

$$x^2 \geq (ax)^2 + [(1-a)x]^2 \geq 2x\left(\frac{x}{2}\right)^2, a \in [0, 1]$$