In [ ]:

```python
import pandas as pd
import geopandas as gpd
import matplotlib.pyplot as plt
from matplotlib.patches import Patch
import palettable.colorbrewer.sequential as pcs
import palettable.colorbrewer.diverging as pcd
import palettable.cartocolors.diverging as cartod
import palettable.cartocolors.sequential as cartos

from statsmodels.stats.outliers_influence import variance_inflation_factor
from sklearn.metrics import mean_squared_error as mse

import libpysal

from esda.moran import Moran,Moran_Local

from spreg import OLS, ML_Lag

from mgwr.sel_bw import Sel_BW
from mgwr.gwr import GWR

from splot.esda import lisa_cluster

from patsy import dmatrices
```

In [ ]:

```python
# read in data
pre_df=pd.read_csv('data/lsoa_data.csv')
```

```python
# lsoa geometry
boundary=gpd.read_file('data/LSOA_boundary/LDN-LSOAs.shp')


# merge two dataframes
gdf=gpd.GeoDataFrame(pre_df).merge(boundary,left_on='Codes',right_on='lsoa11cd')
# drop excessive columns
gdf=gdf.drop(['lsoa11cd','lsoa11nm','lsoa11nmw','objectid','st_areasha','st_lengths'],axis=1)
```

In [ ]:

```python
# explore the structure of the data
gdf.describe()
```

Out[ ]:

	MedianHP	MedianIncome	Pct_nonwhite	Pct_CHDC	c_per_hhlds	PTAL_average	Pct_qual_above_l4
count	4.835000e+03	4835.000000	4835.000000	4835.000000	4835.000000	4835.000000	4835.000000
mean	3.294656e+05	35756.460393	0.392889	0.184459	0.847976	3.744442	37.292513
std	2.150839e+05	11459.895607	0.203536	0.061666	0.366748	1.601374	14.542426
min	0.000000e+00	16167.000000	0.018000	0.023000	0.157000	0.300000	8.300000
25%	2.170000e+05	26996.500000	0.224000	0.140000	0.541500	2.593500	25.800000
50%	2.675000e+05	32609.000000	0.369000	0.185000	0.806000	3.339000	34.500000
75%	3.690000e+05	42272.500000	0.541000	0.226000	1.110500	4.662500	47.500000
max	3.377000e+06	92431.000000	0.965000	0.399000	2.216000	8.000000	83.800000

In [ ]:

```python
gdf['kMedianHP']=gdf['MedianHP']/1000
gdf['kMedianHP'].describe()
```

Out[ ]:

```
count    4835.000000
mean      329.465551
std       215.083933
min         0.000000
25%       217.000000
```

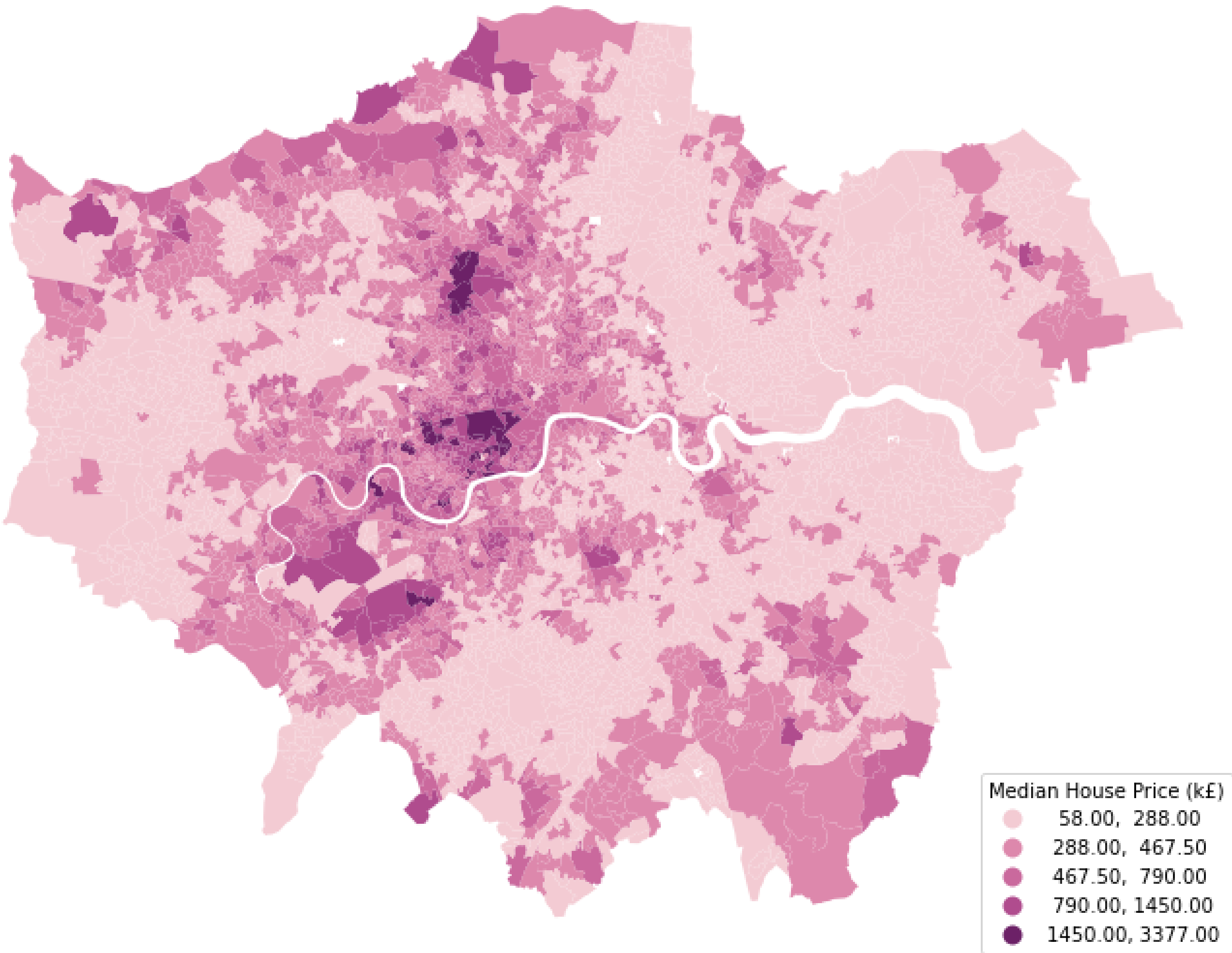
```
50%        267.500000
75%        369.000000
max       3377.000000
Name: kMedianHP, dtype: float64
```

In [ ]:

```
# remove lsoas wth no house price data
gdf=gdf[gdf['kMedianHP']>0].reset_index(drop=True)
```

In [ ]:

```
# Fig 1
fig,ax=plt.subplots(1,figsize=(12,10))

gdf.plot(column='kMedianHP',scheme='fisherjenks',cmap=cartos.Magenta_5.mpl_colormap,ax=ax,
         legend=True,legend_kwds={'loc':'lower right','title':'Median House Price (k£)'})

ax.axis('off')

plt.savefig('graph/fig1.png',dpi=200,bbox_inches='tight',facecolor=None)
plt.show()
```



In [ ]:

```
# check correlation between dependent and independent variables
gdf.corr(method='kendall')['MedianHP']
```

Out[ ]:

```
MedianHP          1.000000
MedianIncome      0.541982
Pct_nonwhite     -0.260607
Pct_CHDC         -0.022876
c_per_hhlds       0.067235
```

```
PTAL_average         0.121890
Pct_qual_above_l4    0.507439
kMedianHP            1.000000
Name: MedianHP, dtype: float64
```

In [ ]:
```
# create a list of all the independent variables
var_n=gdf.columns.tolist()[3:-2]
```

In [ ]:
```
# remove predictors that are not correlated with the dependent variable
var_n.remove('c_per_hhlds')
var_n.remove('Pct_CHDC')

gdf.drop(['Pct_CHDC','c_per_hhlds'],inplace=True,axis=1)
```

In [ ]:
```
def vif(df,dep_var,list):
    form=''
    for i in list:
        form+=i
        if i != list[-1]:
            form+='+'
    y,X=dmatrices(dep_var+ ' ~ ' + form, data=df,return_type='dataframe')

    vif=pd.DataFrame()
    vif['VIF']=[variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
    vif['variable']=X.columns

    return vif
```

In [ ]:
```
# check VIF
```

```python
vif(gdf,'kMedianHP',var_n)
```

Out[ ]:

	VIF	variable
0	35.879212	Intercept
1	3.778308	MedianIncome
2	1.561378	Pct_nonwhite
3	1.496724	PTAL_average
4	3.585510	Pct_qual_above_l4

In [ ]:

```python
# table 2
gdf.describe()[var_n+['kMedianHP']].T
```

Out[ ]:

	count	mean	std	min	25%	50%	75%	max
MedianIncome	4822.0	35796.332020	11448.724483	16167.000	27015.250	32652.000	42290.2500	92431.000
Pct_nonwhite	4822.0	0.392286	0.203428	0.018	0.224	0.368	0.5390	0.965
PTAL_average	4822.0	3.745977	1.602643	0.300	2.595	3.342	4.6645	8.000
Pct_qual_above_l4	4822.0	37.336914	14.534748	8.300	25.825	34.500	47.5000	83.800
kMedianHP	4822.0	330.353782	214.691293	58.000	217.500	268.000	370.0000	3377.000

In [ ]:

```python
# Fig 2
i=1
for var in var_n:
    fig,ax=plt.subplots(1,figsize=(10,8))
    gdf.plot(column=var,ax=ax,scheme='fisherjenks',cmap='viridis',legend=True,
             legend_kwds={'loc':'lower right','fontsize':10,'title':var})
    ax.axis('off')
    plt.text(x=530000,y=153000,s='Figure 2.'+str(i)+': '+var,
```

```
            fontsize=14,color='#c0c0c0',horizontalalignment='center')
    plt.savefig('graph/fig2.'+str(i)+'.png',dpi=200,bbox_inches='tight',facecolor=None)
    i+=1
    plt.show()
```

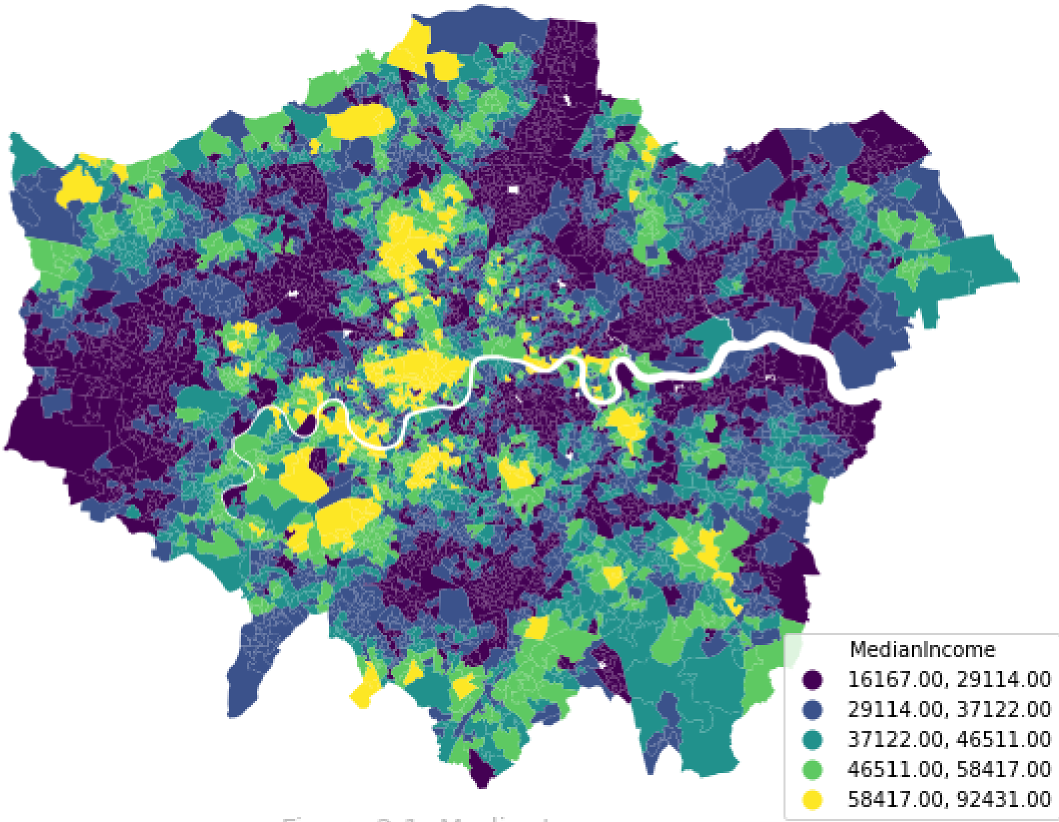


Figure 2.1: MedianIncome

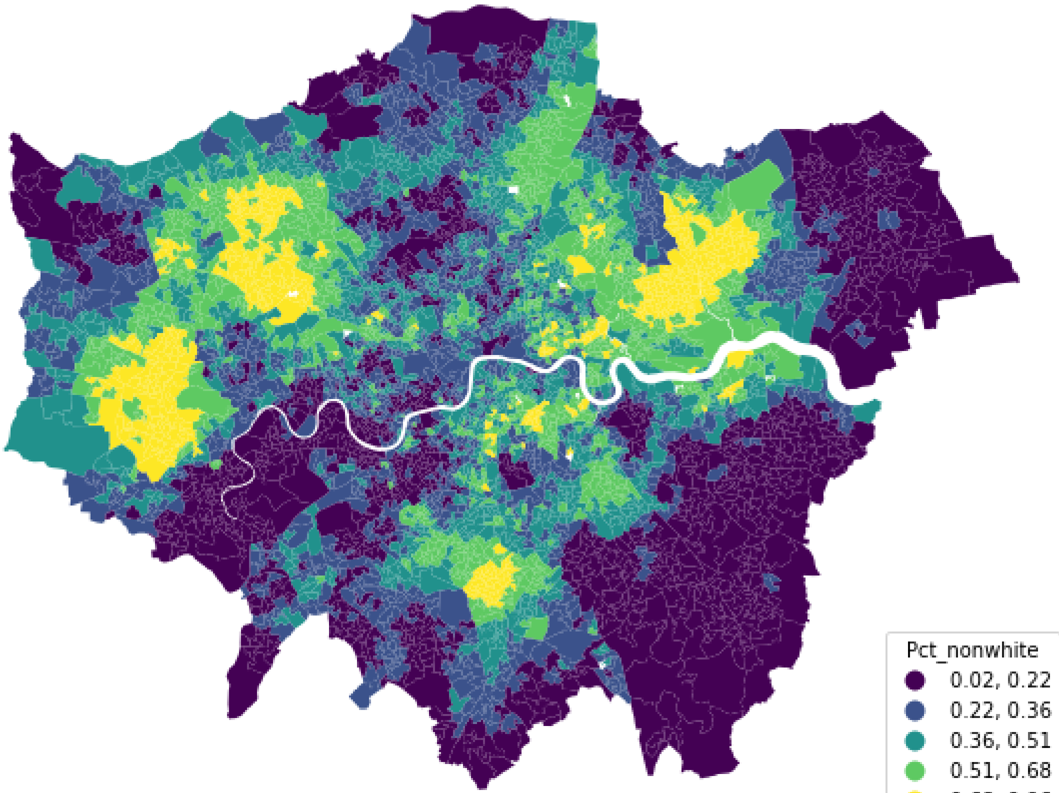


Figure 2.2: Pct_nonwhite

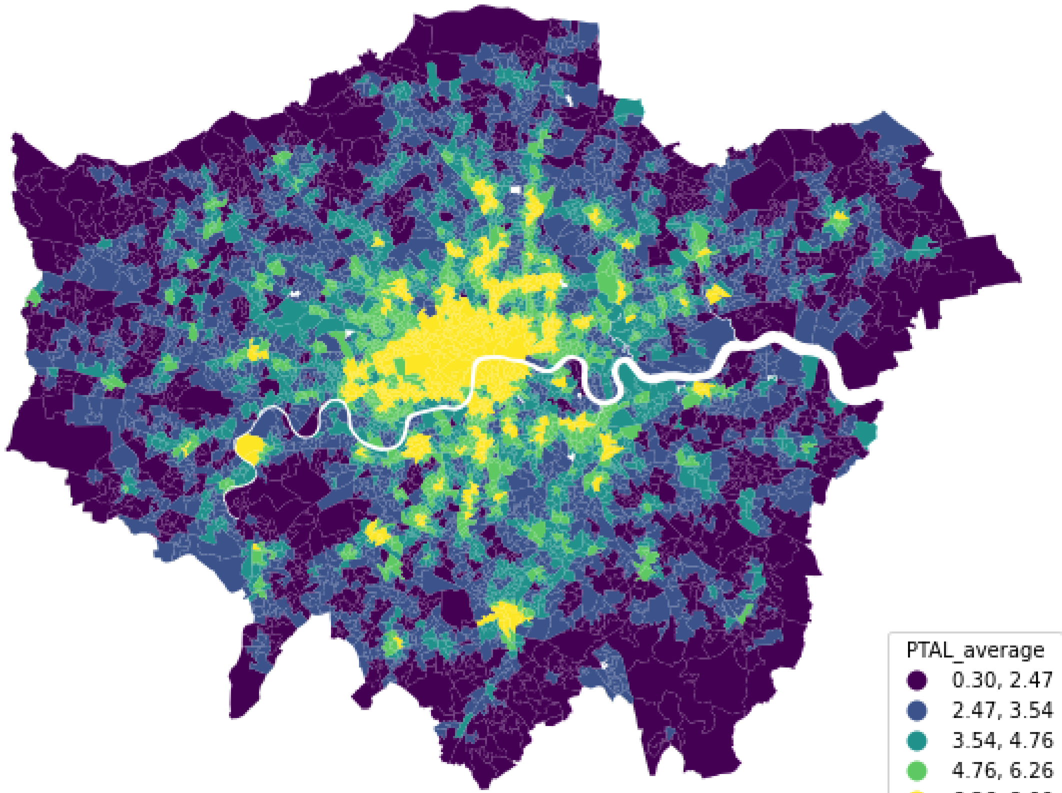


Figure 2.3: PTAL_average

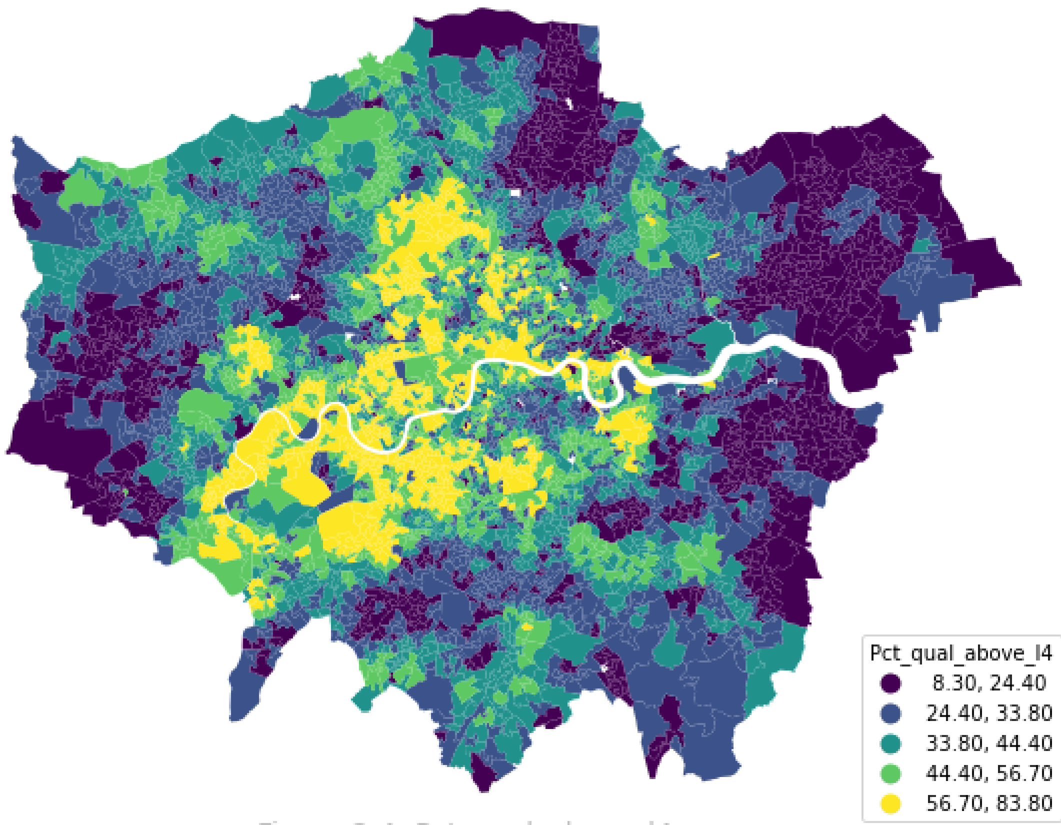


Figure 2.4: Pct_qual_above_l4

In [ ]:
```
# build simple OLS model
m_multi=OLS(gdf[['kMedianHP']].values,
            gdf[var_n].values,
            name_x=var_n,
            name_y='kMedianHP')


print(m_multi.summary)
```

```
REGRESSION
----------
```

```
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES
-----------------------------------------
Data set            :     unknown
Weights matrix      :        None
Dependent Variable  :   kMedianHP                Number of Observations:        4822
Mean dependent var  :    330.3538                Number of Variables   :           5
S.D. dependent var  :    214.6913                Degrees of Freedom    :        4817
R-squared           :      0.5540
Adjusted R-squared  :      0.5537
Sum squared residual:99098546.109                F-statistic           :   1496.0708
Sigma-square        :   20572.669                Prob(F-statistic)     :           0
S.E. of regression  :     143.432                Log likelihood        :  -30784.994
Sigma-square ML     :   20551.337                Akaike info criterion :   61579.988
S.E of regression ML:    143.3574                Schwarz criterion     :   61612.393

------------------------------------------------------------------------------------
            Variable     Coefficient       Std.Error     t-Statistic     Probability
------------------------------------------------------------------------------------
            CONSTANT    -329.6558346      12.3723775     -26.6445017       0.0000000
        MedianIncome       0.0167930       0.0003507      47.8806019       0.0000000
        Pct_nonwhite     112.2304971      12.6887552       8.8448784       0.0000000
        PTAL_average      29.2361276       1.5769237      18.5399763       0.0000000
   Pct_qual_above_l4      -2.5353457       0.2691192      -9.4209022       0.0000000
------------------------------------------------------------------------------------

REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER           19.494

TEST ON NORMALITY OF ERRORS
TEST                             DF        VALUE           PROB
Jarque-Bera                       2      312794.138           0.0000

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                             DF        VALUE           PROB
Breusch-Pagan test                4       13528.673           0.0000
Koenker-Bassett test              4         664.675           0.0000
================================ END OF REPORT =====================================
```

In [ ]:

```
# spatial weights matrix
weights=libpysal.weights.KNN.from_dataframe(gdf,k=6)
weights.transform='r'
```

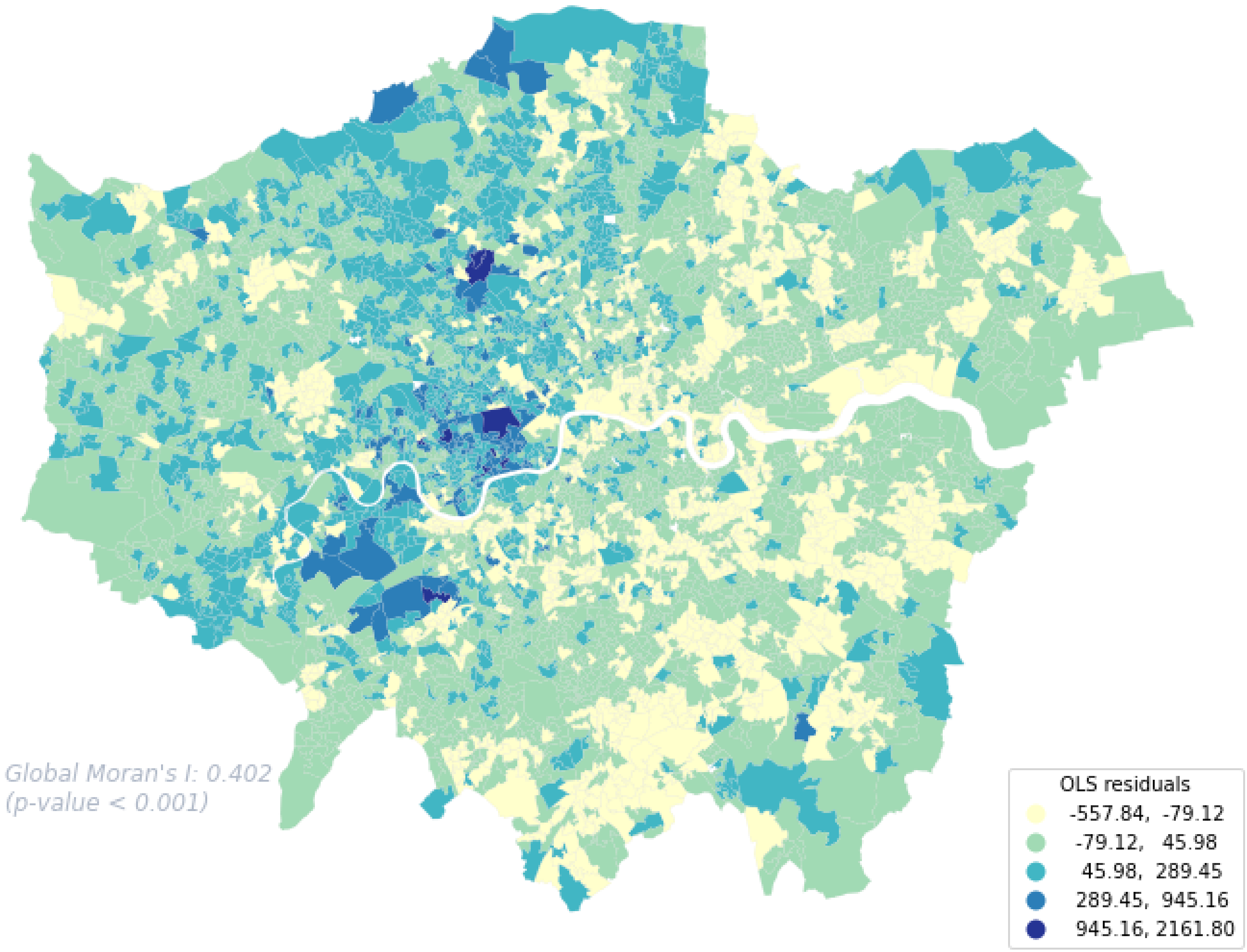
In [ ]:

```
# spatial autocorrelation of the OLS residuals
gdf['multi_res']=m_multi.u
multi_res_moran=Moran(m_multi.u,weights)
print(multi_res_moran.I)
print(multi_res_moran.p_sim)
```

```
0.40186914690491266
0.001
```

In [ ]:

```
# Fig 3
fig,ax=plt.subplots(1,figsize=(12,10))

gdf.plot(column='multi_res',scheme='FisherJenks',ax=ax,
         cmap=pcs.YlGnBu_5.mpl_colormap,edgecolor='lightgrey',linewidth=0.1,
         legend=True,legend_kwds={'loc':'lower right','title':'OLS residuals'})

plt.figtext(x=0.15,y=0.25,
            s="Global Moran's I: "+str(round(multi_res_moran.I,3))\
                +"\n(p-value < "+str(multi_res_moran.p_sim)+")",
            fontsize=12,fontstyle='italic',color='#afb8c7')
ax.axis('off')

plt.savefig('graph/fig3.png',dpi=200, bbox_inches='tight', facecolor=None)
plt.show()
```



In [ ]:

```
# prepare data
g_y = gdf['kMedianHP'].values.reshape((-1,1))
g_X = gdf[var_n].values
g_coords=[]
for i in gdf['geometry'].centroid.tolist():
    g_coords.append((i.x,i.y))
```

```python
# standardise
g_X = (g_X - g_X.mean(axis=0)) / g_X.std(axis=0)

g_y = g_y.reshape((-1,1))

g_y = (g_y - g_y.mean(axis=0)) / g_y.std(axis=0)
```

In [ ]:

```python
# standardised OLS (for comparison)
m_multi=OLS(g_y,g_X,name_x=var_n,name_y='kMedianHP')
print(m_multi.summary)
```

```
REGRESSION
----------
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES
-----------------------------------------
Data set            :     unknown
Weights matrix      :        None
Dependent Variable  :   kMedianHP                Number of Observations:        4822
Mean dependent var  :      0.0000                Number of Variables   :           5
S.D. dependent var  :      1.0001                Degrees of Freedom    :        4817
R-squared           :      0.5540
Adjusted R-squared  :      0.5537
Sum squared residual:    2150.446                F-statistic           :   1496.0708
Sigma-square        :       0.446                Prob(F-statistic)     :           0
S.E. of regression  :       0.668                Log likelihood        :   -4895.206
Sigma-square ML     :       0.446                Akaike info criterion :    9800.413
S.E of regression ML:      0.6678                Schwarz criterion     :    9832.817

------------------------------------------------------------------------------------
            Variable     Coefficient       Std.Error     t-Statistic     Probability
------------------------------------------------------------------------------------
            CONSTANT       0.0000000       0.0096219       0.0000000       1.0000000
        MedianIncome       0.8955103       0.0187030      47.8806019       0.0000000
```

```
       Pct_nonwhite       0.1063428       0.0120231       8.8448784       0.0000000
       PTAL_average       0.2182440       0.0117715      18.5399763       0.0000000
  Pct_qual_above_l4      -0.1716446       0.0182196      -9.4209022       0.0000000
------------------------------------------------------------------------------------

REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER           3.912

TEST ON NORMALITY OF ERRORS
TEST                             DF        VALUE           PROB
Jarque-Bera                       2      312794.138           0.0000

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                             DF        VALUE           PROB
Breusch-Pagan test                4       20973.334           0.0000
Koenker-Bassett test              4        1030.438           0.0000
================================ END OF REPORT =====================================
```

In [ ]:

```python
# Spatial lag model
m_lag=ML_Lag(g_y,g_X,weights,name_x=var_n,name_y='kMedianHP')
print(m_lag.summary)
```

```
C:\Users\Yulun\anaconda3\envs\sds2021\lib\site-packages\scipy\optimize\_minimize.py:779: RuntimeWarning: Method 'bou
nded' does not support relative tolerance in x; defaulting to absolute tolerance.
  warn("Method 'bounded' does not support relative tolerance in x; "
REGRESSION
----------
SUMMARY OF OUTPUT: MAXIMUM LIKELIHOOD SPATIAL LAG (METHOD = FULL)
-----------------------------------------------------------------
Data set            :     unknown
Weights matrix      :     unknown
Dependent Variable  :   kMedianHP                Number of Observations:        4822
Mean dependent var  :      0.0000                Number of Variables   :           6
S.D. dependent var  :      1.0001                Degrees of Freedom    :        4816
Pseudo R-squared    :      0.7114
Spatial Pseudo R-squared:  0.5418
```

```
Sigma-square ML      :       0.289              Log likelihood          :   -4008.245
S.E of regression    :       0.538              Akaike info criterion   :    8028.491
                                                Schwarz criterion       :    8067.377

------------------------------------------------------------------------------------
            Variable     Coefficient       Std.Error     z-Statistic     Probability
------------------------------------------------------------------------------------
            CONSTANT       0.0146925       0.0077473       1.8964647       0.0578986
        MedianIncome       0.6866607       0.0166259      41.3007156       0.0000000
        Pct_nonwhite       0.1269163       0.0096814      13.1093227       0.0000000
        PTAL_average       0.1091396       0.0096403      11.3211226       0.0000000
   Pct_qual_above_l4      -0.2320701       0.0147032     -15.7836786       0.0000000
         W_kMedianHP       0.5935792       0.0117161      50.6634736       0.0000000
------------------------------------------------------------------------------------
================================= END OF REPORT ====================================
```

In [ ]:

```python
# select bandwidth
bw=Sel_BW(g_coords,g_y,g_X,fixed=False,kernel='gaussian',spherical=False)
bw.search()
```

Out[ ]:

```
51.0
```

In [ ]:

```python
# build GWR model
m_gwr=GWR(g_coords,g_y,g_X,bw.bw[0],kernel='gaussian')
m_gwr_fit=m_gwr.fit()
m_gwr_fit.summary()
```

```
===========================================================================
Model type                                                         Gaussian
Number of observations:                                                4822
Number of covariates:                                                     5

Global Regression Results
---------------------------------------------------------------------------
Residual sum of squares:                                           2150.446
Log-likelihood:                                                   -4895.206
```

AIC:	9800.413
AICc:	9802.430
BIC:	-38702.262
R2:	0.554
Adj. R2:	0.554

Variable	Est.	SE	t(Est/SE)	p-value
X0	0.000	0.010	0.000	1.000
X1	0.896	0.019	47.881	0.000
X2	0.106	0.012	8.845	0.000
X3	0.218	0.012	18.540	0.000
X4	-0.172	0.018	-9.421	0.000

Geographically Weighted Regression (GWR) Results

Spatial kernel:	Adaptive gaussian
Bandwidth used:	51.000

Diagnostic information

Residual sum of squares:	952.168
Effective number of parameters (trace(S)):	194.609
Degree of freedom (n - trace(S)):	4627.391
Sigma estimate:	0.454
Log-likelihood:	-2930.990
AIC:	6253.198
AICc:	6269.827
BIC:	7520.929
R2:	0.803
Adjusted R2:	0.794
Adj. alpha (95%):	0.001
Adj. critical t value (95%):	3.221

Summary Statistics For GWR Parameter Estimates

Variable	Mean	STD	Min	Median	Max

```
X0                        0.028      0.465     -0.671     -0.104      1.436
X1                        0.707      0.408      0.224      0.546      1.962
X2                       -0.003      0.189     -0.938      0.022      0.525
X3                        0.040      0.101     -0.153      0.016      0.426
X4                       -0.279      0.361     -1.583     -0.152      0.210
==========================================================================
```

In [ ]:

```
# mean squared error
mp=pd.Series({'OLS':mse(g_y,m_multi.predy.flatten()),
              'Lag':mse(g_y,m_lag.predy.flatten()),
              'GWR':mse(g_y,m_gwr_fit.predy.flatten())})


mp.sort_values()
```

Out[ ]:

```
GWR     0.197463
Lag     0.289345
OLS     0.445966
dtype: float64
```

In [ ]:

```
# filter all the statistically insignificant (at 0.05 level) results
f_est=m_gwr_fit.filter_tvals(alpha=.05)
```

In [ ]:

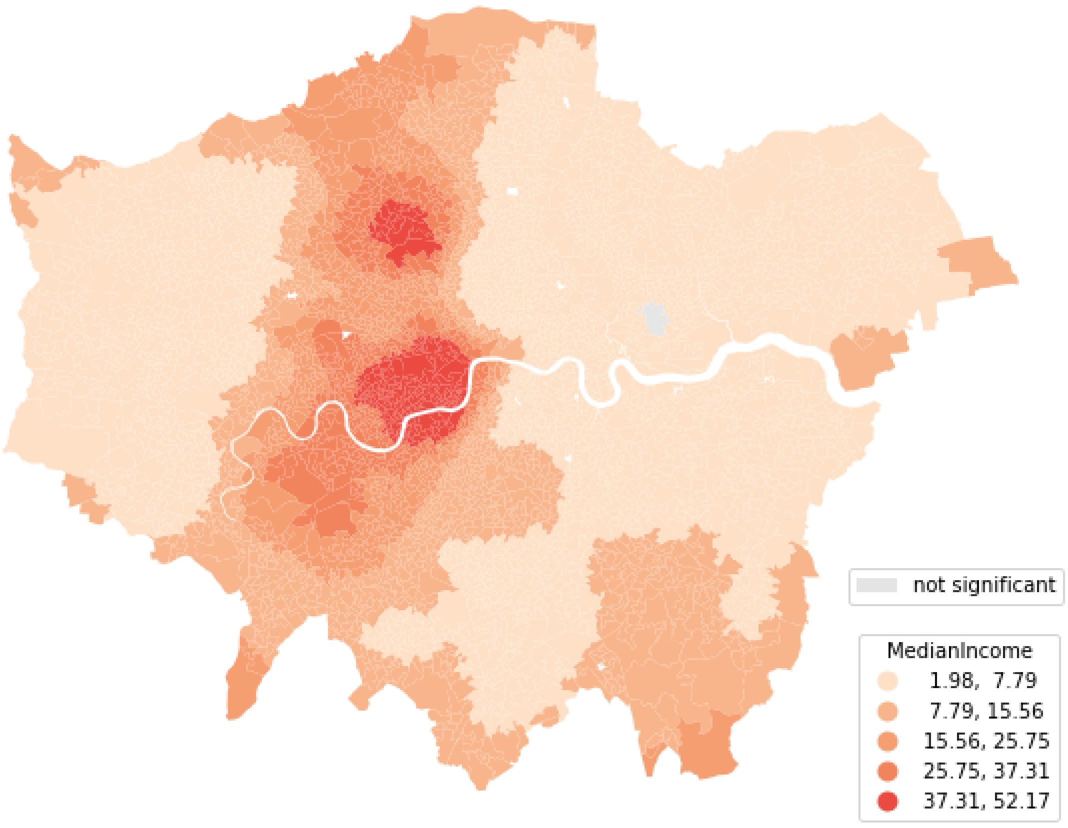
```
# merge the model result with the original dataframe
data_params=pd.DataFrame(f_est)
data_localR2=pd.DataFrame(m_gwr_fit.localR2)

tem_df=pd.DataFrame(gdf[['Codes','Names','kMedianHP','geometry']])
result_df=tem_df.assign(intercept=data_params[0],
                        MedianIncome=data_params[1],
                        Pct_nonwhite=data_params[2],
                        PTAL_average=data_params[3],
```

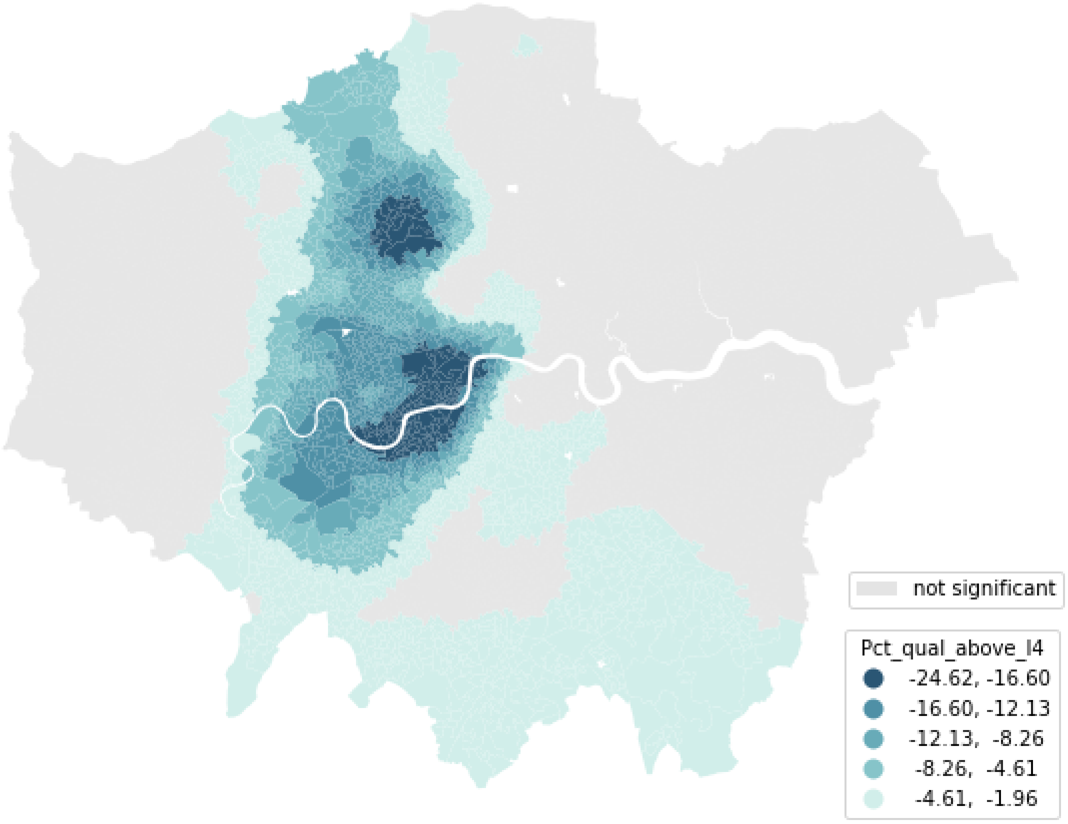
```
                         Pct_qual_above_l4=data_params[4],
                         localR2=data_localR2[0])

result_gdf=gpd.GeoDataFrame(result_df,geometry='geometry')
```

In [ ]:

```
insig=[Patch(facecolor='grey',alpha=.2,label='not significant')]

# Fig 4 and fig 7 (sequential)
# Fig 4
fig,ax=plt.subplots(1,figsize=(10,8))
result_gdf[result_gdf['MedianIncome']==0].plot(color='grey',ax=ax,alpha=.2)

result_gdf[result_gdf['MedianIncome']!=0].\
    plot(column='MedianIncome',cmap=cartos.Peach_5.mpl_colormap,scheme='fisherjenks',
         legend=True,legend_kwds={'loc':'lower right','title':'MedianIncome'},ax=ax)

ax.axis('off')
fig.legend(handles=insig,loc='center',bbox_to_anchor=(0.821,0.34))

plt.savefig('graph/fig4.png',dpi=200,bbox_inches='tight',facecolor=None)
plt.show()

# Fig 7
fig,ax=plt.subplots(1,figsize=(10,8))
result_gdf[result_gdf['Pct_qual_above_l4']==0].plot(color='grey',ax=ax,alpha=.2)

result_gdf[result_gdf['Pct_qual_above_l4']!=0].\
    plot(column='Pct_qual_above_l4',cmap=cartos.Teal_5_r.mpl_colormap,scheme='fisherjenks',
```

```
        legend=True,legend_kwds={'loc':'lower right','title':'Pct_qual_above_l4'},ax=ax)

ax.axis('off')
fig.legend(handles=insig,loc='center',bbox_to_anchor=(0.821,0.335))

plt.savefig('graph/fig7.png',dpi=200,bbox_inches='tight',facecolor=None)
plt.show()
```

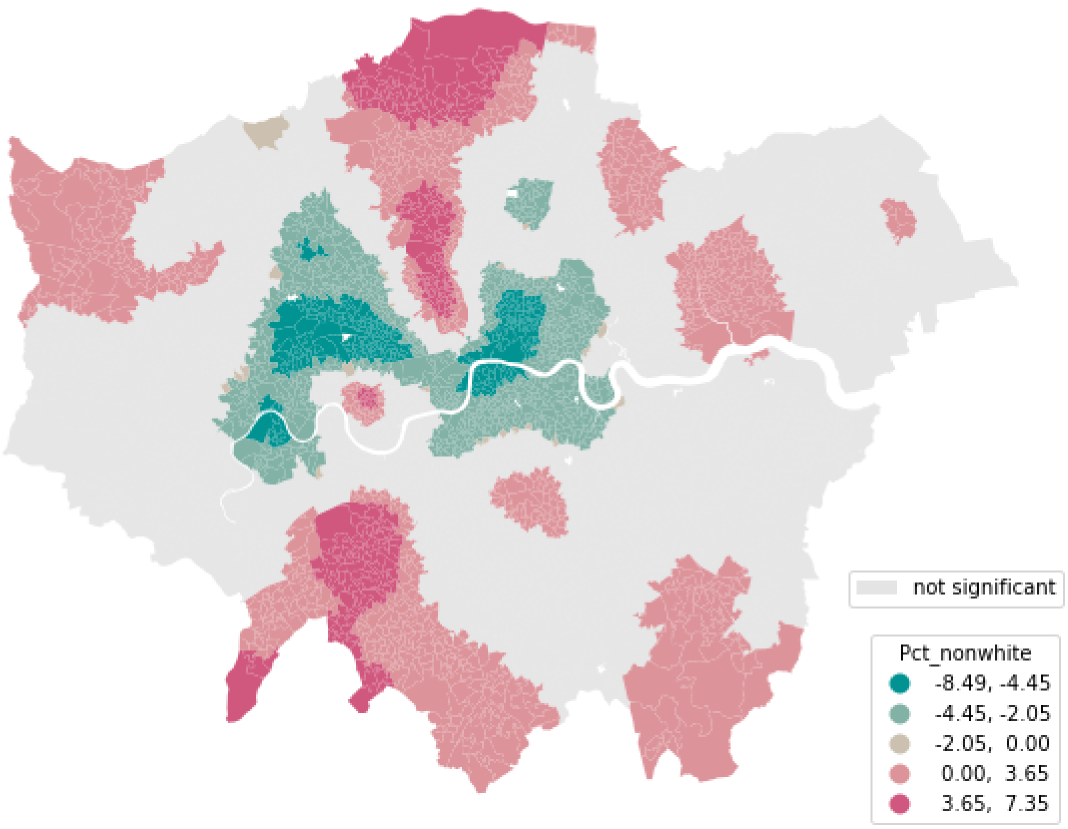


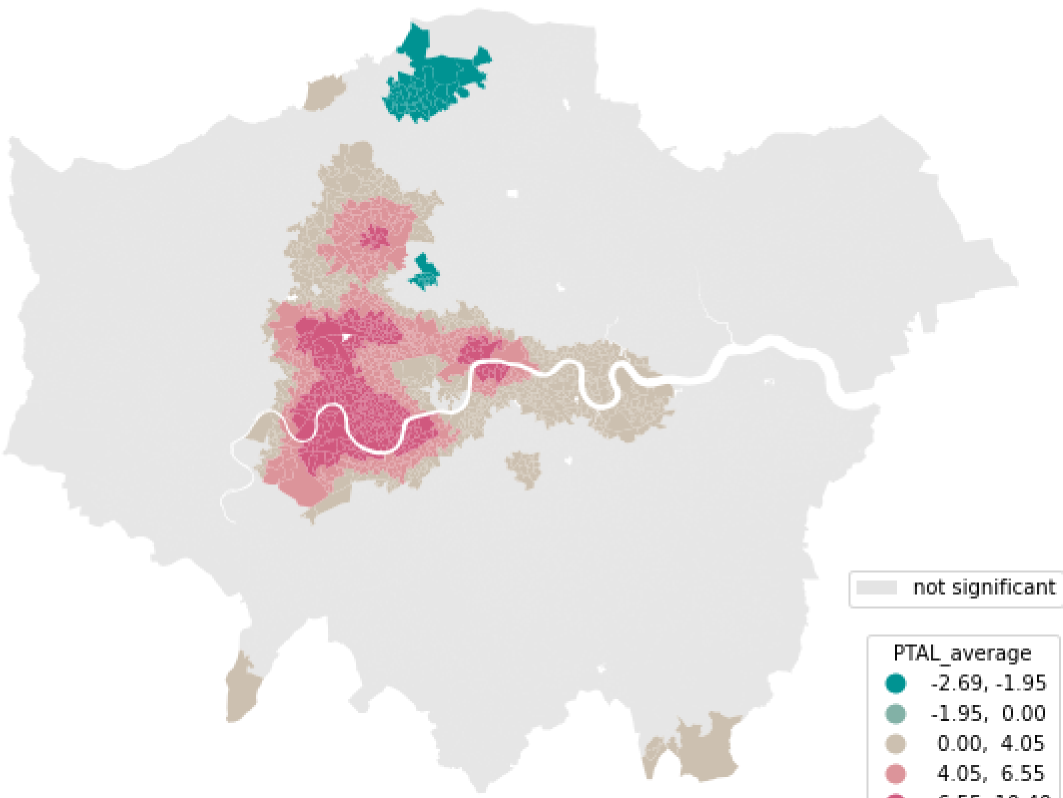


In [ ]:

```
# Fig 5 and fig 6 (diverging)
ud_bins={0:[-4.45,-2.05,0,3.65,7.35],
         1:[-1.95,0,4.05,6.55,10.4]}

for i,row in enumerate(f_est.T[2:-1]):
    fig,ax=plt.subplots(1,figsize=(10,8))
    temp = result_gdf.assign(toplot=f_est.T[i+2])
    temp.query('toplot==0').sort_values('toplot').\
        plot(color='grey',ax=ax,alpha=.2)
```

```
temp.query('toplot≠0').sort_values('toplot').\
    plot('toplot',cmap=cartod.TealRose_6.mpl_colormap,ax=ax,legend=True,
         legend_kwds={'loc':'lower right','title':var_n[i+1]},
         scheme='userdefined',classification_kwds={'bins':ud_bins[i]})

ax.axis('off')
fig.legend(handles=insig,loc='center',bbox_to_anchor=(0.821,0.34))

plt.savefig('graph/fig'+str(i+5)+'.png',dpi=200,bbox_inches='tight',facecolor=None)
plt.show()
```




not significant
PTAL_average
-2.69, -1.95
-1.95, 0.00
0.00, 4.05
4.05, 6.55
6.55, 10.40