ELEC ENG 3CL4: Introduction to Control Systems Lab 4: Phase Lead Compensation Using Root Locus Method

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Objective

To use the root locus technique to design a phase lead compensator for a marginally-stable servomotor.

Context

In Lab. 3, we observed that the gain of a proportional controller could be chosen to achieve a desired overshoot in the step response of the closed-loop servomechanism control system. However, we were unable to adjust the settling time of the closed-loop by manipulating the value of the gain. Later, we designed a proportional controller with velocity feedback to eliminate this limitation of the proportional controller.

In this lab, we will design a phase lead compensator that will enable us to place the dominant poles of the closed-loop system at some desired locations in the s-plane. In particular, we will choose the locations of the poles to achieve a desired overshoot and settling for the step response of the closed-loop control system. We will also examine the velocity error constant obtained in this design.

Assessment

This laboratory is conducted in groups of no more than two students. You are required to attend your assigned lab section. The assessment of this lab will occur based on your pre-lab design report, in-lab design and experiment activities, and a final lab report. Each group is required to submit their pre-lab report electronically through the course webpage by 12:01pm on the day of the lab. Pre-labs submitted after 12:01pm but before 2:30pm will be subject to a penalty of 50%. No marks will be awarded to pre-labs submitted after 2:30pm. You will earn a maximum of 100 marks from Lab 4 activities. Lab 4 will contribute to a maximum of 25% of your total lab grade for this course. The components of the assessment are:

- Pre-lab root locus based phase lead control design and evaluation (36 marks);
- In-lab design and experiment activities (39 marks);
- Final laboratory report (25 marks).

Your performance of the experiments will be evaluated during the lab by the TA's. The marks for each component are clearly indicated in this document.

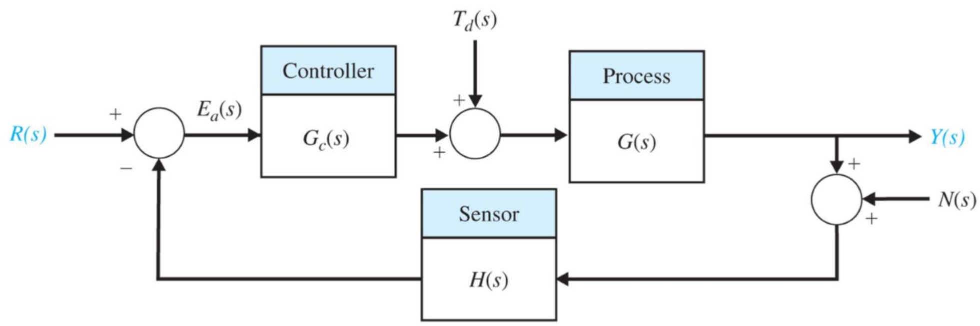

Figure 1: Block diagram of a generic feedback control system. We will focus on the case in which $H(s) = 1$. (Figure 10.1 of Dorf and Bishop, *Modern Control Systems*, 11th edition, Prentice Hall, 2008.)

1 Pre-lab Design Exercise

The purpose of the pre-lab design exercise to prepare you for the actual phase lead controller design during the lab. The plant model parameters and design requirements in the pre-lab are somewhat different from those in the actual lab, but the design steps are essentially the same.

Consider the feedback control system in Figure 1 in the case in which $H(s) = 1$ and $G(s) = \frac{4.7}{s(s+3.2)}$. The goal is to design a phase lead compensator of the form $G_c(s) = k_c \frac{s+z}{s+p}$, $p > z$ so that for a unit step input the percentage overshoot of the output $y(t)$ is 30% and its 2% settling time is 0.75 sec. Follow the the control design steps below to achieve these objectives:

1. Compute the desired closed-loop poles based on the overshoot and settling time requirements given above. **(2 marks)**

2. Assuming a proportional controller, i.e., $G_c(s) = k_c$, sketch the root locus of the closed-loop control system for $0 < k_c < +\infty$. Is there a value of k_c that would place the closed-loop poles at the desired locations? **(6 marks)**

3. Let s_0 be one of the desired dominant closed-loop poles. Compute the angle of $G(s_0)$ and from it the required angle contribution for the controller, $\phi_c = \angle G_c(s_0)$, so that s_0 can actually be a closed-loop pole for some value of k_c. Explain how a phase lead compensator can help provide this required phase. **(4 marks)**

4. Compute the the values of z and p of the phase lead compensator based on ϕ_c in the previous step. Note that you have two free design parameters, but only one condition to satisfy. Therefore, there are many solutions. You will need to choose the value for one of the parameters, and then calculate the value of the other so that the criterion is satisfied. In the design of phase lead compensators, the designer usually selects the value of z and computes p accordingly. Let $s_0 = -\sigma + j\omega$ denote one of the desired dominant closed-loop poles. A typical choice for z is $-\sigma$. Now find the value of p such that

$$\angle G_c(s_0) = \angle(s_0 + z) - \angle(s_0 + p) = \phi_c. \quad \textbf{(6 marks)} \tag{1}$$

5. Sketch the root locus of the closed-loop system as a function of the lead compensator gain k_c. Note that in this case the open-loop transfer function without the gains (refer to the lecture slides) is $P(s) = \frac{s+z}{s+p} \cdot \frac{1}{s(s+3.2)}$. **(8 marks)**

6. Use the magnitude condition, i.e., $|G_c(s_0)G(s_0)| = 1$, to compute the value of k_c that would place two of the closed-loop poles at the desired locations. **(4 marks)**

7. Compute the closed-loop transfer function $T(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)}$ and comment on its form. Do you expect any deviation from the design objectives? Explain. **(4 marks)**

8. Compute the velocity error constant for the feedback control system that you have designed. Recall that for a stable feedback control system, the velocity error constant is defined as $k_v = \lim_{s\to 0} sG_c(s)G(s)$. The steady-state error to a unit ramp input is given by $e_{ss} = \frac{1}{k_v}$. **(2 marks)**

Your pre-lab report must include all relevant "by-hand" computations and plots.

Having done those computations by hand, we now explore the use of computational tools to automate the analysis and design processes. There are no marks for this part of the pre-lab, but this work will be useful for the design work in Section 2.

a) In Matlab type `help tf` and use that function to build a Matlab model for the transfer function $\frac{1}{4.7}G(s) = \frac{1}{s(s+3.2)}$.

b) Type `help rlocus` and use that function to draw a picture of the root locus for the uncompensated motor.

c) Use `tf` to build a transfer function model for your designed compensator function $\frac{s+z}{s+p}$.

d) Type `help series` and use that function to build a model for the compensated open loop without the gains $P(s) = \frac{s+z}{s+p} \cdot \frac{1}{s(s+3.2)}$.

e) Type `figure` and in the new window use `rlocus` to sketch the root locus of the compensated open loop. Type `hold on` because we will be putting more items on this figure.

f) Use `series` to construct a model for $4.7k_cP(s)$ for your chosen value of k_c.

g) Type `help feedback` and use that function to build a model for the closed loop that includes your chosen pole, zero and gain values for your compensator.

h) Type `help pole` and use that function to find the poles of the closed loop.

i) Observe the real part of those poles and confirm that the dominant poles will satisfy the settling time criterion.

j) Compute the angle that the dominant poles make with the negative real axis, and confirm that the cosine of that angle is the desired damping ratio.

k) Using the `plot` function, plot squares at the positions of the closed loop poles on the root locus for the compensated open loop transfer function. By design, these squares should lie on the root locus.

l) Based on observations of the positions of the squares, estimate the extent to which you would expect the step response of the closed loop to match the specified performance requirements.

m) Type `figure`, then type `help step` and use that function to plot the step response of the closed loop with your designed controller. Does it meet your design specifications?

n) Type `figure`, then type `help lsim` and use that function to plot the response to a ramp input of unit slope. Set the time base to be long enough that the response will converge (use the settling time of the step response as a guide), and not that if the signal value is equal to the time value, then you have produced a ramp with unit slope. Does the steady-state error match the predictions that are generated from your calculation of the velocity error constant in Question 8 of the prelab?

o) *This item is optional, as it was added in rev 1 of the lab. However, you might find it useful.* Type `figure`, then use `lsim` to plot the response of the system to two cycles of a triangular wave in which the upslope in the first half of the period looks like the input signal that you used in part n) above, and the downslope in the second half of the period is the reflection of that. Does the behaviour towards the end of each of the sloped sections match the predictions from Question 8?

2 Design of Phase-Lead Compensator

In this section we will design a phase-lead compensator for the model of the Quanser servomotor. You should use the values of A and τ_m that you identified in Lab. 2. If you are unsure that you did that process correctly, or sufficiently accurately, you may wish to perform that parameter identification process again.

1. Follow the procedure in the pre-lab exercise to design a phase lead compensator for the Quanser servomotor to achieve a 20% overshoot and a 2% settling time of 0.5 sec in the step response of the closed-loop control system. **(10 marks)**

2. Compute the velocity error constant, k_v from the open loop system that includes your designed controller. **(2 marks)**

3. With the help of Matlab, compute the closed-loop transfer function $T(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)}$ and its poles and zeros. **(3 marks)**

4. Using Matlab, plot the step response of the closed-loop system. Does your control system satisfy the design objectives? Comment on any potential discrepancy in the results. **(2 marks)**

5. Using Matlab, plot the response of the closed-loop system to a unit ramp input. Does this response satisfy the predictions made by your computation of the velocity error constant? **(2 marks)**

Remark: To obtain marks for this section, you must show a TA your worked solution, including the computed position for the poles, the final form of the controller, and the computed value of k_v. You also need to demonstrate the closed-loop transfer function and its poles and zeros, as well as the closed-loop step response. These results must be included in the final report.

3 Experiment with Phase Lead Compensator (20 marks)

The experiments in this section will examine the step and ramp responses of the closed-loop system with the designed controller in previous section. You should follow the steps below to carry out the experiments.

For the step response:

i) Open Matlab, and allow it to fully open.

ii) Download the simulink file `qube_servo2_EE3CL4_Lab4.slx` from Avenue-to-Learn, and open it. Once having completed those two steps, open Quanser Interactive Labs, and proceed to the "Servo Workspace" as described in Lab. 1.

iii) Make sure that the amplitude of the disturbance signal is set to zero.

iv) Enter the coefficients of the numerator polynomial and the denominator polynomial for the controller that you designed for the model of the Quanser servomotor in Section 2. (Do not implement the controller that you designed in the pre-lab in Section 1, as that controller was designed for a different motor model.)

v) Run the simulation and measure the peak overshoot and the 2% settling time.

Next the ramp response of the closed-loop control system will be examined. For this case:

i) Close and re-open Quanser Interactive Labs.

ii) Set the input signal to a triangular wave, but keep the amplitude and frequency the same.

iii) Run the simulation and measure the steady-state error due to the ramp input.

Remark: To earn marks for the performance of the experiment. You must show to your TA the step and ramp responses of the closed-loop control system.

4 Laboratory Report (25 marks)

Each group must submit an electronic report through Avenue to Learn due by midnight one week from the day of the lab. For example if your lab is on Monday, your report would be due by next Monday at midnight. Reports that are late up to 24 hours will receive a penalty of 50. No marks will be awarded to reports that are more than 24 hours late. The report should be formatted in single-column, single-spaced, using Times New Roman 12 or equivalent font. The group members should clearly state their individual contributions to the report in a statement in the beginning of the report. The laboratory report must include the design calculations and plots of the results of the experiments, followed with a brief analysis of these results. You must compute the values of settling time, percentage overshoot, and steady-state error to a ramp input from the results of the experiments and compare those to the corresponding theoretical values. You should discuss any potential discrepancies between the experimental and theoretical results.