Effects of Fuel on Boeing 737-800 Aircraft Flight Cost, Efficiency, and Operation Seen Through a Thermodynamic Cycle

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Executive Summary

On account of economic, environmental, and safety advantages, the combustion of ATJ, an ethanol based fuel, is the best fuel to use to power the Boeing 737-800. Comparatively to other common jet fuels such as Jet A and SAF, ATJ is the cheapest fuel to burn with a net cost of combustion of \$36.18/GJ. Despite releasing CO_2 into the atmosphere, ATJ doesn't come with a carbon tax unlike Jet A, and it also releases 1 kg CO_2/GJ less than SAF and Jet A, making it the most sustainable fuel. In terms of safety, ATJ has a lower adiabatic flame temperature and autoignition temperature, so it is safer to handle within turbine components. Using ATJ as an ideal fuel, we designed a jet turbine that optimizes for efficiency and found the ideal pressure and fuel to air ratios to be 25 and 0.2% respectively. Under these conditions, it would cost approximately \$2300 in fuel to fly the 737-800 from LAX to JFK.

Introduction

Fuel efficiencies and costs are a massive area in which companies look to maximize their profit. Airplanes serve an important role in traveling and transportation for humans, therefore, optimizing every aspect of the airplane is important. Jet engines are one of the most essential areas to explore and specifically, we want to maximize the efficiency of the turbine as well as the cost of the engine by carefully choosing the fuel, the fuel to air ratio, the pressure ratio, and the flight speed. Jet turbine engines mainly consist of having a diffuser, compressor and turbine with a common shaft, a combustion chamber, and a nozzle. The diffuser slows down the incoming air which is relative to the speed of the aircraft to near negligible levels, and the nozzle does the opposite. The compressor increases the pressure of the gas for a certain amount of work and the turbine expands the gas for a certain amount of work out. The combustion chamber incorporates the fuel and the incoming air to heat the air up using a combustion reaction to generate energy which will then be converted into the air that will generate force for the aircraft. We can vary the pressure ratio which tells us how much the compressor can compress the gas, and fuel to air ratio which tells us the amount of fuel we need to put in. Structural constraints that are put in place are that the pressure ratio can't be greater than 50 and the temperature of the air coming out of the combustion chamber can't be greater than 1200 °C which could possibly melt our turbine blades. We have 3 possible fuel choices: Jet A, containing n-dodecane included with carbon tax; SAF, also containing n-dodecane with no carbon tax, and ATJ, an ethanol based fuel.

Fuel Selection

The first step of optimizing our jet engine is to look at what fuel we can use to reduce the cost. Different fuels have their own advantages and disadvantages as well as their respective physical and chemical properties.

Fuel	Chemical Formula	Molar Mass	Base Cost	Carbon Tax	Standard Reaction Enthalpy	Net Cost for Heat	Energy Density
Jet A	$C_{12}H_{26}$	170.34 g/mol	$5.0 /gallon	$40 / metric ton CO_2 produced	-7512242 J/mol	$42.60 / GJ	0.126 GJ / gallon
SAF	$C_{12}H_{26}$	170.34 g/mol	$6.5 / gallon	none (renewable fuel)	-7512242 J/mol	$51.95 / GJ	0.126 GJ / gallon
ATJ	C_2H_6O	46.06 g/mol	$3.0 / gallon	none (renewable fuel)	-1277372 J/mol	$36.18 / GJ	0.083 GJ/ gallon

Fig. 1. Table of physical properties and prices for different fuels. [1], Appendix 1

First, Jet A and SAF are n-dodecane based fuels with a fuel to air ratio of 6.7% by mass corresponding to its stoichiometric combustion reaction. The combustion of n-dodecane is described by the following,

$$C_{12}H_{26}\,(g) \;+\; 37/2\; O_2\,(g) \;\rightarrow\; 12\; CO_2\,(g) \;+\; 13\; H_2O\,(g)$$

The specific enthalpy of combustion is -7.5 MJ/mol which corresponds to -44.1 MJ/kg of dodecane if we use its molar mass. Considering Jet A has a base cost of $5 per gallon, it would cost $40.0/GJ released by the fuel, but with the added carbon tax of $40/metric ton CO_2 released, it brings the price up to $42.60/GJ. The carbon tax was incorporated by looking at how many metric tons of CO_2 are released by 1 gallon of Jet A and adding it on top of the base cost. It would add an additional $0.325/gallon, yielding a new base cost of $5.325/gallon. If we consider

SAF with a base cost of $6.5/gallon, we find that it would cost $51.95 / GJ. Despite the added carbon tax, Jet A is still cheaper to buy and use since the amount of carbon released per gallon combusted doesn't cause a significant increase in the price.

ATJ is an alternative fuel we can use in a similar combustion reaction described by the following,

$$C_2H_6O\ (g)\ +\ 3\ O_2\ (g)\ \rightarrow\ 2\ CO_2\ (g)\ +3\ H_2O\ (g)$$

Based on its stoichiometric ratios, the reaction has a fuel to air ratio of 11.1% by mass. The specific enthalpy of combustion is -1.28 MJ/mol which corresponds to -27.2 MJ/kg of ethanol. Using the base price of ATJ being $3.0/gallon, we find that it costs $36.18/GJ by using values of its density and molar mass.

As listed in Figure 1, despite the energy density of ATJ being the lowest, the cheaper base price and lower molecular weight make up for its lack of energy content, making it the cheapest per GJ released. Just by comparing purely by prices, ATJ has the lowest cost per GJ released, making it an ideal target for companies that aim to maximize their profits.

Fuel	Adiabatic Flame Temperature	Autoiginition Temperature	Carbon Density	Melting Point	Boiling Point
Jet A	2360.2 K	210 °C	69.68 kg CO_2 / GJ	-9.55 °C	216.3 °C
SAF	2360.2 K	210 °C	69.68 kg CO_2 / GJ	-9.55 °C	216.3 °C
ATJ	2149.4 K	363 °C	68.93 kg CO_2 / GJ	-114.14 °C	78.24 °C

Fig. 2. A table of the safety and environmental concerns in regards to the fuel. [1] [2] Appendix 1

For Jet A and SAF, the adiabatic flame temperatures are identical at 2360.2 K, which suggests that they have similar combustion performance under the given combustion conditions. Their autoignition temperatures are also identical at 210 °C, which indicates that the two fuels have the same susceptibility to ignite spontaneously under the same conditions. On the other hand, ATJ, has a significantly lower adiabatic flame temperature of 2145.5 K, which is about 9.10% lower than the adiabatic flame temperatures of Jet A and SAF. This suggests that ATJ fuel produces less heat during combustion per unit basis. Additionally, ATJ's autoignition temperature is 363 °C, much higher than those of Jet A and SAF.

When analyzing turbojets, higher adiabatic flame temperature often represents a larger amount of heat released during the combustion process, which then leads to more power output and higher engine efficiency. Using fuels that burn under high adiabatic flame temperature usually requires highly compatible materials that can sustain such high temperatures. Moreover, in terms of safety, a low autoignition temperature could be dangerous as the fuel can catch on fire very easily and could damage the aircraft mid-flight, therefore Jet A and SAF wouldn't be the safest fuel to work with. While being a significantly safer fuel due to its high autoignition temperature, ATJ has a lower power output than the other two types of fuels. This showcases the tradeoff between energy efficiency and fuel safety when designing jet engines. Thus, as ATJ also has a lower net cost per unit of energy produced from combustion and a lower carbon density, indicating a smaller amount of carbon dioxide is emitted per unit of energy produced, we recommend ATJ as the top fuel choice.

On a side note, as shown in Appendix VI, SAF, including ATJ, is significantly more expensive than the conventional Jet-A fuel, while having larger market fluctuations due to production costs, supply chain, and relevant research and development expenses. Even from a snippet of the fuels' market performance, we could determine that the conventional Jet A is the most likely to have more stable costs, explaining why the aviation industry undergoes a rather slow transition to the renewable alternatives.

Jet Turbine Design

In the process of designing the jet turbine, the fed air and gaseous fuels are assumed to be ideal, which would be warranted only at certain conditions and at different parts of the engine. Initially, the fed air into the diffuser at high altitudes is at 18 kPa (0.178 atm) and -53.7 °C. The reduced pressure and temperature are 0.0048 and 1.66 respectively, referencing Appendix IV. If we analyze the generalized compressibility chart for z, all the reduced temperature lines all lead to 1 as reduced pressure gets close to zero. Because our reduced pressure is 0.0048 which is close to zero, we can assume air is ideal. Because air is ideal, we can also take advantage of its constant Cp value which is 7/2*R. If we calculate deltaH in two different ways, the two ways give a similar value as displayed in Appendix V. This demonstrates that we can use a constant 7/2*R for our calculations involving air.

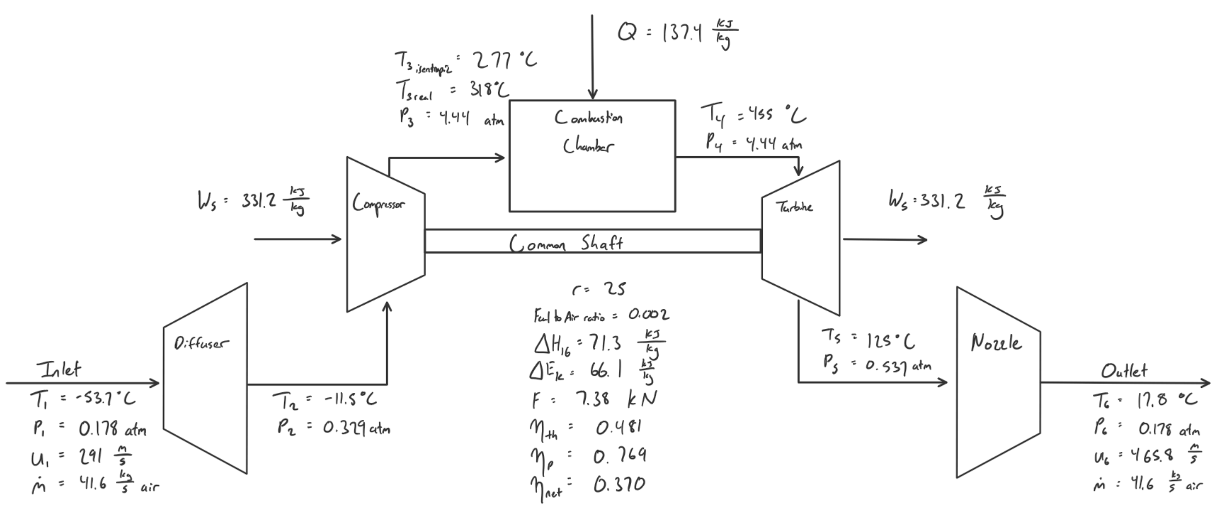


Fig. 3. Jet turbine design for ATJ with optimized pressure and fuel to air ratios of 25 and 0.002 respectively [1]

The maximum possible Carnot thermal efficiency is determined to be 85.1%, as temperature of the cold reservoir (T_C) is -53.7 °C and the temperature of the hot reservoir (T_H) is 1200 °C. Since Carnot engines assume perfect thermal insulation of its components, where no

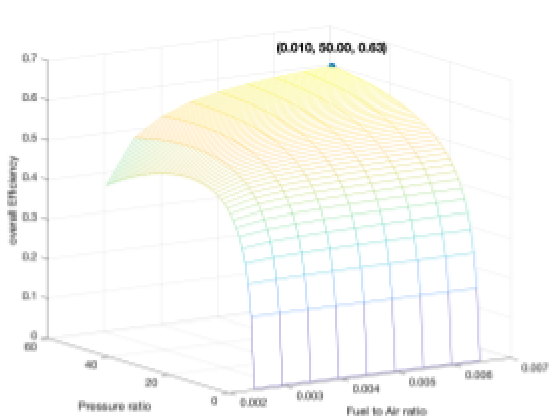


Fig. 4 Thermal Efficiency as a function of the compression and fuel to air ratios. The peak thermal efficiency is at 0.63 at pressure ratio = 50 and fuel to air ratio = 0.01

heat exchange with surroundings takes place, it is impossible for us to reach the Carnot thermal efficiency when designing turbojets under realistic conditions. However, if we operate under a fuel to air ratio of 1.0% and a pressure ratio (r) of 50 at 650 mph, the thermal efficiency of the turbojet can be maximized to 0.63 as seen in Figure 4.

To achieve the maximum overall turbojet efficiency, we found that the optimal conditions of operation are a pressure ratio of 25 and a fuel to air ratio of 0.2% at 650 mph. The efficiency of the turbine is graphed as a function of the pressure ratio and fuel to air ratio as seen in Figure 5, indicating a maximum efficiency of 0.37. Based on Figure 5 and the calculations to find the efficiency (Appendix III), it can be concluded the overall efficiency is a stronger function of the pressure ratio. The slope is larger in the graph going about the pressure ratio axis showing the change in efficiency is the greatest when the pressure ratio changes. This is supported by the equations used to derive the efficiency, where many of the values end up depending on pressure ratio more often than they depend on the fuel to air ratio as seen in Appendix 1.

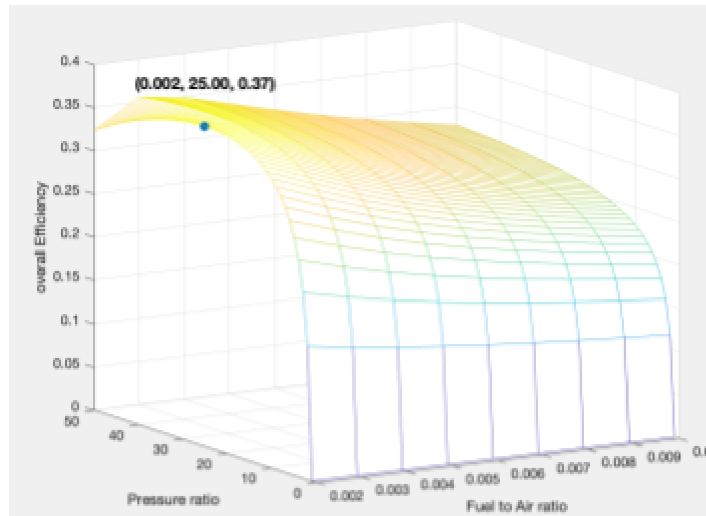


Fig. 5 Efficiency as a function of the compression and fuel to air ratios. The peak efficiency is at 0.37 at pressure ratio = 25 and fuel to air ratio = 0.002

Operating Cost	Fuel Delivery Rate	Net Fuel Cost (LAX to JFK)	Turbojet Efficiency
\$0.0836 / s	0.0279 gal / s	\$2,311.70	37%

Fig. 6. A table regarding turbojet efficiency and operating cost.

At our recommended conditions, each turbojet would consume our recommended ATJ fuel at a rate of 0.0279 gallons per second. Therefore, considering the two turbojets operating at a 37% efficiency, the duration of our ~2500 miles flight from LAX to JFK would be approximately 3.8 hours if we traveled at the max regulated speed of 650 mph, and the net fuel cost would be approximately \$2,311.70 (Appendix IX).

If we were to decrease the speed we traveled at while keeping our pressure and fuel to air ratios constant, we would find a decrease in the net efficiency. The graph in Appendix VII reveals a near linear relationship between speed and efficiency where efficiency decreases as speed decreases. This could be explained by a strong dependence of efficiency on the air's entering speed (u_1) which is the same as the plane travel speed as shown in Appendix VII and VIII. The change in velocity and heat should be nearly the same in all cases with speed decreasing, so the efficiency is more proportional to u_1.

Recommendations and Discussions

After conducting both qualitative and quantitative analysis, our team discovered fuel to air ratio and pressure ratio to be the two critical parameters one should look at when optimizing the turbojet. While an ideal fuel to air ratio helps us to ensure a complete combustion of the selected fuel, it also plays an important role helping us evaluate and optimize the engine's thermal efficiency, which impacts our fuel consumption and therefore profitability. Moreover, as the pressure ratio is defined as the ratio of the compressor pressure to the inlet pressure, we found that thrusting can significantly increase as the pressure ratio becomes higher. We found that the market indicates that the customers prefer shorter trips, so going at the max speed of 650 mph is ideal as it also is the most efficient speed to operate at.

However, these parameters all have restrictions due to the limit of materials and therefore, the optimization process is vitally important, as it not only helps us understand and improve engine efficiency, but also provides valuable insight into the flight's profitability. The efficiency of our turbojet design can be further enhanced by taking either of the two approaches: improving the compressor efficiency or increasing the maximum temperature that the turbine can withstand. While both can improve efficiency, the two approaches do so through different mechanisms. Higher compressor efficiency means that less work is required for compressing the air entering the combustion chamber, which can then be reflected by an increased overall efficiency of the entire turbojet. On the other hand, since the max power output of engines is often restricted by the material's ability to sustain high temperatures, increasing the maximum temperature that the turbine can withstand allows for more power generated from units of the same size. However, the research and development of the desired high-temperature operating materials often involves a tremendous effort, no matter money wise or timewise, and such materials are sometimes not as durable or cost-effective for applications at an industrial scale. Therefore, as each approach would be a great way to increase the overall turbojet efficiency, we recommend the developing teams to repetitively evaluate the cost-effectiveness of each approach and decide which approach would help meet the specific goals accordingly.

EcoThrust can further look to improve their environmental considerations by taking into account their carbon footprint in the process of flight operation. For example, the vehicles used to transport the ethanol to the planes could be run by renewable sources, or investing in more efficient ways to produce more ethanol, or even finding a more sustainable fuel. In terms of the turbine design, EcoThrust could look to improve the material capabilities of the turbine components which would allow for closer to ideal efficiencies so it would take less fuel to travel the same distance.

References:

[1] https://pubchem.ncbi.nlm.nih.gov/

[2] https://skybrary.aero/articles/ignition-fuels

[3] https://www.reuters.com/sustainability/us-sustainable-aviation-fuel-production-target-faces-cost-margin-challenges-2023-11-01/

[4] https://commons.wikimedia.org/wiki/File:Compressibility_factor_generalized_diagram.png

Appendix I.

Turbine Calculations, Calculations of ΔH_{rxn}, Fuel-air Ratio (mass basis), Energy Density, Carbon Density

Diffuser

Given isentropic : $\Delta S = 0$

Isentropic adiabatic

No work

$$1^{\circ} = \cancelto{0}{\frac{d(mU)}{dt}} + \Delta\left(\dot{m}\left(H + \frac{1}{2}u^2 + \cancelto{0}{gz}\right)\right) = \cancelto{0}{Q} + \cancelto{0}{W}$$

$$\Delta H^{t} = -\Delta \frac{1}{2} u^2$$

$$\Delta H^{t} = -\frac{1}{2}\left(0 - (u_1)^2\right)$$

$$\left(\frac{1}{mm}\right) m \Delta H_{12} = \frac{1}{2} u_1^2 m \left(\frac{1}{mm}\right)$$

$$n \Delta H_{12} = \frac{1}{2} u_1^2 n$$

$$n \Delta H = n \int C_p \, dT$$

$$\Delta H = C_{p_{air}} (T_2 - T_1)$$

$$\Delta H = \frac{7}{2} R (T_2 - T_1)$$

$$\frac{u^2}{2} = \frac{7}{2} R (T_2 - T_1)$$

$$T_2 = \frac{u_1^2}{7R\left(\frac{1}{mm}\right)} + T_1$$

$$\Delta S = 0 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{C_p}}$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{C_p}{R}} = \frac{P_2}{P_1}$$

$$P_2 = \left(\frac{T_2}{T_1}\right)^{\frac{7}{2}} P_1$$

$$P_2 = \left(\frac{u_1^2}{7RT_1} + 1\right)^{\frac{7}{2}} P_1$$

Compressor

$$r \equiv \frac{P_3}{P_1}$$

$$P_3 = r P_1$$

In isentropic case: $\Delta S = 0 = C_p \ln\left(\frac{T_3}{T_2}\right) - R \ln\left(\frac{P_3}{P_2}\right)$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{R}{C_p}} \qquad C_p = \frac{7}{2}R$$

$$T_3 = \left(\frac{rP_1}{P_2}\right)^{\frac{2}{7}} T_2$$

$$T_3 = \left(\frac{r}{\left(\frac{u_1^2}{7RT_1} + 1\right)^{\frac{7}{2}}}\right)^{\frac{2}{7}} \left(\frac{u_1^2}{7R} + T_1\right)$$

Isentropic ideal

$$1^{\circ} \quad \cancelto{0}{\frac{d(mU)}{dt}} + \Delta\left(\dot{m}\left(H + \cancelto{0}{\frac{1}{2}u^2} + \cancelto{0}{gz}\right)\right) = \cancelto{0}{Q} + W$$

$$\Delta H = \frac{W}{n}$$

$$\int C_p \, dT = \frac{W}{n}$$

$$\frac{7}{2} R (T_3 - T_2) = \frac{W_{isentropic}}{n}$$

$$\eta_{comp} = \frac{W_{isen}}{W_{real}} = 0.95 - 0.003r$$

$$W_{real} = \frac{W_{isen}}{0.95 - 0.003r} = \frac{\frac{7}{2} R (T_3 - T_2) n}{0.95 - 0.003r}$$

$$\Delta H_{real,23} = \frac{\Delta H_{isen,23}}{0.95 - 0.003r}$$

$$n C_p (T_{3real} - T_2) = \frac{n C_p (T_3 - T_2)}{0.95 - 0.003r}$$

$$T_{3real} = \frac{(T_3 - T_2)}{0.95 - 0.003r} + T_2$$

Combustion Chamber ATJ

$$C_2H_6O(g) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(g)$$

$$1°: \cancel{\frac{d(mU)}{dt}}^{0} + \Delta(\dot{m}(H + \cancel{\tfrac{1}{2}u^2}^{0} + \cancel{gz}^{0})) = Q + \cancel{W}^{0}$$

$$\Delta\dot{H}_{rxn}(1400k) = Q = n\,C_p dT$$

$$\Delta\dot{H}_{rxn}(1400k) = \underbrace{\sum_{\nu_i} n_i R \int_{T_3}^{298} \frac{C_{pi}}{R} dT}_{\text{Reactants}} + \underbrace{\Delta H_{rxn}(298k)}_{\text{Reaction}} + \underbrace{\sum_{\nu_i} n_i R \int_{298}^{T_4} \frac{C_{pi}}{R} dT}_{\text{Product}}$$

$$\Delta H_{rxn}(298k) = \Delta H_{f,product}(298k) - \Delta H_{f,reactant}(298k)$$

$$H_{rxn}(298k) = \left[2(-393509 \tfrac{J}{mol}) + 3(-241818 \tfrac{J}{mol})\right] - \left[1(-235100) + 3(0 \tfrac{J}{mol})\right]$$

$$\Delta H_{rxn}(298k) = -1277372 \ \tfrac{J}{mol}$$

$$\frac{C_p}{R}\left[\begin{array}{llllllll} & & A & & B & & C & D \\ C_2H_6O: & & 3.518 & + & 20.001\,T & - & 6.002\,T^2 & + 0 \\ CO_2: & & 5.457 & + & 1.045\,T & + & 0 & - \frac{1.157}{T^2} \\ H_2O: & & 3.470 & + & 1.450\,T & + & 0 & + \frac{0.121}{T^2} \end{array}\right]$$

$$O_2 : \tfrac{7}{2}R$$

$$N_2 : \tfrac{7}{2}R$$

$$\dot{m}\Delta H = Q$$

$$(kg)\left(\tfrac{mol}{kg}\right)C_p(T_4 - T_3) = \Delta H_{rxn}(T_4)$$

$$(kg)\left(\tfrac{mol}{kg}\right)\tfrac{7}{2}R(T_4 - T_3) = \Delta H_{rxn}(T_4)$$

Turbine

$$1°: \Delta H = -\frac{W_{real}}{n} = -\frac{W_{isen}}{(0.95 - 0.003c)n} = \int C_p dT = \tfrac{7}{2}R(T_5 - T_4)$$

$$-\frac{\frac{7}{2}R(T_3 - T_2)}{(0.95 - 0.003c)} = \tfrac{7}{2}R(T_5 - T_4)$$

$$-\frac{(T_3 - T_2)}{(0.95 - 0.003c)} = T_5 - T_4$$

$$T_5 = -\frac{(T_3 - T_2)}{(0.95 - 0.003c)} + T_4$$

$$2°: \Delta S = 0 = C_p \ln\left(\frac{T_5}{T_4}\right) - R\ln\left(\frac{P_5}{P_4}\right)$$

$$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{R/C_p}$$

$$\left(\frac{T_5}{T_4}\right)^{\frac{7}{2}} = \frac{P_5}{P_4}$$

$$P_5 = \left(\frac{T_5}{T_4}\right)^{\frac{7}{2}} P_4$$

Nozzle

$$1°: \cancel{\frac{d(mU)}{dt}}^{0} + \Delta(\dot{m}(H + \tfrac{1}{2}u^2 + \cancel{gz}^{0})) = \cancel{Q}^{0} + \cancel{W}^{0}$$

$$\Delta H = -\tfrac{1}{2}(u_6^2 - \cancel{u_5^2}^{0})$$

$$\Delta H = -\frac{u_6^2}{2}$$

$$C_p(T_6 - T_5) = -\frac{u_6^2}{2}$$

$$\tfrac{7}{2}R(T_6 - T_5) = -\frac{u_6^2}{2}$$

$$\sqrt{-7R(T_6 - T_5)\left(\tfrac{1}{mm}\right)} = u_6$$

$$2°: \Delta S = 0 = C_p \ln\left(\frac{T_6}{T_5}\right) - R\ln\left(\frac{P_6}{P_5}\right)$$

$$P_6 = P_1$$

$$\frac{T_6}{T_5} = \left(\frac{P_6}{P_5}\right)^{R/C_p}$$

$$T_6 = \left(\frac{P_6}{P_5}\right)^{\frac{2}{7}} T_5$$

$$T_6 = \left(\frac{P_1}{P_5}\right)^{\frac{2}{7}} T_5$$

Overall

$$1°: \cancel{\frac{d(mU)}{dt}}^{0} + \Delta(\dot{m}(H + \tfrac{1}{2}u^2 + \cancel{gz}^{0})) = Q + \cancel{W}$$

$$\Delta H_{61} + \tfrac{1}{2}\Delta u^2 = \frac{Q}{m}$$

$$\left(\tfrac{1}{mm}\right)\tfrac{7}{2}R(T_6 - T_1) + \tfrac{1}{2}(u_6^2 - u_1^2) = \frac{Q}{m}$$

Fuel-air ratio:

$$3 \text{ mol } O_2 \times \frac{1 \text{ mol air}}{0.21 \text{ mol } O_2} \times \frac{28.97 \text{ g}}{1 \text{ mol air}} = 413.86 \text{ g air}$$

$$1 \text{ mol ethanol} \times \frac{46.07 \text{ g}}{\text{mol ethanol}} = 46.07 \text{ g ethanol}$$

$$F \text{ to } A = \frac{46.07}{413.86} \times 100 = 11.13\%$$

$$\frac{\dot{n}_{C_{12}H_{26}}}{\dot{n}_{O_2}} = \frac{1}{\frac{37}{2}\left(\frac{1 \text{ mol air}}{0.21 \text{ mol } O_2}\right)} = \frac{1 \text{ mol}}{88.1 \text{ mol}}\left(\frac{170.33 \text{ g/mol}}{28.97 \text{ g/mol}}\right) = 6.7\% =$$

ΔH_{rxn}:

- Jet A & SAF

$$\Delta H_{Rxn} = 12(-393509) + 13(-241818) - 1(-288.1 \times 10^3) - \frac{37}{2}(0)$$
$$= -7577642 \text{ J/mol}$$

- ATJ

$$\Delta H_{rxn}(298k) = \Delta H_{f,\text{product}}(298k) - \Delta H_{f,\text{reactant}}(298k)$$
$$\Delta H_{rxn}(298k) = \left[2\left(-393509 \tfrac{J}{mol}\right) + 3\left(-241818 \tfrac{J}{mol}\right)\right] - \left[1(-235100) + 3\left(0 \tfrac{J}{mol}\right)\right]$$
$$\Delta H_{rxn}(298k) = -1277372 \frac{J}{mol}$$

Energy Density:

- Jet A

Unit conversions:

$$\left(1\ \text{mol}\ C_{12}H_{26}\right)\left(170.34\ \frac{g}{mol}\right)\left(\frac{1000g}{1kg}\right)\left(750\ \frac{kg}{m^3}\right)\left(\frac{1m^3}{1000L}\right)\left(\frac{1\ gal}{3.785L}\right) = 0.0600\ gal := 1\ mol\ C_{12}H_{26}$$

$$\text{energy density} \Rightarrow \frac{\left|-7577642\ \frac{J}{mol}\right|\left(\frac{1\ GJ}{10^9 J}\right)}{0.0600\ \frac{gal}{mol}} = 0.126294\ \frac{GJ}{gal}$$

Jet A ⇒ carbon tax:

$$\text{Carbon tax}: (12\ mol)\left(44\ \frac{g}{mol}\right)\left(\frac{1kg}{1000g}\right)\left(\frac{1\ ton}{1000kg}\right)(\$40)$$

$$= \$0.02112 / 1\ \text{mol of Jet A used}$$

$$\left(\frac{\$0.02112}{mol}\right)\left(\frac{1\ mol}{0.06\ Gal}\right) = \$0.325\ \frac{\text{Carbon tax}}{Gal}$$

$$\$5 + \$0.325 = \$5.325/Gal$$

$$\Rightarrow \frac{\$5.325/gal}{0.126294\ GJ/gal} = \$42.16\ /GJ$$

- SAF

Unit conversions:

$$\left(1\ \text{mol}\ C_{12}H_{26}\right)\left(170.34\ \frac{g}{mol}\right)\left(\frac{1000g}{1kg}\right)\left(750\ \frac{kg}{m^3}\right)\left(\frac{1m^3}{1000L}\right)\left(\frac{1\ gal}{3.785L}\right) = 0.0600\ gal := 1\ mol\ C_{12}H_{26}$$

$$\text{energy density} \Rightarrow \frac{\left|-7577642\ \frac{J}{mol}\right|\left(\frac{1\ GJ}{10^9 J}\right)}{0.0600\ \frac{gal}{mol}} = 0.126294\ \frac{GJ}{gal}$$

$$\text{SAF} \Rightarrow \frac{\$6.5/gal}{0.126294\ GJ/gal} = \$51.47\ /GJ$$

- ATJ

$$ATJ \Rightarrow \Delta H_{rxn} = -127372\,J.$$

$$(1\,mol\ C_2H_6O)\left(46.068\,\frac{g}{mol}\right)\left(\frac{1}{0.78945\,\frac{g}{cm^3}}\right)\left(\frac{0.001\,L}{1\,cm^3}\right)\left(\frac{1\,gal}{3.785\,L}\right)$$

$$= 0.0154\,gal/mol$$

$$\frac{|-127372\,J/\cancel{mol}|\left(\frac{1\,GJ}{10^9\,J}\right)}{0.0154\,gal/mol} = 0.0829\ GJ/gal$$

$$\Rightarrow \frac{\$3.0/gal}{0.0829\ GJ/gal} = \$36.21/GJ$$

Carbon Density:

- Jet A & SAF:

$$\underline{\text{Carbon density}}:$$

Jet A & SAF :

$$\left(0.126294\,\frac{GJ}{\cancel{gal}}\right)\left(\frac{0.06\,\cancel{gal}}{1\,\cancel{mol}\ fuel}\right)\left(\frac{1\,\cancel{mol\ fuel}}{12\,\cancel{mol\ CO_2}}\right)$$

$$\left(\frac{1\,\cancel{mol}\,CO_2}{44\,g}\right) = \frac{0.0000143\,52\ GJ}{g\ CO_2}$$

$$\left(\frac{0.0000143\,52\,GJ}{g\ CO_2}\right)\left(\frac{1000\,g}{1\,kg}\right) = 0.014352\ \frac{GJ}{kg\,CO_2}$$

$$\Rightarrow 1/0.0014352\,\frac{GJ}{kg\,CO_2} = 69.677\ \frac{kg\,CO_2}{GJ}$$

- ATJ

ATJ:

$$\Rightarrow \left(0.0829\ \frac{GJ}{\cancel{gal}}\right)\left(0.0154\ \frac{\cancel{gal}}{smol\ fuel}\right)\left(\frac{1\ mol\ fuel}{2\ mol\ CO_2}\right)$$

$$= 0.00063833\ \frac{GJ}{smol\ CO_2}$$

$$\left(0.00063833\ \frac{GJ}{smol\ CO_2}\right)\left(\frac{1\ mol}{44g}\right)\left(\frac{1000g}{1kg}\right) = 0.01450775\ \frac{GJ}{kg\ CO_2}$$

$$\Rightarrow 1/0.0145075\ \frac{GJ}{kg\ CO_2} = 68.930\ kg\ CO_2/GJ$$

Appendix II.

Calculations for Adiabatic Flame Temperatures (10% excess air) of Different Fuels

```
% define calculation parameters
syms T Tf

Tref = 298.15;
R = 8.314;

% heat capacities of products
Cp_CO2 = 5.457 + 1.045*10^-3*T -1.157*10^5/T^2;
Cp_H2O = 3.470 + 1.450*10^-3*T + 0.121*10^5/T^2;
Cp_O2 = 3.639 + 0.506*10^-3*T -0.227*10^5/T^2;
Cp_N2 = 3.280 + 0.593*10^-3*T +0.040*10^5/T^2;

% JetA & SAF Fuel
H_298_dodecane = 7577642;
n_o2_JetAexcess = 37/2*0.1;
n_H2O_jeta = 13;
n_CO2_jeta = 12;
n_N2_jeta = 37/2/0.21*0.79;

% obtaining solution
Q_jeta = R*int(n_CO2_jeta*Cp_CO2,T,Tref,Tf) + R*int(n_H2O_jeta*Cp_H2O,T,Tref,Tf)...
    + R*int(n_o2_JetAexcess *Cp_O2,T,Tref,Tf) + R*int(n_N2_jeta*Cp_N2,T,Tref,Tf);
solution_JetA_SAF = vpasolve(Q_jeta == H_298_dodecane, Tf);

% ATJ Fuel
H_298_ethanol = 1277372;
n_o2_ATJexcess = 3*0.1;
n_H2O_atj = 3;
n_CO2_atj = 2;
n_N2_atj = 7/2/0.21*0.79;

% obtaining solution
Q_atj = R*int(n_CO2_atj*Cp_CO2,T,Tref,Tf) + R*int(n_H2O_atj*Cp_H2O,T,Tref,Tf)...
    + R*int(n_o2_ATJexcess *Cp_O2,T,Tref,Tf) + R*int(n_N2_atj*Cp_N2,T,Tref,Tf);
solution_ATJ = vpasolve(Q_atj == H_298_ethanol, Tf);

```

```
solution_JetA_SAF =

-11752.922207000707503255545553594
0.97450063092162334432195018599362
 2360.1825786222919350264293092579

solution_ATJ =

-11261.962047902882115014266702243
0.85631854219723560960981348801271
 2149.4339043836598629239662579479
```

Appendix III.

Calculations for optimizing pressure ratio and fuel to air ratio at 291 m/s

```
syms x;
syms T;
syms T4;
syms FtoA;
syms T5
syms P5
syms u2
lowerspeedlimit = 195;
upperspeedlimit = 205;
counter = 1;
vector = zeros(1, 9);
%vector2 = zeros(1, 4);
u = 291;
i = 1;
%while(u <= upperspeedlimit)
disp(u)
tempvector = zeros(50, 9);
tempvector2 = zeros(1, 9);
tempvector3 = zeros(50, 9);
tempvector4 = zeros(50, 9);
tempvector5 = zeros(50, 9);
tempvector6 = zeros(50, 9);
tempvector7 = zeros(50, 9);
%mass of air in kg/s
m = 0.143*u;
%moles/s of oxygen
nO2 = m/0.02897 * 0.21;
%Diffuser calculation
T2 = u^2*0.02897/(7*8.314) + 219.3;
P2 = (T2/219.3)^(7/2) * 18;
   for x = 1:50
       %compressor calculation
       P3 = 18*x
       T3 = (P3/P2)^(2/7) * T2
       T3real = (T3-T2)/(0.95-0.003*x)+T2;
       if(P3 < P2)
           continue
       else
           index2 = 1;
           for FtoA = 0.002:0.001:0.01
               %n = FtoA*0.81*nO2;
               n = FtoA/0.046068*m;
               H1 = 8.314*int(3.518 + 20.001*10^-2*T - 6.002*10^-6*T^2, [T3real
298]);
```

```
               %O2
               H2 = 7/2*8.314*(298 - T3real);
               H2excess = 7/2*8.314*(T4 - 298);
               %CO2
               H3 = 8.314*int(5.457 + 1.045*10^-3*T - 1.157*10^5*T^-2, [298 T4]);
               %H2O
               H4 = 8.314*int(3.47 + 1.45*10^-3*T + 0.121*10^5*T^-2, [298 T4]);
               %N2
               H5 = 7/2*8.314*(T4 - T3real);

               Hrxn = n*H1 + 37/2*n*H2 + (nO2 - 37/2*n)*H2excess + 12*n*H3 +
13*n*H4 + (m/0.02897*0.79)*H5 - n*7577642 == (m/0.02897)*7/2*8.314*(T4 - T3real);
               Q = n*H1 + 37/2*n*H2 + (nO2 - 37/2*n)*H2excess + 12*n*H3 + 13*n*H4 +
(m/0.02897*0.79)*H5 - n*7577642;

               S = vpasolve(T4 == -Q/(m/0.02897*7/2*8.314) + T3real, T4);
               solution = S(3);

               H2excessfinal = 7/2*8.314*(solution - 298);
               %CO2
               H3final = 8.314*int(5.457 + 1.045*10^-3*T - 1.157*10^5*T^-2, [298
solution]);
               %H2O
               H4final = 8.314*int(3.47 + 1.45*10^-3*T + 0.121*10^5*T^-2, [298
solution]);
               %N2
               H5final = 7/2*8.314*(solution - T3real);
               Qprod = n*H1 + 37/2*n*H2 + (nO2 - 37/2*n)*H2excessfinal +
12*n*H3final + 13*n*H4final + (m/0.02897*0.79)*H5final - n*7577642
               Qprod2 = (m/0.02897)*7/2*8.314*(solution - T3real)
               %cost only depends on FtoA. The amount of air coming in
               %doesn't affect how much fuel we should use
               cost = n*0.0463;
               %cost for Jet A
               %cost for SAF
               %cost = n*0.39;
               %{
               if(double(solution) > 1473 || double(solution) < 0)
                   continue
               else
               %}

                   if(double(solution) < T3real)
                       index2 = index2 + 1;
                       continue
                   else
```

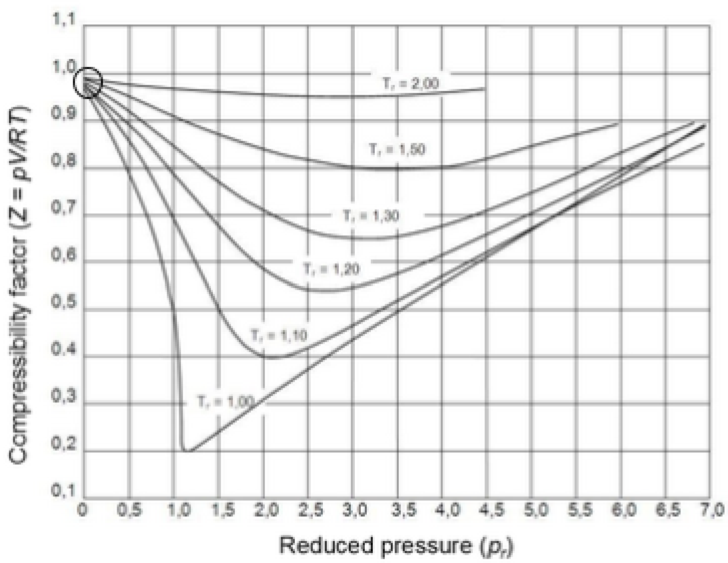
```
                        T5 = -(T3 - T2)/(0.95-0.003*x) + solution;
                        P5 = (T5/solution)^(7/2)*P3; %P4 = P3;
                        P6 = 18;
                        T6 = (P6/P5)^(2/7)*T5;
                        u6 = sqrt(2*7/2*8.314*(T5-T6)/0.02897);
                        deltaT = solution - T3real;
                        -Qprod/m == 7/2*8.314*(T6 - 219.3)/0.02897 + 1/2*(u6^2 -
u^2)
                        %u62 = sqrt((Qprod -
m/0.02897*7/2*8.314*(T6-219.3))/(m/2)+u^2)
                        efficiency = 1/2*(u6^2);
                        tempvector(x, index2) = deltaT;
                        tempvector2(x, index2) = cost;
                        tempvector3(x, index2) = solution;
                        %tempvector4(x, index2) = T5;
                        tempvector5(x, index2) = u6;
                        EK = 1/2*(u6^2 - u^2);
                        overallEff = (m*EK)/(-Qprod) * u*(u6 - u)/EK
                        tempvector6(x, index2) = overallEff;
                        tempvector4(x, index2) = (m*EK)/(-Qprod);
                        tempvector7(x, index2) = u*(u6 - u)/EK;
                        %tempvector6(x, index2) = finalVelocity;
                        %tempvector6(x, index2) = finalV;
                        %tempvector6(x, index2) = Q/1000;
                        index2 = index2 + 1;
                    end
                end
            %end
        end
    end
    maxOverallEff = max(tempvector6, [], "all")
    maxThermalEff = max(tempvector4, [], "all")
    graph = mesh(tempvector6);
    xaxis = get(gca, 'XTickLabel');
    newXAxis = set(gca, 'XTickLabel', [0.002:0.001:0.01]);
    xlabel('Fuel to Air ratio')
    ylabel('Pressure ratio')
    zlabel('overall Efficiency')
    hold on
    scatter3(1, 25, tempvector6(25, 1), 'filled', 'SizeData', 50); % Adjust size as
needed
    text(1, 25, tempvector6(25, 1) + 0.05, sprintf('(%.3f, %.2f, %.2f)', 0.002, 25,
tempvector6(25, 1)), ...
    'FontSize', 12, 'FontWeight', 'bold', 'HorizontalAlignment', 'center',
'VerticalAlignment', 'middle');
    hold off
```

```
maxOverallEff =

    0.3701

maxThermalEff =

    0.6291
```

Appendix IV: Calculation of Reduced Pressure and Reduced Temperature, with the Generalized Compressibility chart

$P_r = \frac{P}{Pc} = \frac{0.18\ bar}{37.45\ bar} = 0.0048$

$T_r = \frac{T}{Tc} = \frac{219.3K}{132.2} = 1.66$

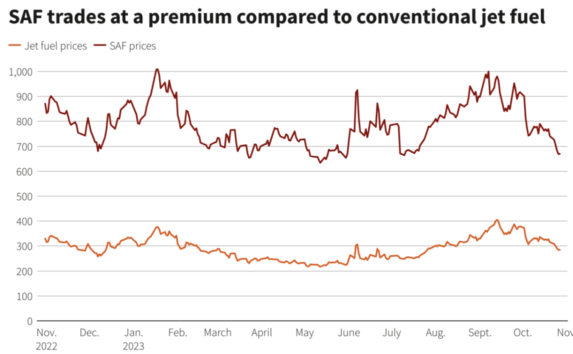


Appendix V: Calculation for Cp of air

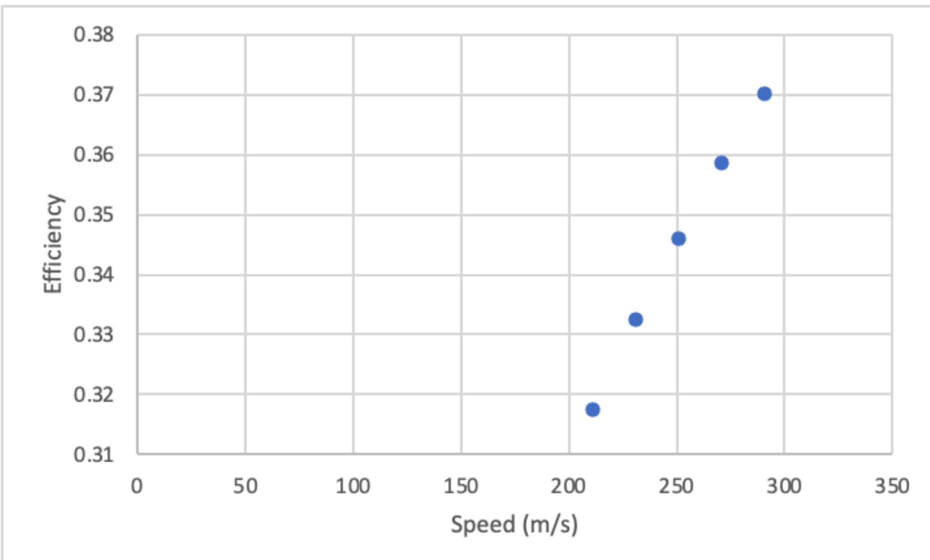
```
7/2*8.314*(261.5 - 219.3)
cp = (3.355 + 0.575*10^-3*T - 0.016*10^5*T^-2);
H = vpa(8.314*int(cp, [219.3, 261.5]))
```

```
ans =

   1.2280e+03


H =

1215.8136798220331473600601252551
```

Appendix VI: Market fluctuation of different fuels [3]



Appendix VII: Graph of relationship between Efficiency and flight speed



Appendix VIII: Efficiency calculations and velocity relation.

Efficiencies

$$\eta_{th} = \frac{\Delta E_k}{Q} = 1 - \frac{\Delta H_{61}}{Q}$$

$$\eta_{th} = 1 - \frac{\dot{m} C_p (T_6 - T_1)}{Q}$$

$$\eta_p = \frac{F u_1}{\dot{m} \Delta E_k} = \frac{\dot{m}(u_6 - u_1) u_1}{\dot{m}\left(\frac{1}{2}(u_6^2 - u_1^2)\right)}$$

$$\eta_p = \frac{u_1(u_6 - u_1)}{\frac{1}{2}(u_6^2 - u_1^2)}$$

$$\eta_{net} = \eta_p \cdot \eta_{th}$$

$$\eta_{net} = \frac{u_1(U_6 - u_1)}{Q} = \frac{u_1 \Delta U_{16}}{Q}$$

Appendix IX

Calculations for Operating Costs (fuel cost per time), the Net Fuel Cost for a LAX-to-JFK Trip, the Required Fuel Delivery Rate, the Turbojet Efficiency

$v = 291 \ m/s$

$r = 25$

f to A = 0.002

$(291 \ \frac{m}{s})(0.5 \ m^2) = 145.5 \ \frac{m^3}{s}$

$$\hat{V} = \frac{RT}{P} = \frac{(0.0821 \frac{L \cdot atm}{mol \cdot \cancel{K}})(219.261 \ \cancel{K})}{(0.178 \ \cancel{atm})}$$

$$= (101.131 \ \frac{L}{mol})(\frac{1 \ m^3}{10^3 \ L})$$

$$= (0.101 \ \frac{m^3}{\cancel{mol}})(\frac{1 \ \cancel{mol}}{28.979 \ g})$$

$$= (0.00349 \ \frac{m^3}{\cancel{g}})(\frac{1000 \ \cancel{g}}{1 \ kg})$$

$$= 3.491 \ \frac{m^3}{1 \ kg}$$

0.37

turbojet efficiency

$\frac{50}{1}$

$$\Rightarrow \dot{m}_{air} = (145.5 \ \frac{\cancel{m^3}}{s})(\frac{1}{3.491 \frac{\cancel{m^3}}{kg}})$$

$$= 41.680 \ \frac{kg}{s}$$

$$\Rightarrow \dot{m}_{fuel} = (41.680 \ \frac{kg}{s})(0.002)$$

$$= 0.0834 \ \frac{kg}{s}$$

$$(0.0834 \ \frac{\cancel{kg}}{s})(\frac{1000 \ \cancel{g}}{1 \ \cancel{kg}})(\frac{1 \ \cancel{mol}}{46.0683 \ \cancel{g}})(0.0154 \ \frac{gal}{\cancel{mol}})$$

$$= 0.0279 \ \frac{gal}{s}$$

fuel delivery rate

$$(0.0279 \ \frac{\cancel{gal}}{s})(\frac{3.0 \ \$}{\cancel{gal}}) = 0.0836 \ \$/s$$

fuel cost

$2500 \text{ miles} = 4023.36 \text{ km}$

$$3.84 \text{ hrs} \Leftarrow \frac{\left(4023.36 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}}\right)}{291 \frac{\text{m}}{\text{s}}} \times 0.0836 \frac{\$}{\text{s}}$$

$= 1155.852 \$$ for 1 engine

$$1155.852 \times 2 = \boxed{2311.704 \$}$$

net fuel cost

LAX – JFK.