Appendix

Normal Dist
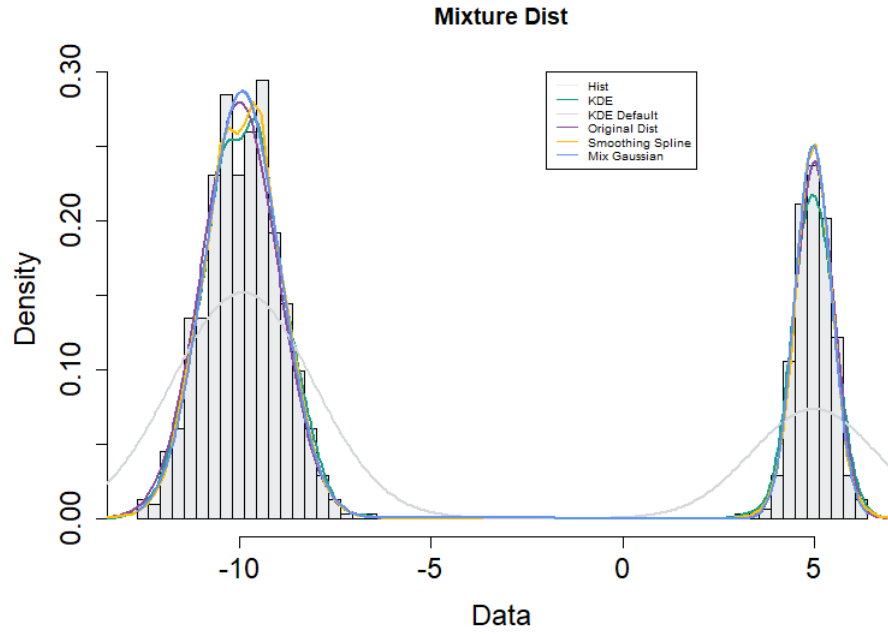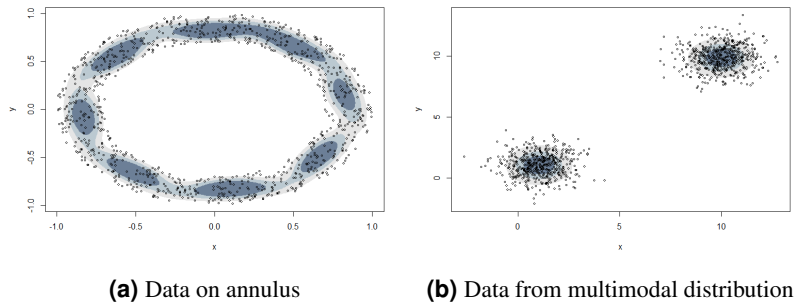

Figure 1. Density estimation of normal distribution.
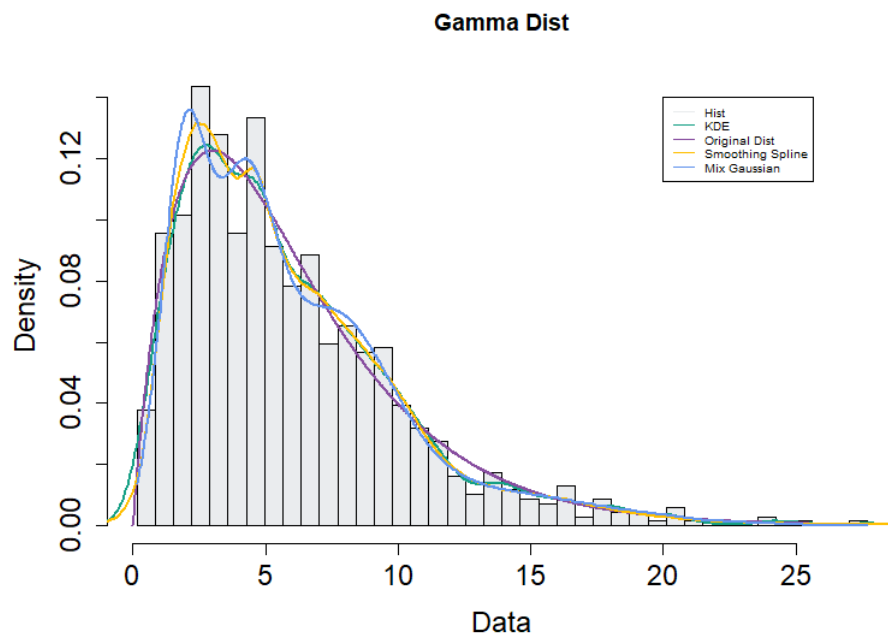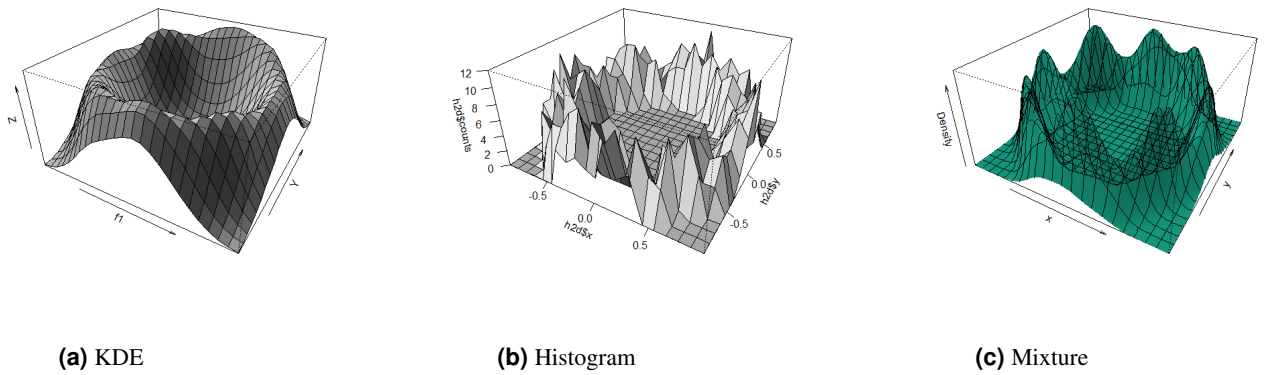
Gamma Dist

Figure 2. Density estimation of gamma distribution.

Mixture Dist

Figure 3. Density estimation of mixture distribution.

(a) Data on annulus   **(b)** Data from multimodal distribution

Figure 4. Scatter plot of 2D data generated and contour plot of mixture model

(a) KDE   **(b)** Histogram   **(c)** Mixture
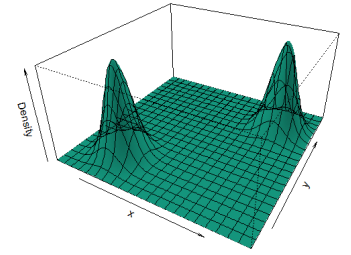
Figure 5. Density estimation of data on annulus
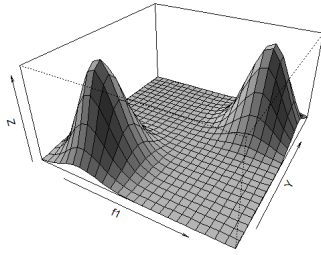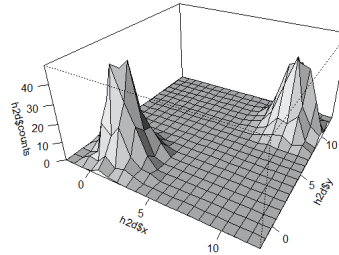
(a) KDE   **(b)** Histogram   **(c)** Mixture

Figure 6. Density estimation of data on 2D two-peak normal distribution

Value	Dist	KDE			Histogram		
		250	500	1000	250	500	1000
Mean	Norm Dist	0.00330655	0.001856879	0.001114913	0.01240743	0.007784372	0.004902357
	Gamma Dist	0.001623677	0.001008014	0.0006296534	0.004633285	0.002939214	0.001778537
	Mix Model	0.001173814	0.0006751656	0.0003913412	0.002911143	0.001713241	0.00119162
Sd	Norm Dist	0.00227001	0.001188117	0.000675403	0.008996144	0.005580659	0.002062684
	Gamma Dist	0.0007094382	0.0004059617	0.0002323864	0.006736886	0.001968476	0.0009768748
	Mix Model	0.001851897	0.0010681	0.000613399	0.1347564	0.0853433	0.0453686

Table 1. Mean and Sd of ISE under KDE and Histogram.

Value	Dist	Mixture Model			Smooth Spline Model		
		250	500	1000	250	500	1000
Mean	Norm Dist	0.001055327	0.0004914699	0.0002539465	0.001876304	0.001013376	0.0005855561
	Gamma Dist	0.003698371	0.002122818	0.001354644	0.001906122	0.001244318	0.0008339185
	Mix Model	9.050128e-5	6.053535e-5	3.624322e-05	0.002837427	0.001638725	0.0009512964
Sd	Norm Dist	0.001320288	0.000552482	0.000269173	0.001520139	0.0008141249	0.0004678311
	Gamma Dist	0.00156573	0.0007910385	0.000468714	0.0009451824	0.0005409505	0.0003333594
	Mix Model	0.00162663	0.001151384	0.0006057706	0.001718881	0.0009530741	0.0005065663

Table 2. Mean and Sd of ISE under Mixture Model and Smooth Spline Model.