答案

一、单项选择题（每小题只有一个选项是正确的，每小题 2 分，共 20 分）

1 C　2 B　3 C　4 D　5 C　6 C　7C　8 A　9　C 10 D

二、单项选择题（每小题只有一个选项是正确的，每小题 2 分，共 20 分）

1 2π　2 $\frac{1}{4}$　3 $\leq$或$<$　4 $0 \le I \le 16$　5 0　6 $\int_0^{2\pi} d\theta \int_1^e r \ln r \, dr$　7 $\frac{4}{3}\pi R^3$　8 $f(0,0)$　9 $\int_0^{\frac{\pi}{2}} d\theta \int_0^{2\cos r} r^2 dr \int_0^a z dz$　10 $c\pi R^2$

三、计算题（每题 4 分，共 20 分）

1. 解　$D: \frac{1}{2}x \le y \le 3-x, 0 \le x \le 2$　　$\therefore \int_0^1 dy \int_0^{2y} f(x,y)dx + \int_1^3 dy \int_0^{3-y} f(x,y)dx = \int_0^2 dx \int_{\frac{1}{2}x}^{3-x} f(x,y)dy$

2.解 $I = \iint_D xydxdy = \int_0^1 xdx \int_0^x ydy = \int_0^1 x[\frac{y^2}{2}]_0^x dx = \frac{1}{2}\int_0^1 x^3 dx = \frac{1}{8}$

3.解 $D: y_2 \le x \le \sqrt{y}, \quad 0 \le y \le 1$

$\therefore \iint_D x\sqrt{y}dxdy = \int_0^1 \sqrt{y}dy \int_{y^2}^{\sqrt{y}} xdx = \frac{1}{2}\int_0^1 \sqrt{y}(y-y^4)dy = \frac{6}{55}$

4. 解 $\iint_D \sin\sqrt{x^2+y^2}\,\mathrm{d}x\mathrm{d}y = \int_0^{2\pi} \mathrm{d}\theta \int_\pi^{2\pi} \sin r \cdot r dr = 2\pi \cdot (-3\pi) = -6\pi$

5.解 换序 $D: 1 \le x \le 1+y, \quad 0 \le y \le 2 \therefore I = \int_1^2 dy \int_0^{1+y} e^{y^2} dx = \int_0^2 e^{y^2} y dy = \frac{1}{2}e^{y^2}\Big|_0^2 = \frac{1}{2}(e^4-1)$

四（6 分）

$$\iint_D |y-x^2| d\sigma$$

$$= \iint_{D_1+D_2} (x^2-y)d\sigma + \iint_{D_3} (y-x^2)d\sigma$$

$$= \int_{-1}^1 dx \int_0^{x^2} (x^2-y)dy + \int_{-1}^1 dx \int_{x^2}^1 (y-x^2)dy = \frac{11}{15}$$

五（8 分）解 Ω：$0 \le x \le 1, 0 \le y \le \frac{1-x}{2}, 0 \le z \le 1-x-2y$

$$\therefore \iiint_\Omega xdxdydz = \int_0^1 dx \int_0^{\frac{1-x}{2}} dy \int_0^{1-x-2y} xdz = \int_0^1 xdx \int_0^{\frac{1-x}{2}} (1-x-2y)dz = \frac{1}{4}\int_0^1 [x - \frac{x^2}{2} + x^3]dx = \frac{1}{48}$$

六（10 分）. 解 选取柱面坐标系计算方便,此时，　$\Omega: \begin{cases} 0 \le z \le 4-r^2 \\ 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$

所以 $\iiint_\Omega \sqrt{x^2+y^2}dV = \int_0^{2\pi} d\theta \int_0^2 r^3 dr \int_0^{4-r^2} dz = 2\pi \int_0^2 r^3(4-r^2)dr = \frac{32}{3}\pi$

七（8 分）

解：先求投影 D_{xy}：联立 $\begin{cases} z = \sqrt{x^2+y^2} \\ z^2 = 2x \end{cases}$　得 $x^2+y^2 = 2x \Rightarrow D_{xy}: (x-1)^2+y^2 \le 1$　$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}, \frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}},$

$$A = \iint_{D_{xy}} \sqrt{1+(\tfrac{\partial z}{\partial x})^2+(\tfrac{\partial z}{\partial y})^2}\,dxdy = \iint_{D_{xy}} \sqrt{2}dxdy \qquad = \sqrt{2}\pi 1^2 = \sqrt{2}\pi$$

八（8 分）

D_{xy}：　$\begin{cases} 0 \le \theta \le \pi \\ 0 \le \rho \le a \end{cases}$　$I_x = \iint_D y^2 \mu d\sigma = \mu \int_0^\pi d\theta \int_0^a \rho^2 \sin^2\theta \rho d\rho = \frac{\mu a^4}{4}\int_0^\pi \sin^2\theta d\theta = \frac{\mu a^4}{4} \cdot \frac{\pi}{2}$