \cdot \sin B$$
$$= \sin A \cdot \cos B + \sin\left[\tfrac{\pi}{2} - \left(\tfrac{\pi}{2} - A\right)\right] \cdot \sin B$$
$$= \sin A \cdot \cos B + \sin A \cdot \cos B$$

Tip : Focus on the formular we can use, then you will get the idea.

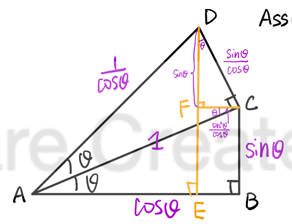
# Prove double angle formula

Recall : $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\sin 2\theta = 2\sin\theta \cdot \cos\theta$

Way 1 : Based on : $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$
$\sin(A+B) = \sin A \cdot \cos B + \sin B \cdot \cos A$

$$\cos 2\theta = \cos(\theta + \theta) = \cos\theta \cdot \cos\theta - \sin\theta \cdot \sin\theta$$
$$= \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = \sin(\theta + \theta) = \sin\theta \cdot \cos\theta + \sin\theta \cdot \cos\theta$$
$$= 2\sin\theta \cdot \cos\theta$$

Way 2 : Geometric Proof




Assume $AC = 1$, $\angle DAC = \angle CAB = \theta$,
$\angle ACD = \angle ABC = \frac{\pi}{2}$

$\cos\theta = \frac{AC}{AD} = \frac{1}{AD}$   $\sin\theta = \frac{CD}{AD} = \frac{CD}{\frac{1}{\cos\theta}} = CD \cdot \cos\theta$

$AB = \cos\theta$ , $BC = \sin\theta$ , $AD = \frac{1}{\cos\theta}$ , $CD = \frac{\sin\theta}{\cos\theta}$

$\because CF \parallel AB$

$\therefore \angle FCA = \angle CAB = \theta$

$\because \angle FDC + \angle DCF = \angle DCF + \angle FCA = \frac{\pi}{2}$

$\therefore \angle FDC = \angle FCA = \theta$

$\therefore \sin\theta = \frac{CF}{CD} = \frac{CF}{\frac{\sin\theta}{\cos\theta}} \Rightarrow CF = \sin\theta \cdot \frac{\sin\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$

$\cos\theta = \frac{DF}{CD} = \frac{DF}{\frac{\sin\theta}{\cos\theta}} \Rightarrow DF = \cos\theta \cdot \frac{\sin\theta}{\cos\theta} = \sin\theta$

$\therefore AE = AB - BE = AB - CF = \cos\theta - \frac{\sin^2\theta}{\cos\theta}$

$DE = DF + FE = DF + BC = \sin\theta + \sin\theta = 2\sin\theta$

$$\therefore \sin 2\theta = \sin\angle DAE = \frac{DE}{AD} = \frac{2\sin\theta}{\frac{1}{\cos\theta}} = 2\sin\theta \cdot \cos\theta$$

$$\cos 2\theta = \cos\angle DAE = \frac{AE}{AD} = \frac{\cos\theta - \frac{\sin^2\theta}{\cos\theta}\ {\scriptstyle\times\cos\theta}}{\frac{1}{\cos\theta}\ {\scriptstyle\times\cos\theta}} = \frac{\cos^2\theta - \sin^2\theta}{1}$$
$$= \cos^2\theta - \sin^2\theta$$