

## **Computational Cognitive Modeling**

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# **Probabilistic programming, program induction, and language of thought models**

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Brenden Lake & Todd Gureckis

**course website:**

<https://brendenlake.github.io/CCM-site/>

# Probabilistic programs / probabilistic programming

```
class world():
    def __init__(self):
        self.dict_strength = {}
    def clear(self): # used when sampling over possible world
        self.dict_strength = {}

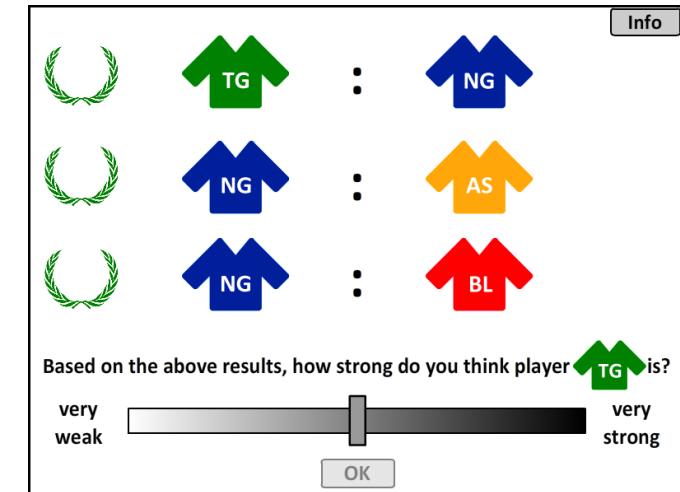
    def strength(name):
        if name not in W.dict_strength:
            return random.random()

    def lazy(name):
        return random.random() < 0.1

def team_strength(team):
    # team : list of names
    mysum = 0.
    for name in team:
        if lazy(name):
            mysum += (strength(name) / 2.)
        else:
            mysum += strength(name)
    return mysum
```

```
def winner(team1,team2):
    # team1 : list of names
    # team2 : list of names
    if team_strength(team1) > team_strength(team2):
        return team1
    else:
        return team2

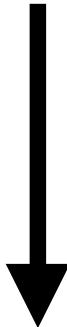
def beat(team1,team2):
    return winner(team1,team2) == team1
```



(example from homework; Goodman et al., 2015)

# Probabilistic programs / probabilistic programming

Stochastic program  
(known structure)



Data

(e.g., game matches  
and who won)



```
class world():
    def __init__(self):
        self.dict_strength = {}
    def clear(self): # used when sampling over possible world
        self.dict_strength = {}
```

```
def strength(name):
    if name not in W.dict_strength:
        return random.random() < 0.1

def lazy(name):
    return random.random() < 0.1
```

How strong is Bob?

The interface shows three game results:

W	TG	:	NG
W	NG	:	AS
W	NG	:	BL

Based on the above results, how strong do you think player TG is?

very weak  very strong

OK

# Probabilistic programs / probabilistic programming

- Probabilistic program: A probabilistic model defined in a structured description language (much like a programming language) using random programming primitives.
- Due to random primitives, every time the program executes it returns a different output.
- Probabilistic programs are a generalization of Bayesian networks, and many of the other Bayesian models we have discussed.
- Especially convenient when the prior is too complex to write down as a set of hypotheses, or the model is awkward or impossible to write as a Bayesian network.

# Probabilistic programs: A simple example

## Preliminary definitions

```
def flip(theta=0.5):          (flip a coin with 'theta' chance of heads)
    return random.random() < theta
```

## Simple probabilistic program

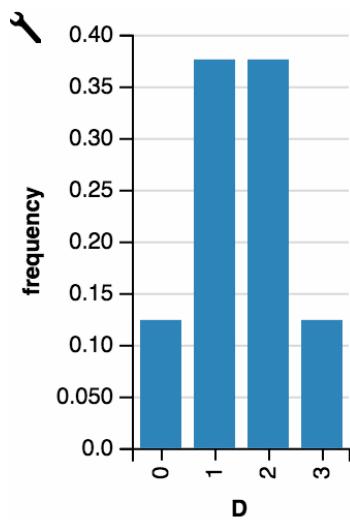
```
A = flip()
B = flip()
C = flip()
D = A + B + C
```

**Key idea:** A probabilistic program is a generative process for producing data

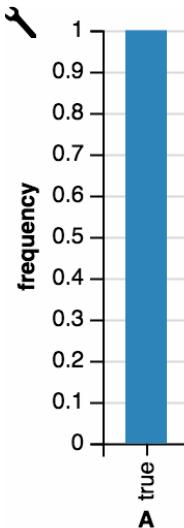
## Bayesian inference

(again, notice productivity reasoning)

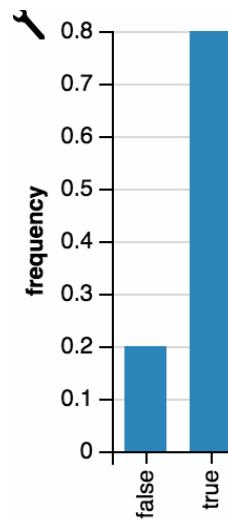
$$P(D)$$



$$P(A|D = 3)$$



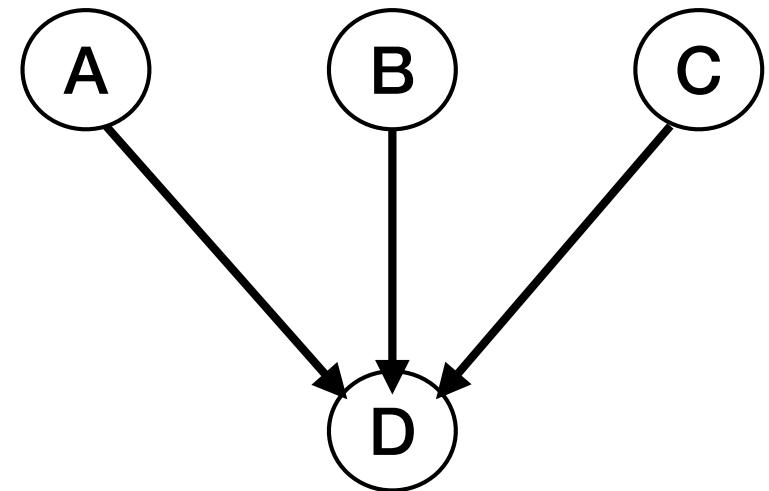
$$P(A|D \geq 2)$$



Example from Noah Goodman and Josh Tenenbaum  
<https://probmods.org/>

# Probabilistic program or Bayesian network?

```
A = flip()  
B = flip()  
C = flip()  
D = A + B + C
```



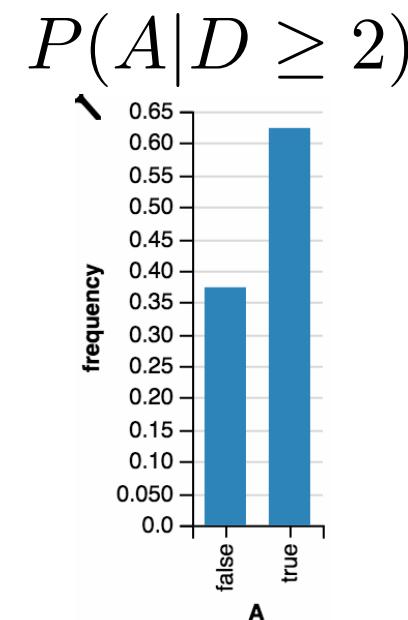
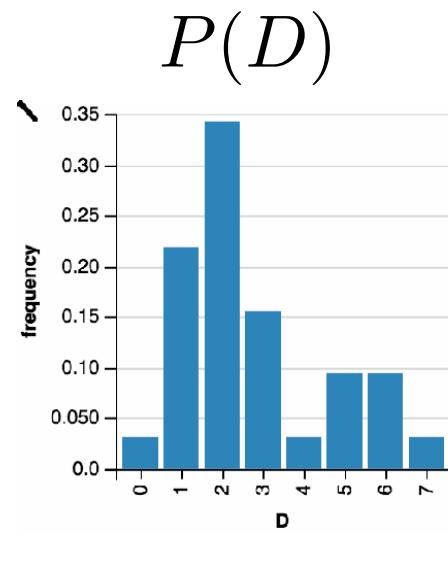
In this case, the probabilistic program can be straightforwardly represented as a Bayesian network, although the program representation conveys more information.

# Probabilistic programs: Another example

Simple probabilistic program (yet more complex than before)

```
A = flip()  
B = flip()  
C = flip()  
if C:  
    D = A + B + C  
else:  
    E = flip()  
    F = (2*flip())**2  
    D = A + B + C +E + F
```

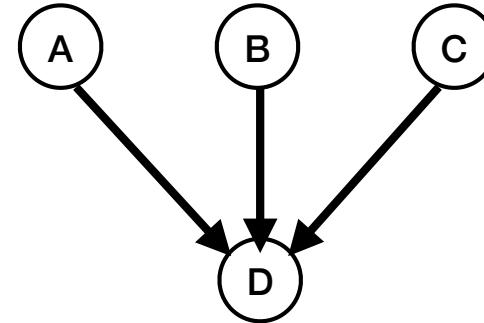
Bayesian inference



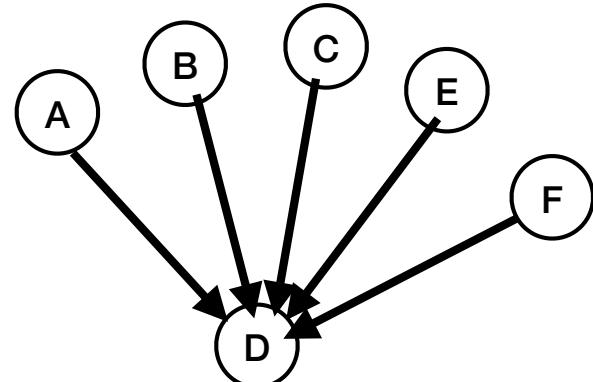
# Probabilistic program or Bayesian network?

if C is True:

```
A = flip()  
B = flip()  
C = flip()  
if C:  
    D = A + B + C  
else:  
    E = flip()  
    F = (2*flip())**2  
    D = A + B + C +E + F
```



if C is False:



Bayesian networks (graphical models) do not have a mechanism for adding additional variables, and they lack general control structures that are relevant in both cognitive science and data science applications (if statements, for loops, while loops, recursion, etc.)

# From Homework: Reasoning about tennis with probabilistic programs

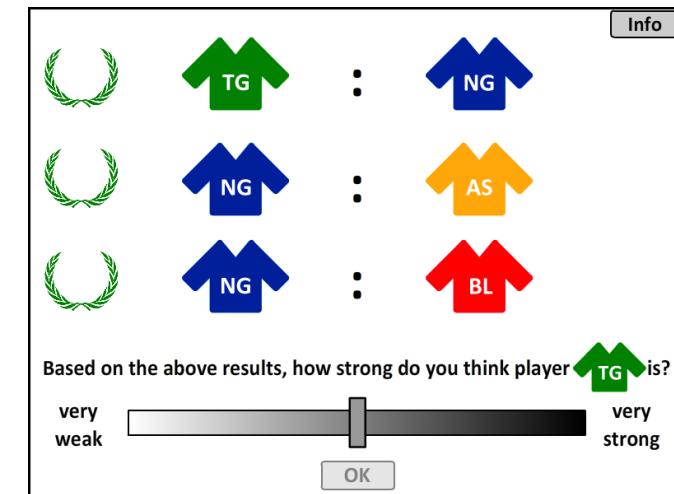
```
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    def clear(self): # used when sampling over possible world
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    def strength(name):
        if name not in W.dict_strength:
            return lazy(name)

    def team_strength(team):
        # team : list of names
        mysum = 0.
        for name in team:
            if lazy(name):
                mysum += (strength(name) / 2.)
            else:
                mysum += strength(name)
        return mysum
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```
def winner(team1,team2):
    # team1 : list of names
    # team2 : list of names
    if team_strength(team1) > team_strength(team2):
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        return team2

def beat(team1,team2):
    return winner(team1,team2) == team1
```



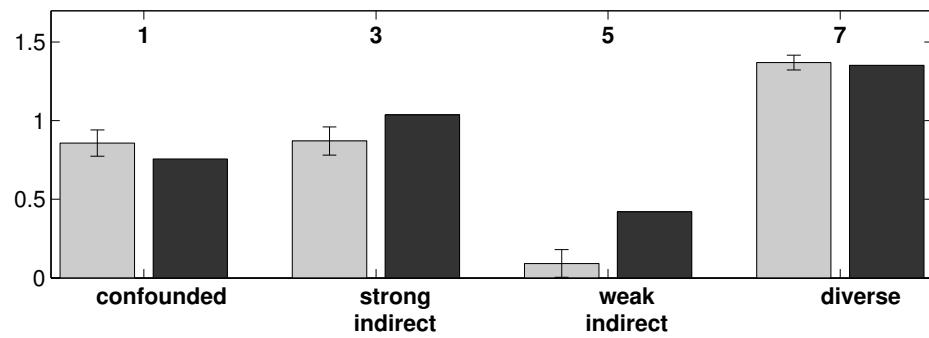
(see Goodman et al., 2015)

# Reasoning about tennis with probabilistic programs

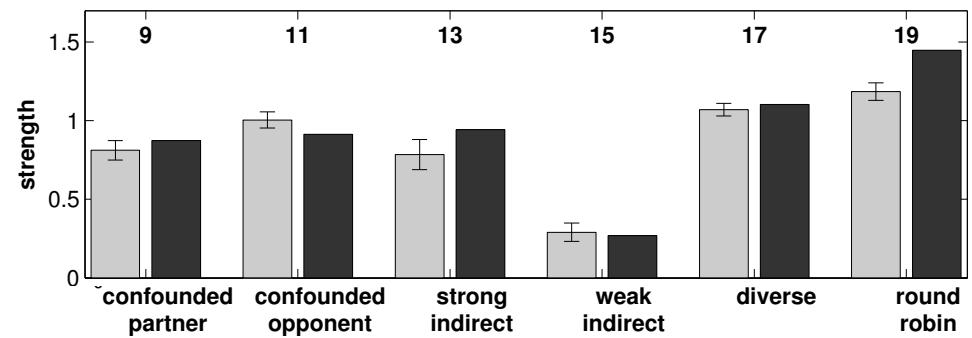
confounded evidence (1,2)	strong indirect evidence (3,4)	weak indirect evidence (5,6)	diverse evidence (7,8)
A > B	A > B	A > B	A > B
A > B	B > C	B < C	A > C
A > B	B > D	B < D	A > D

strength of player A

singles games



doubles games

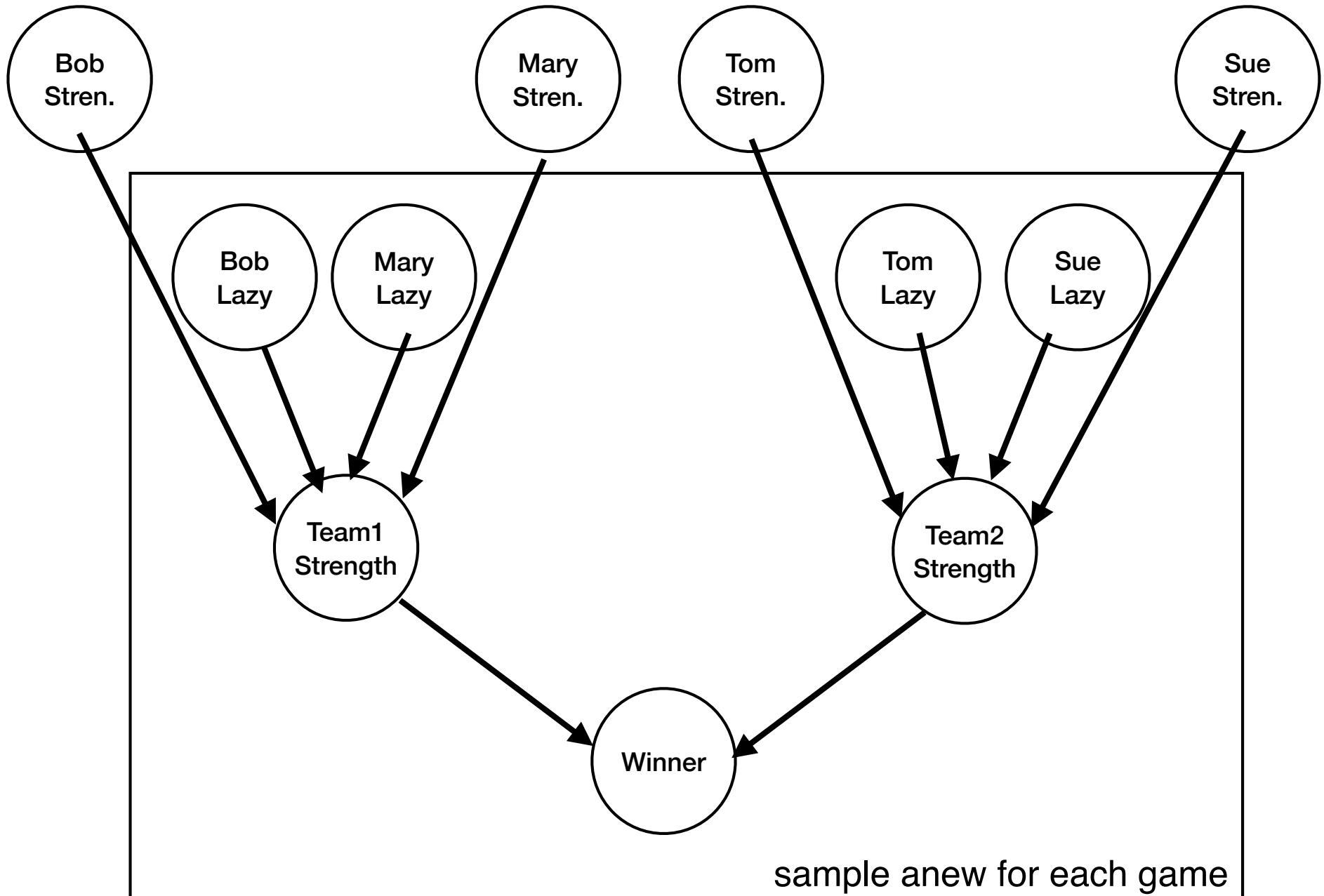


human judgements  
model judgements

(see Gerstenberg et al., 2012; Goodman et al., 2015)

# Bayesian network formulation depends on set of players

Each specification of the teams changes the Bayes net structure, and thus an additional modeling mechanism is needed to construct the Bayes nets.



# Example probabilistic programming languages and software

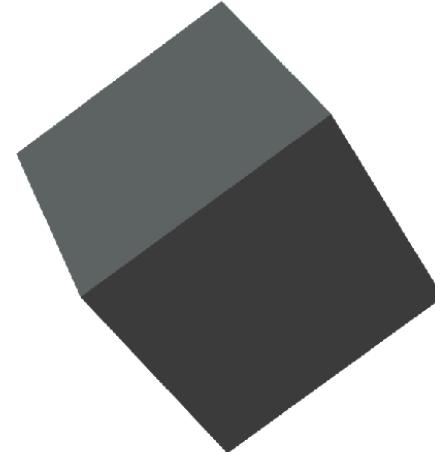
*WebPPL*



Church



pyro



Edward



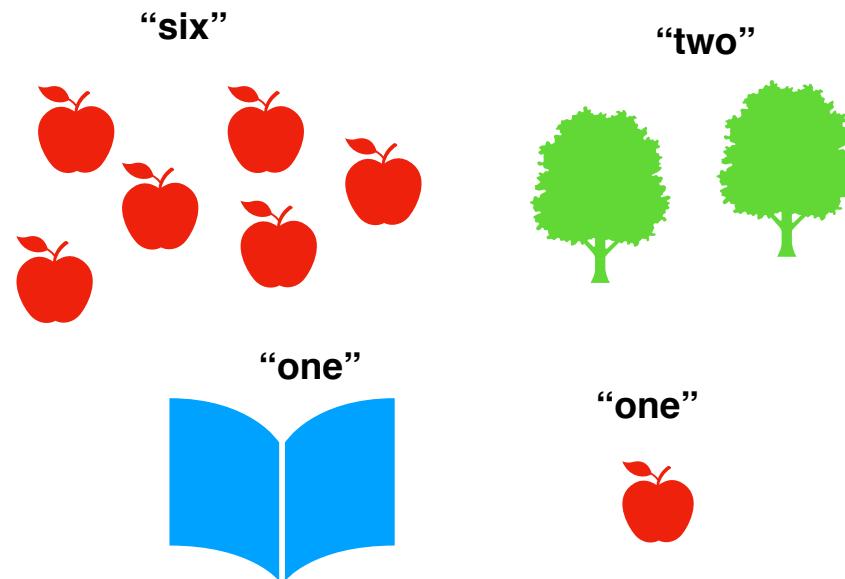
Stan

# Program induction

Prior distribution over (unknown) possible programs



Data ( $D$ )



Hypothesis 1

$$\lambda S . \left( \begin{array}{l} \text{if } (\text{singleton? } S) \\ \quad \text{"one"} \\ \text{(next } (L \text{ (set-difference } S \\ \quad \text{(select } S)))) \end{array} \right)$$

Hypothesis 2

$$\lambda S . \left( \begin{array}{l} \text{if } (\text{singleton? } S) \\ \quad \text{"one"} \\ \text{(if } (\text{doubleton? } S) \\ \quad \text{"two"} \\ \quad \text{undef})) \end{array} \right)$$

Hypothesis 3

$$\lambda S . \left( \begin{array}{l} \text{if } (\text{singleton? } S) \\ \quad \text{"one"} \\ \quad \text{undef} \end{array} \right)$$

Hypothesis N ...



Which is the right program?

inference

Most likely to have generated the data?

# Program induction

- Data is generated from an unknown program, where unlike standard probabilistic programming, we don't know the structure of the program.
- Prior over programs is usually defined by assuming a set of programming primitives and combination operations, which is also referred to as a “Language of thought” model in cognitive science (a la Jerry Fodor)
- More analogous to “structure learning” for Bayesian networks, where we are searching for the right causal model that generated the data.

# Language of thought / program induction in Python

[piantado / LOTlib](#)

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Lanugage of Thought (LOT) models in Python.

1,303 commits    3 branches    0 releases    10 contributors    GPL-3.0

Branch: master ▾    New pull request    Create new file    Upload files    Find file    Clone or download ▾

**piantado** added languages; revised search and model    Latest commit 03b9933 on Jul 4, 2017

<a href="#">Documentation</a>	updated input to example data	2 years ago
<a href="#">LOTlib</a>	added languages; revised search and model	10 months ago
<a href="#">.gitignore</a>	update gitignore with double-star syntax	2 years ago
<a href="#">LICENSE</a>	initial commit	5 years ago
<a href="#">MAJOR-CHANGES.txt</a>	extreme simplification of proposals	2 years ago
<a href="#">README.md</a>	updated Readme	a year ago
<a href="#">Test.sh</a>	Revert "Revert "Merge remote-tracking branch 'origin/master"""	2 years ago

[README.md](#)

## LOTlib

LOTlib is a Python 2 library for implementing "language of thought" models. A LOTlib model specifies a set of primitives and captures learning as inference over compositions of those primitives in order to express complex concepts. LOTlib permits lambda expressions, meaning that learners can come up with abstractions over compositions and define new

# Motivation: We need more than Bayesian networks to represent complex, real causal processes for generating data

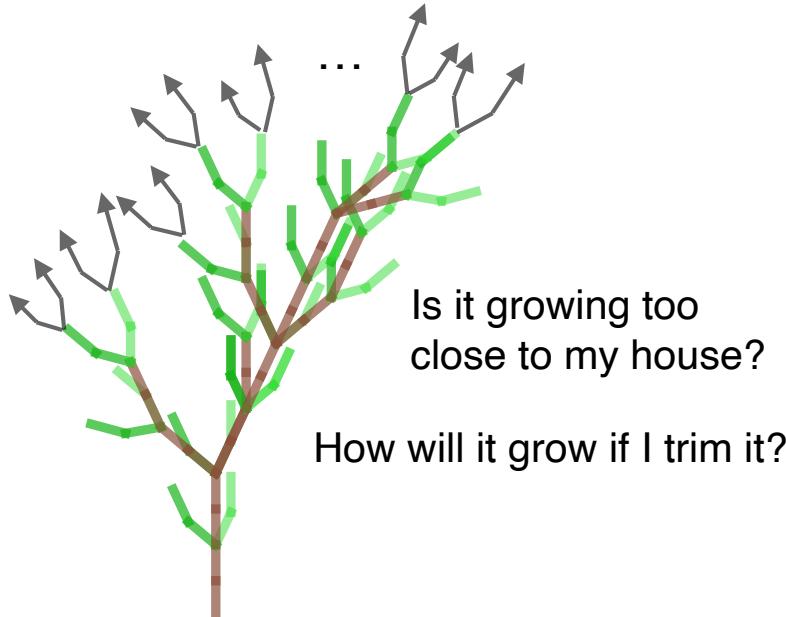
same causal process



different examples

2	2	2
2	2	2
2	2	2

(Figure credit: Hinton & Nair, 2006)



state-of-the-art neural net caption generation:  
“A group of people standing on top of a beach”

# Case study: Learning new handwritten letters

RESEARCH

## RESEARCH ARTICLES

COGNITIVE SCIENCE

### Human-level concept learning through probabilistic program induction

Brenden M. Lake,<sup>1,\*</sup> Ruslan Salakhutdinov,<sup>2</sup> Joshua B. Tenenbaum<sup>3</sup>

People learning new concepts can often generalize successfully from just a single example, yet machine learning algorithms typically require tens or hundreds of examples to perform with similar accuracy. People can also use learned concepts in richer ways than conventional algorithms—for action, imagination, and explanation. We present a computational model that captures these human learning abilities for a large class of simple visual concepts: handwritten characters from the world's alphabets. The model represents concepts as simple programs that best explain observed examples under a Bayesian criterion. On a challenging one-shot classification task, the model achieves human-level performance while outperforming recent deep learning approaches. We also present several “visual Turing tests” probing the model's creative generalization abilities, which in many cases are indistinguishable from human behavior.

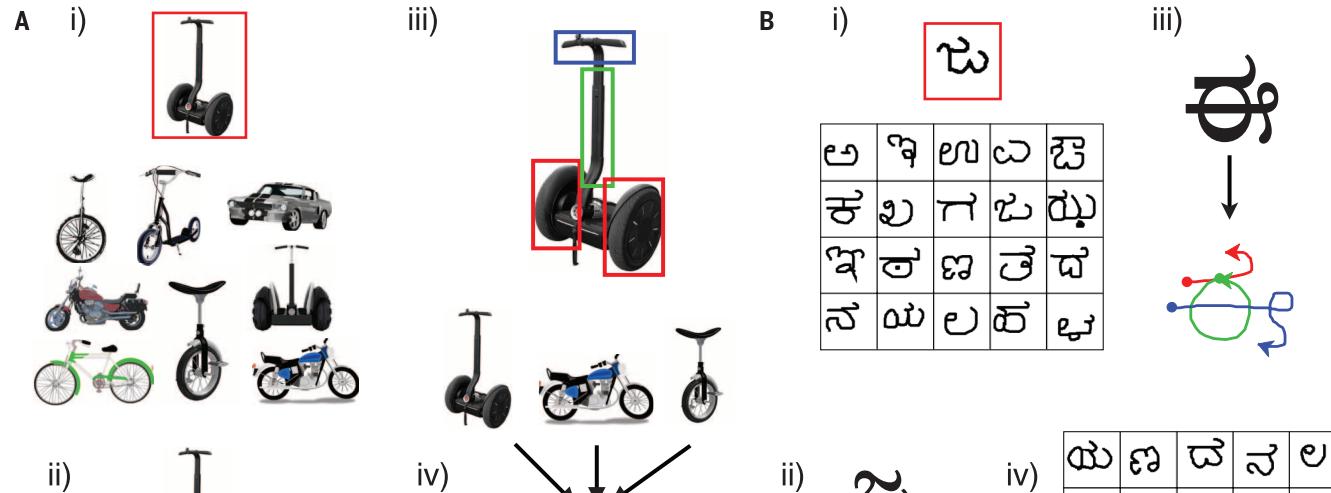
Despite remarkable advances in artificial intelligence and machine learning, two aspects of human conceptual knowledge have eluded machine systems. First, for most interesting kinds of natural and man-made categories, people can learn a new concept

from just one or a handful of examples, whereas standard algorithms in machine learning require tens or hundreds of examples to perform similarly. For instance, people may only need to see one example of a novel two-wheeled vehicle (Fig. 1A) in order to grasp the boundaries of the

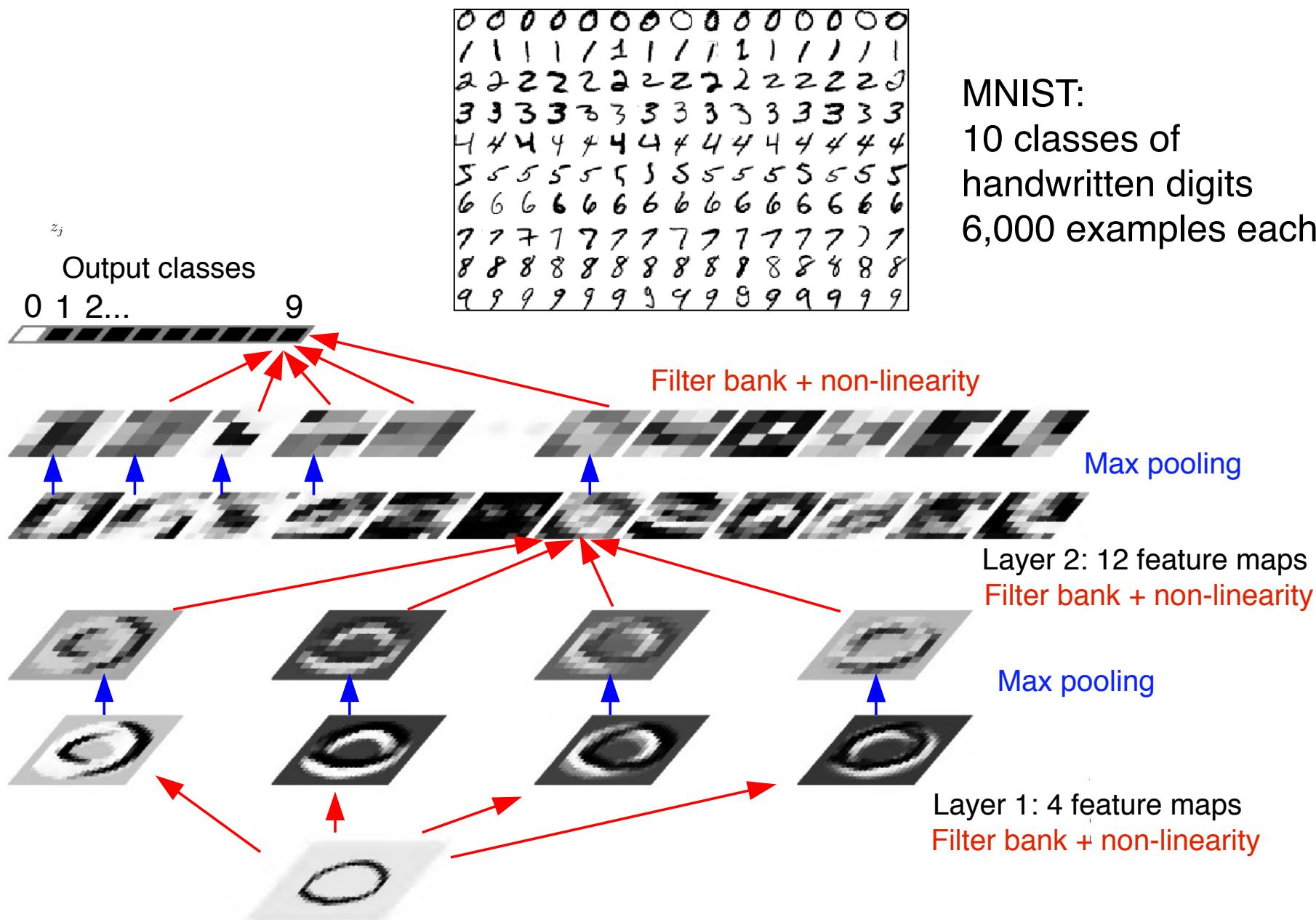
new concept, and even children can make meaningful generalizations via “one-shot learning” (1–3). In contrast, many of the leading approaches in machine learning are also the most data-hungry, especially “deep learning” models that have achieved new levels of performance on object and speech recognition benchmarks (4–9). Second, people learn richer representations than machines do, even for simple concepts (Fig. 1B), using them for a wider range of functions, including (Fig. 1, ii) creating new exemplars (10), (Fig. 1, iii) parsing objects into parts and relations (11), and (Fig. 1, iv) creating new abstract categories of objects based on existing categories (12, 13). In contrast, the best machine classifiers do not perform these additional functions, which are rarely studied and usually require specialized algorithms. A central challenge is to explain these two aspects of human-level concept learning: How do people learn new concepts from just one or a few examples? And how do people learn such abstract, rich, and flexible representations? An even greater challenge arises when putting them together: How can learning succeed from such sparse data yet also produce such rich representations? For any theory of

<sup>1</sup>Center for Data Science, New York University, 726 Broadway, New York, NY 10003, USA. <sup>2</sup>Department of Computer Science and Department of Statistics, University of Toronto, 6 King's College Road, Toronto, ON M5S 3G4, Canada. <sup>3</sup>Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA.

\*Corresponding author. E-mail: brenden@nyu.edu



# Standard machine learning approach: Deep neural network with large amounts of data



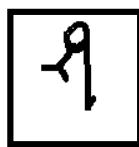
Slide credit: Yann LeCun

# People learn knowledge from less data

## less data

People can learn a new concept from a single image.

ಅ	ಣ	ಳ	ಯ	ಡ
ಕ	ನ	ಗ	ಬ	ರು
ಇ	ಟ	ಟೆ	ತೆ	ದೆ
ರ	ಯ	ಪ	ಹು	ಹು

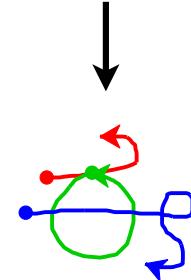


ಅ	ಂ	ಆ	ಸ	ರ
ಕ	ನ	ತ್ತ	ಷ	ಡ್
ಇ	ಟ	ಷ	ಷ	ಷ
ರ	ಯ	ಸ	ಸ	ಸ್

## more knowledge

People can apply their knowledge flexibly to new tasks.

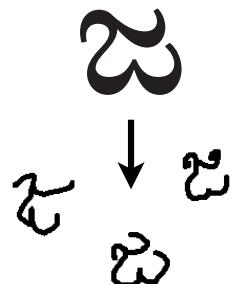
parsing



generating  
new concepts

ಯ	ಣ	ದ	ನ	ಲ
ಯ	ಣ	ದ	ನ	ಲು

generating  
new examples



# People learn knowledge from less data

## less data

People can learn a new concept from a single image.



ಅ	ಇ	ಉ	ಯ	ಡ್
ಕು	ನ	ಗ	ಒ	ರ್ಮು
ಇ	ರ	ಣ	ತೆ	ದ್
ನ್	ಯ್	ಲ	ಹ್	ಬ್

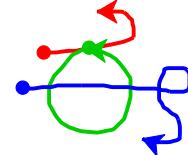


ಗ	ಂ	ಷ	ಸ	ರ
ಹ	ಂ	ತ್ರ	ಷ	ಕ್
ನ	ತ್	ಷ	ಸ	ಹ್
ಬ್	ಯ	ಸ್	ನ	ಜ್

## more knowledge

People can apply their knowledge flexibly to new tasks.

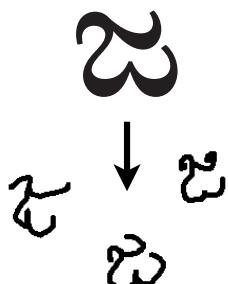
parsing



generating  
new concepts

ಯಣದನಲ		
ಯಣಪಿಸ್ತಳ್		

generating  
new examples



# Omniglot stimulus set

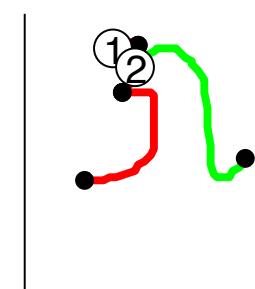
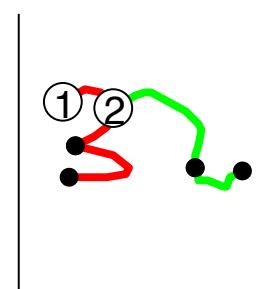
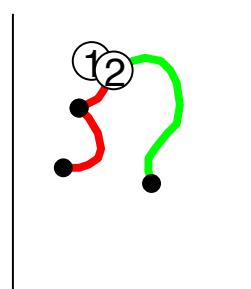
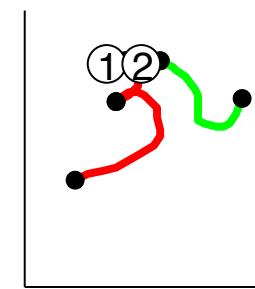
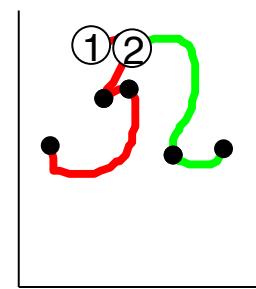
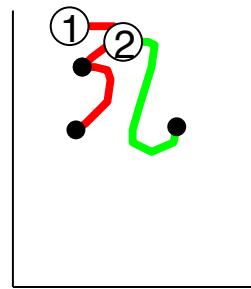
(<https://github.com/brendenlake/omniglot>)

1600+ concepts  
20 examples each

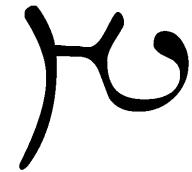
32

*π*

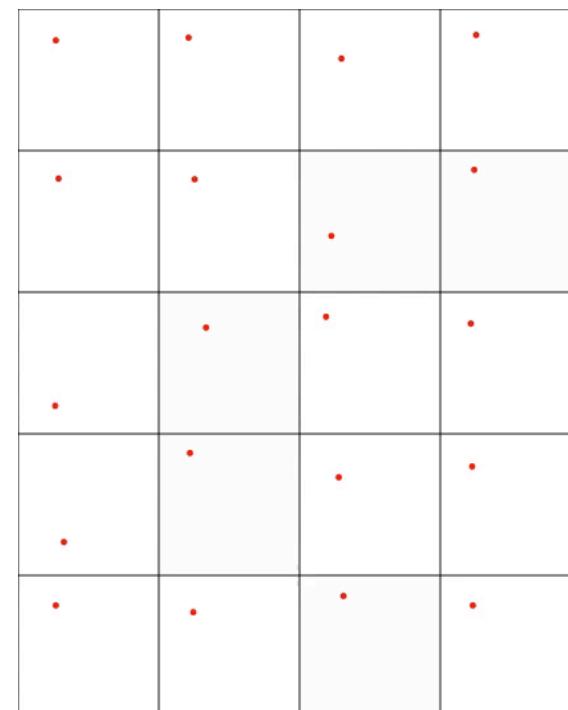
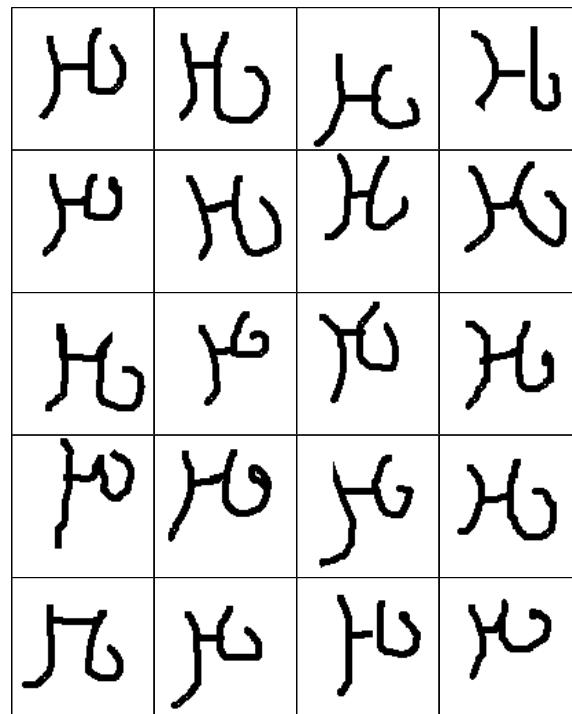
## Human drawings



Original

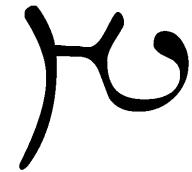


Human drawings

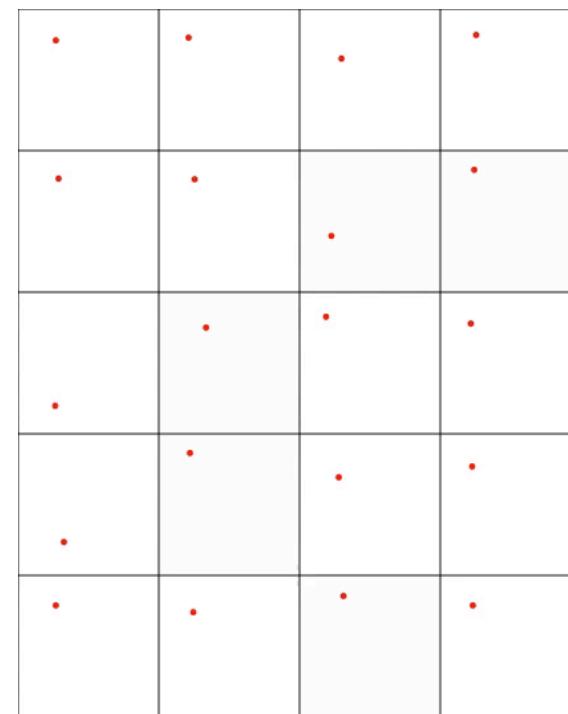
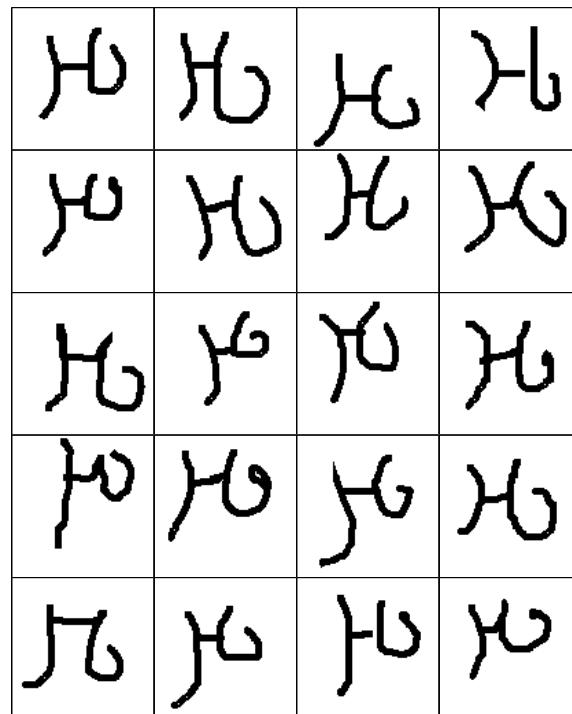


stroke order: — 1 — 2 — 3 — 4 — 5

Original



Human drawings

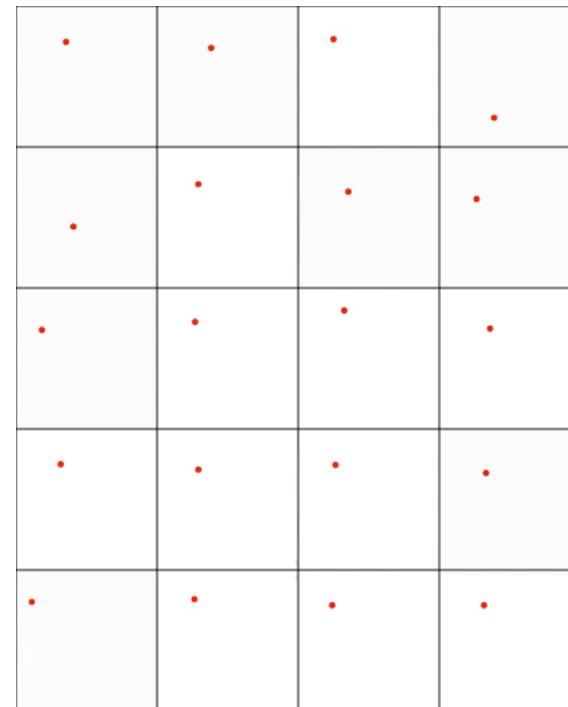
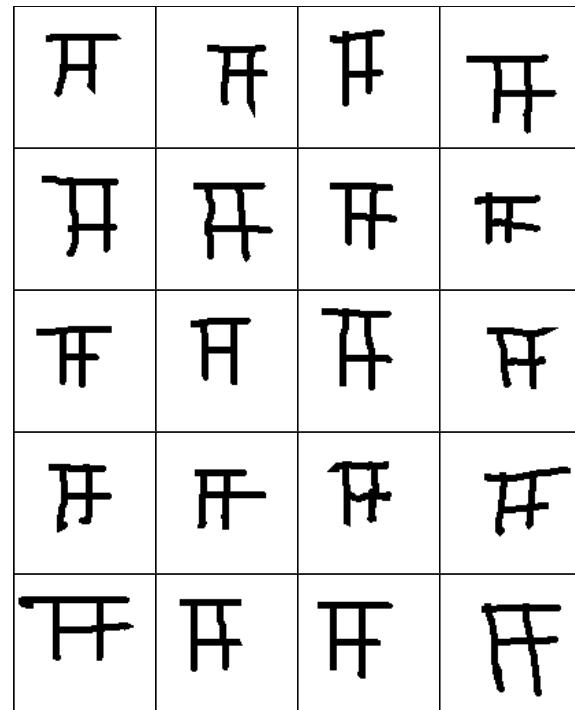


stroke order: — 1 — 2 — 3 — 4 — 5

Original



Human drawings

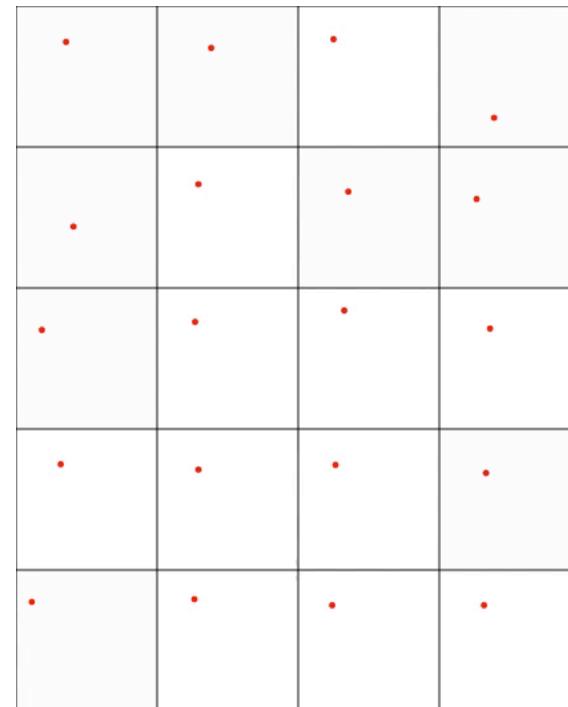
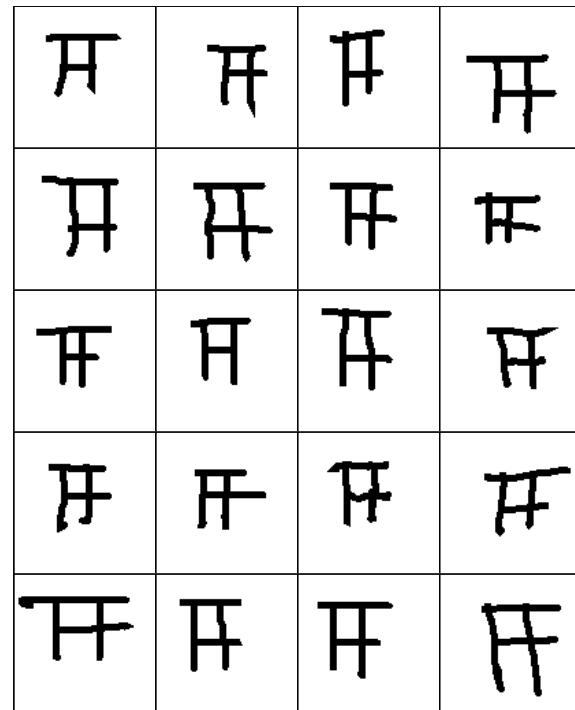


stroke order: — 1 — 2 — 3 — 4 — 5

Original



Human drawings

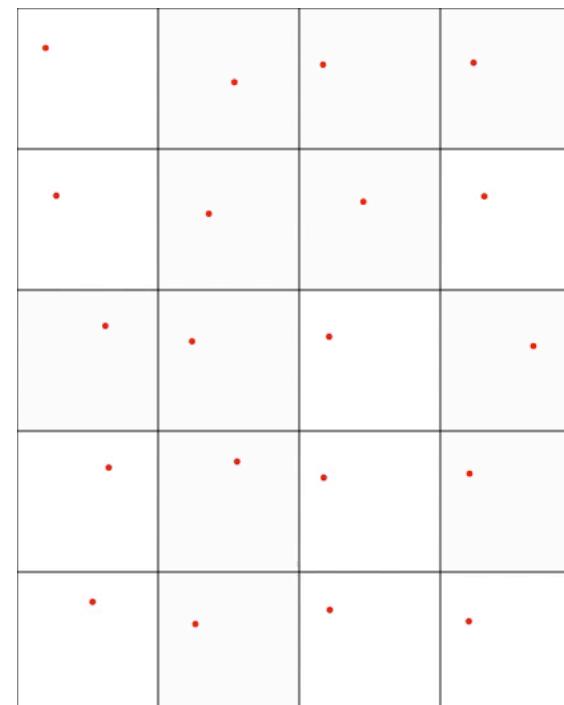


stroke order: — 1 — 2 — 3 — 4 — 5

Original

ର୍ହ

Human drawings

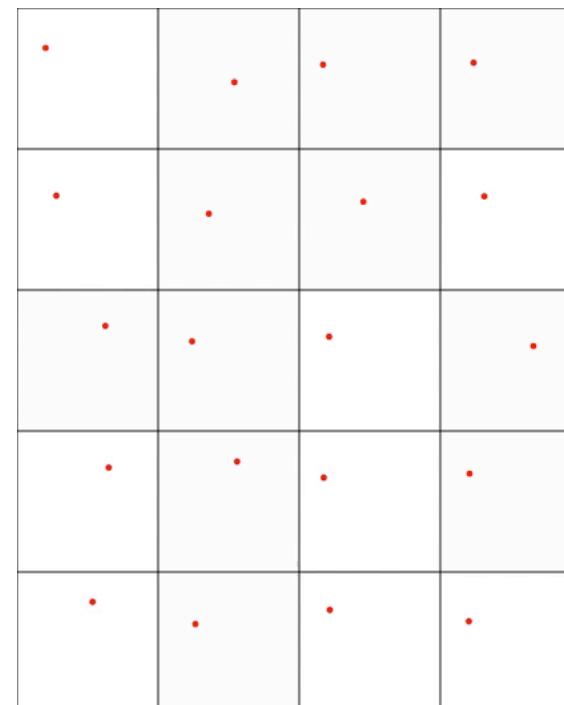


stroke order: — 1 — 2 — 3 — 4 — 5

Original

ର୍ହ

Human drawings



stroke order: — 1 — 2 — 3 — 4 — 5

# Probabilistic program induction model of concept learning

primitives  
(1D curvelets, 2D patches, 3D geons, actions, sounds, etc.)

sub-parts

parts

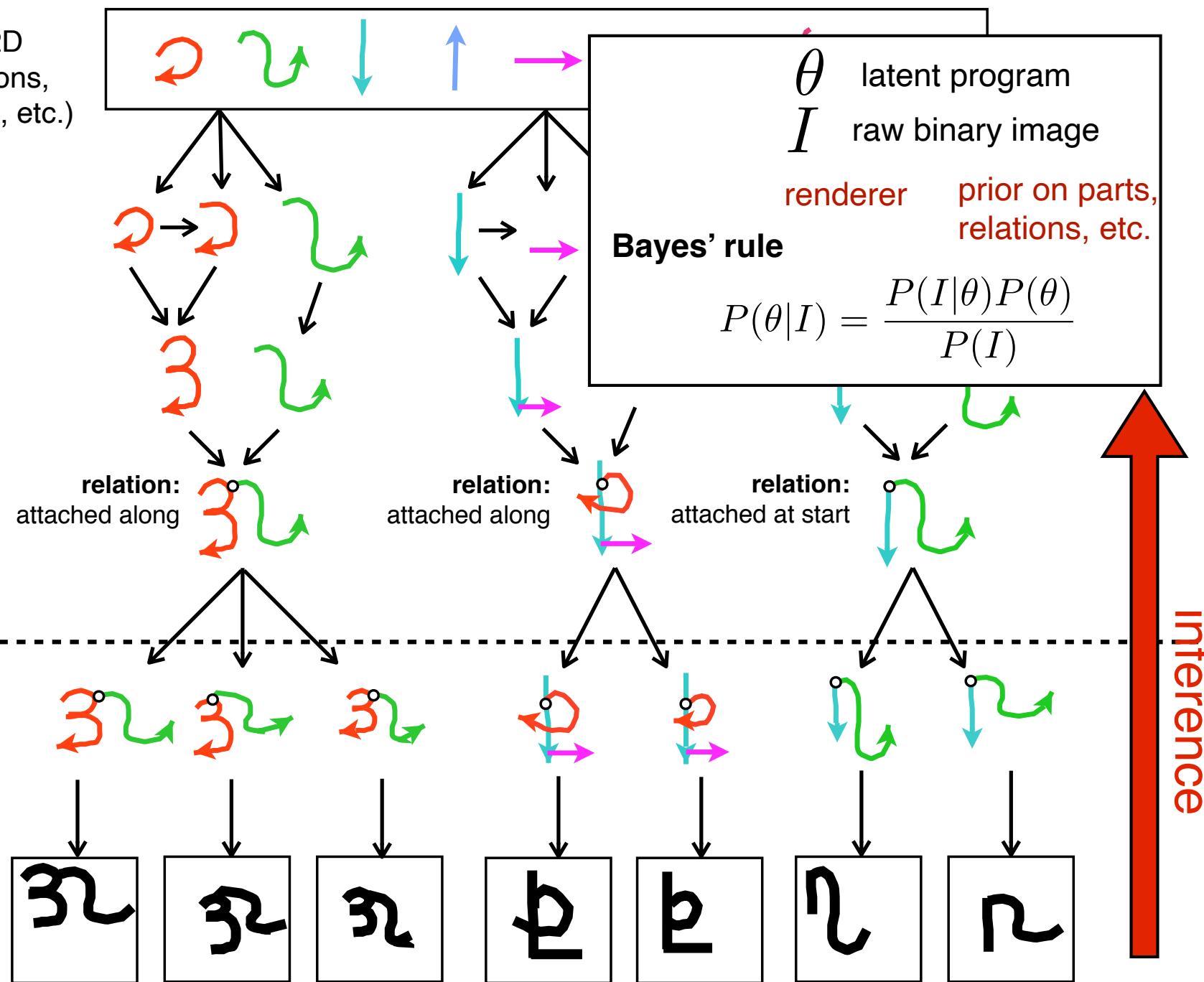
object template

**type level**

**token level**

exemplars

raw data



# Task: “Generate a new example”

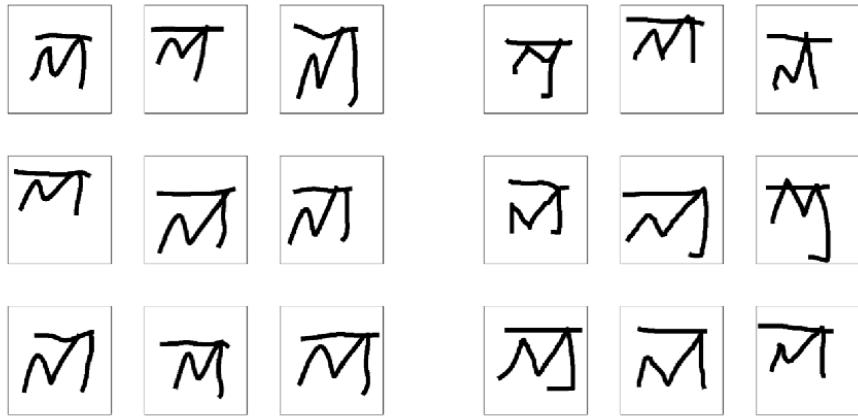


# A “visual Turing test” for generating new examples

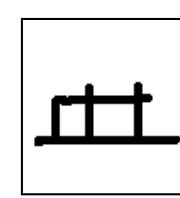
A



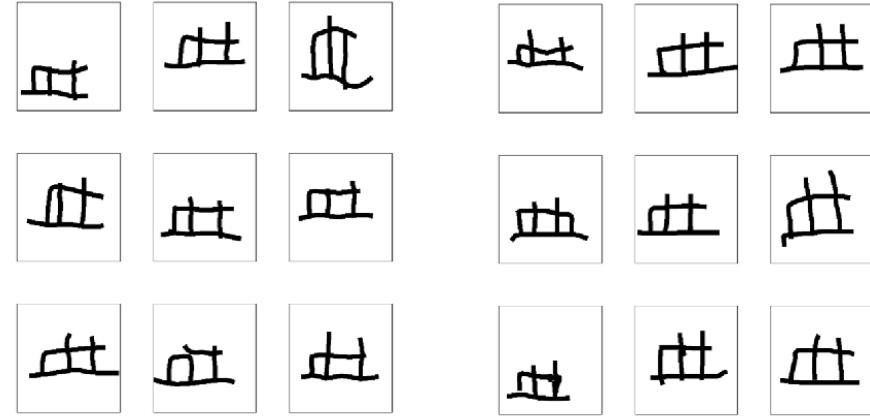
B



A



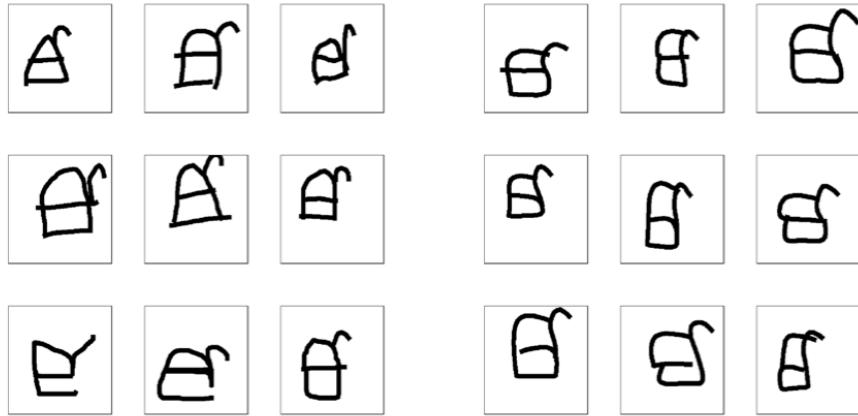
B



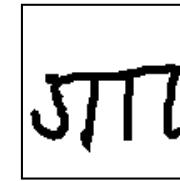
A



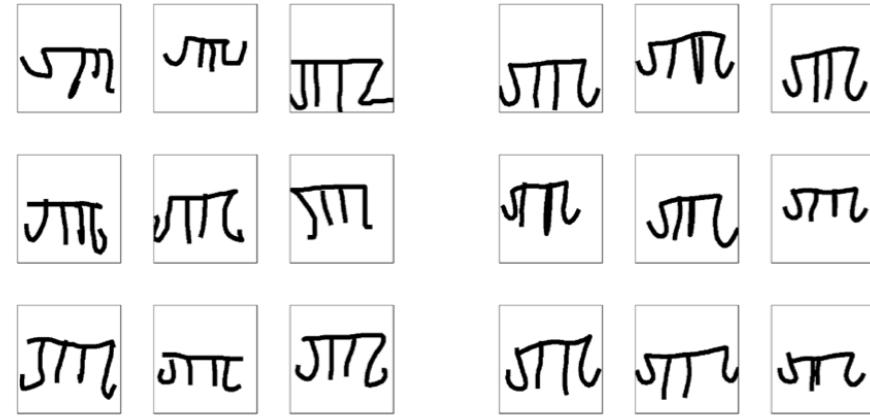
B



A



B



# A “visual Turing test” for generating new examples

machine generated

A



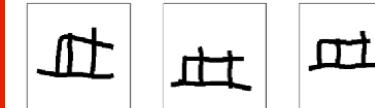
B



A



B



A



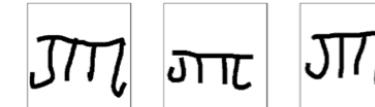
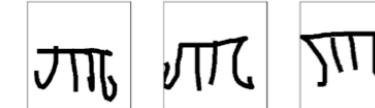
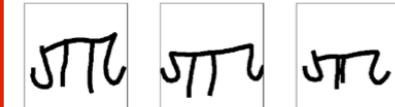
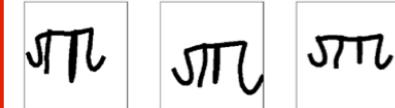
B



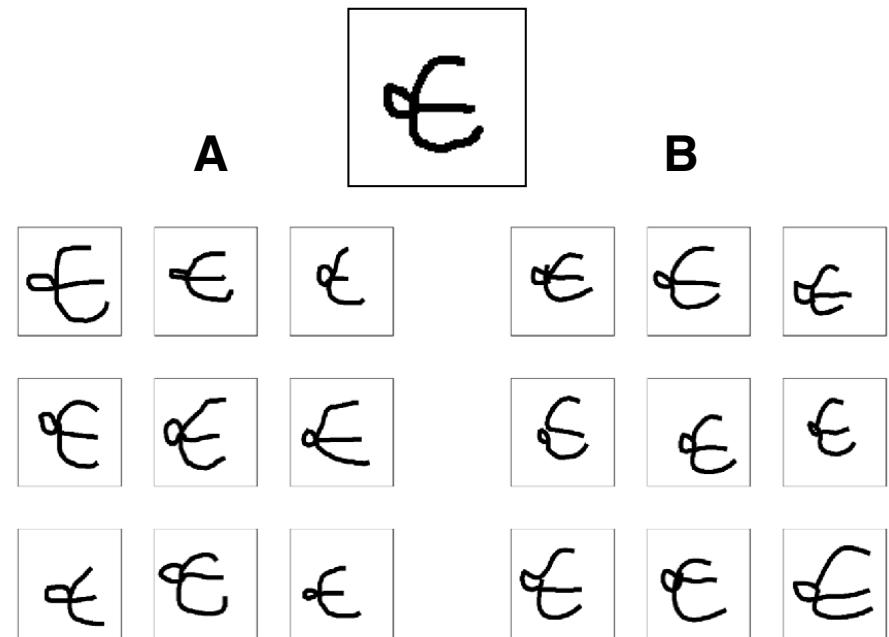
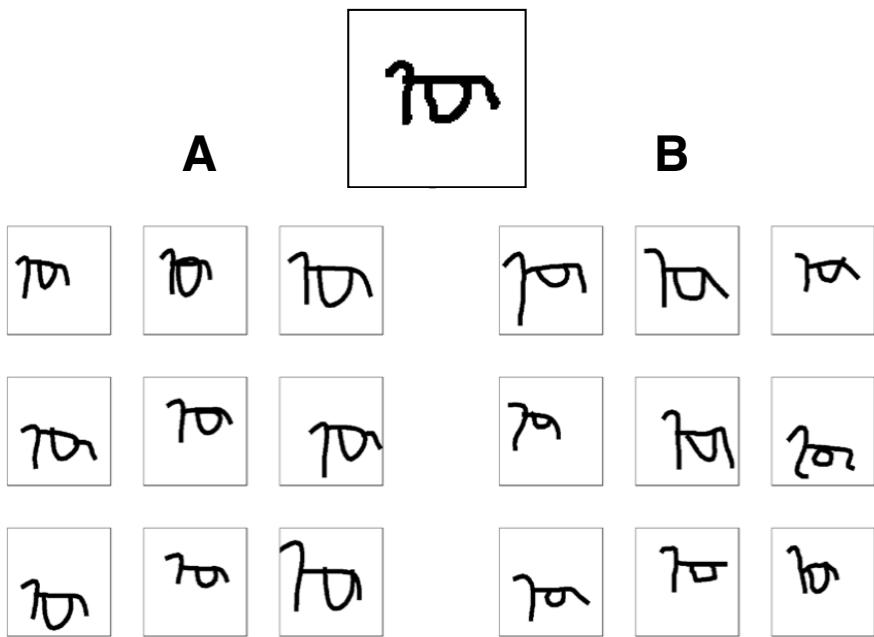
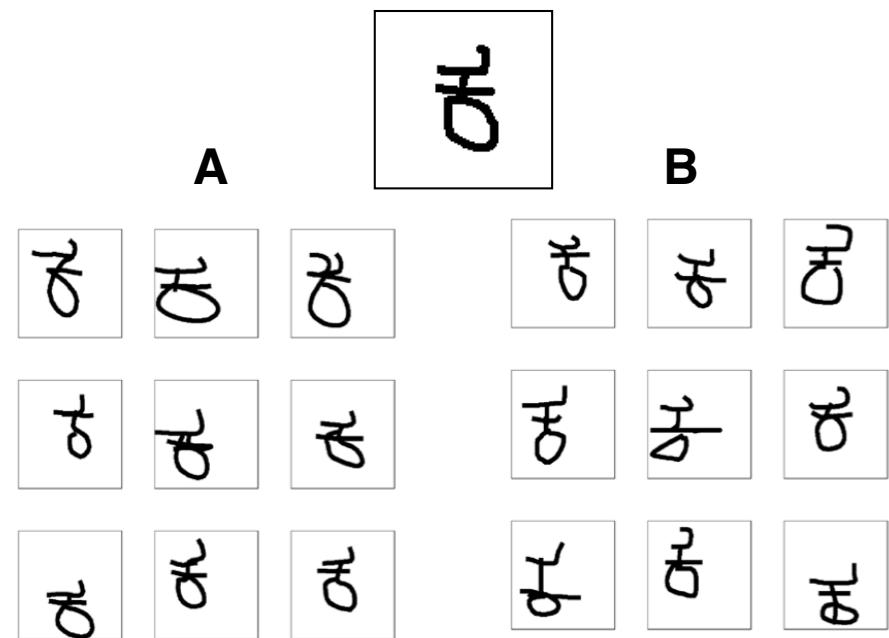
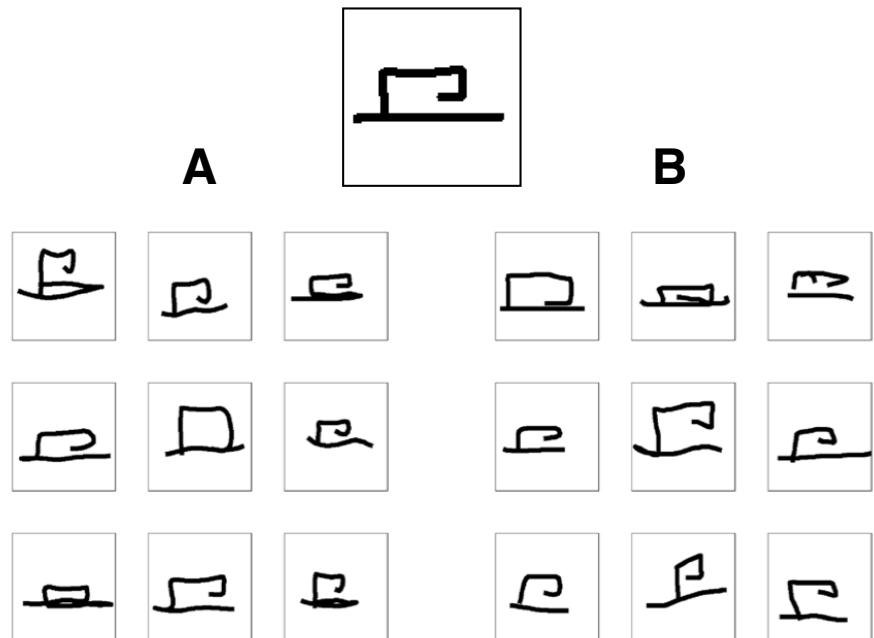
A



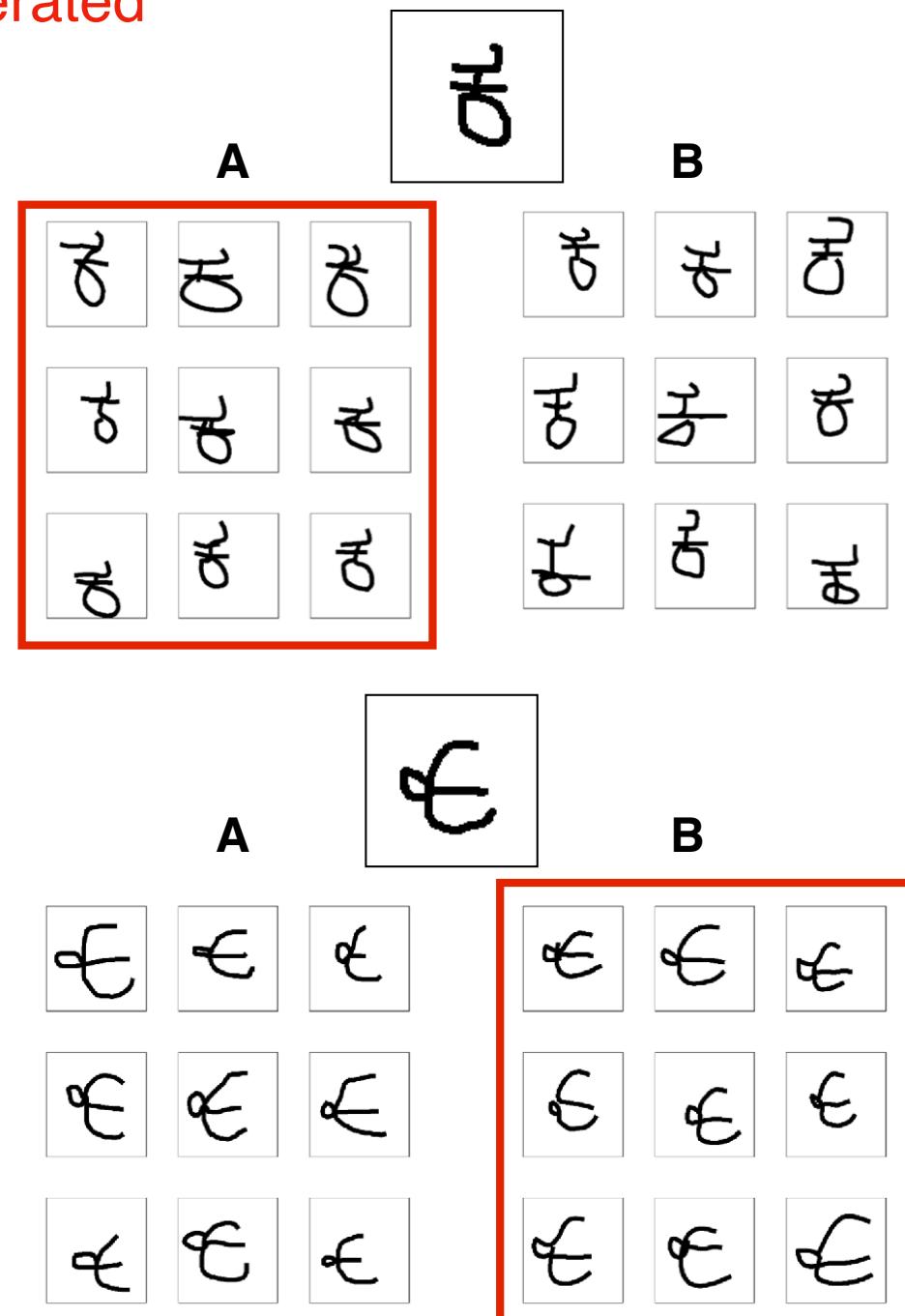
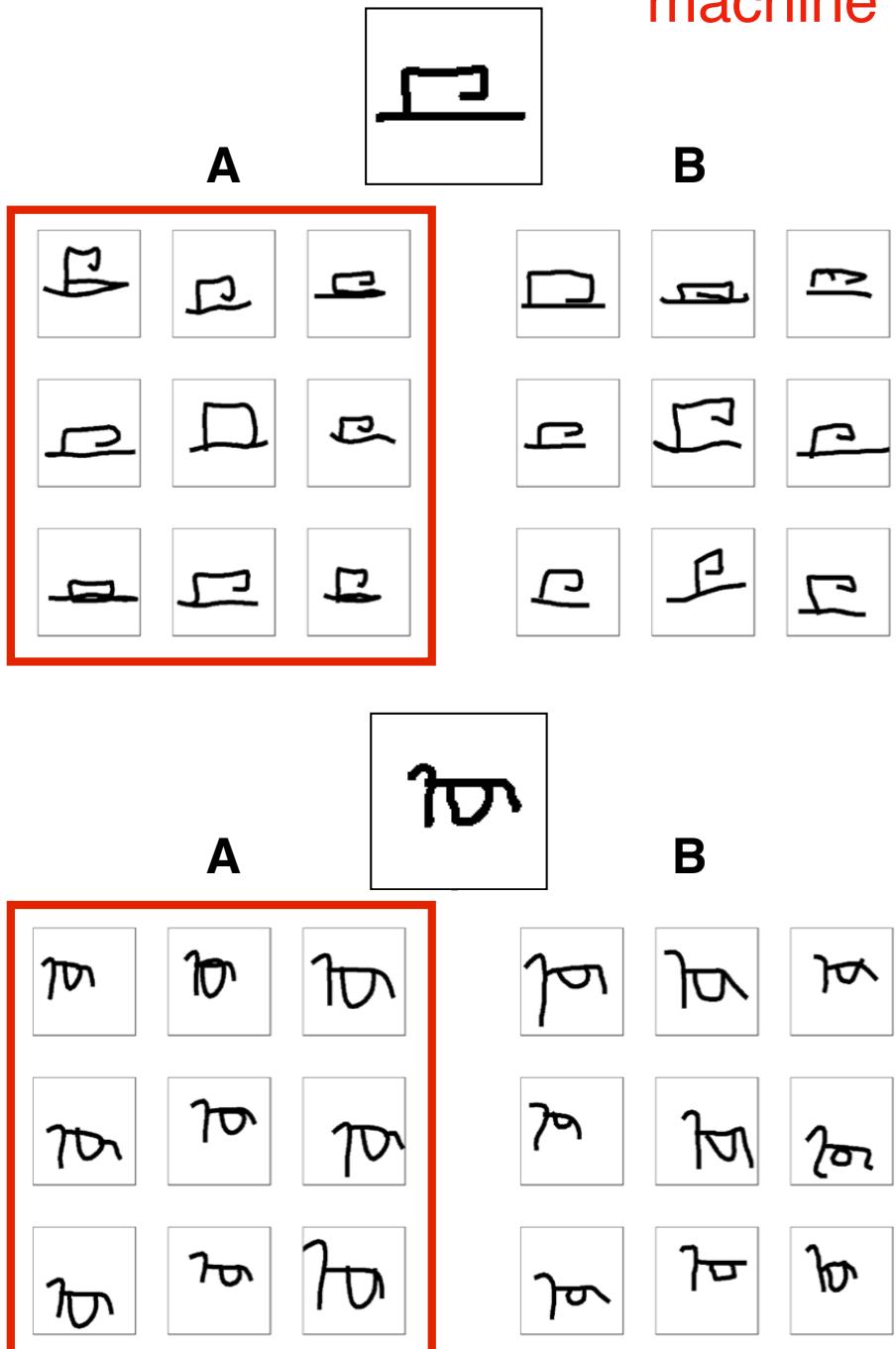
B



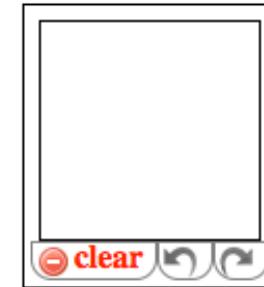
# A “visual Turing test” for generating new examples



# A “visual Turing test” for generating new examples

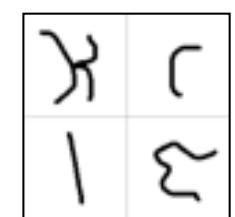
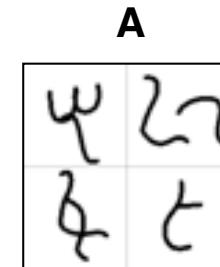
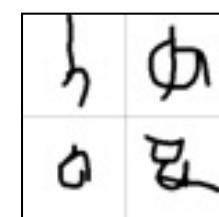
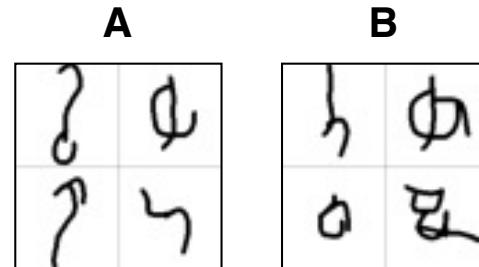
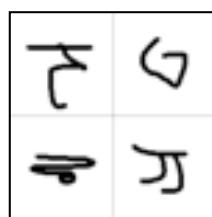
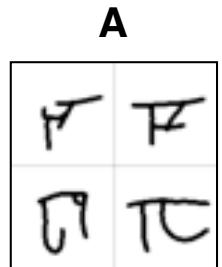
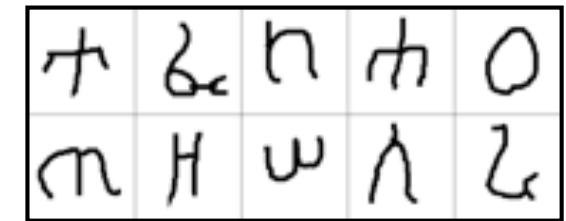
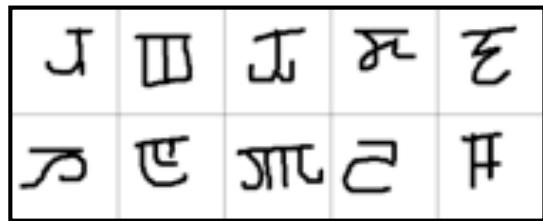


Task: “Generate a new character from the same alphabet”



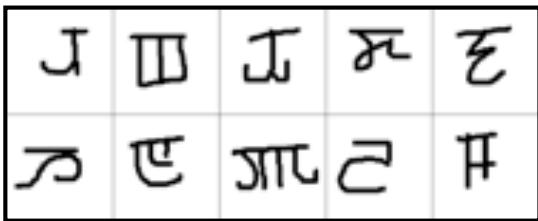
3 seconds  
remaining

# A “visual Turing test” for generating new concepts

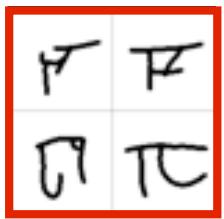


# A “visual Turing test” for generating new concepts

machine generated



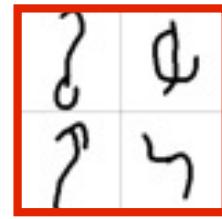
A



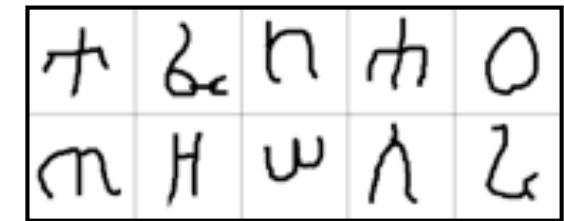
B



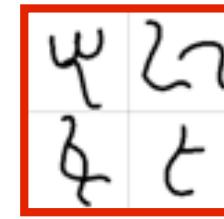
A



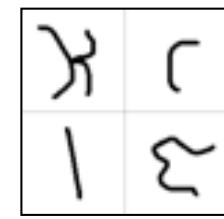
B



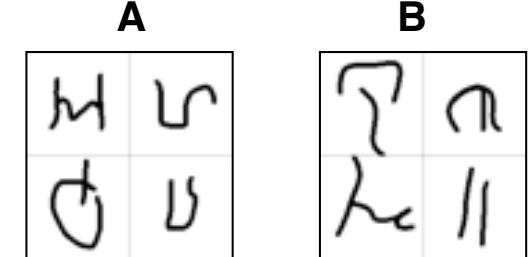
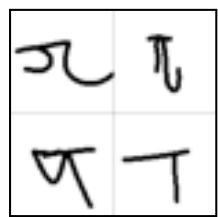
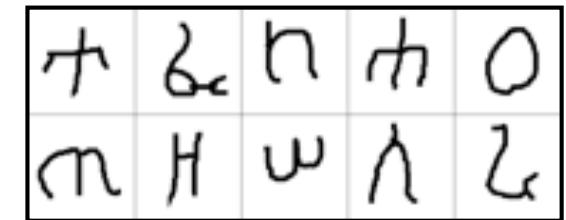
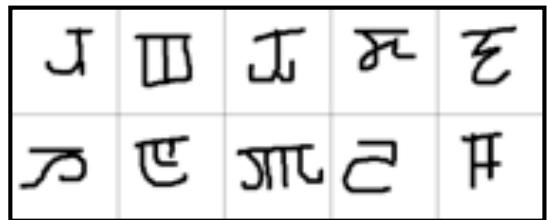
A



B

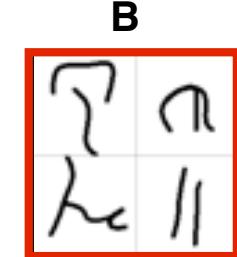
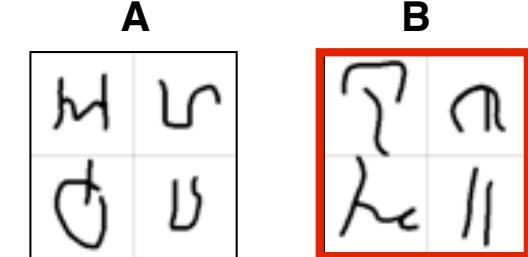
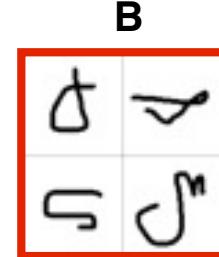
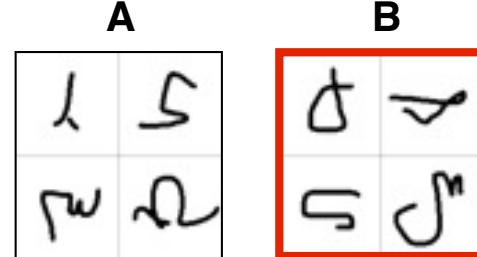
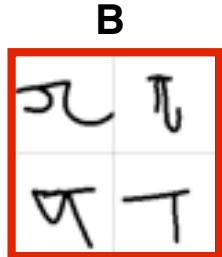
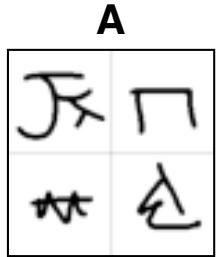
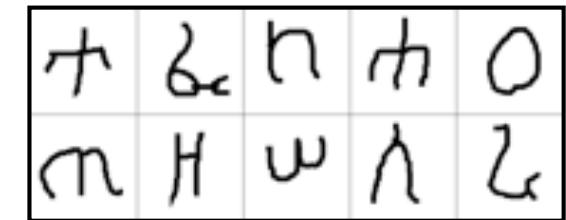
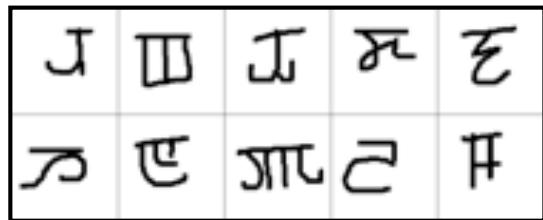


# A “visual Turing test” for generating new concepts

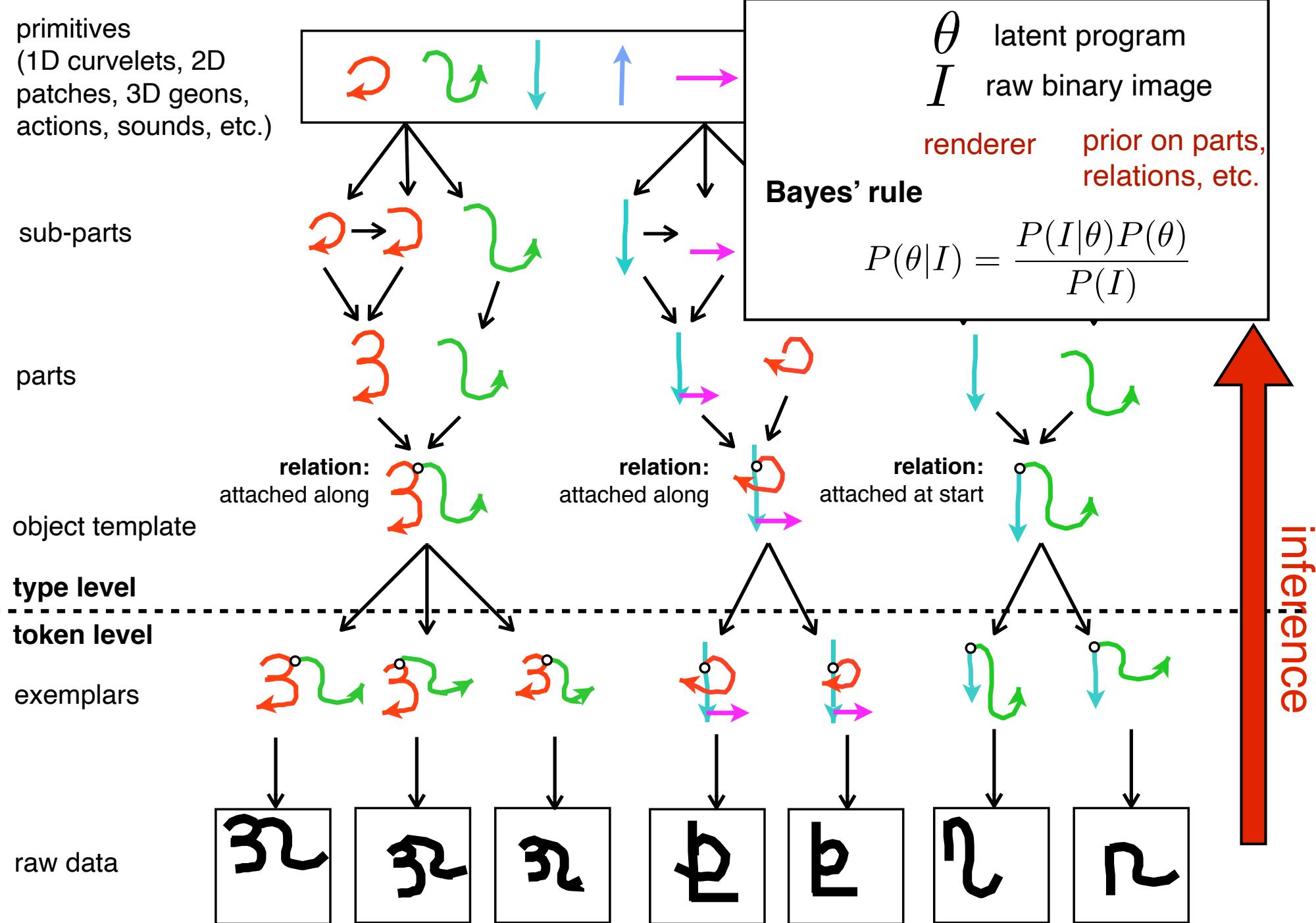


# A “visual Turing test” for generating new concepts

machine generated



# Probabilistic program induction



# Probabilistic program induction

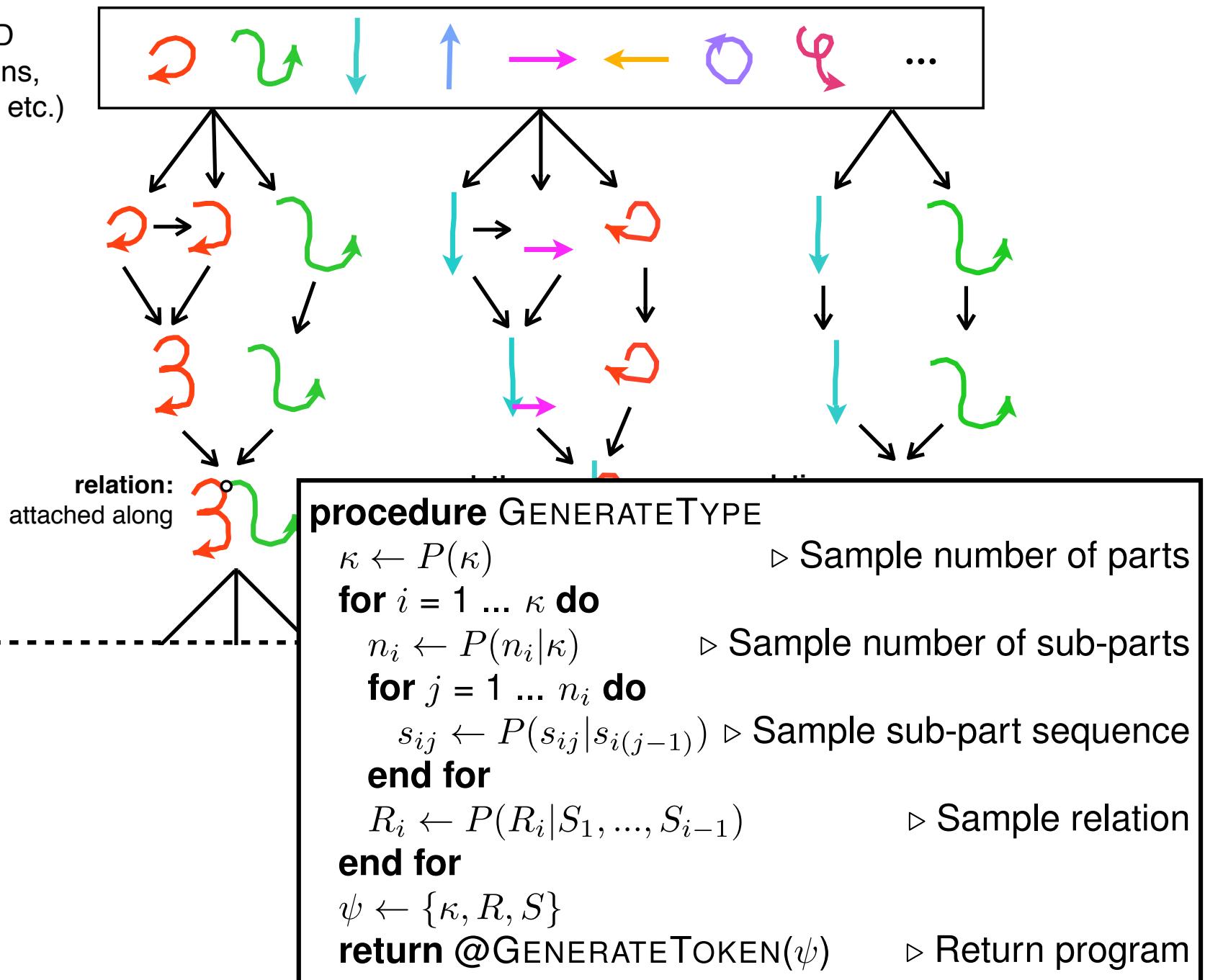
primitives  
 (1D curvelets, 2D patches, 3D geons, actions, sounds, etc.)

sub-parts

parts

object template

**type level**



# Probabilistic program induction

primitives  
 (1D curvelets, 2D patches, 3D geons, actions, sounds, etc.)

sub-parts

parts

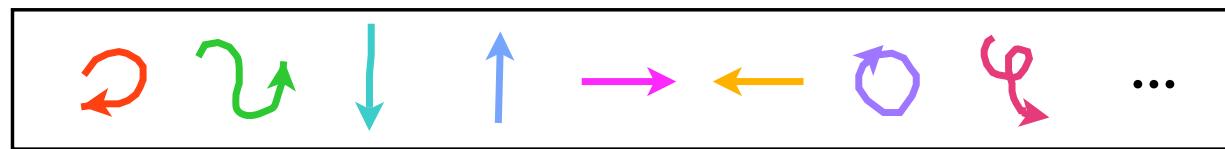
object template

**type level**

**token level**

exemplars

raw data



**procedure GENERATETOKEN( $\psi$ )**

**for**  $i = 1 \dots \kappa$  **do**

$S_i^{(m)} \leftarrow P(S_i^{(m)} | S_i)$

▷ Add motor variance

$L_i^{(m)} \leftarrow P(L_i^{(m)} | R_i, T_1^{(m)}, \dots, T_{i-1}^{(m)})$

▷ Sample part's start location

$T_i^{(m)} \leftarrow f(L_i^{(m)}, S_i^{(m)})$  ▷ Compose a part's trajectory

**end for**

$A^{(m)} \leftarrow P(A^{(m)})$

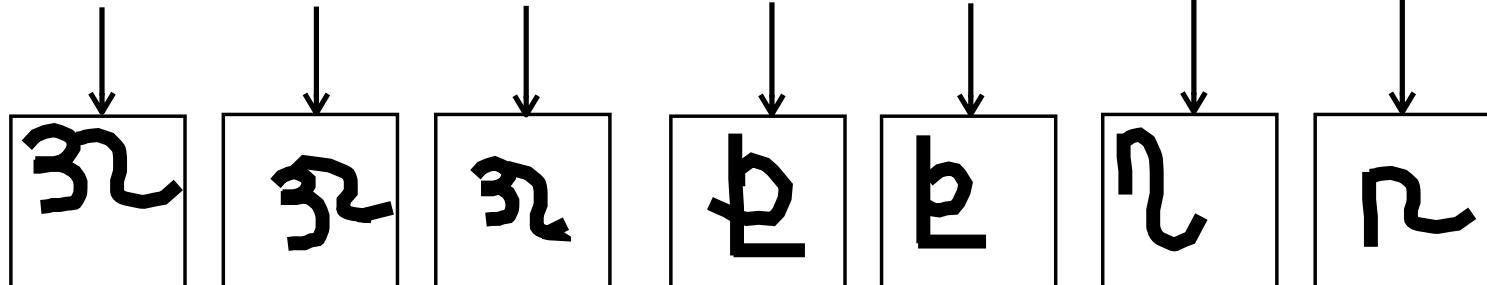
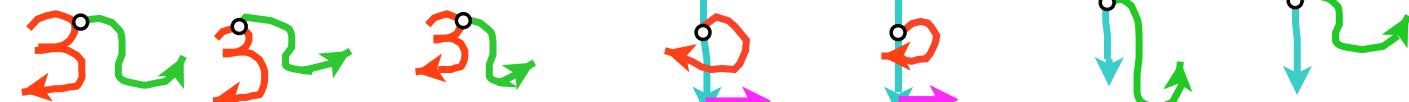
▷ Sample affine transform

$I^{(m)} \leftarrow P(I^{(m)} | T^{(m)}, A^{(m)})$

▷ Sample image

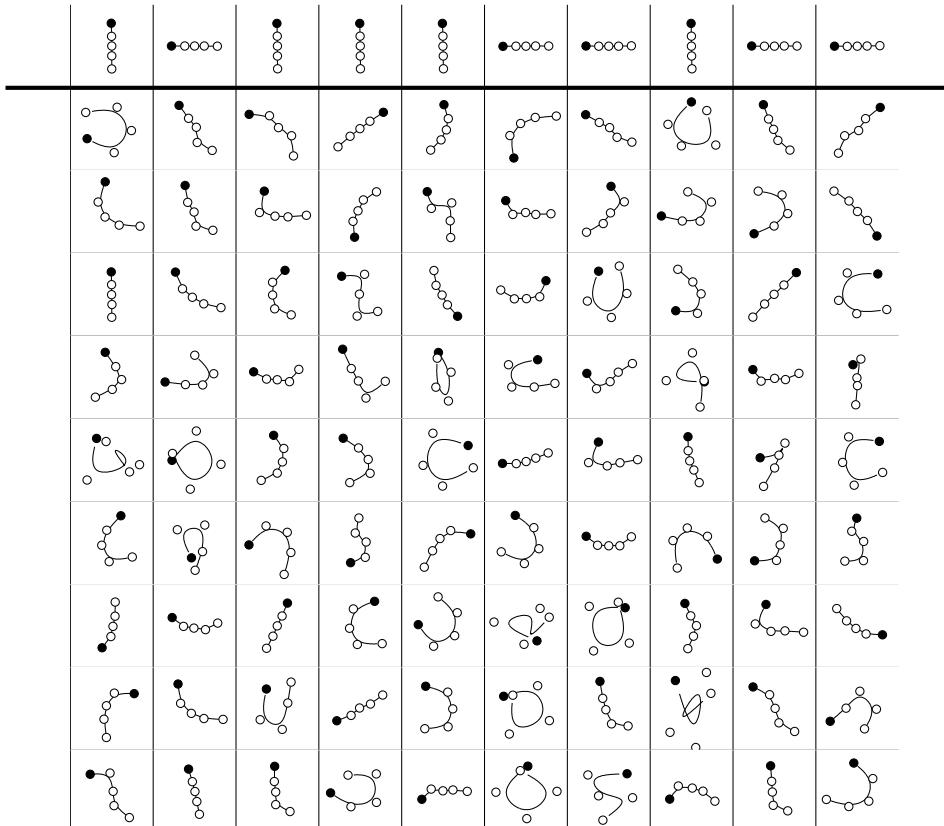
**return**  $I^{(m)}$

relation:  
 attached along



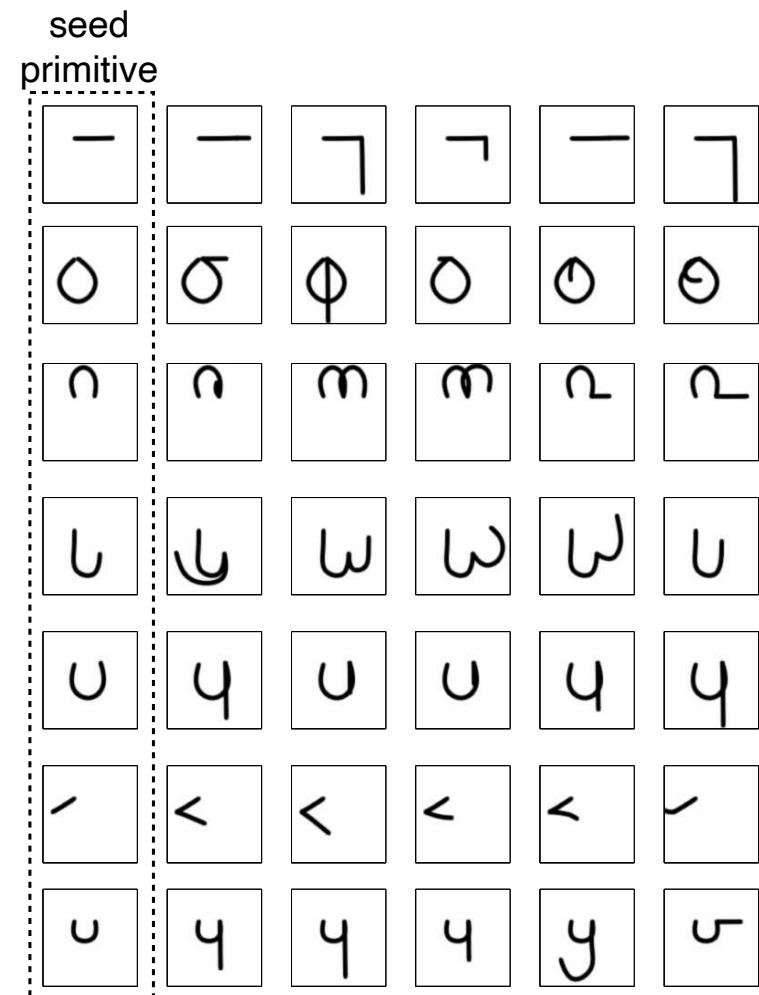
# Learning a prior distribution over programs

# learned action primitives



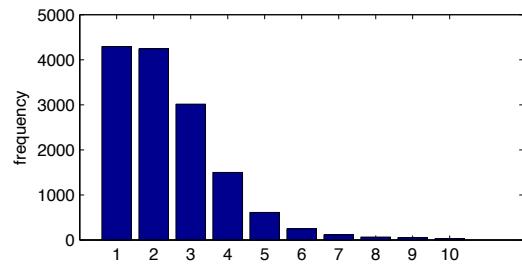
1250 primitives  
scale selective  
translation invariant

## **learned primitive transitions**

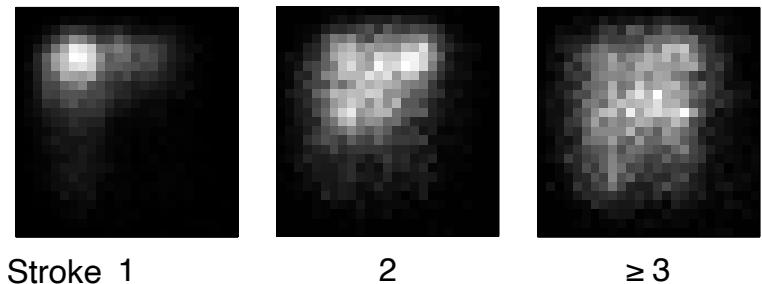


# Learning a prior distribution over programs

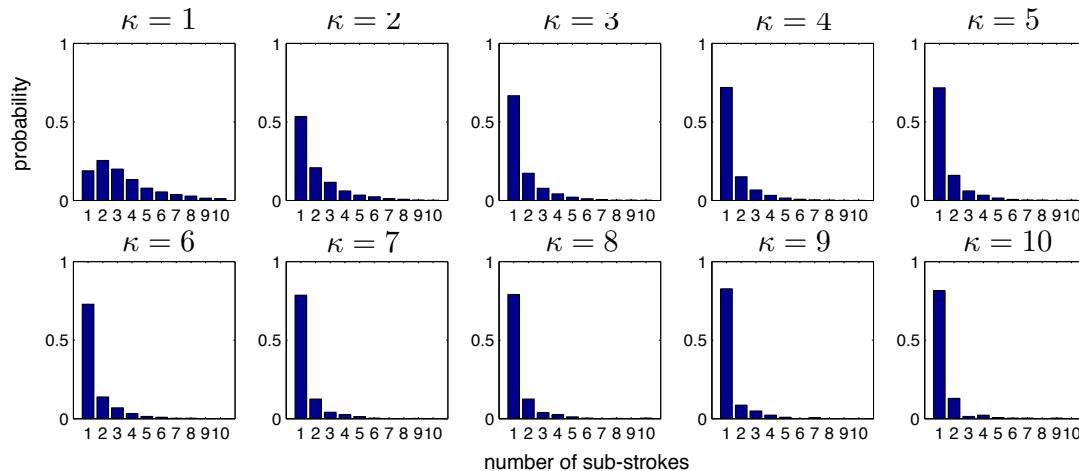
number of strokes



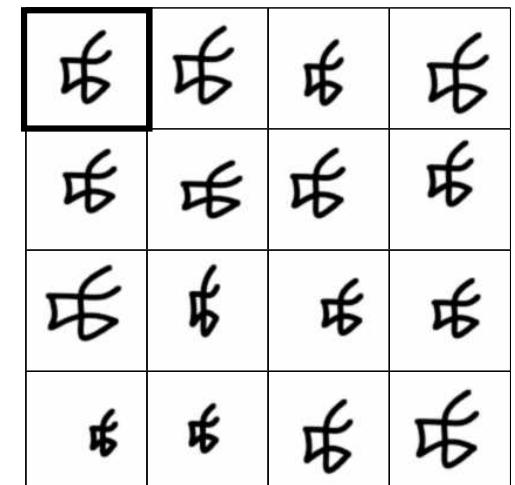
stroke start positions



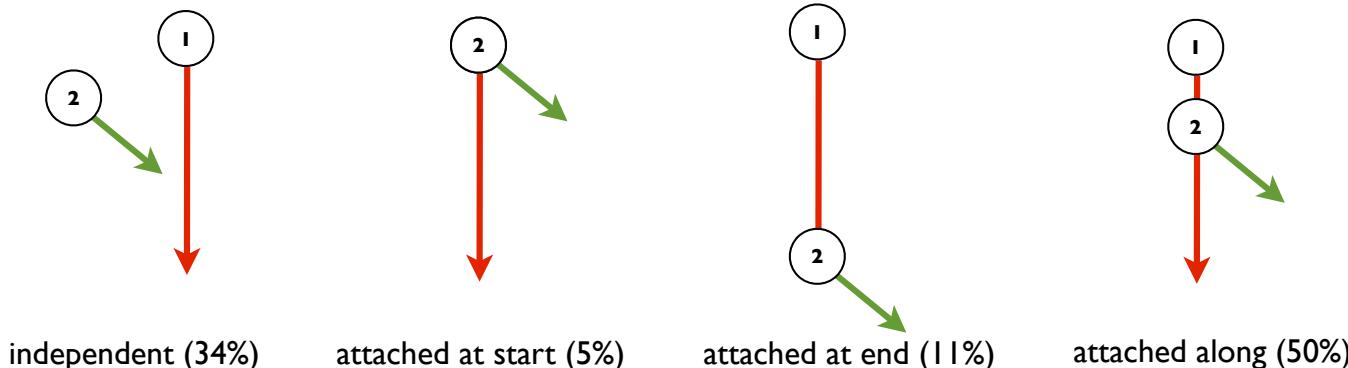
number of sub-strokes for a character with  $\kappa$  strokes



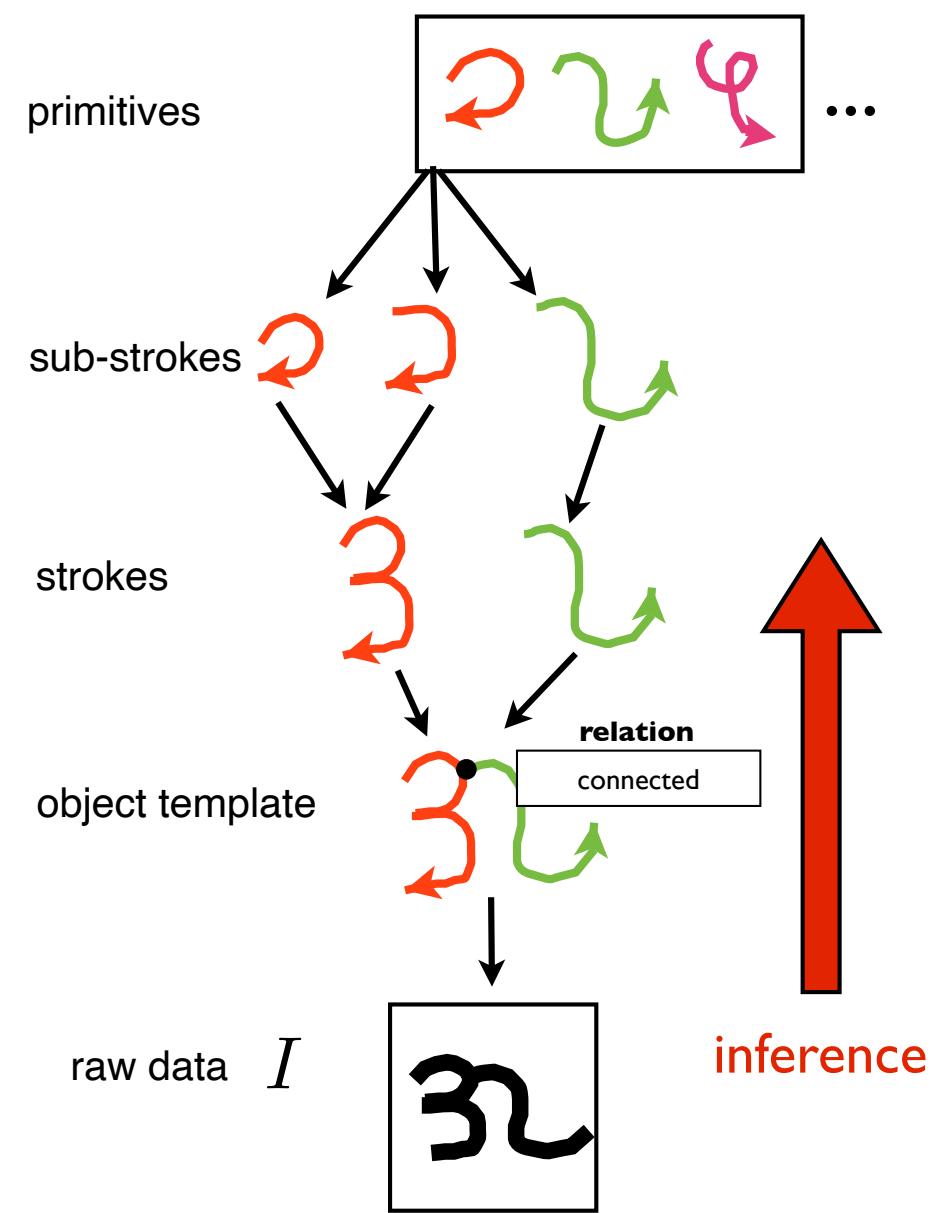
global transformations



relations between strokes



# Approximate probabilistic inference



$\theta$  latent program

$I$  raw binary image

Bayes' rule

$$P(\theta|I) = \frac{P(I|\theta)P(\theta)}{P(I)}$$

renderer prior on programs

Discrete ( $K=5$ ) approximation to posterior

$$P(\theta|I) \approx \frac{\sum_{i=1}^K w_i \delta(\theta - \theta^{[i]})}{\sum_{i=1}^K w_i}$$

such that

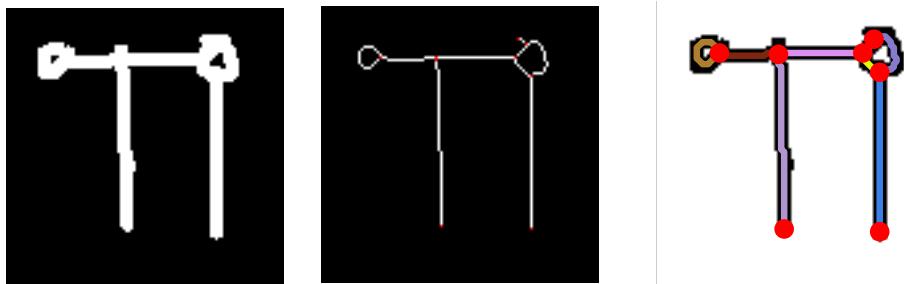
$$w_i \propto P(\theta^{[i]}|I)$$

Intuition: Fit strokes to the observed pixels as closely as possible, with these constraints:

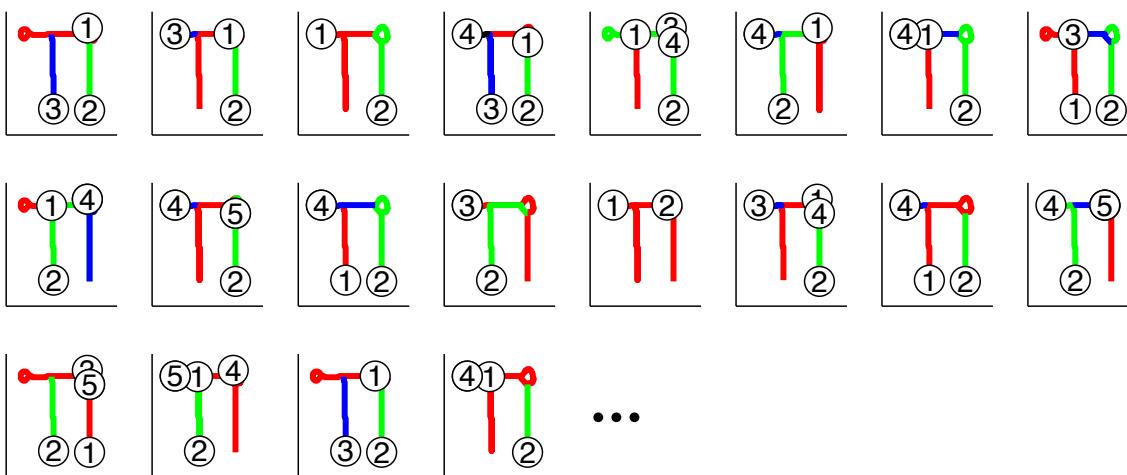
- fewer strokes
- high-probability primitive sequence
- use relations
- stroke order
- stroke directions

# Approximate probabilistic inference

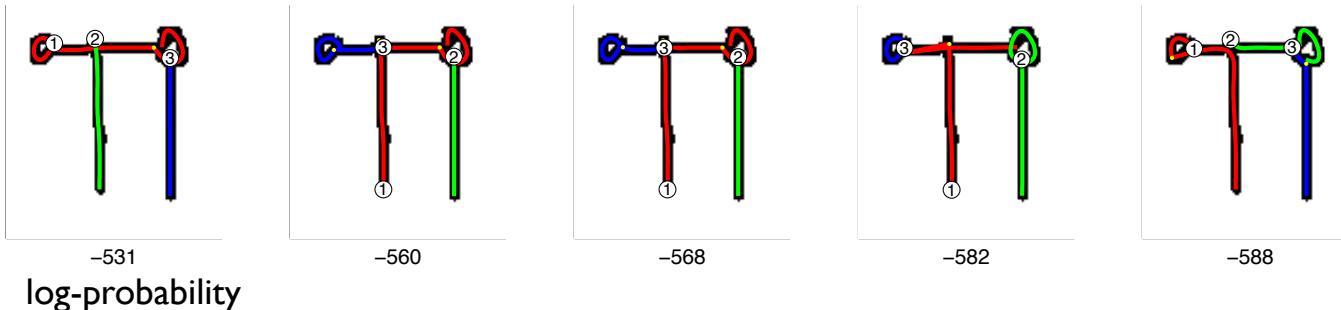
Step 1: characters as undirected graphs



Step 2: guided random parses



Step 3: Top-down fitting with gradient-based optimization



# Human-level concept learning

## the speed of learning

ମ	ବ	ଶ	ସ	ର
କ	ଟ	ତୁ	ପ୍ର	କ୍ଷ
ନ୍ତ	ଟ୍ଟ	ଫୁ	ପ୍ର୍ଯୁ	ମୁ
ବୁ	ଲ୍ଲ	ମୁ	ନ୍ତୁ	ମୁଖ

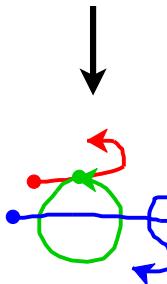
ଚିତ୍ର

ଗ	ବୀ	ବୁ	ଦୁ	ଦ୍ଵୀ
କୁ	ବୁନ୍ଦୁ	ଗୁରୁ	ଦୁର୍ବୁଲୁ	ଦ୍ଵୀରୁଳୁ
ଶୁଣୁ	କୁଣୁ	ତେବୁ	ଦୁର୍ବୁଲୁ	ଦ୍ଵୀରୁଳୁ
ନୁ	ବୁନ୍ଦୁ	କୁଣୁ	ଦୁର୍ବୁଲୁ	ଦ୍ଵୀରୁଳୁ

## the richness of representation

parsing

ଶବ୍ଦ



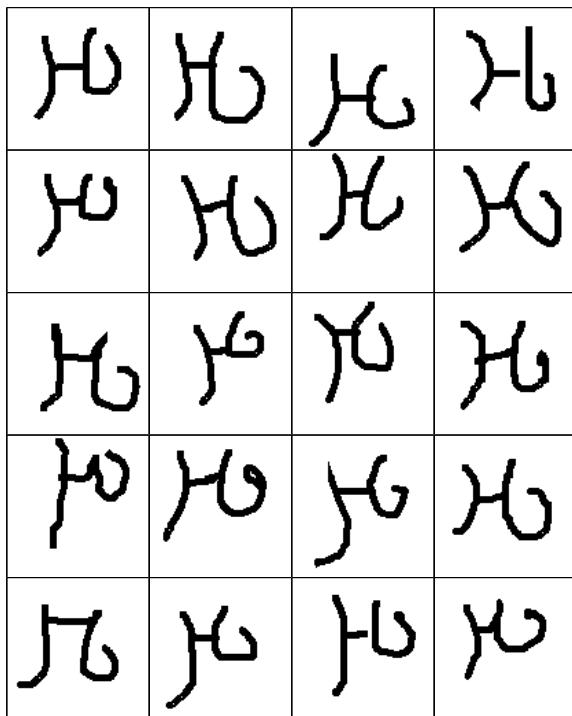
generating  
new concepts

ଯୁଦ୍ଧାଦ୍ୱାରାଲ	ନେତ୍ରାଲ
ଯୁଦ୍ଧାଦ୍ୱାରାଲ	ନେତ୍ରାଲ

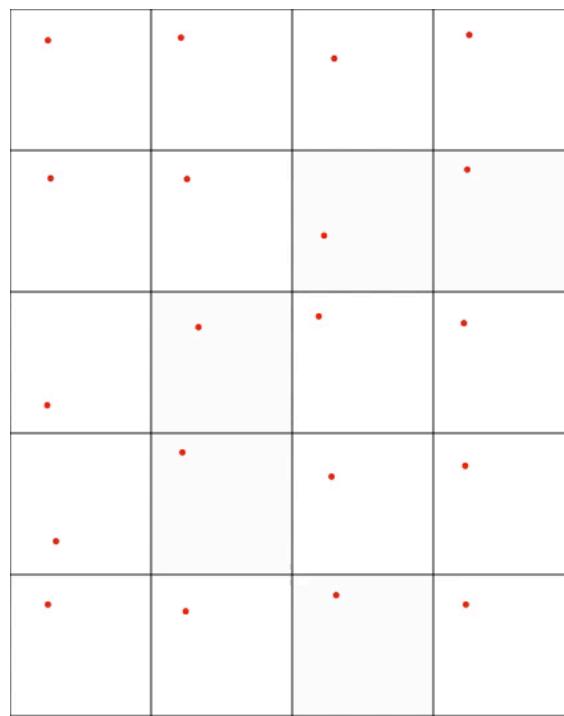
generating  
new examples

ଚିତ୍ର  
ଚିତ୍ର  
ଚିତ୍ର

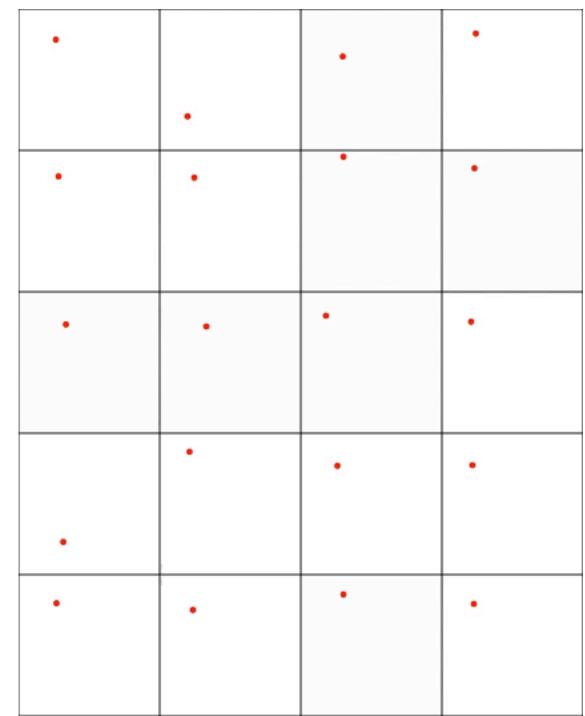
Human drawings



Human parses

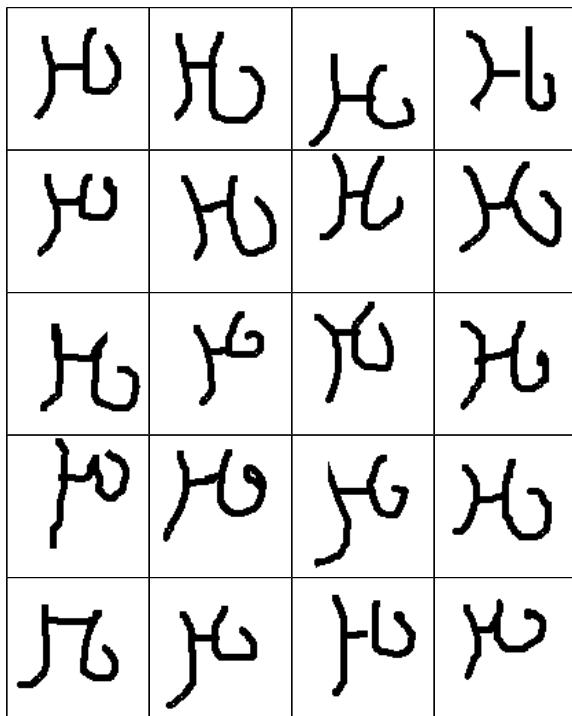


Machine parses

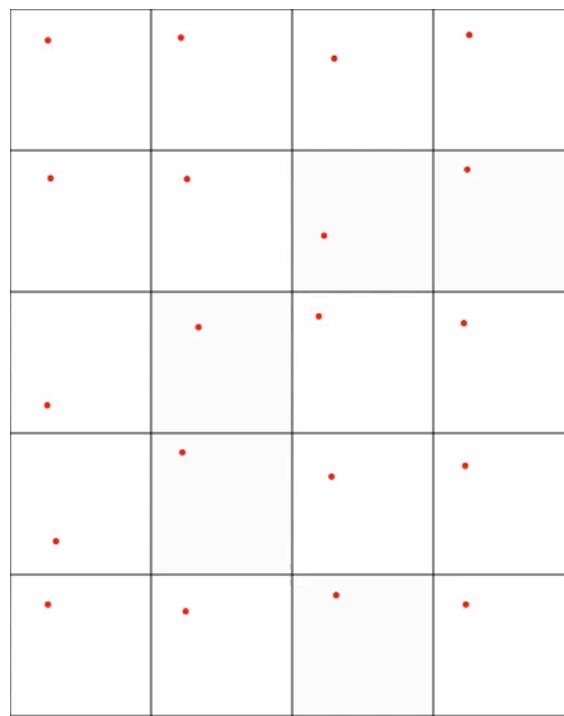


stroke order: — 1 — 2 — 3 — 4 — 5

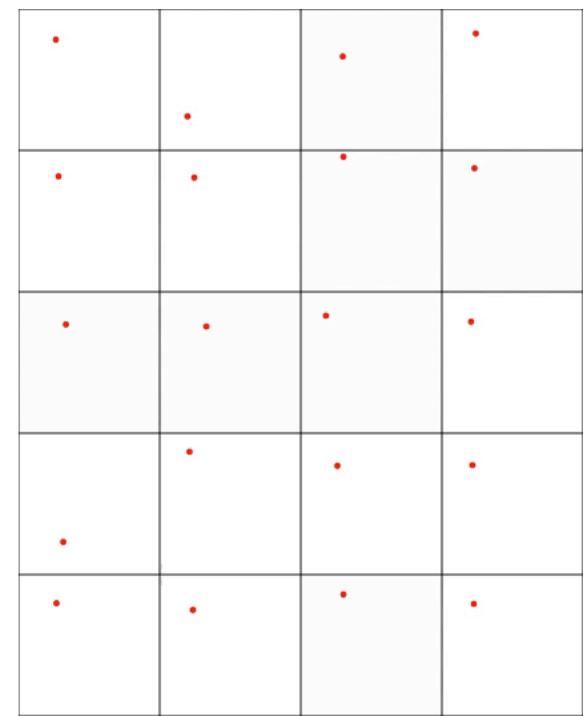
Human drawings



Human parses

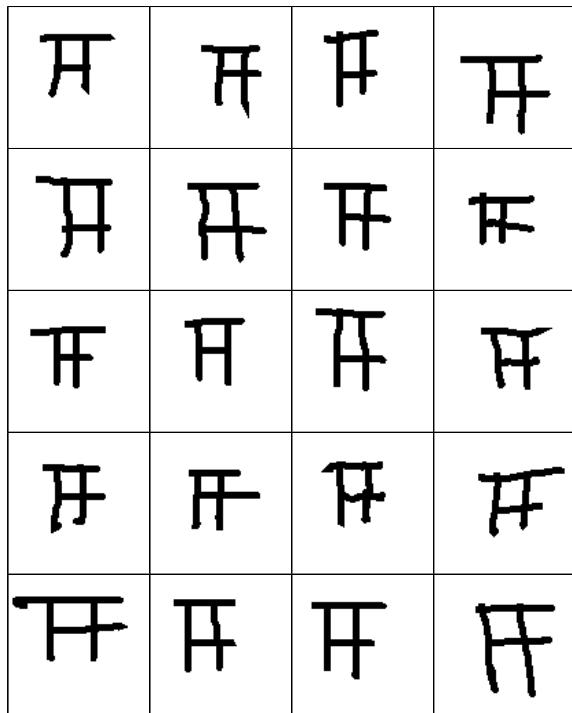


Machine parses

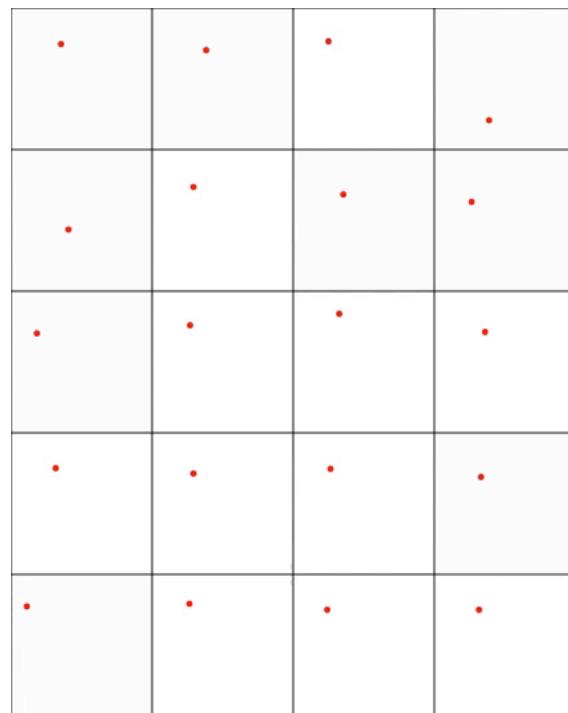


stroke order: — 1 — 2 — 3 — 4 — 5

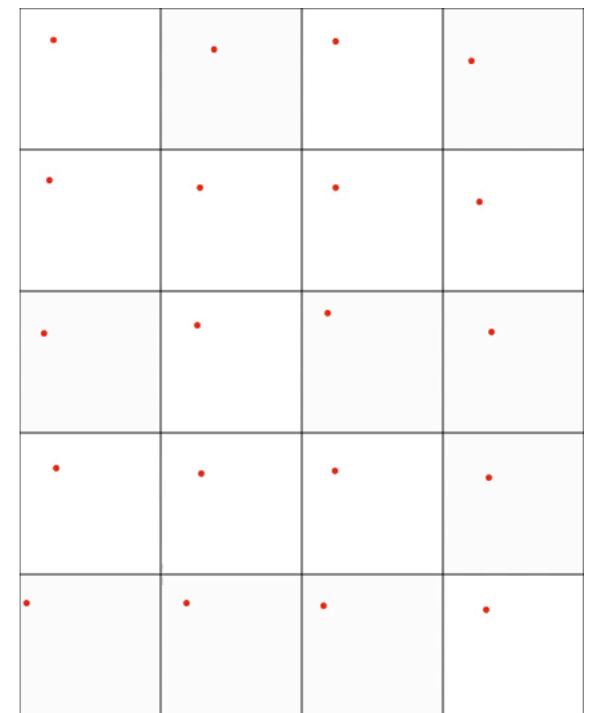
Human drawings



Human parses

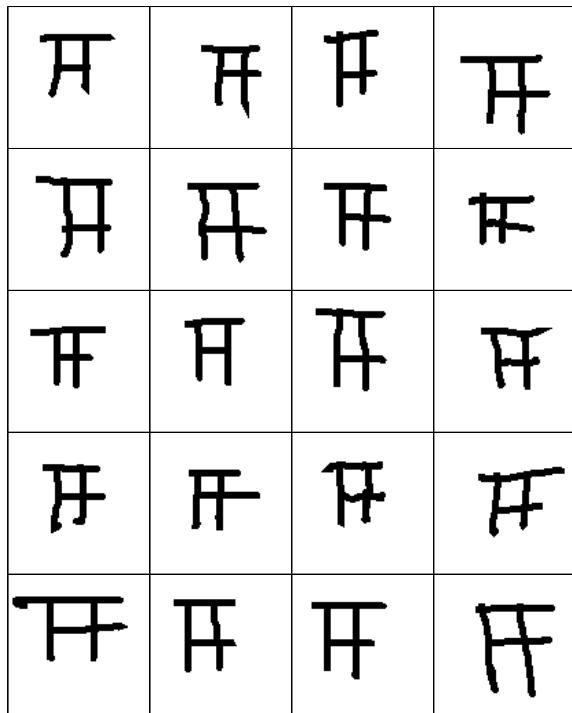


Machine parses

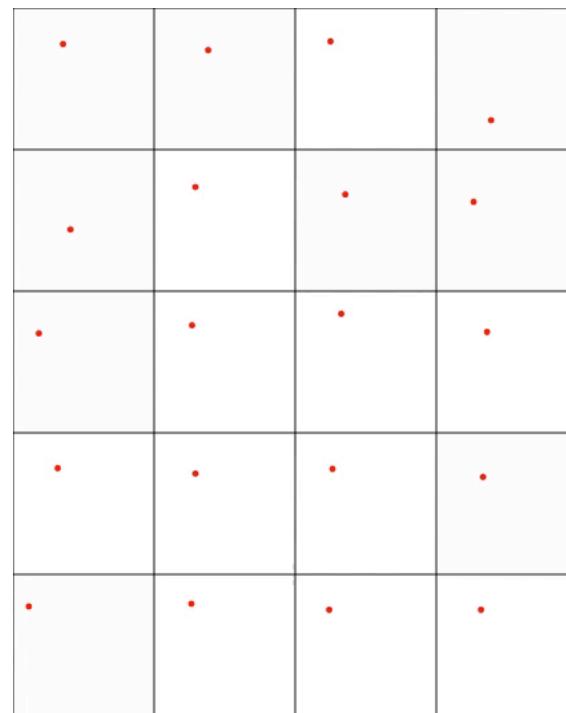


stroke order: — 1 — 2 — 3 — 4 — 5

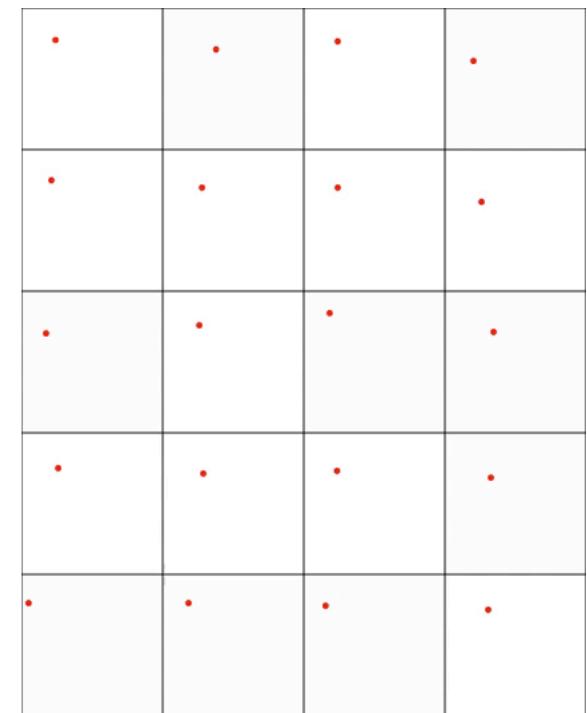
Human drawings



Human parses

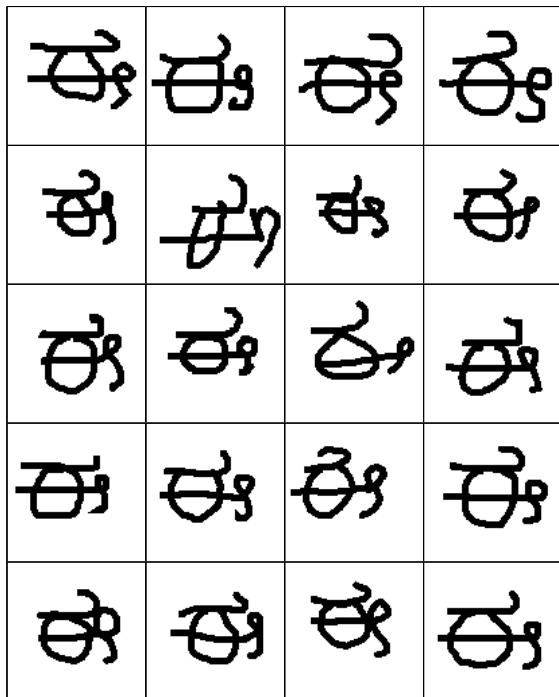


Machine parses

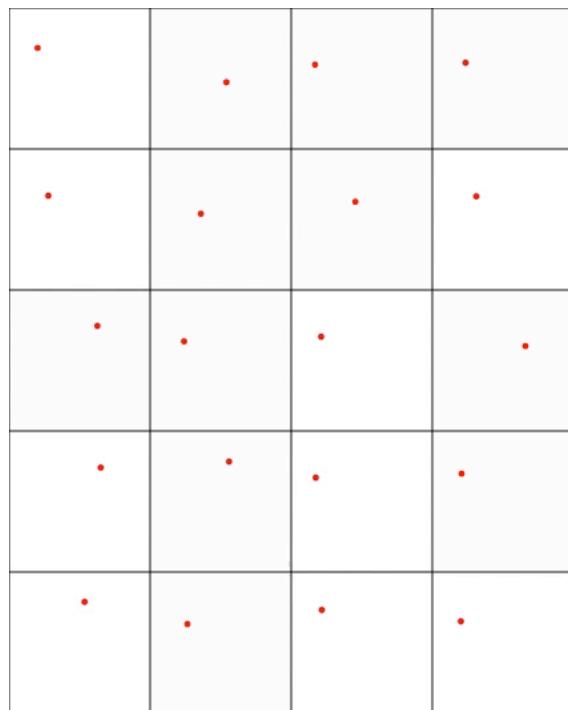


stroke order: — 1 — 2 — 3 — 4 — 5

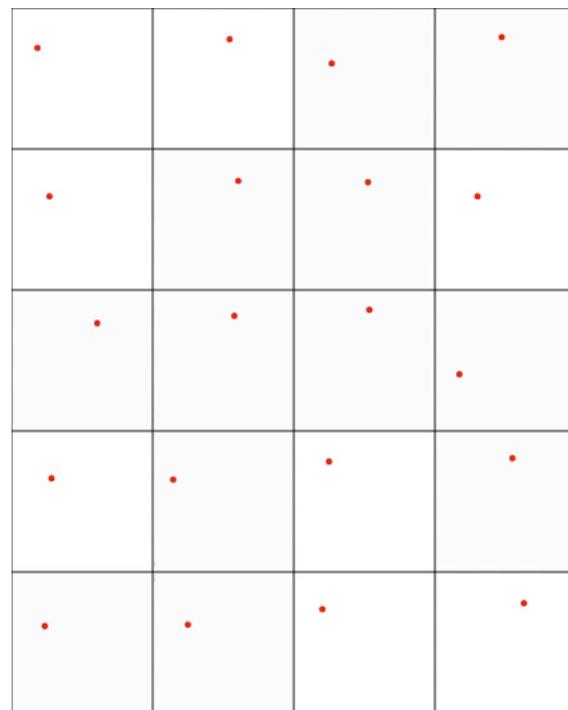
Human drawings



Human parses

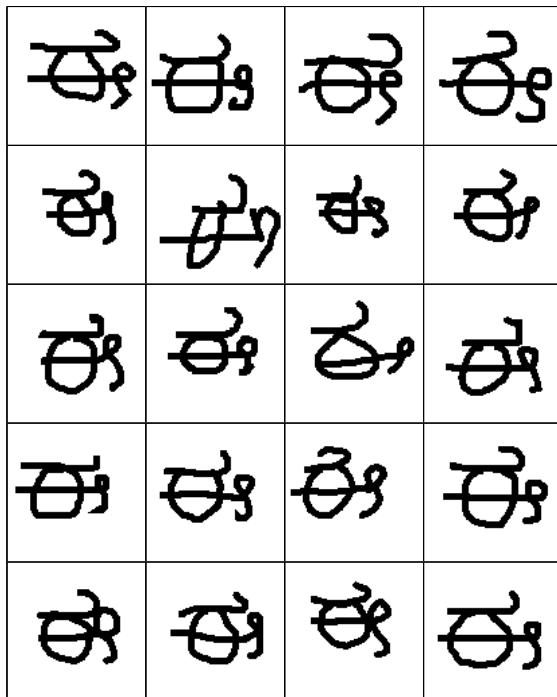


Machine parses

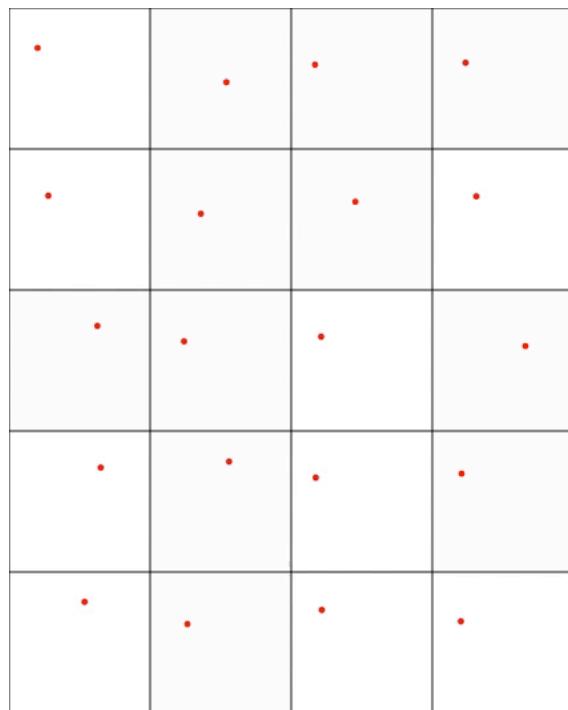


stroke order: — 1 — 2 — 3 — 4 — 5

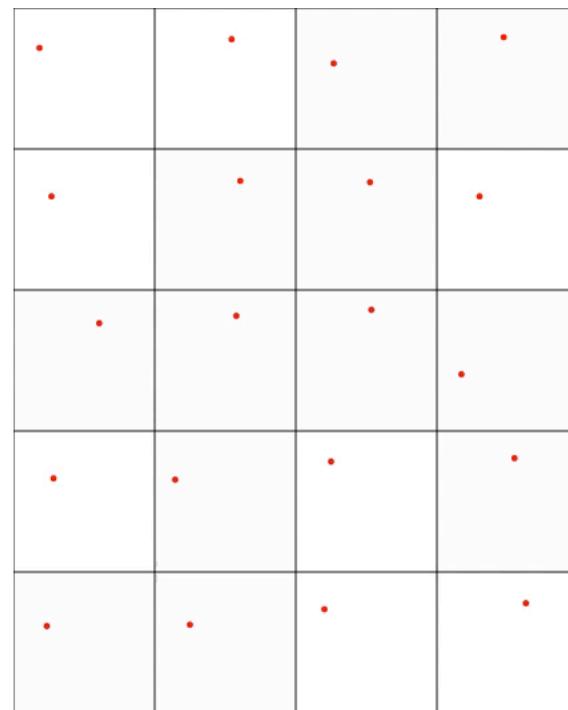
Human drawings



Human parses



Machine parses



stroke order: — 1 — 2 — 3 — 4 — 5

# Human-level concept learning

the speed of learning

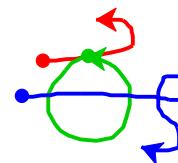
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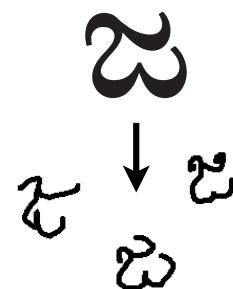
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the richness of representation

parsing



generating  
new examples



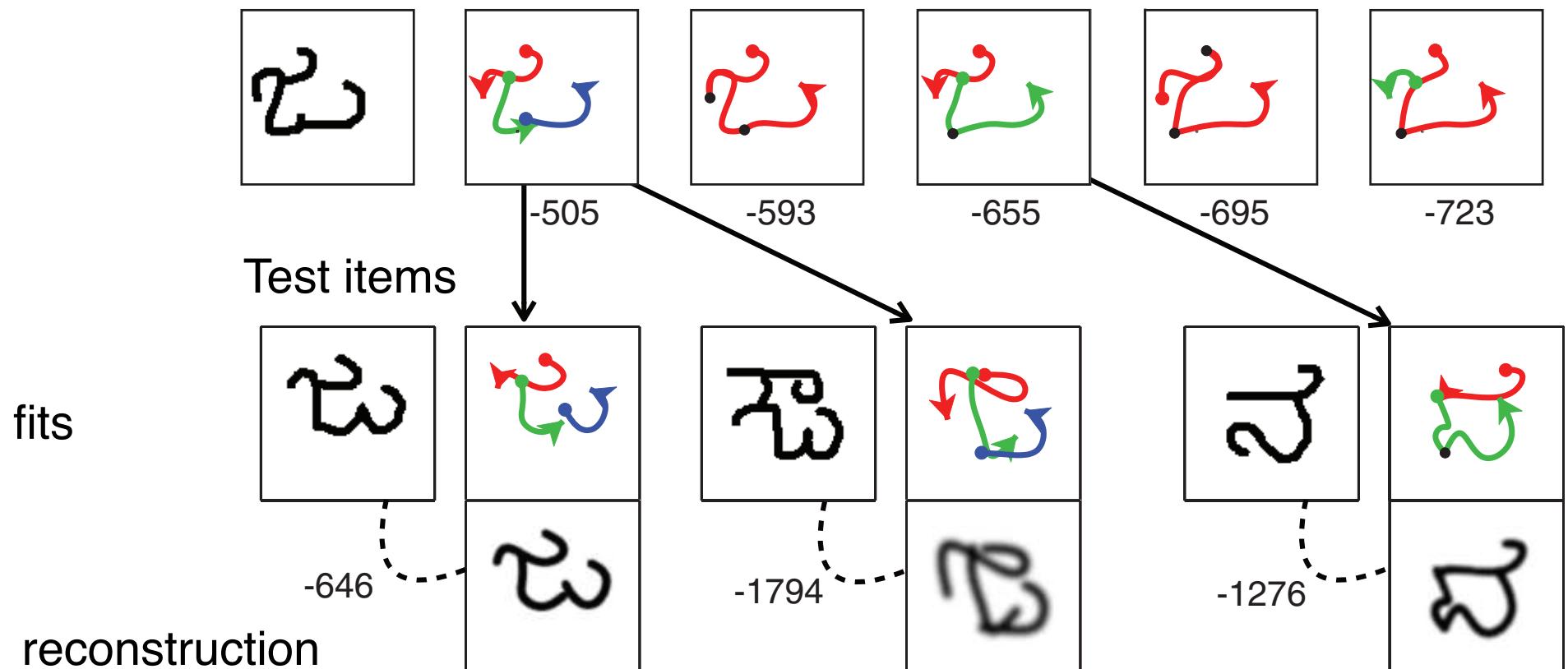
generating  
new concepts

ଯୁଦ୍ଧାଚନ୍ଦ୍ର	ନେଲ
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# One-shot classification

Training item with model's five best parses

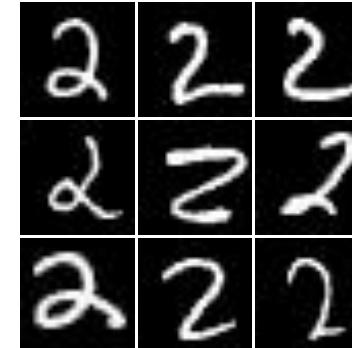


# Do people represent *static characters* by their causal dynamics?

same causal process



different examples



(Figure credit: Hinton & Nair, 2006)

## Behavioral evidence

- Writing experience influences perception

(Freyd, 1983; Tse & Cavanagh, 2000; Knoblich & Prinz, 2001; James & Gauthier, 2009).

- Inferring the dynamics from static letters.

(Babcock & Freyd, 1988)



## Neuroimaging evidence

- Writing experience changes the functional specialization of visual cortex for letters.

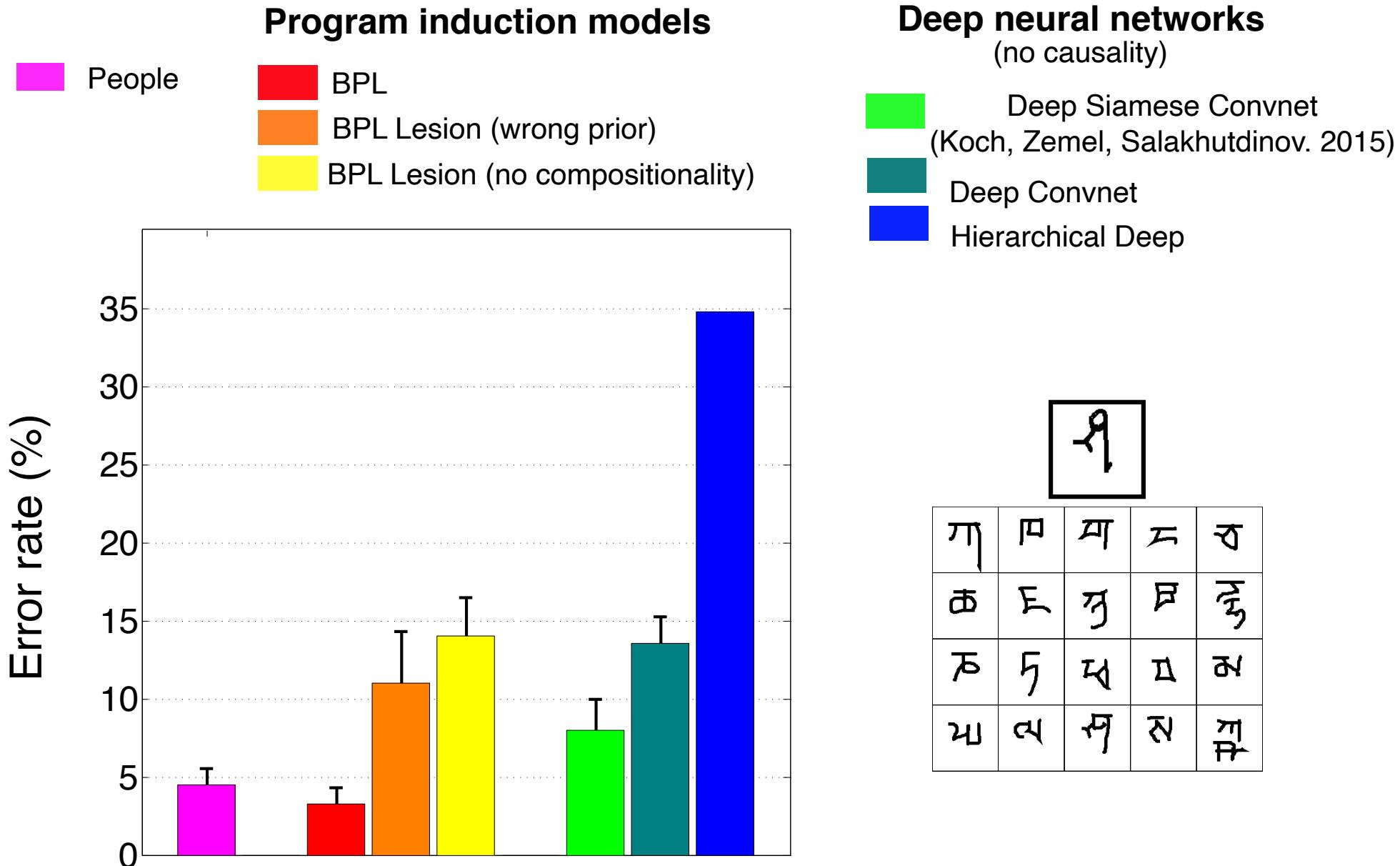
(James & Atwood, 2009; James, 2010)

- Motor areas of cortex respond to *static* letters.

(Anderson et al., 1990; Longcamp et al., 2003; James & Gauthier, 2006; Longcamp et al., 2006; Longcamp et al., 2010 )

# One-shot classification performance

After all models pre-trained on 30 alphabets of characters.



# Human-level concept learning

the speed of learning

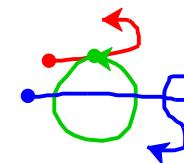
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ନ୍ତ	ଟ୍ଟ	ଫୁ	ପ୍ରୁ	ମୁ
ବୁ	ଲୁ	ମୁ	ନୁ	ମୁଖ



ଚ	ବୁ	ଲୁ	ଦୁ	ଦୁଃ
କୁ	ଲୁ	ଗୁ	ଭୁରୁ	
ହୁ	କୁ	ଛୁ	ତେବୁ	
ନୁ	ଯୁ	ଲୁ	କୁହ	

the richness of representation

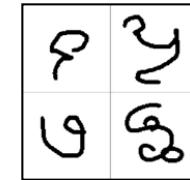
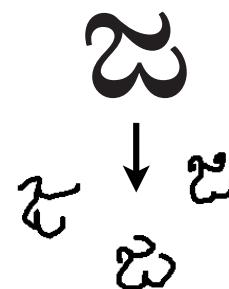
parsing



generating  
new concepts

ଯୁଦ୍ଧାଦ୍ର	ନେଲ
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generating  
new examples

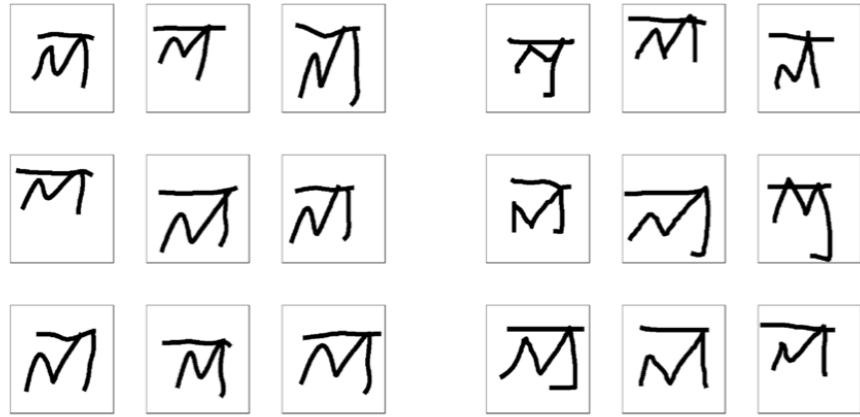


# A “visual Turing test” for generating new concepts

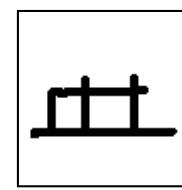
A



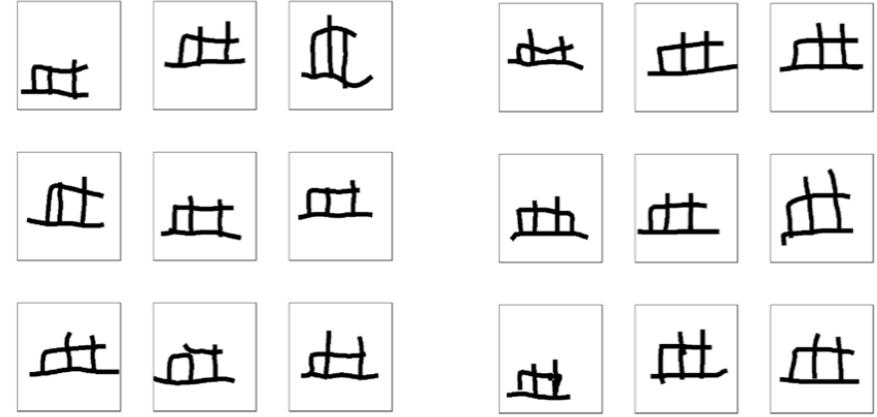
B



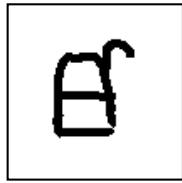
A



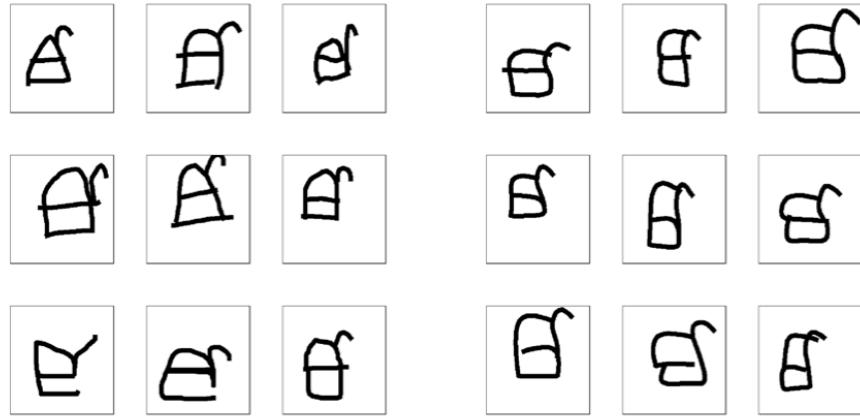
B



A



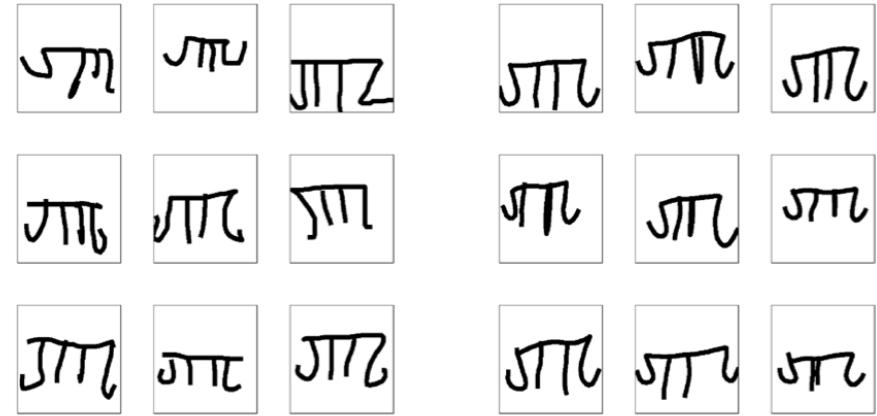
B



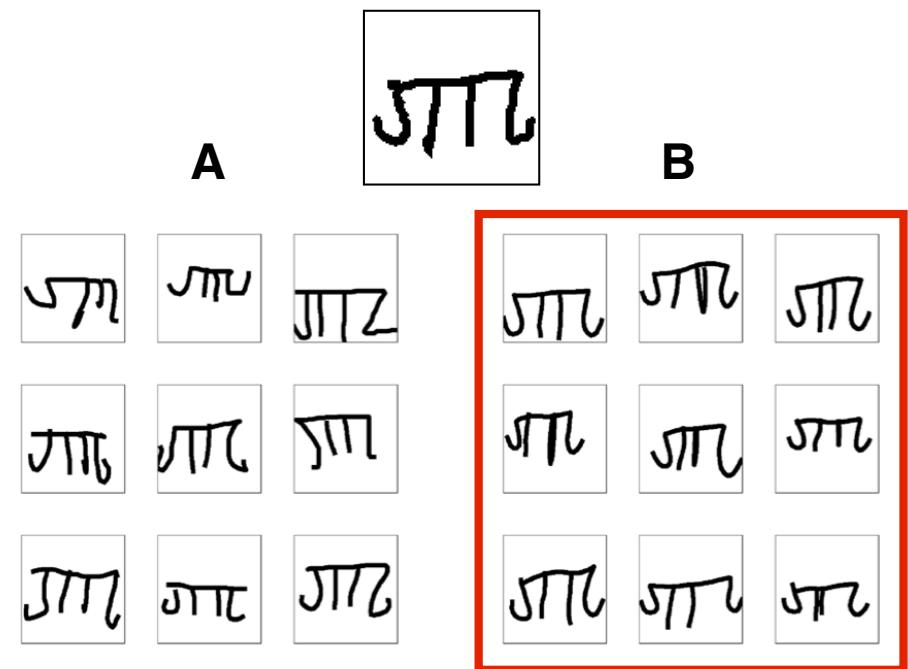
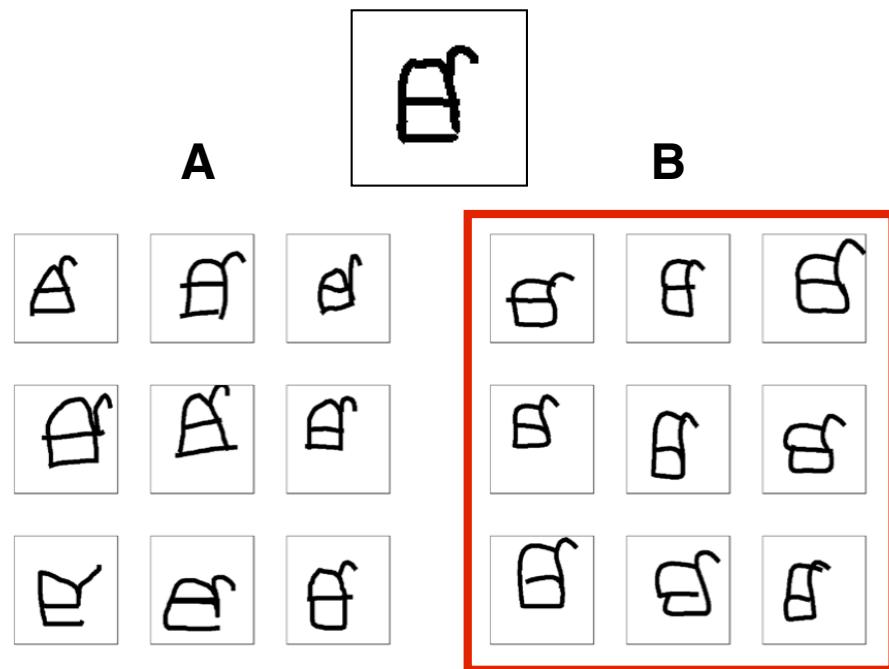
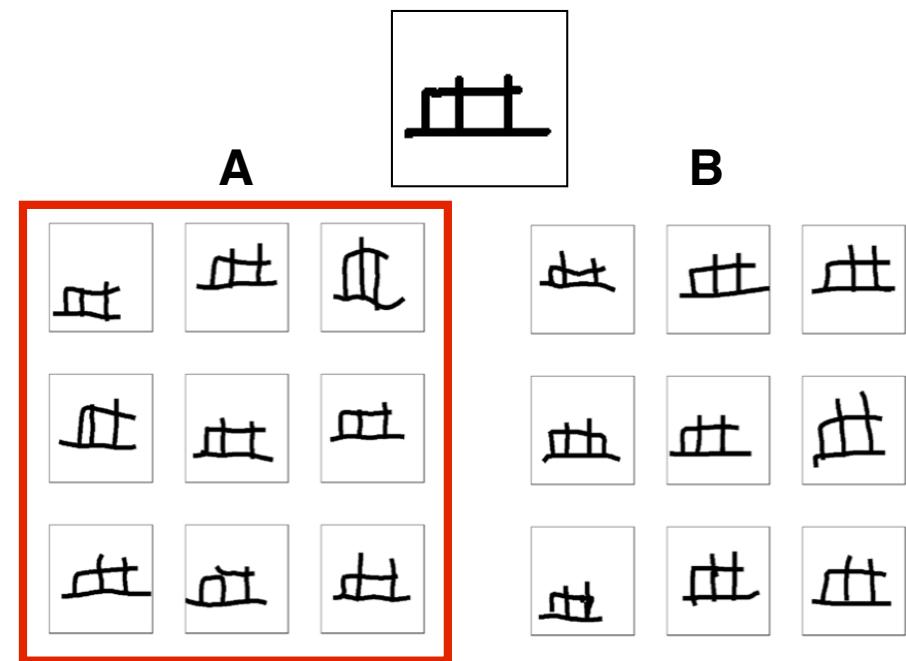
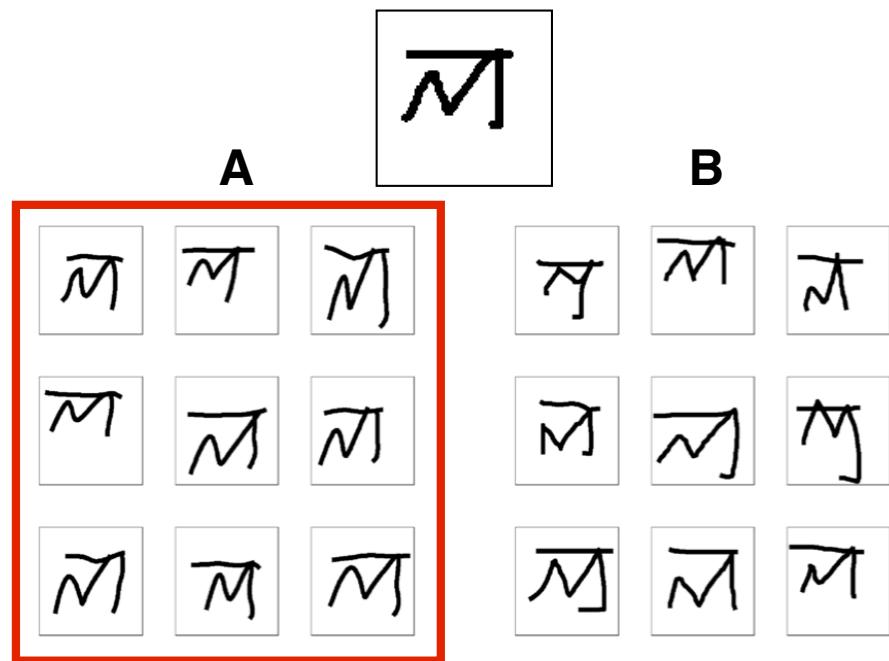
A



B

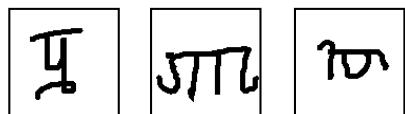


# A “visual Turing test” for generating new concepts



# More large-scale behavioral experiments

## Generating new examples (dynamic)

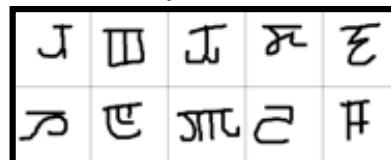


Human  
or Machine?

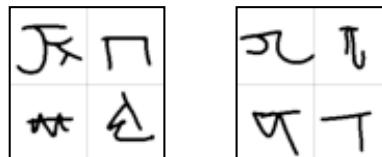
59% correct in visual Turing test  
6 of 30 judges above chance

## Generating new concepts (from type)

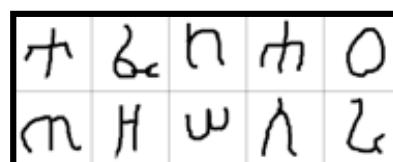
Alphabet



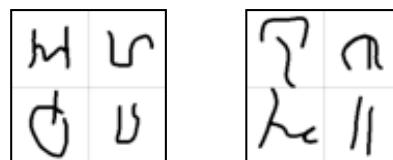
Human or Machine?



Alphabet



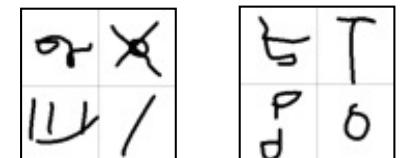
Human or Machine?



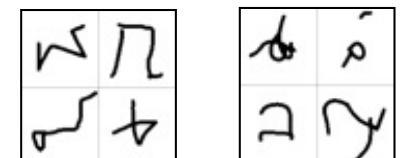
49% correct in visual Turing test  
8 of 35 judges above chance

## Generating new concepts (unconstrained)

Human or Machine?



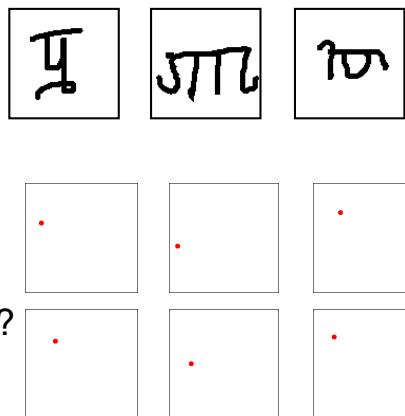
Human or Machine?



51% correct in  
visual Turing test  
2 of 25 judges  
above chance

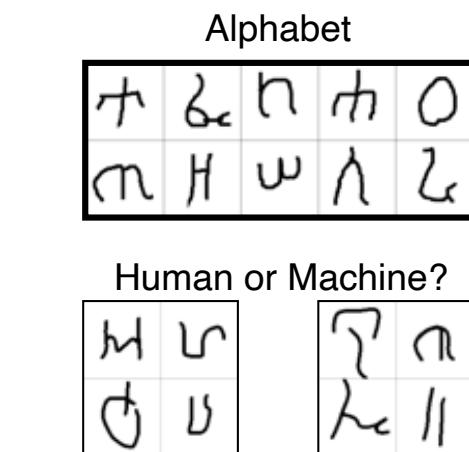
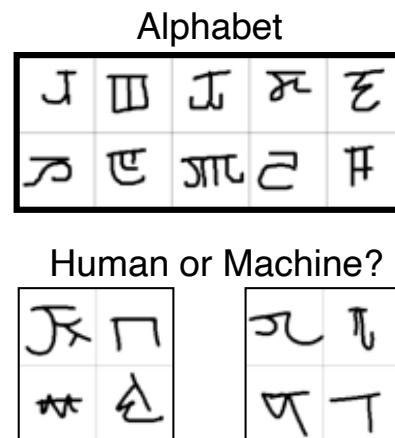
# More large-scale behavioral experiments

## Generating new examples (dynamic)



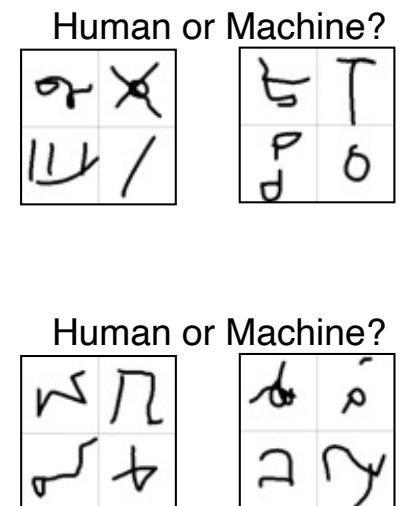
59% correct in visual Turing test  
6 of 30 judges above chance

## Generating new concepts (from type)



49% correct in visual Turing test  
8 of 35 judges above chance

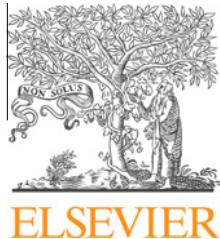
## Generating new concepts (unconstrained)



51% correct in  
visual Turing test  
2 of 25 judges  
above chance

# Case study: Numerical concept learning and cognitive development

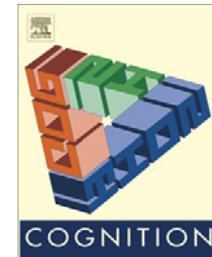
Cognition 123 (2012) 199–217



Contents lists available at SciVerse ScienceDirect

Cognition

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## Bootstrapping in a language of thought: A formal model of numerical concept learning

Steven T. Piantadosi <sup>a,\*</sup>, Joshua B. Tenenbaum <sup>b</sup>, Noah D. Goodman <sup>c</sup>

<sup>a</sup> Department of Brain and Cognitive Sciences, University of Rochester, United States

<sup>b</sup> Department of Brain and Cognitive Sciences, MIT, United States

<sup>c</sup> Department of Psychology, Stanford University, United States

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### ARTICLE INFO

#### Article history:

Available online 26 January 2012

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#### Keywords:

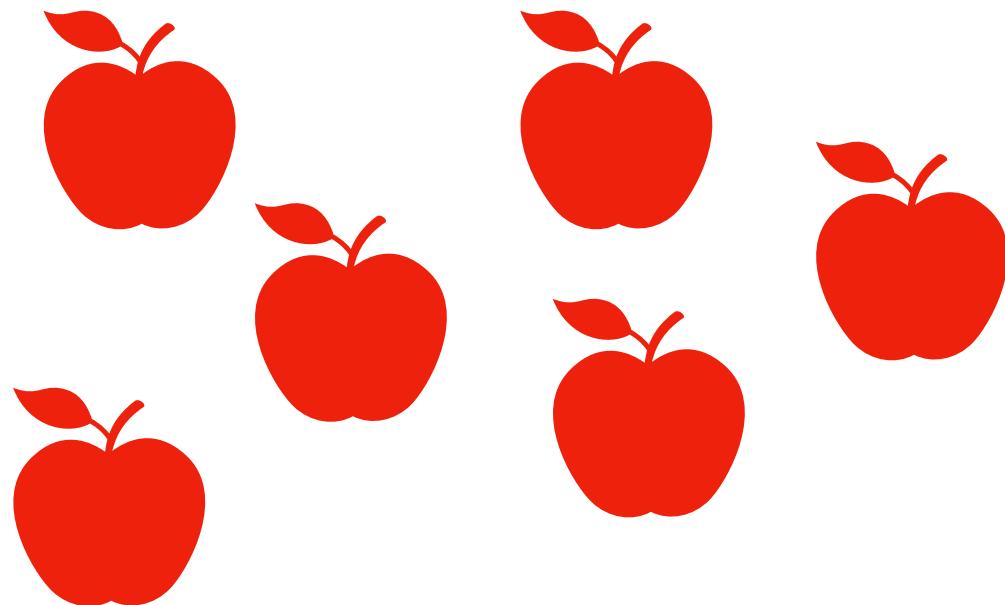
Number word learning  
Bootstrapping  
Cognitive development  
Bayesian model  
Language of thought  
CP transition

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### ABSTRACT

In acquiring number words, children exhibit a qualitative leap in which they transition from understanding a few number words, to possessing a rich system of interrelated numerical concepts. We present a computational framework for understanding this inductive leap as the consequence of statistical inference over a sufficiently powerful representational system. We provide an implemented model that is powerful enough to learn number word meanings and other related conceptual systems from naturalistic data. The model shows that bootstrapping can be made computationally and philosophically well-founded as a theory of number learning. Our approach demonstrates how learners may combine core cognitive operations to build sophisticated representations during the course of development, and how this process explains observed developmental patterns in number word learning.

# Children’s development of numerical concepts



**“Give-a-number” task**

“Give me two”

“Give me three”

(Wynn, 1990; Wynn, 1992)

# Children’s development of numerical concepts

Children progress through a series of stages

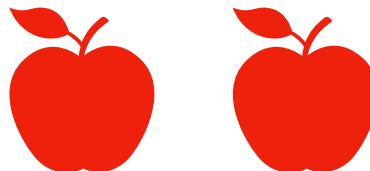
- “one-knower”, “two-knower,” “three-known,” “four-knower” (sometimes), and then “cardinal-principle knower”

## Example: “two knower”

give me one:



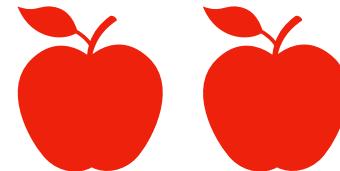
give me two:



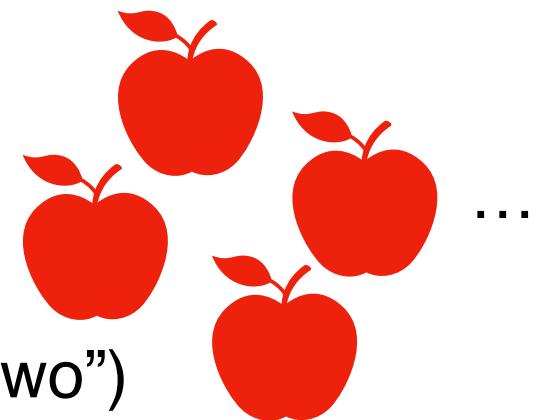
~~give me three:~~



OR



OR



(inconsistent; arbitrary response beyond “two”)

# Children's development of numerical concepts

- Critically, children can count well-beyond the range of their “knower” status, yet they don’t understand the meaning of the numbers.
- Transition from “N-knower” to “CP-knower” happens roughly between ages 2.5 and 3.5

## *Patterns of success in give-a-number task in Experiment 3*

	Success pattern						Number of children	Mean age	Counting ability	
	1	2	3	4	5	6			Mean	(range)
one-knower	-	-	-	-	-	-	1	2:8	3.00	(3-3)
two-knower	+	-	-	-	-	-	3	3:0	4.67	(3-6)
three-knower	+	+	-	-	-	-	2	2:11	4.50	(3-6)
CP-knower	+	+	+	+	+	+	4	3:5	5.75	(5-6)
							7	3:7	6.00	(6-6)

Note: “+” indicates success on a numerosity; “-” indicates failure.

(data from Wynn, 1990)

# Programming primitives allowed in the language of thought model

## Functions mapping sets to truth values

*(singleton? X)*

*(doubleton? X)*

*(tripleton? X)*

Returns true iff the set  $X$  has exactly one element

Returns true iff the set  $X$  has exactly two elements

Returns true iff the set  $X$  has exactly three elements

## Functions on sets

*(set-difference X Y)*

*(union X Y)*

*(intersection X Y)*

*(select X)*

Returns the set that results from removing  $Y$  from  $X$

Returns the union of sets  $X$  and  $Y$

Returns the intersect of sets  $X$  and  $Y$

Returns a set containing a single element from  $X$

## Logical functions

*(and P Q)*

*(or P Q)*

*(not P)*

*(if P X Y)*

Returns TRUE if  $P$  and  $Q$  are both true

Returns TRUE if either  $P$  or  $Q$  is true

Returns TRUE iff  $P$  is false

Returns  $X$  iff  $P$  is true,  $Y$  otherwise

## Functions on the counting routine

*(next W)*

*(prev W)*

*(equal-word? W V)*

Returns the word after  $W$  in the counting routine

Returns the word before  $W$  in the counting routine

Returns TRUE if  $W$  and  $V$  are the same word

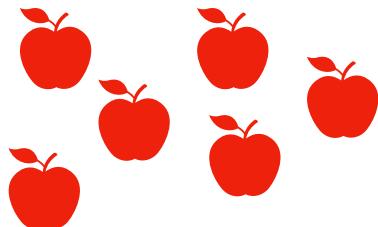
## Recursion

*(L S)*

Returns the result of evaluating the entire current lambda expression on  $S$

# Example hypotheses in a language of thought

example set  $S$



One-knower

$$\lambda S . (if (\text{singleton? } S) \\ \quad \text{"one"} \\ \quad \text{undef})$$

Two-knower

$$\lambda S . (if (\text{singleton? } S) \\ \quad \text{"one"} \\ \quad (if (\text{doubleton? } S) \\ \quad \quad \text{"two"} \\ \quad \quad \text{undef}))$$

Three-knower

$$\lambda S . (if (\text{singleton? } S) \\ \quad \text{"one"} \\ \quad (if (\text{doubleton? } S) \\ \quad \quad \text{"two"} \\ \quad (if (\text{tripleton? } S) \\ \quad \quad \text{"three"} \\ \quad \quad \text{undef}))$$

CP-knower

$$\lambda S . (if (\text{singleton? } S) \\ \quad \text{"one"} \\ \quad (next (L (\text{set-difference } S \\ \quad \quad \text{(select } S))))))$$

$\lambda S .$   
indicates a function  
that takes a set  $S$  as  
an argument.

Singular-Plural

$$\lambda S . (if (\text{singleton? } S) \\ \quad \text{"one"} \\ \quad \text{"two"})$$

Mod-5

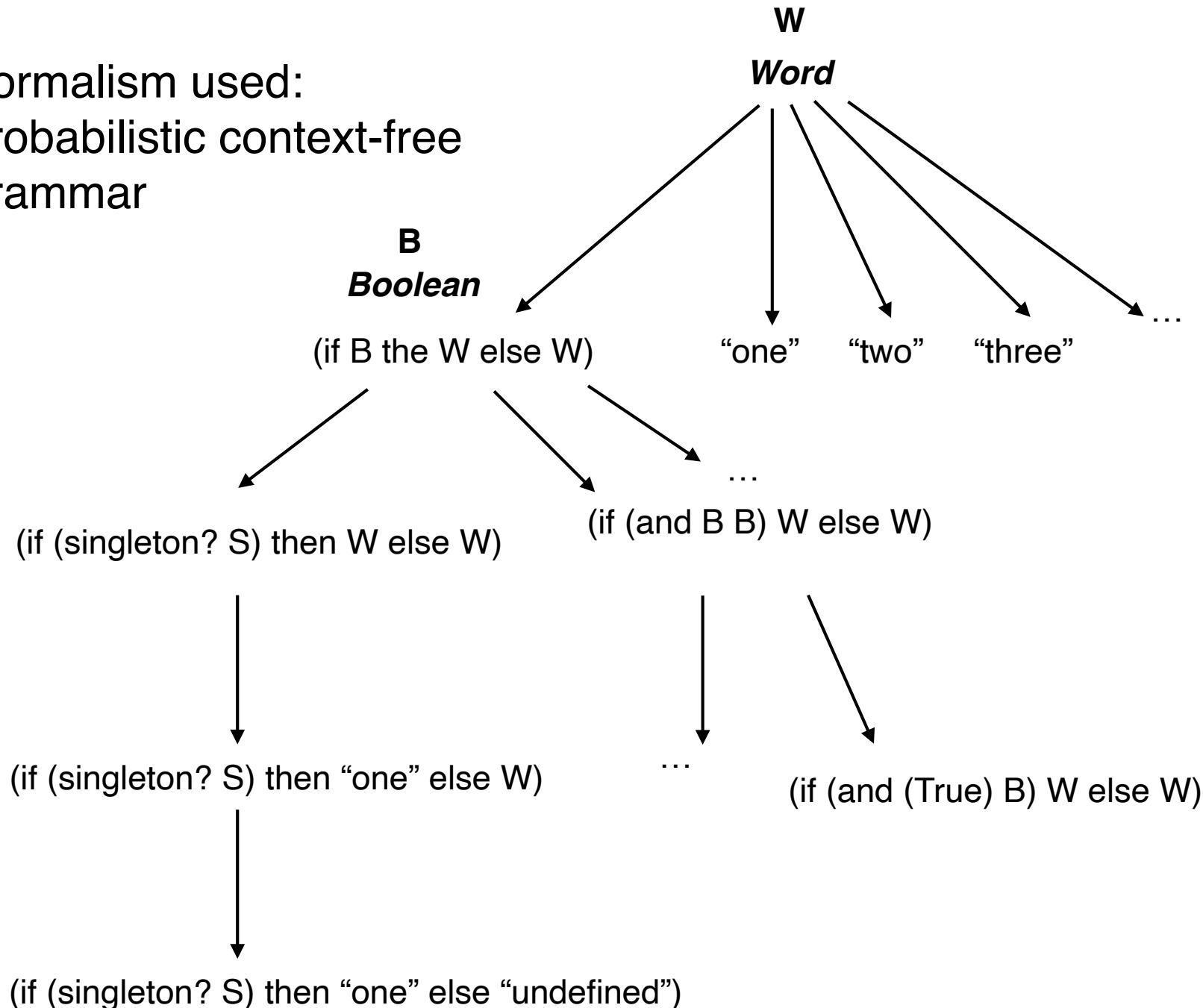
$$\lambda S . (if (\text{or} (\text{singleton? } S) \\ \quad (\text{equal-word? } (L (\text{set-difference } S \\ \quad \quad \text{(select } S))) \\ \quad \quad \text{"five"}))) \\ \quad \text{"one"} \\ \quad (next (L (\text{set-difference } S \\ \quad \quad \text{(select } S))))))$$

# Defining a prior distribution over programs

(a “Probabilistic Language of Thought”)

Formalism used:

probabilistic context-free  
grammar



# Probabilistic model over programs

## Example $L$

$$\lambda S . (\text{if } (\text{singleton? } S) \\ \quad \quad \quad \text{“one”} \\ \quad \quad \quad \text{“two”} \\ \quad \quad \quad \text{“six”} \\ \quad \quad \quad \text{“undef”})$$

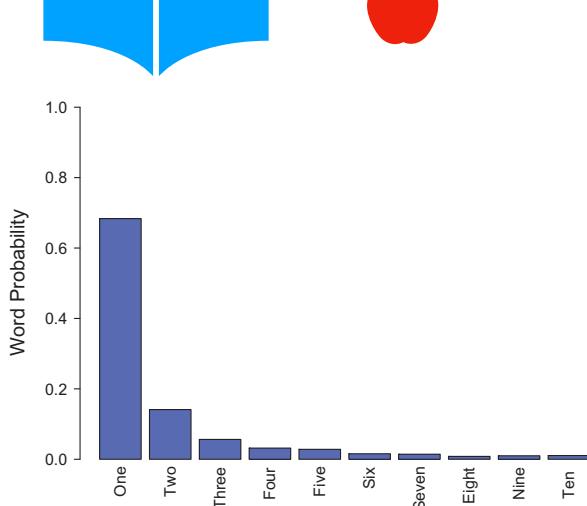
## Probabilistic model

 $P(L)$ 

prior on programs  $L$   
(defined with probabilistic  
grammar)

 $P(D|L)$ 

Noisy likelihood where  
right number is usually returned,  
but with some noise



## Bayes' rule for learning programs

$$P(L|D) = \frac{P(D|L)P(L)}{P(D)}$$

# Learning as program induction

**Program ( $L$ )**     $\lambda S . (\text{if } (\text{singleton? } S)$

“one”

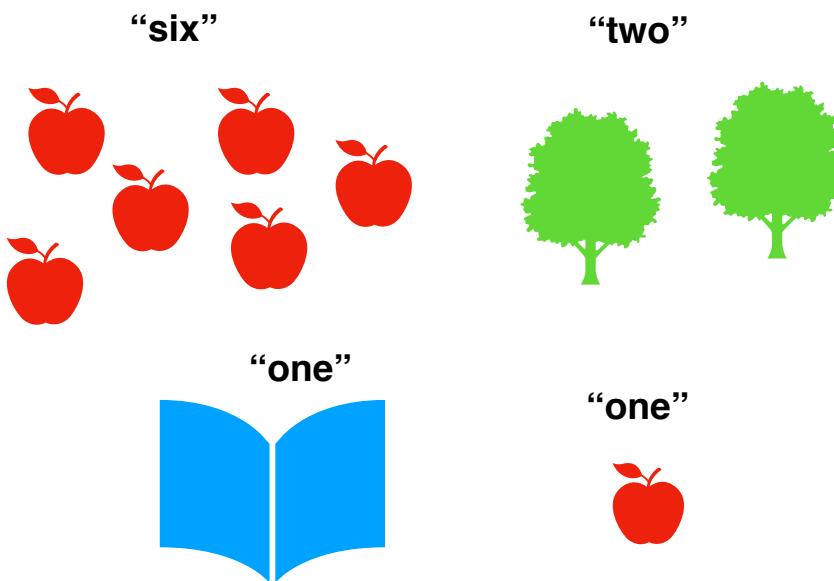
( $\text{next} (L (\text{set-difference } S$

( $\text{select } S)))))$

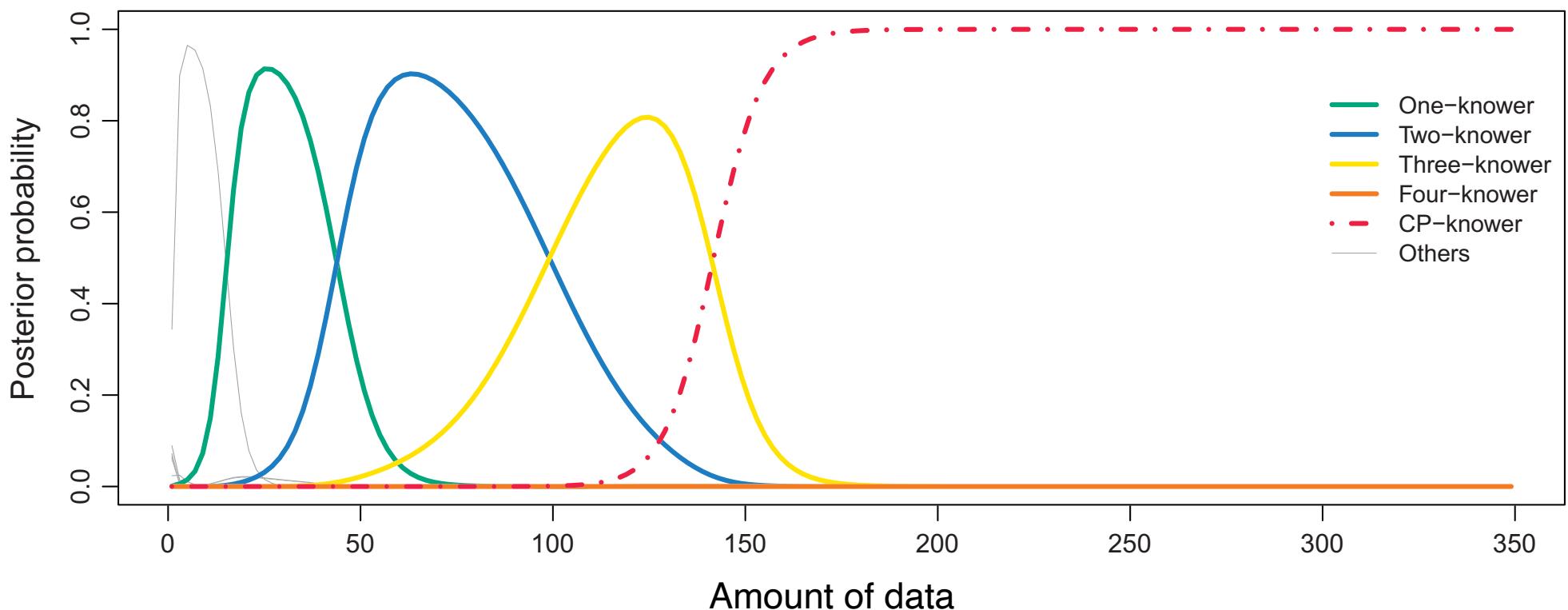


$$P(L|D) = \frac{P(D|L)P(L)}{P(D)}$$

**Data ( $D$ )**



# Results: Program induction model follows a similar developmental trajectory



## One-knower

```
 $\lambda S . (if (\text{singleton? } S)$ 
 $\quad \quad \quad \text{"one"}$ 
 $\quad \quad \quad \text{undef})$ 
```

## Two-knower

```
 $\lambda S . (if (\text{singleton? } S)$ 
 $\quad \quad \quad \text{"one"}$ 
 $\quad \quad \quad (if (\text{doubleton? } S)$ 
 $\quad \quad \quad \quad \quad \text{"two"}$ 
 $\quad \quad \quad \quad \quad \text{undef}))$ 
```

## Three-knower

```
 $\lambda S . (if (\text{singleton? } S)$ 
 $\quad \quad \quad \text{"one"}$ 
 $\quad \quad \quad (if (\text{doubleton? } S)$ 
 $\quad \quad \quad \quad \quad \text{"two"}$ 
 $\quad \quad \quad (if (\text{tripleton? } S)$ 
 $\quad \quad \quad \quad \quad \text{"three"}$ 
 $\quad \quad \quad \quad \quad \text{undef}))$ 
```

## CP-knower

```
 $\lambda S . (if (\text{singleton? } S)$ 
 $\quad \quad \quad \text{"one"}$ 
 $\quad \quad \quad (next (L (\text{set-difference } S$ 
 $\quad \quad \quad \quad \quad (\text{select } S))))))$ 
```

# Case study: Learning by asking questions

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## Question Asking as Program Generation

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**Anselm Rothe<sup>1</sup>**

anselm@nyu.edu

**Brenden M. Lake<sup>1,2</sup>**

brenden@nyu.edu

**Todd M. Gureckis<sup>1</sup>**

todd.gureckis@nyu.edu

<sup>1</sup>Department of Psychology      <sup>2</sup>Center for Data Science  
New York University

### Abstract

A hallmark of human intelligence is the ability to ask rich, creative, and revealing questions. Here we introduce a cognitive model capable of constructing human-like questions. Our approach treats questions as formal programs that, when executed on the state of the world, output an answer. The model specifies a probability distribution over a complex, compositional space of programs, favoring concise programs that help the agent learn in the current context. We evaluate our approach by modeling the types of open-ended questions generated by humans who were attempting to learn about an ambiguous situation in a game. We find that our model predicts what questions people will ask, and can creatively produce novel questions that were not present in the training set. In addition, we compare a number of model variants, finding that both question informativeness and complexity are important for producing human-like questions.

### 1 Introduction

In active machine learning, a learner is able to query an oracle in order to obtain information that is expected to improve performance. Theoretical and empirical results show that active learning can speed acquisition for a variety of learning tasks [see 21 for a review]. Although impressive, most

# active learning for people and machines



## rich human questions



How do they grow their babies?

Why is he up in the tree?

What is the difference between a shark and a fish?

## simple machine questions



What is the category label of this object?

What is the category label of this object?

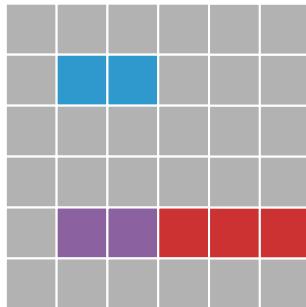
What is the category label of this object?

# Experiment: Free-form question asking



(e.g., Markant & Gureckis, 2012, 2014)

**Hidden configuration of ships**



**3 ships (blue, purple, red)**

**3 possible sizes (2-4 tiles)**

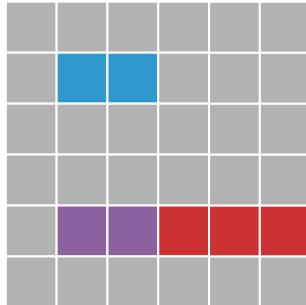
**1.6 million possible configurations**

# Experiment: Free-form question asking



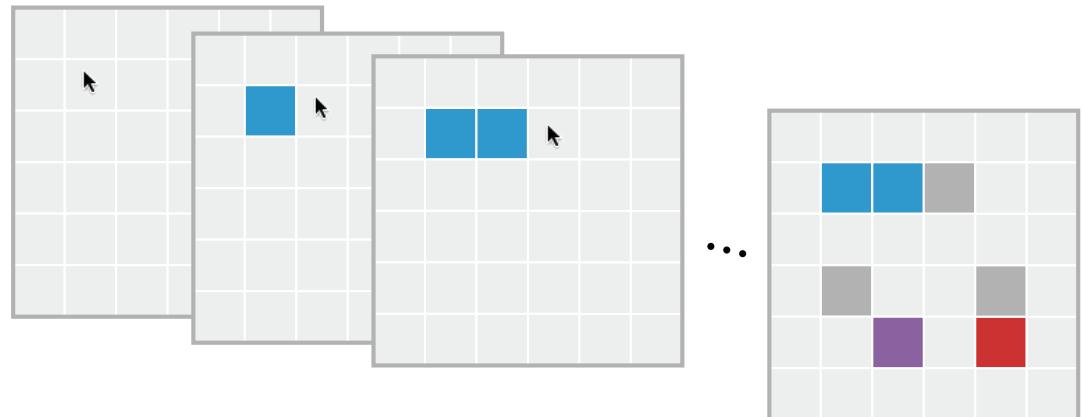
(e.g., Markant & Gureckis, 2012, 2014)

Hidden configuration of ships



3 ships (blue, purple, red)  
3 possible sizes (2-4 tiles)  
1.6 million possible configurations

## Phase 1: Sampling



## Phase 2: Question asking

Is the red ship horizontal? |

### Constraints

- one word answers
- no combinations

Repeated for 18 different hidden configurations

<b>N</b>	<b>Location/standard queries</b>
24	What color is at [row][column]?
24	Is there a ship at [row][column]?
31	Is there a [color_incl_water] tile at [row][column]?
	<b>Region queries</b>
4	Is there any ship in row [row]?
9	Is there any part of the [color] ship in row [row]?
5	How many tiles in row [row] are occupied by ships?
1	Are there any ships in the bottom half of the grid?
10	Is there any ship in column [column]?
10	Is there any part of the [color] ship in column [column]?
3	Are all parts of the [color] ship in column [column]?
2	How many tiles in column [column] are occupied by ships?
1	Is any part of the [color] ship in the left half of the grid?
	<b>Ship size queries</b>
185	How many tiles is the [color] ship?
71	Is the [color] ship [size] tiles long?
8	Is the [color] ship [size] or more tiles long?
5	How many ships are [size] tiles long?
8	Are any ships [size] tiles long?
2	Are all ships [size] tiles long?
2	Are all ships the same size?
2	Do the [color1] ship and the [color2] ship have the same size?
3	Is the [color1] ship longer than the [color2] ship?
3	How many tiles are occupied by ships?
	<b>Ship orientation queries</b>
94	Is the [color] ship horizontal?
7	How many ships are horizontal?
3	Are there more horizontal ships than vertical ships?
1	Are all ships horizontal?
4	Are all ships vertical?
7	Are the [color1] ship and the [color2] ship parallel?
	<b>Adjacency queries</b>
12	Do the [color1] ship and the [color2] ship touch?
6	Are any of the ships touching?
9	Does the [color] ship touch any other ship?
2	Does the [color] ship touch both other ships?
	<b>Demonstration queries</b>
14	What is the location of one [color] tile?
28	At what location is the top left part of the [color] ship?
5	At what location is the bottom right part of the [color] ship?

---

## N Location/standard queries

- 24 What color is at [row][column]?
- 24 Is there a ship at [row][column]?
- 31 Is there a [color\_incl\_water] tile at [row][column]?

- 
- 10 Is there any ship in column [column]?
  - 10 Is there any part of the [color] ship in column [column]?
  - 3 Are all parts of the [color] ship in column [column]?
  - 2 How many tiles in column [column] are occupied by ships?
  - 1 Is any part of the [color] ship in the left half of the grid?

---

### Ship size queries

- 185 How many tiles is the [color] ship?
- 71 Is the [color] ship [size] tiles long?
- 8 Is the [color] ship [size] or more tiles long?
- 5 How many ships are [size] tiles long?
- 8 Are any ships [size] tiles long?
- 2 Are all ships [size] tiles long?
- 2 Are all ships the same size?
- 2 Do the [color1] ship and the [color2] ship have the same size?
- 3 Is the [color1] ship longer than the [color2] ship?
- 3 How many tiles are occupied by ships?

---

### Ship orientation queries

- 94 Is the [color] ship horizontal?
- 7 How many ships are horizontal?
- 3 Are there more horizontal ships than vertical ships?
- 1 Are all ships horizontal?
- 4 Are all ships vertical?
- 7 Are the [color1] ship and the [color2] ship parallel?

---

### Adjacency queries

- 12 Do the [color1] ship and the [color2] ship touch?
- 6 Are any of the ships touching?
- 9 Does the [color] ship touch any other ship?
- 2 Does the [color] ship touch both other ships?

---

### Demonstration queries

- 14 What is the location of one [color] tile?
- 28 At what location is the top left part of the [color] ship?
- 5 At what location is the bottom right part of the [color] ship?

<b>N</b>	<b>Location/standard queries</b>
24	What color is at [row][column]?
24	Is there a ship at [row][column]?
31	Is there a [color_incl_water] tile at [row][column]?

---

#### **Region queries**

- 4 Is there any ship in row [row]?
- 9 Is there any part of the [color] ship in row [row]?
- 5 How many tiles in row [row] are occupied by ships?
- 1 Are there any ships in the bottom half of the grid?
- 10 Is there any ship in column [column]?
- 10 Is there any part of the [color] ship in column [column]?
- 3 Are all parts of the [color] ship in column [column]?
- 2 ~~How many tiles in column [column] are surrounded by ships?~~

## **Ship size queries**

- 185 How many tiles is the [color] ship?
- 71 Is the [color] ship [size] tiles long?
- 8 Is the [color] ship [size] or more tiles long?
- 5 How many ships are [size] tiles long?
- 8 Are any ships [size] tiles long?
- 2 Are all ships [size] tiles long?

- 
- 94 Is the [color] ship horizontal?
  - 7 How many ships are horizontal?
  - 3 Are there more horizontal ships than vertical ships?
  - 1 Are all ships horizontal?
  - 4 Are all ships vertical?
  - 7 Are the [color1] ship and the [color2] ship parallel?

---

#### **Adjacency queries**

- 12 Do the [color1] ship and the [color2] ship touch?
- 6 Are any of the ships touching?
- 9 Does the [color] ship touch any other ship?
- 2 Does the [color] ship touch both other ships?

---

#### **Demonstration queries**

- 14 What is the location of one [color] tile?
- 28 At what location is the top left part of the [color] ship?
- 5 At what location is the bottom right part of the [color] ship?

<b>N</b>	<b>Location/standard queries</b>
24	What color is at [row][column]?
24	Is there a ship at [row][column]?
31	Is there a [color_incl_water] tile at [row][column]?
	<b>Region queries</b>
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7	Are the [color1] ship and the [color2] ship parallel?
	<b>Adjacency queries</b>

## Demonstration queries

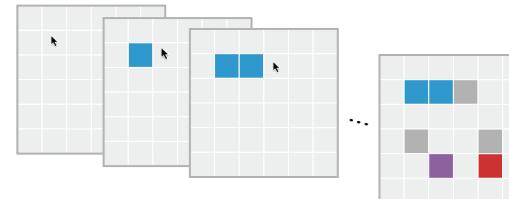
- 14 What is the location of one [color] tile?
- 28 At what location is the top left part of the [color] ship?
- 5 At what location is the bottom right part of the [color] ship?

# How do people think of a question to ask? question asking as program generation

## Game primitives

“Size of the blue ship?”  
( size Blue )

“Color at tile A1?”  
( color A1 )



“Orientation of the blue ship?”  
( orient Blue )

## Primitive operators

( + X X )

( = X X )

## Novel questions

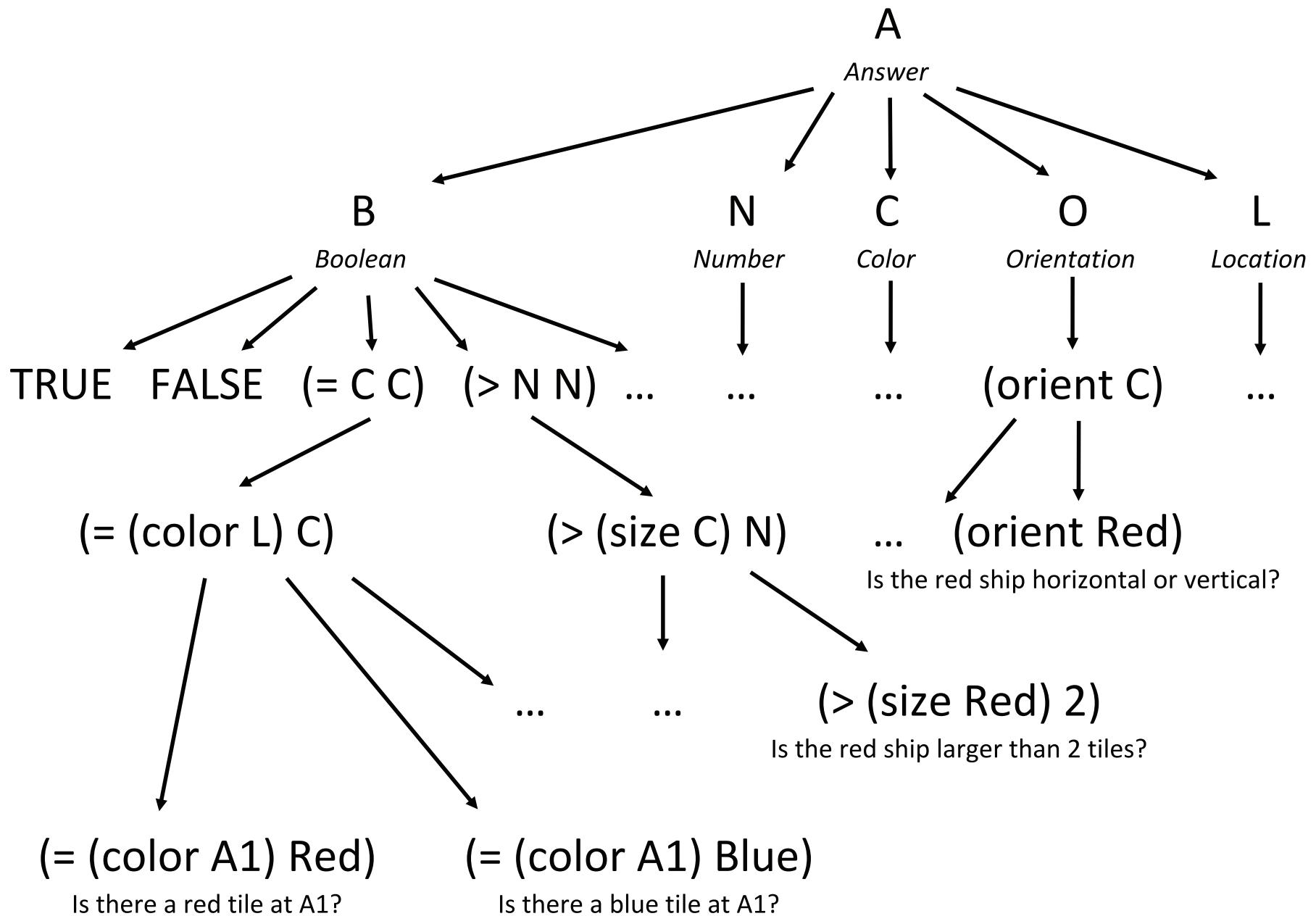
“What is the total size  
of all the ships?”  
(  
  (+  
    (+  
      ( size Blue )  
      ( size Red )  
    )  
    ( size Purple )  
)

“Are the blue ship and the red  
ship **parallel**? ”  
( =  
  ( orient Blue )  
  ( orient Red )  
)

# Questions as programs

GROUP	QUESTION	FUNCTION	EXPRESSION
location	What color is at A1? Is there a ship at A1? Is there a blue tile at A1?	location locationA locationD	(color A1) (not (= (color A1) Water)) (= (color A1) Blue)
segmentation	Is there any ship in row 1? Is there any part of the blue ship in row 1? Are all parts of the blue ship in row 1? How many tiles in row 1 are occupied by ships? Are there any ships in the bottom half of the grid? Is there any ship in column 1? Is there any part of the blue ship in column 1? Are all parts of the blue ship in column 1? How many tiles in column 1 are occupied by ships? Is any part of the blue ship in the left half of the grid?	row rowD rowDL rowNA rowX2 col colD colDL colNA colX1 shipsize shipsizeD shipsizeM shipsizeN shipsizeDA shipsizeDL shipsizeL shipsizeX1 shipsizeX2 totalshipsize horizontal	(> (+ (map (λ x (and (= (row x) 1) (not (= (color x) Water)))) (set A1 ... F6))) 0) (> (+ (map (λ x (and (= (row x) 1) (= (color x) Blue))) (set A1 ... F6))) 0) (> (+ (map (λ x (and (= (row x) 1) (= (color x) Blue))) (set A1 ... F6))) 1) (+ (map (λ x (and (= (row x) 1) (not (= (color x) Water)))) (set A1 ... F6))) ... (> (+ (map (λ x (and (= (col x) 1) (not (= (color x) Water)))) (set A1 ... F6))) 0) (> (+ (map (λ x (and (= (col x) 1) (= (color x) Blue))) (set A1 ... F6))) 0) (> (+ (map (λ x (and (= (col x) 1) (= (color x) Blue))) (set A1 ... F6))) 1) (+ (map (λ x (and (= (col x) 1) (not (= (color x) Water)))) (set A1 ... F6))) ... (size Blue) (= (size Blue) 3) (or (= (size Blue) 3) (> (size Blue) 3)) (+ (map (λ x (= (size x) 3)) (set Blue Red Purple))) (> (+ (map (λ x (= (size x) 3)) (set Blue Red Purple))) 0) (= (+ (map (λ x (= (size x) 3)) (set Blue Red Purple))) 3) (= (map (λ x (size x)) (set Blue Red Purple))) (= (size Blue) (size Red)) (> (size Blue) (size Red)) (+ (map (λ x (size x)) (set Blue Red Purple))) (= (orient Blue) H)
ship size	How many tiles is the blue ship? Is the blue ship 3 tiles long? Is the blue ship 3 or more tiles long? How many ships are 3 tiles long? Are any ships 3 tiles long? Are all ships 3 tiles long? Are all ships the same size? Do the blue ship and the red ship have the same size? Is the blue ship longer than the red ship? How many tiles are occupied by ships?	horizontalN horizontalM horizontalL verticalL parallel touching touchingA touchingXA touchingX1 demonstration topleft bottomright	(+ (map (λ x (= (orient x) H)) (set Blue Red Purple))) (> (+ (map (λ x (= (orient x) H)) (set Blue Red Purple))) 1) (= (+ (map (λ x (= (orient x) H)) (set Blue Red Purple))) 3) (= (+ (map (λ x (= (orient x) H)) (set Blue Red Purple))) 0) (= (orient Blue) (orient Red)) (touch Blue Red) (or (touch Blue Red) (or (touch Blue Purple) (touch Red Purple))) (or (touch Blue Red) (touch Blue Purple)) (and (touch Blue Red) (touch Blue Purple)) (draw (select (set A1 ... F6) Blue)) (topleft Blue) (bottomright Blue)
orientation	Is the blue ship horizontal? How many ships are horizontal? Are there more horizontal ships than vertical ships? Are all ships horizontal? Are all ships vertical? Are the blue ship and the red ship parallel? Do the blue ship and the red ship touch? Are any of the ships touching? Does the blue ship touch any other ship? Does the blue ship touch both other ships?		
touching	What is the location of one blue tile? At what location is the top left part of the blue ship? At what location is the bottom right part of the blue ship?		
demonstration			

# Defining an infinite set of questions through compositionality



# **Question asking as program generation**

**Example for ideal observer finding the optimal/most informative question:**

# Question asking as program generation

Example for ideal observer finding the optimal/most informative question:

```
(+
  (
    +
      (* 100 (size Purple) )
      (* 10 (size Blue) )
  )
  (size Red)
)
```

# Question asking as program generation

Example for ideal observer finding the optimal/most informative question:

```
(+
  (
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      (* 100 (size Purple) )
      (* 10 (size Blue) )
    )
    (size Red)
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)
```

**Learning a probabilistic generative model of questions:**

Goal: predict human questions in novel scenarios

$x$  : question

$f(\cdot)$  : features (Expected Info. Gain, length, answer type, etc.)

$\theta$  : trainable parameters

energy:

$$\mathcal{E}(x) = \theta_1 f_1(x) + \theta_2 f_2(x) + \cdots + \theta_K f_K(x)$$

generative model:

$$P(x; \theta) = \frac{\exp^{-\mathcal{E}(x)}}{\sum_{x' \in X} \exp^{-\mathcal{E}(x')}}$$

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**Compositionality is key!**

energy:

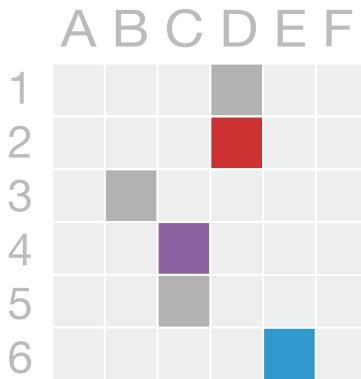
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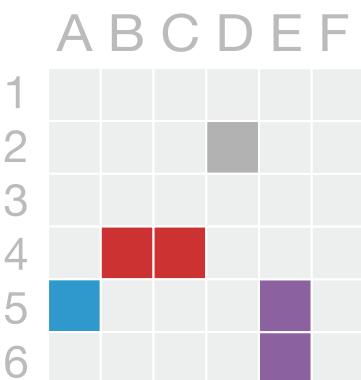
# Asking novel questions through program generation

## Context 7



EIG	Question/Program	Human
2.44	How many tiles are occupied by ships? (++ (map (lambda x (size x)) (set Blue Red Purple)))	
1.79	How many ships are 4 tiles long? (++ (map (lambda x (== (size x) 4)) (set Blue Red Purple)))	
Energy	Question/Program	
6.53	What is the column of the bottom right water tile? (coll (bottomright (coloredTiles Water)))	
7.88	What is the row of the top left purple tile? (rowL (topleft (coloredTiles Purple)))	
8.90	Are all the ships horizontal? (all (map (lambda x (== H (orient x))) (set Blue Red Purple)))	
10.51	What is the column of the bottom right of the tiles with the same color as tile 3E? (coll (bottomright (coloredTiles (color 3E))))	
12.89	Are any of the ship sizes greater than 2? (any (map (lambda x (> (size x) 2)) (set Blue Red Purple)))	

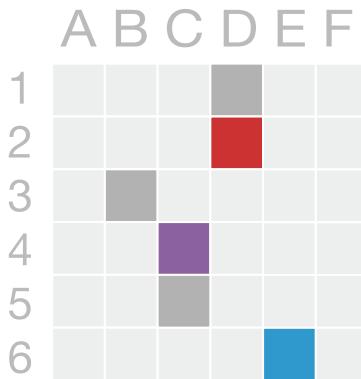
## Context 9



EIG	Question/Program	Human
1.59	How many tiles in row 4 are occupied by ships? (++ (map (lambda y (and (== (rowL y) 4) (not (== (color y) Water)))) (set 1A ... 6F)))	
1.56	How many tiles is the purple ship? (size Purple)	
Energy	Question/Program	
7.48	What is the column of the bottom right blue tile? (coll (bottomright (coloredTiles Blue)))	
8.74	How many tiles have the same color as tile 4A? (setSize (coloredTiles (color 4A)))	
9.94	What is the top left of all the ship tiles? (topleft (setDifference (set 1A ... 6F) (coloredTiles Water)))	
10.98	What is the color of the top left of the tiles that have the same color as 5C? (color (topleft (coloredTiles (color 5C))))	
16.34	Are blue and purple ships touching and red and purple not touching (or vice versa)? (== (touch Blue Purple) (not (touch Red Purple)))	

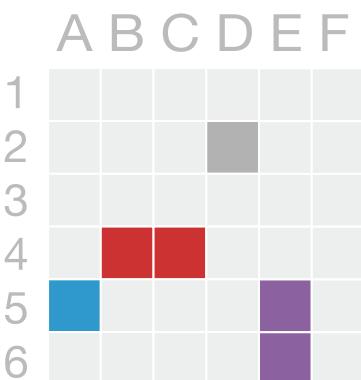
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## **Computational Cognitive Modeling**

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# **Probabilistic programming, program induction, and language of thought models**

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Brenden Lake & Todd Gureckis

**course website:**

<https://brendenlake.github.io/CCM-site/>