Project 1

The report includes five parts, including the most consistent models for each client in part 1, the optimal allocation in part 2 (it is also the statement of the optimal allocation weights for each client), the most restrictive primary characteristic (shadow price of RHS) in part 3, objective function change rate in part 4 (reduced cost) and other valuable insights.

(1)
After the regression, we can get the objective value of each clients for different models with the command “mipObjValGLPK(lp)”.

Home office model results

	ZACHARY ZENIC	YOLANDA YEATS
MODEL 1	200.8	393.1
MODEL 2	171	319.75
MODEL 3	144.9	290.05

Therefore, model 3 is requires the smallest changes for two clients. And the minimum penalty for Zachary Zenic is 144.9 while the minimum penalty for Yolanda Yeats is 290.05.

(2)
Similarly, we can receive the allocations for each client with model 3 with command “mipColsValGLPK(lp)”. The first 15 values are the weights of the portfolio.

Optimal allocation weights for each client

	ZACHARY ZENIC	YOLANDA YEATS
U.S. LARGE CAP EQUITY	28.85%	0
U.S. MID CAP EQUITY	0	0
U.S. SMALL CAP EQUITY	0	4.3%
DEVELOPED NON-U.S. EQUITY	20.15%	0
EMERGING MARKET EQUITY	0	28.7%
GLOBAL EQUITY	0	0
GLOBAL SMALL CAP EQUITY	0	2%
REITS	4%	11%
LISTED INFRASTRUCTURE	0	1%
COMMODITIES	0	0
EMERGING MARKET DEBT	0	2%
HIGH YIELD BONDS	0	4%
CORE U.S. BONDS	35%	47%
SHORT DURATION CREDIT BONDS	4%	0
TIPS	8%	0

(3)
The shadow price indicates the value of allowing a 1% deviation between the client portfolio and the Home Office model for primary characteristic. Therefore, we can fix the binary variables to compute the linear programming. The dual values of rows are the shadow prices. By the way, the first primary characteristic is totally equivalent to the second primary characteristic so the following

chart only shows the shadow prices of first and third characteristics. The command is "getRowsDualGLPK(lp)".

MODEL 3	ZACHARY ZENIC	YOLANDA YEATS
FIRST/ SECOND	0	0
THIRD	-200	300

From above results, third characteristic is most restrictive for Zachary Zenic as well as Yolanda Yeats. The values of allowing a 1% deviation between the client portfolio and the Home Office model for the third characteristic are -200 and 300 respectively.

(4)
If the penalty on secondary characteristics is increased, the coefficients of objective function will increase. The reduced costs will tell the answer of the change rates of objective functions. Recalling that there are two variables (Δ_i^+ *and* $\Delta_i^+, i = 1,2,3,4,5$) for each characteristic due to the absolute value in the objective function. The command "getColsDualGLPK(lp)" will show the reduced costs.

MODEL 3	ZACHARY ZENIC		YOLANDA YEATS	
1	300	300	0	600
2	0	600	600	0
3	100	500	600	0
4	0	600	500	100
5	500	100	0	600

The sum of the pair result is the rate. Therefore, if the penalty on secondary characteristics is increased, the rate of the objective function changes would be 600.

(5)
According shadow prices of secondary characteristics of each client, we can conclude that:

MODEL 3	ZACHARY ZENIC	YOLANDA YEATS
1	0	300
2	300	-300
3	200	-300
4	300	-200
5	-200	300

Therefore, the least restrict secondary characteristic for Zachary Zenic is the first one while the least restrict secondary characteristic for Yolanda Yeats is the fourth one.

Another interesting point is that before I conducted the linear programming, I guessed Model 3 was most suitable for Zachary Zenic and Model 1 was most suitable for Yolanda Yeats because they need the least change if the client changes to the weights which are exactly the same as the model. However, actually, the model 3 is the right answer in part 1. The reason is that when I made a guess, the LP for me is to minimize the change of the client portfolio and the model without constraints. The actual problem is much more complicated and data and quantitative analysis are more convincing. My guess is similar as a first impression of the optimization problem, but the actual solutions are far from the initial guess. It implies the power of quantitative analysis.

Appendix: LP result for first client

After inputting the value of objective function and constraints, we can conduct linear program, and the codes and results for LP is shown below. Here is the first client with third model. Other models and second client are similar.

```
>
> solveMIPGLPK(lp)
GLPK Integer Optimizer, v4.65
39 rows, 70 columns, 145 non-zeros
15 integer variables, 10 of which are binary
Integer optimization begins...
Long-step dual simplex will be used
+    35: mip =     not found yet >=              -inf        (1; 0)
+    57: >>>>>   1.469000000e+02 >=   4.380000000e+01  70.2% (5; 0)
+    63: >>>>>   1.460000000e+02 >=   6.400000000e+01  56.2% (3; 2)
+    66: >>>>>   1.449000000e+02 >=   7.600000000e+01  47.6% (3; 3)
+    78: mip =   1.449000000e+02 >=     tree is empty   0.0% (0; 19)
INTEGER OPTIMAL SOLUTION FOUND
[1] 0
>
> # get retsults of MIP solution
> mipStatusGLPK(lp)
[1] 5
> status_codeGLPK(mipStatusGLPK(lp))
[1] "solution is optimal"
>
> # report results
> mipObjValGLPK(lp)
[1] 144.9
> mipColsValGLPK(lp)
 [1] 0.2885 0.0000 0.0000 0.2015 0.0000 0.0000 0.0000 0.0400 0.0000 0.0000 0.0000 0.0000 0.3500 0.0400 0.0800 0.0000 0.0630
[18] 0.0000 0.0600 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0600 0.0400 0.0000 0.0400 0.0300 0.0200 0.0000 0.0100
[35] 0.0100 0.0200 0.0400 0.0000 0.0000 0.0000 0.1285 0.0000 0.0000 0.1215 0.0000 0.0000 0.0000 0.0200 0.0000 0.0000 0.0000
[52] 0.0000 0.0000 0.0000 0.0000 1.0000 0.0000 0.0000 1.0000 0.0000 0.0000 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 1.0000
[69] 1.0000 1.0000
```

```
> OptimalCols = mipColsValGLPK(lp)
>
> reclass = OptimalCols[56:70]
> print(OptimalCols[56:70])
 [1] 1 0 0 1 0 0 0 1 0 0 0 0 1 1 1
> # reset the bound on the u and run the simplex method again
> clower<-c(rep(0,55),reclass)
> cupper<-c(rep(1,55),reclass)
> setColsBndsGLPK(lp,c(1:ncols),clower,cupper)
>
> solveSimplexGLPK(lp)
GLPK Simplex Optimizer, v4.65
39 rows, 70 columns, 145 non-zeros
     78: obj =   0.000000000e+00 inf =   7.460e+00 (14)
     99: obj =   1.449000000e+02 inf =   7.633e-17 (0)
*   107: obj =   1.449000000e+02 inf =   4.857e-17 (0)
OPTIMAL LP SOLUTION FOUND
[1] 0
> #printRangesGLPK(lp,fname="sensitivity.txt")
> getRowsDualGLPK(lp)
 [1]  200    0 -200    0  300  200  300 -200 -200  200  200 -200  200  200  200 -200  200  200  200  200 -200    0    0    0
[25] -400 -700    0 -400 -430 -700    0 -400 -400 -700 -700    0    0    0    0
> getColsDualGLPK(lp)
 [1]    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0  300    0  100    0  500  300  600  500  600
[25]  100  400    0    0  400    0    0    0  400    0    0    0    0  400  200  200    0  400  400    0  400  400  400    0
[49]  400  400  400  400    0  200  200    0 -400 -700    0 -400 -430 -700    0 -400 -400 -700 -700    0    0    0
> getObjValGLPK(lp)
[1] 144.9
```