ELEC ENG 3CL4: Introduction to Control Systems
Lab 2: Closed-loop System Identification

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Objective

To identify the plant model of a marginally-stable servomotor.

Assessment

This laboratory is conducted in groups of no more than two students. Students are required to attend their assigned lab section. The assessment of this lab will occur based on your answers to the pre-lab questions, on your in-lab activities, and on a written laboratory report. Each group is required to submit their answers to the pre-lab questions electronically to the appropriate submission box on Avenue-to-Learn by 12:01pm on the day of the lab. (That is, just after noon.) Pre-labs submitted after 12:01pm but before 2:30pm will be subject to a penalty of 50%. No marks will be awarded to pre-labs submitted after 2:30pm. You will earn a maximum of 100 marks from Lab 2 activities. Lab 2 will contribute to a maximum of 20% of your total lab grade for this course. The components of the assessment are:

- Pre-lab Questions 1-10, which must be completed before the lab;
- Two experiments (see Section 3);
- A laboratory report (see Section 4).

Your performance of the experiments will be evaluated during the lab by the TA's. The marks for each component are clearly indicated in this document.

1 Description of Laboratory Equipment

As explained in Lab 1, in these laboratories we will deal with (a simulation of) a closed-loop angular positioning system based around a DC motor. Such systems are often used to position heavy or difficult to move objects using a 'command tool' that is easy to move, in which case they are often called servomechanisms. One example of a servomechanism is that involved in moving the control surfaces of an aircraft using a lever in the cockpit. The goal of this lab is to identify the plant model for the subsequent experiments.

In our system, the plant is the motor and its associated electronics. As illustrated in Figure 1, the input to the process is a control voltage, denoted $x(t)$, and the output of the

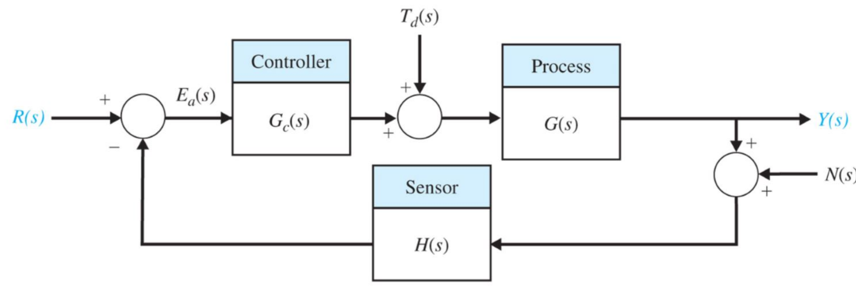

Figure 1: Feedback system with $y(t) = \theta(t)$. We will consider the case in which $H(s) = 1$ and $G_c(s) = K$. We neglect the effects of the disturbance $T_d(s)$ and the noise $N(s)$ in the identification of the plant model. (Figure 4.3 of Dorf and Bishop, *Modern Control Systems*, 11th edition, Prentice Hall, 2008.)

process is the angular position of the shaft, denoted by $\theta(t)$, and measured by an optical encoder. For the purposes of our ELEC ENG 3CL4 labs, we will presume a linear model for the plant. Using information about the structure of the motor and rotational Newtonian mechanics, the linear model for the operation of the motor can be described by the following differential equation:

$$J\frac{d^2\theta(t)}{dt^2} + b\frac{d\theta(t)}{dt} = K_m x(t), \tag{1}$$

where J is the rotational inertia of the motor, b is the coefficient of viscous friction in the motor structure, and K_m is the (internal) gain of the motor. Taking Laplace transforms of both sides of (1) we obtain the transfer function of the plant:

$$s^2 J\Theta(s) + sb\Theta(s) = K_m X(s) \tag{2}$$

$$\implies G(s) = \frac{\Theta(s)}{X(s)} = \frac{A}{s(s\tau_m + 1)}, \tag{3}$$

where $A = K_m/b$ and $\tau_m = J/b$. In this lab we will identify A and τ_m, as these are not known in advance in typical industrial applications. We will do that in two ways:

- The first is based on a time-domain analysis of the step response of a proportionally-controlled closed-loop system with an appropriately chosen gain.

- The second is based on an analysis of the frequency response of a proportionally-controlled closed-loop system with an appropriately chosen gain.

In the following labs we will use the model in (3), with the parameters identified in this laboratory, to design more sophisticated controllers for the servomechanism.

The labs in this course are based around an understanding of the model $G(s)$ in (3). To help develop that understanding, please answer the following questions.

Pre-lab Question 1 (2 marks) *Provide a complete derivation of the step response of the model $G(s)$.*

Pre-lab Question 2 (2 marks) *Is that step response bounded? Justify your answer.*

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2 Closed Loop System Identification

Since it has a pole at $s = 0$, the system $G(s)$ in (3) is only 'marginally stable,' and this can make it very difficult to identify A and τ_m directly. In this section we will show how to set up a stable closed loop system, that can be used to identify A and τ_m.

We will construct a stable closed loop by using a simple proportional controller, $G_c(s) = K$, in the configuration in Figure 1, with $H(s) = 1$.

Pre-lab Question 3 (6 marks) *Provide detailed derivations that show that when $H(s) = 1$,*

(i) the closed loop transfer function from $R(s)$ to $Y(s) = \Theta(s)$ in Figure 1 can be written in the generic form in (4a); and

(ii) when $G(s) = \frac{A}{s(s\tau_m+1)}$ and $G_c(s) = K$ the generic form can be re-written in the form in (4b).

Here,

$$T(s) = \frac{\Theta(s)}{R(s)} = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)} \tag{4a}$$

$$= \frac{KA/\tau_m}{s^2 + (1/\tau_m)s + KA/\tau_m} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{4b}$$

where $\omega_n = \sqrt{\frac{KA}{\tau_m}}$ and $\zeta = \frac{1}{2\omega_n\tau_m}$.

Pre-lab Question 4 (3 marks) *Derive expressions that would enable you to determine A and τ_m if you were given ζ and ω_n.*

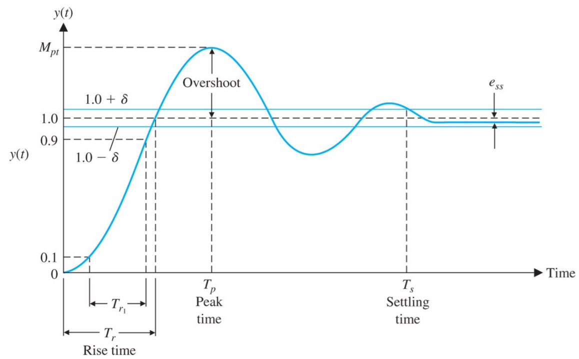

Figure 2: Generic step response of an under-damped second-order system (Figure 5.7 of Dorf and Bishop, *Modern Control Systems*, 11th edition, Prentice Hall, 2008).

2.1 Closed-loop System Identification from the Step Response

If the input $r(t)$ is a unit step function, then the output of the closed-loop system in Figure 1 is called the step response, and can be computed using

$$\theta_{\text{step}}(t) = \mathscr{L}_t^{-1}\left\{\frac{T(s)}{s}\right\}, \tag{5}$$

where $w(t) = \mathscr{L}_t^{-1}\{W(s)\}$ denotes the inverse Laplace Transform of $W(s)$ written as a function of t. Now assume that K is chosen so that the closed-loop system $T(s)$ is under-damped. That is, K is chosen such that $s^2 + (1/\tau_m)s + KA/\tau_m$ has complex roots. Equivalently, K is chosen such that $0 < \zeta < 1$. In that case, it was shown in class and tutorial that for $t \geq 0$,

$$\theta_{\text{step}}(t) = 1 - \tfrac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t + \phi\right), \tag{6}$$

where $\phi = \text{atan}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) = \text{acos}(\zeta)$. A plot of a generic step response from this standard under-damped second-order system is given in Figure 2.

Pre-lab Question 5 (4 marks) *Consider the generic step response from Figure 2. Show that the percent overshoot is*

$$P.O. = 100 \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \tag{7}$$

Pre-lab Question 6 (4 marks) *Show that the peak time is determined by:*

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{8}$$

Pre-lab Question 7 (3 marks) *Derive expressions that would enable you to compute ζ and ω_n if you were given P.O. and T_p.*

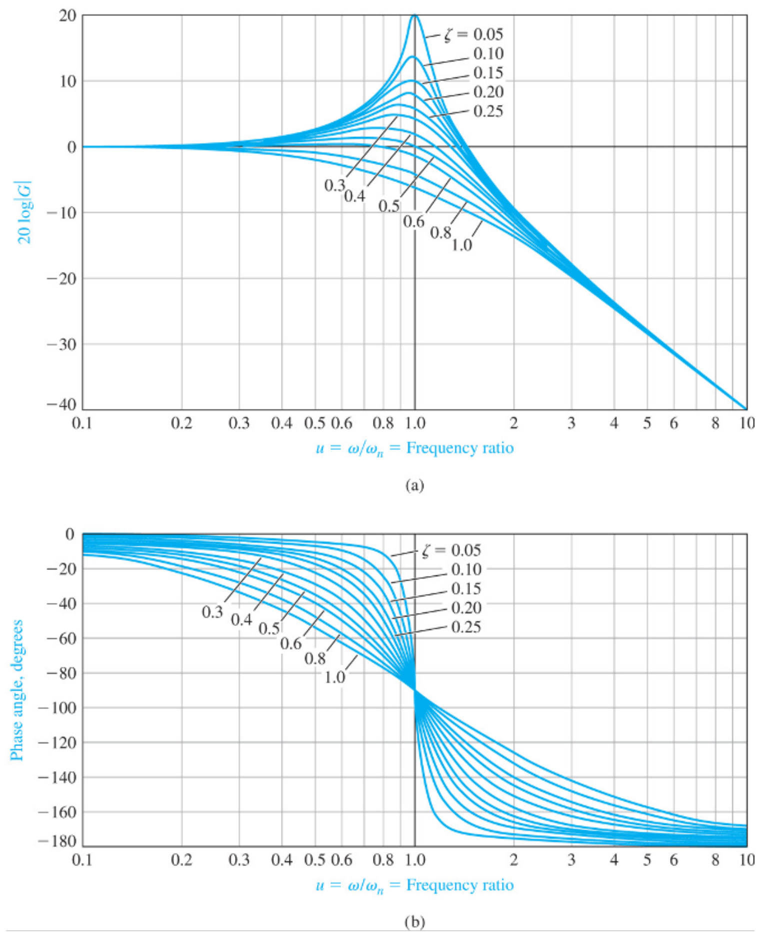

Figure 3: Bode plots of $|T(j\omega)|$ in (9) for various values of ζ with a normalized frequency axis (Figure 8.10 of Dorf and Bishop, *Modern Control Systems*, 11th edition, Prentice Hall, 2008).

2.2 Closed-Loop System Identification using the Frequency Response

If the closed loop is appropriately under-damped, the values of A and τ_m can be identified from the 'peak' of the frequency response of $T(s)$ in (4) in the following way: Observe that

$$|T(j\omega)|^2 = \frac{\omega_n^4}{|\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega|^2}. \tag{9}$$

Sketches of $|T(j\omega)|$ on a log-log scale for different values of ζ are provided in Figure 3.

Pre-lab Question 8 **(4 marks)** *By differentiating the denominator with respect to ω and setting the derivative to zero, show that for $\zeta \leq 1/\sqrt{2}$ the denominator reaches a minimum, and hence $|T(j\omega)|^2$ reaches a maximum, when $\omega = \omega_p$, where*

$$\omega_p = \omega_n\sqrt{1 - 2\zeta^2}. \tag{10}$$

Hint: Recall that for a complex number $a + jb$, the square of the magnitude is $|a + jb|^2 = a^2 + b^2$.

Pre-lab Question 9 **(4 marks)** *Show that the value of the peak is*

$$M_p^2 = \max_{\omega} |T(j\omega)|^2 = |T(j\omega_p)|^2 = \frac{1}{4\zeta^2(1 - \zeta^2)}. \tag{11}$$

Pre-lab Question 10 **(3 marks)** *Derive expressions that would enable you to calculate ζ and ω_n if you were given ω_p and M_p*

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3 Perform Closed Loop Identification

We will perform the above closed loop identification using two approaches:

- Time domain identification: First we will observe the step response of the system using an appropriately-chosen proportional feedback gain. We will measure peak time and percentage overshoot, as defined in Figure 2, based on which we can calculate the unknown plant parameters.

- Frequency domain identification: First we will observe the frequency response of the system using an appropriately-chosen proportional feedback gain. We will measure peak frequency and peak gain, based on which we can calculate the unknown plant parameters.

3.1 Experiment 1: Time Domain Identification (20 marks)

You must demonstrate the closed-loop step response to your TA in order to obtain the marks for this part.

The following steps are recommended in order to measure the overshoot and peak time of the closed-loop system from which the motor transfer function parameters A and τ_m will be calculated.

i) Open Matlab, and allow it to fully open.

ii) Download the simulink file `qube_servo2_EE3CL4_Lab2.slx` from Avenue-to-Learn, and open it. Once having completed those two steps, open Quanser Interactive Labs, and proceed to the "Servo Workspace" as desribed in Lab. 1.

iii) Run the Simulink file, and observe the small non-linear effects that become apparent as the motor slows down. (You also did that in Lab. 1.)

iv) Without stopping the simulation, set the proportional gain to $K = 2$, and observe that the amount of overshoot has increased, and that the impact of the non-linear behaviour of the model is diminished. This makes the choice of $K = 2$ a better choice for closed-loop identification. We will select $K = 2$ for this part of the experiment.

v) Now we need to make the appropriate measurements. We need to:

- Measure the switching times of the input square wave, and the amplitude of the input square wave.
- Measure the height of the first overshoot peak.
- Measure the time of the first overshoot peak.

vi) Pause the simulation, and adjust the scale of the "Sensor Angle" plot so that you can measure the amplitude of the step input, and the amplitude of the first peak. You may wish to use some of the available cursor tools. From these measurements, calculate the percentage overshoot.

vii) Then measure the time difference between the edge of the input step function and the time at which the first peak occurs. This will be the peak time. You should adjust the scale of the time base to simplify this measurement.

viii) Record your measurements and save a copy of this plot by using an appropriate export function so that you can include it in your report.

ix) Use your measurements to calculate the motor parameters A and τ_m. Include yours measurements, your calculations, and your estimates of the motor parameters in your report.

3.1.1 Experiment 1, optional extension: Using automated measurement tools

The experiments in this section are optional, but they will help to introduce you to some of the automated measurement tools that are available in the Simscape toolbox for Matlab.

i) Close Quanser Interactive Tools.

ii) Check to see if you have the Simscape toolbox from Matlab installed.

iii) You can do that by going to the “Add-Ons” icon in the “Home” ribbon in the Matlab window. If you can’t see this, click the word “Home” in the top left of the Matlab window and it should appear. Click “Get Add-Ons” under the “Add-Ons” menu, and in the window that opens, to the right of “MathWorks Toolboxes and Products”, click “Show All”. Find Simscape, and click through the steps to install it. This will involve Matlab being shut down, so be sure to save any work that you have done.

iv) Once Simscape is installed, open Matlab and allow it to fully open.

v) Open Quanser Interactive Tools.

vi) Reload the simulink file `qube_servo2_EE3CL4_Lab2.slx`, and set the proportional gain to $K = 2$.

vii) Restart the simulation

viii) Some tools to help you automate the measurement process will now be available in the “Sensor Angle” display window, under the ruler icon.

- First, there are magnification and scaling tools which will help you display the output at an appropriate scale. These are under the menu with the magnifying glass icon.
- Second, there are tools to help you measure things. These are under the menu with the ruler icon.
- Under the ruler menu, begin by exploring the basic cursor tools. Note that these are only applied to one channel at a time. Make sure that you select the channel that you are interested in.

ix) Repeat your measurements using the basic cursor tools.

x) Compare the results that you have obtained, with the results that are provided by the “Peak Finder” tool that is under the menu with the ruler icon.

xi) While the simulation is still running, compare both sets of measurements with those that are provided by the “Bi-Level Measurements” tool under the ruler icon.

xii) Record all the necessary measurements so that you can discuss them in your report. That discussion would include the impact on the calculation of the motor parameters A and τ_m.

3.2 Experiment 2: Frequency Domain Identification (20 marks)

You must demonstrate the closed-loop frequency response to your TA in order to obtain the marks for this part.

The following steps need to be followed in order to measure the peak frequency ω_p and peak gain M_p of the closed-loop system from which the motor transfer function parameters

A and τ_m will be calculated. From Figure 3 it is apparent that we will need a reasonably underdamped system to be able to measure these parameters effectively.

i) Select an appropriate value for the proportional gain. Seeing as we have already made an attempt at estimating A and τ_m using the time-domain method, one way you could do this is to verify that the choice of $K = 2$ gives a value for ζ that leads to a reasonable peak in Figure 3.

ii) If necessary, close and restart Quanser Interactive Labs.

iii) Open the simulink model for the lab, and set the proportional gain to 2.

iv) Open the signal generator box, and change the signal type to "sine" and set the frequency to 0.5 Hz. Leave the amplitude at 0.6923, which corresponds to commands to move $\pm 45°$ from the straight-ahead position.

v) Run the simulation.

vi) Observe that the signal at the output is about the same size as the signal at the input. This is what we would expect from the magnitude of the frequency response in Figure 3. (Recall that 0 dB corresponds to a gain of 1.) Observe that the "delay" in the signal is quite small. Of course the motor is lagging behind the command, but it is now lagging by much. This is what we would expect from the phase of the frequency response in Figure 3 (i.e., small amounts of negative phase.) However, we should also notice that there are some small non-linear effects where the motor changes direction. (These occur because the motor is moving slowly at those points.) This means the output is not quite sinusoidal.

vii) Without stopping or pausing the simulation, change the input frequency to 1 Hz. After the transients have settled, observe that the gain of the system is now slightly greater than one, that there is a slightly longer delay, and that the non-linear effects have been diminished, but are still there.

viii) Without stopping or pausing the simulation, change the input frequency to 2 Hz. After the transients have settled, observe that the gain of the system is now significantly greater than one, that there is a moderate delay, and that the non-linear effects are negligible. Compare your observations to the magnitude and phase responses in Figure 3.

ix) Without stopping or pausing the simulation, change the input frequency to 3 Hz. After the transients have settled, observe that the gain of the system is now significantly greater than one, that there is a significant delay, and that the non-linear effects are negligible. Compare your observations to the magnitude and phase responses in Figure 3.

x) Without stopping or pausing the simulation, change the input frequency to 4 Hz. After the transients have settled, observe that the gain of the system is now smaller than at 3 Hz, and that the output is almost completely out of phase with the input. That is, there is close to 180° of phase shift from input to output. Compare your observations to the magnitude and phase responses in Figure 3.

xi) Without stopping or pausing the simulation, change the input frequency to 5 Hz. After the transients have settled, observe from the visualization tool that the sinusoid is now at a frequency where the motor cannot follow the command. The output swing is now much less than ±45°. Also observe the phase delay. Compare your observations to the magnitude and phase responses in Figure 3.

xii) Finally, without stopping or pausing the simulation, change the input frequency to 6 Hz, and make some appropriate observations.

xiii) Having made the observations above, begin a search for the frequency at which the magnitude of the frequency response reaches a peak. You can use the cursors on the "Sensor Angle" display to help you, and you may choose to use the peak detection tool as well. Think carefully about your strategy regarding which frequencies to test. Simply gridding up the space is very inefficient. Some of your algorithms from 2SI4 and other courses, including variations on the concept of bisection search may greatly reduce the amount of work that you need to perform in this step. Try to determine the peak frequency with ±0.02 Hz.

xiv) Save a copy of an appropriate display from the "Sensor Angle" display for use in your report, and record your measurements of ω_p and M_p. Use those measurements to calculate A and τ_m.

4 Laboratory report (25 marks)

Each group must submit an electronic report through the course webpage due by 11:59pm one week from the day of the lab. (That is, just before midnight.) For example if your lab is on Monday, your report would be due by 11:59pm the next Monday night. Reports that are late up to 24 hours will receive a penalty of 50%. No marks will be awarded to reports that are more than 24 hours late. The report should be formatted in single-column, single-spaced, using Times New Roman 12 or equivalent font. The group members should clearly state their individual contributions to the report in a statement in the beginning of the report. The laboratory report must include the following items:

- A brief description of the objective of the lab.
- A discussion of the time domain approach for the identification of the motor transfer function parameters. You should include all relevant data plots and calculations.
- A discussion of the frequency domain approach for the identification of the motor transfer function parameters. You should include all relevant data plots and calculations.
- A comparison of the results and discussion of any potential discrepancies.
- A discussion on how we might consider improving the accuracy of the estimation of the model parameters.