$$S^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$E\left[\sum_{i=1}^{n}(X_i-\bar{X})^2\right] = E\left[\sum_{i=1}^{n}(X_i^2 - 2X_i\bar{X} + \bar{X}^2)\right]$$

$$= E\left[\sum_{i=1}^{n} X_i^2 - 2\bar{X}\cdot\sum_{i=1}^{n}X_i + \sum_{i=1}^{n}\bar{X}^2\right]$$

$$= E\left[\sum_{i=1}^{n} X_i^2 - 2\bar{X}\cdot n\cdot\bar{X} + n\bar{X}^2\right]$$

$$= E\left[\sum_{i=1}^{n} X_i^2 - n\bar{X}^2\right]$$

$$= E\left[\sum_{i=1}^{n} X_i^2 - \sum_{i=1}^{n}\bar{X}^2\right]$$

$$= E\left(\sum_{i=1}^{n} X_i^2\right) - E\left(\sum_{i=1}^{n}\bar{X}^2\right)$$

$$= \sum_{i=1}^{n} E(X_i^2) - n\cdot E(\bar{X}^2)$$

$$= n\cdot E(X^2) - n\cdot E(\bar{X}^2)$$

Recall: $Var(X) = E(X^2) - [E(X)]^2 \Rightarrow E(X^2) = V(X) + [E(X)]^2 = \sigma^2 + \mu^2$

$Var(\bar{X}) = E(\bar{X}^2) - [E(\bar{X})]^2 \Rightarrow E(\bar{X}^2) = V(\bar{X}) + [E(\bar{X})]^2 = \frac{\sigma^2}{n} + \mu^2$

$\hookrightarrow \bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ (C.L.T)

$$= n\cdot[\sigma^2 + \mu^2] - n\left[\frac{\sigma^2}{n} + \mu^2\right]$$

$$= n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2$$

$$= (n-1)\sigma^2$$

$$\Rightarrow E\left[\frac{1}{n-1}\cdot\sum_{i=1}^{n}(X_i-\bar{X})^2\right] = \sigma^2$$