ON A THEOREM OF MINGFENG CHEN, AND THE PROOF OF THE RIEMANN HYPOTHESIS

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Dedicated to Yiyu Wang on his 25th birthday

ABSTRACT. We give a quick application for Chen's structure theorem of abelian groups, to attack the building block of mathematics.

CONTENTS

1. INTRODUCTION

In his celebrated homework [1], Chen proved the following result:

Theorem 1.1. [1] *Given finite abelian groups G and H, we have $G \simeq H$ if and only if $|G| = |H|$.*[1]

In this paper, we will give a quick application for this theorem:

Theorem 1.2. *Given any prime number p and q, one has $p = q$. In particular, one has $2 = 3$.*

Corollary 1.3. *Riemann zeta function has trivial analytic continuation, and Riemann hypothesis holds.*

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[1]Zero points for Q1. - Simon Marshall Sep 20 2023

2. p-adic number revisite

Theorem 2.1. *$\mathbb{Z}_p$ is pure of characteristic p.*

Proof. Recall that $\mathbb{Z}_p = \varprojlim \mathbb{Z}/p^n$. By Theorem 1.1, one has $\mathbb{Z}/p^n \simeq (\mathbb{Z}/p)^n$, hence $p\mathbb{Z}_p = 0$. ☐

3. Proof of main theorem

An immediate consequence of Theorem 2.1 is the following:

Corollary 3.1. *$\mathbb{Q}_p$ is pure of characteristic p.*

Corollary 3.2. *$\mathbb{C}$ is pure of characteristic p.*

Proof. By the classical identification $\mathbb{C} \simeq \overline{\mathbb{Q}_p}$. ☐

Corollary 3.3. *For any prime numbers p and q, one has $p = q$.*

Proof. One has $\overline{\mathbb{Q}_p} \simeq \overline{\mathbb{Q}_q}$, hence $p = q$ by comparing the characteristic. ☐

4. Proof of Riemann hypothesis

By Corollary 3.2, the Riemann zeta function is equal to the local zeta function for any prime p, in particular, one has

$$\zeta(s) = \frac{1}{1 - 2^{-s}}. \tag{4.0.1}$$

Hence it has trivial analytic continuation, and there is no zeros in the plane. As a result, all nontrivial zeros lie in $\mathrm{Re}(s) = \frac{1}{2}$. ☐

References

[1] Mingfeng, Chen, Math 749 Analytic number theory Homework 1, unpublished note. 1

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