EE3CL4 – Lab 3 Prelab Report
L04 - Group 06 - Tuesday

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Contribution:
Yiming Chen takes charge of Q1-8
Ruiyi Deng takes charge of Q9-15

Q1: $G(s) = \frac{A}{s(s\tau_m + 1)}$

$G_c(s) = k_p$

$H(s) = 1$ $N(s)$ can be neglected

$$Y(s) = [-Y(s)H(s) + R(s)]\, G(s)G_c(s)$$

$$[1 + G(s)G_c(s)]\, Y(s) = G(s)G_c(s)R(s)$$

$$\therefore T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)} = \frac{k_p G(s)}{1 + k_p G(s)}$$

$$1 + k_p G(s) = 0$$

$$1 + \frac{A k_p}{s(s\tau_m + 1)} = 0$$

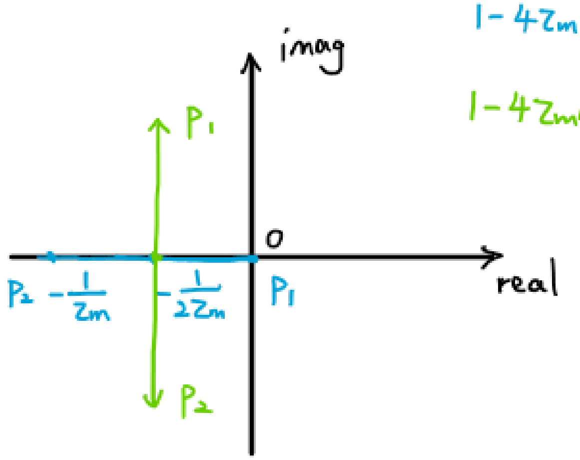
$$\tau_m s^2 + s + A k_p = 0$$

$$P_1, P_2 = \frac{-1 \pm \sqrt{1 - 4\tau_m A k_p}}{2\tau_m} \qquad P_1 = \frac{-1 + \sqrt{1 - 4\tau_m A k_p}}{2\tau_m}$$

$$P_2 = \frac{-1 - \sqrt{1 - 4\tau_m A k_p}}{2\tau_m}$$

The process above is for the deriviation of poles.

Q2:



$1 - 4\tau_m A k_p > 0$ $\quad k_p < \frac{1}{4\tau_m A}$

$1 - 4\tau_m A k_p < 0$ $\quad k_p > \frac{1}{4\tau_m A}$

Q3: $T(s) = \dfrac{k_p G(s)}{1 + k_p G(s)} = \dfrac{Ak_p}{Z_m s^2 + s + Ak_p}$

$$= \frac{\frac{Ak_p}{Z_m}}{s^2 + \frac{1}{Z_m}s + \frac{Ak_p}{Z_m}}$$

Second Order System:

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\therefore \quad \omega_n = \sqrt{\dfrac{Ak_p}{Z_m}}, \quad \zeta\omega_n = \dfrac{1}{2Z_m}, \quad \zeta = \dfrac{1}{2\sqrt{k_p A Z_m}}$

Q4: If $k_p = \dfrac{1}{4AZ_m}$.

damping ratio. $\zeta = \dfrac{1}{2 \cdot \sqrt{\dfrac{AZ_m}{4AZ_m}}} = 1$.

Thus, the system is critically damped.

Q5: $k_p > \dfrac{1}{4AZ_m} \qquad 1 - 4Z_m A k_p < 0 \qquad \zeta = \dfrac{1}{\sqrt{4k_p A Z_m}}$

$$P_1, P_2 = \frac{-1 \pm \sqrt{1 - 4Z_m A k_p}}{2Z_m}$$

$$= \frac{-1 \pm j\sqrt{4Z_m A k_p - 1}}{2Z_m}$$

$$= \frac{1}{2Z_m}\left[-1 \pm j\sqrt{\frac{1 - \frac{1}{4Z_m A k_p}}{\frac{1}{4Z_m A k_p}}}\right]$$

$$= \frac{1}{2Z_m}\left(-1 \pm \frac{j\sqrt{1-\zeta^2}}{\zeta}\right) \qquad \phi = \cos^{-1}(\zeta)$$

$\sqrt{1-\zeta^2} \iff \tan^{-1}\phi = \dfrac{\sqrt{1-\zeta^2}}{\zeta}$

$\therefore \quad P_1 \cdot P_2 = \dfrac{1}{2Z_m}(-1 \pm j\tan^{-1}\phi)$

Q6: $\xi \omega_n = \frac{1}{2 Z_m}$

$$T_s \approx \frac{4}{\xi \omega_n} = 8 Z_m$$

$$P.O. = 100 \exp\left(-\frac{\pi \xi}{\sqrt{1-\xi^2}}\right)$$

$$= 100 \exp\left(-\frac{\frac{\pi}{2\omega_n Z_m}}{\sqrt{1-\frac{1}{4\omega_n^2 Z_m^2}}}\right)$$

$$= 100 \exp\left(-\frac{\pi}{\sqrt{4\omega_n^2 Z_m^2 - 1}}\right)$$

$$= 100 \exp\left(-\frac{\pi}{\sqrt{4 \cdot \frac{k_p A}{Z_m} Z_m^2 - 1}}\right)$$

$$= 100 \exp\left(-\frac{\pi}{\sqrt{4 k_p A Z_m - 1}}\right)$$

$$T_{r1} \approx \frac{2.16 \xi + 0.6}{\omega_n}$$

$$= \frac{2.16 \cdot \frac{1}{2\omega_n Z_m} + 0.6}{\omega_n}$$

$$= \frac{2.16 + 1.2 \omega_n Z_m}{2 Z_m} = \frac{2.16 + 1.2\sqrt{k_p A Z_m}}{2 k_p A}$$

Q7: With increasing k_p, we will have:

T_s does not change;

P.O. increases and approaches a horizontal asymptote of 100%. It reflectes on the behavior of under-damped system;

T_r decreases and approaches a horizontal asymptote of 0%.

Q8: The settling time T_s cannot be controlled by k_p.
Through k_p we can only control the percentage of overshoot as well as the 10% to 90% rise time T_r.

Q9: $\theta(s) = T(s)\left(R(s) + T_d(s)/G_c\right)$

$$\Rightarrow \theta(s) = \frac{\frac{k_p A}{\tau_m}}{s^2 + \frac{1}{\tau_m} s + \frac{k_p A}{\tau_m}} \cdot R(s) + \frac{\frac{A}{\tau_m}}{s^2 + \frac{1}{\tau_m} s + \frac{k_p}{\tau_m}} \cdot T_d(s)$$

Q_{10}: $E(s) = R(s) - Y(s)$

$$= R(s) - R(s) \cdot T(s)$$

$$= R(s) \cdot \left[1 - \frac{G_c(s) \cdot G(s)}{1 + G_c(s) \cdot G(s)}\right]$$

$$= \frac{R(s)}{1 + G_c(s) \cdot G(s)}$$

$$\because R(s) = \frac{\theta d}{s}$$

$$\therefore \Rightarrow \frac{\frac{\theta d}{s}}{1 + G_c(s) \cdot G(s)}$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot \frac{\frac{\theta d}{s}}{1 + G_c(s) G(s)} = \frac{\theta d}{1 + G_c(0) \cdot G(0)}$$

$$G_c(0) = kp$$
$$G(0) = \infty$$

$$\Rightarrow e_{ss} \approx 0$$

Q11: $E(s) = R(s) - Y(s)$

$$= R(s) - T(s)\left[R(s) + T_d(s)/G_c\right]$$

$$= R(s)\cdot\left[1 - \frac{G_c(s)\cdot G(s)}{1+G_c(s)\cdot G(s)}\right] + \frac{G(s)}{1+G_c(s)G(s)}\cdot T_d(s)$$

$$= \frac{R(s) + T_d(s)\,G(s)}{1+G_c(s)\cdot G(s)}$$

$$= \frac{R(s) + T_d(s)\frac{A}{s(\tau_m s+1)}}{1+\frac{k_p A}{s(\tau_m s+1)}}$$

$$= \frac{R(s)\cdot s(\tau_m s+1) + T_d(s)\cdot A}{K_p A}$$

$$e_{ss} = \lim_{s\to 0} s\cdot \frac{\frac{\theta_d}{s}\cdot s(\tau_m s+1) + T_d(s)\cdot A}{K_p A}$$

$$= \lim_{s\to 0} \frac{\theta_d\cdot s\cdot(\tau_m s+1) + T_d\cdot A}{K_p A} = \frac{T_d}{K_p}$$

Q_{12}: $\theta(s) = T(s) \cdot \left[R(s) + \frac{T_d(s)}{k_p} - \frac{\theta(s) \cdot k_v s}{k_p}\right]$

$$\theta(s) = \frac{k_p G(s)}{1+G(s)k_p} \cdot R(s) + \frac{G(s)}{1+G(s)k_p} \cdot T_d(s) - \frac{G(s) \cdot \theta(s) \cdot k_v s}{1+G(s)k_p} \cdot \theta(s) +$$

$$\theta(s) \cdot \frac{G(s) k_v s}{1+G(s)k_p}$$

$$\theta(s) \, \frac{G(s) k_v s + 1 + G(s) k_p}{1+G(s)k_p} = \frac{k_p G(s)}{1+G(s)k_p} \cdot R(s) + \frac{G(s)}{1+G(s)k_p} \cdot T_d(s)$$

$$\theta(s) = \frac{k_p G(s)}{G(s) k_v s + G(s) k_p + 1} \cdot R(s) + \frac{G(s)}{G(s) k_v s + G(s) k_p + 1} \cdot T_d(s)$$

$$\Rightarrow = \frac{\frac{k_p A}{\tau_m}}{s^2 + s \cdot \frac{k_v A + 1}{\tau_m} + \frac{k_p A}{\tau_m}} \cdot R(s) + \frac{\frac{A}{\tau_m}}{s^2 + s \cdot \frac{k_v A + 1}{\tau_m} + \frac{k_p A}{\tau_m}} \cdot T_d(s)$$

Q_{13}: $E(s) = R(s) - \theta(s)$

$$= \left[1 - \frac{\frac{kPA}{\tau m}}{s^2 + \frac{1+kvA}{\tau m} s + \frac{kPA}{\tau m}}\right] \cdot R(s) - \frac{\frac{A}{\tau m}}{s^2 + s\frac{kvA+1}{\tau m} + \frac{kPA}{\tau m}} \cdot Td(s)$$

$$= \frac{\left(s^2 + \frac{1+kvA}{\tau m} \cdot s\right) R(s) - \frac{A}{\tau m} Td(s)}{s^2 + \frac{1+kvA}{\tau m} \cdot s + \frac{kpA}{\tau m}}$$

$$ess = \lim_{s \to 0} s \cdot \frac{\left(s^2 + \frac{1+kvA}{\tau m} \cdot s\right) R(s) - \frac{A}{\tau m} \cdot Td(s)}{s^2 + \frac{1+kvA}{\tau m} \cdot s + \frac{kpA}{\tau m}}$$

$$= - \frac{Td \cdot \frac{A}{\tau m}}{\frac{Akp}{\tau m}}$$

$$|ess| = \frac{Td}{kp}$$

$$Q_{14}: \quad \Theta(s) = T(s) \cdot \left[R(s) + \frac{T_d(s)}{k_p} \right]$$

$$\Rightarrow \quad T(s) = \frac{\frac{k_pA}{\tau_m}}{s^2 + \frac{1+k_vA}{\tau_m} s + \frac{k_pA}{\tau_m}}$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{k_pA}{\tau_m}}$$

$$2 \xi \omega_n = \frac{1+k_vA}{\tau_m}$$

$$\xi = \frac{1+k_vA}{2\tau_m} \cdot \frac{\sqrt{\tau_m}}{\sqrt{k_pA}}$$

$$\xi = \frac{1+k_vA}{2\sqrt{k_pA \cdot \tau_m}}$$

Q15: $T_s = \frac{4}{\xi \omega_n} = \frac{4}{\frac{1+k_v A}{2\tau_m}} = \frac{8\tau_m}{1+k_v A}$

percentage overshoot $= 100 \cdot \exp\left(-\frac{\pi \xi}{\sqrt{1-\xi^2}}\right)$

$$= 100 \exp\left(-\pi \frac{\frac{1+k_v A}{2\sqrt{k_p A \cdot \tau_m}}}{\sqrt{1-\frac{(1+k_v A)^2}{4 k_p A \cdot \tau_m}}}\right)$$

$$= 100 \exp\left(\frac{-\pi}{\sqrt{\frac{4 k_p A \cdot \tau_m}{(1+k_v A)^2}-1}}\right)$$

$$T_{r_1} = \frac{2.16\xi + 0.6}{\omega_n} = \frac{2.16 \cdot \frac{1+k_v A}{2\sqrt{k_p A \cdot \tau_m}} + 0.6}{\sqrt{\frac{k_p A}{\tau_m}}}$$

$$= 2.16 \frac{(1+k_v A)}{2 k_p A} + 0.6 \frac{\sqrt{\tau_m}}{\sqrt{k_p A}} = \frac{2.16(1+k_v A) + 1.2\sqrt{k_p A \cdot \tau_m}}{2 k_p A}$$

when k_p increases, the settling time will not affect, rise time decrease and maximum overshoot increase.

when k_v increases, the settling time decrease, rice time increase, and maximum overshoot decrease