

# Overview of Quantum Control

## Abstract

In recent years, due to the application of quantum control in metrology, quantum optics, quantum computing, quantum communication, chemical physics, and other quantum technologies, scholars in various fields have shown considerable interest in quantum control [1-17]. The most basic problem of quantum control is the controllability analysis of quantum systems, which involves the corresponding Lie group or Lie algebra structure [18], and the second problem is the stability of the quantum system [19,20] and robustness [21,22], entanglement [23] and other performance requirements [24-26]. Among many quantum systems, linear quantum optical systems have received the most extensive research in the field of control, and the results have been the most fruitful [20].

Quantum control can be divided into open-loop quantum control and quantum feedback control. Open-loop quantum control does not require a continuous feedback mechanism, so it is relatively easy to implement. Optimal control [27-42], Lyapunov control [43-49], and Lyapunov optimal control combining the two have been successfully applied to open-loop quantum systems [50]. Feedback control helps to manipulate quantum systems to achieve some pre-required closed-loop characteristics [1, 51]. However, if the controller itself is a classical system dominated by Newtonian physics, this will often destroy the quantum coherence and quantum coherence of the controlled quantum system. Quantum information. Therefore, the recent quantum feedback control research focuses on the realization of the controller with the quantum system, that is, the quantum system is controlled by the quantum system, which is called coherent control [2, 17, 24]. Compared with feedback control based on measurement, the advantage of coherent control is that it retains quantum coherence and can improve the performance and speed of quantum control [3, 9]. The recent quantum control has taken into consideration the situation that when the quantum system interacts with the outside world, the quantum coherence will inevitably be destroyed at this time, and the coherence control is no longer sufficient to control the quantum system. Incoherent control (incoherent control) is needed. control, or non-homology control) strategy. There are three strategies for incoherent control: (1) adding an incoherent electromagnetic field [52], (2) adding an auxiliary system coupled with the environment [53], (3) adding quantum measurement [54,55]. In addition, quantum control continues to be applied to the processing of quantum information [56], such as quantum error correction [57] or the preparation of quantum states.

This overview covers the four major topics discussed in the field of quantum control in recent years: the establishment of quantum modeling, the realization of the quantum estimator, the establishment of quantum actuators, and the establishment of corresponding controls Function adjustment strategy (quantum controller) and other four steps.

## 1. Establishing a quantum model

Regardless of whether it is in the form of an open-loop or a closed-loop, a quantum model is required before performing quantum control. According to different needs, four commonly used quantum models have been proposed:

## 1.1 Bilinear Model

This quantum model is suitable for closed quantum systems without interference from external environments or the intervention of measuring instruments, such as independent molecular systems in chemical reactions, electron spin systems in nuclear magnetic resonance, and so on. Without control intervention, a closed quantum system is described by Schrodinger's equation:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}_0 |\psi(t)\rangle, \quad |\psi(t=0)\rangle = |\psi_0\rangle, \quad (1.1)$$

where  $\hbar$  is Planck's constant,  $|\psi(t)\rangle$  represents a quantum state, which is an infinite-dimensional vector;  $\hat{H}_0$  is the quantum operator is called the free Hamiltonian operator. The control of Eq.(1.1) forms a controlled quantum system through the intervention of the control function  $u_k(t)$ :

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \left[ \hat{H}_0 + \sum_k u_k(t) \hat{H}_k \right] |\psi(t)\rangle. \quad (1.2)$$

This is the bilinear model of the quantum system. The so-called bilinear means that this model contains the multiplication of the linear  $u_k(t)$  and the linear  $\hat{H}_k$ . This quantum model is only suitable for open-loop quantum control because we noticed that this formula does not involve the measurement and feedback of the system output. The bilinear model is most commonly used in electron spin control in nuclear magnetic resonance (NMR). The purpose of quantum control here is to design the control function  $u_k(t)$  so that the quantum state starts from  $|\psi_0\rangle$  and reaches the specified quantum state  $|\psi_f\rangle$  at time  $t = T$ .

## 1.2 Markovian Master Equations

When the quantum state is a mixed state composed of multiple particles, we usually use the density matrix operator  $\hat{\rho}$  to describe the quantum system. For a closed quantum system, the evolution of  $\hat{\rho}$  overtime satisfies the quantum Liouville equation:

$$i \frac{d}{dt} \hat{\rho} = [\hat{H}(t), \hat{\rho}(t)], \quad (1.3)$$

where  $[\hat{\Omega}, \hat{\Lambda}] = \hat{\Omega}\hat{\Lambda} - \hat{\Lambda}\hat{\Omega}$  is called the commutator between  $\hat{\Omega}$  and  $\hat{\Lambda}$ . The above equation is only applicable to closed quantum systems. When the system interacts with the environment, such as the intervention of measuring instruments, some interaction quantities need to be considered. When the interaction time between the system and the environment is very short (the measurement is completed in an instant, that is, projective measurement), the dynamic effect of the measurement at this time can be ignored (memoryless), then the measured quantum system can be approximated by Markovian to get

$$i \frac{d}{dt} \hat{\rho} = [\hat{H}(t), \hat{\rho}(t)] + \frac{1}{2} \sum_{j,k=0}^{N^2-1} \alpha_{jk} \{[\hat{F}_j \hat{\rho}(t), \hat{F}_k^\dagger] + [\hat{F}_j, \hat{\rho}(t) \hat{F}_k^\dagger]\}, \quad (1.4)$$

where  $\{\hat{F}_j\}_{j=0}^{N^2-1}$  is a set of base operators, the coefficient matrix  $A = (\alpha_{jk})$  is a non-negative definite matrix, Eq. (1.4) is called the Markovian master equation of  $N$  dimension. Note that in the case of finite dimensions, all operators can be expressed in the form of a matrix. Eq. (1.4) is widely used in Markovian-style quantum feedback.

### 1.3 Stochastic Master Equations

In quantum feedback control, we must make continuous measurement and feedback of a certain physical quantity  $X$ . This measurement action will continuously change the quantum system. In contrast, the Markov master equations mentioned in Sec.1.2 are suitable for instantaneous measurements. Under the effect of continuity measurement, the equation describing the quantum system is called Stochastic Master Equation:

$$d\hat{\rho} = -i[\hat{H}, \hat{\rho}]dt - \kappa [\hat{X}, [\hat{X}, \hat{\rho}]]dt + \sqrt{2\kappa}(\hat{X}\hat{\rho} + \hat{\rho}\hat{X} - 2\langle\hat{X}\rangle\hat{\rho})dW, \quad (1.5)$$

where  $X$  is the physical quantity to be measured, and  $\hat{X}$  is its corresponding operator. The parameter  $\kappa$  determines the measurement strength;  $\langle\hat{X}\rangle$  represents the expected value of  $X$ ;  $dW$  represents Wiener noise, the expected value is zero, and the variance is  $\langle dW dW^T \rangle = dt$ . Different from instantaneous measurement, the continuous measurement itself is a dynamic process (or it has memory, which is called a non-Markovian process in statistics), which needs to be described by a dynamic equation:

$$dy = 2\sqrt{\kappa}\langle\hat{X}\rangle dt + dW, \quad (1.6)$$

where  $y$  is the output value of the physical quantity  $X$  by measurement. The above two formulas can be derived from quantum filtering theory. Eq. (1.5) is just one form of Stochastic Master Equation (SME) [58,59], and different measurement processes will result in different forms of SME.

### 1.4 Linear Stochastic Differential Equations

The first three quantum modes are all related to the quantum state  $|\psi\rangle$  or the density matrix of the mixed quantum state  $\hat{\rho}$ . When changing over time, we call this type of equation From Schrödinger's view (Schrödinger picture). Another type of quantum dynamic equation is derived from Heisenberg's view (Heisenberg picture), which is to describe the change of quantum operators over time. Such equations mainly appear in linear quantum optics:

$$dx(t) = Ax(t)dt + BdW(t), \quad x(0) = x_0, \quad (1.7a)$$

$$dy(t) = Cx(t)dt + DdW(t), \quad (1.7b)$$

where  $A, B, C, D$  are known matrices, the vector  $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$  is used to represent the state of the system, and each component element  $x_i(t)$  is an operator. Therefore, Eq. (1.7) is describing the dynamics of a group of operators over time, and its initial conditions satisfy:

$$[x_j(0), x_k(0)] = 2i\Theta_{jk}, \quad j, k = 1, 2, \dots, n \quad (1.8)$$

, where  $\Theta_{jk}$  is a known non-negative definite constant matrix.  $y(t)$  is the output signal of the measurement result,  $w(t)$  is the input signal, which contains two components with different properties:

$$dw(t) = \beta_w(t)dt + d\tilde{w}(t), \quad (1.9)$$

where  $\beta_w(t)dt$  is the exact signal,  $d\tilde{w}(t)$  is Wiener noise, with the following auto-correlation matrix

$$\langle d\tilde{w}(t)d\tilde{w}^T(t) \rangle = Fdt, \quad (1.10)$$

where  $F$  is a given non-negative definite matrix. Eq. (1.7) to Eq. (1.10) form a set of linear quantum stochastic differential equations (LQSDE) [19, 60]. For how to use this set of equations in the control of quantum optics. Note that the mathematical form of Eq. (1.7) is the same as the state-space equation in classical system control.

This model has the form of quantum Kalman decomposition [64], which often appears in the field of quantum linear optics [19] and has been extended to the field of nonlinear quantum optics and superconducting quantum circuits [63]. When using the QSDE model, we must pay attention to whether the matrix in the model (corresponding to classical control) is physically achievable in the quantum system. Ian R. Petersen discussed the QSDE model in detail, studied its physical feasibility [60, 61], and discussed how to obtain the corresponding SLH Quantum System Models from the QSDE model [62].

In addition, in recent years, a new quantum system modeling based on the SLH framework has been proposed:

## 1.5 SLH Quantum System Models

- SLH Quantum System Models: Compared with Stochastic Master Equation (SME) equation or Quantum Stochastic Differential Equation (QSDE), another way to describe an open quantum system is to model the quantum system based on actual physical quantities. This is called SLH Quantum System Models. This model contains three elements:  $S$  represents the scattering matrix (defined in Hilbert space),  $L$  is the coupling operator (vector in Hilbert space), and  $H$  is the system Hamiltonian  $H$  (defined in Hilbert space). The self-adjoint operator of special spaces (self-adjoint operator) [65]. For a given SLH model, standard formulas can be used to calculate the corresponding SME model or QSDE model [19, 66]. Conversely, in some cases, the SLH model can be calculated from the SME model or the QSDE model.

## 2. Closed-loop learning control of quantum system

Due to the inherent uncertainty of quantum systems and the inevitable interference from the external environment, this makes quantum closed-loop control necessary. Since 1993, closed-loop quantum control has gradually sprouted and developed, and the first to be studied is the closed-loop learning control method for quantum systems. In the process of using learning strategies to control

quantum systems, three methods have been proposed: (1) Stochastic Learning Method (random learning method) and genetic algorithm (GA), (2) Gradient-Based Method (gradient learning) Method), (3) Reinforcement Learning Method. For a comprehensive introduction to the learning control method, please refer to the long review article "Control of Quantum Phenomenon: Past, Present, and Future" written by Brif [67], which uses the adaptive feedback control point of view to analyze three types. The advantages and disadvantages of the learning method.

## 2.1 Quantum reinforcement learning

Reinforcement learning (RL) [68] is an important machine learning method that has been used to learn and control quantum systems in recent years. At present, a method based on fidelity probability Q-learning has been proposed, which can naturally solve the problem of balance between exploration and development and apply it to the learning control of quantum systems [69,70]. Dong [71] combined quantum computing and reinforcement learning and proposed a new algorithm called quantum reinforcement learning, which was successfully applied to the incoherent control of quantum systems [72] (incoherent control). Bukov [73] proved that in the task of finding short and high-fidelity protocols, the performance of RL is comparable to the best control method. Niu et al. proposed that deep RL can be used to optimize the speed and fidelity of quantum computing at the same time to prevent leakage and random control errors [74]. Fasel et al. showed how to use RL to find a good quantum error correction strategy to protect qubits from noise [75]. Learning control has always been one of the main control methods of quantum systems. Among them, gradient-based methods have shown their efficiency advantages in numerically solving quantum optimal and robust control problems. In addition, evolutionary learning methods, including genetic algorithms and differential evolution algorithms [76], have the potential to control open quantum systems and quantum control designs.

## 2.2 Quantum robust control based on learning

Because the existence of noise and uncertainty is inevitable, the robust control of quantum systems has been recognized as a key task for the development of practical quantum technology. In recent years, the object of quantum control has expanded from a single particle to a non-uniform quantum ensemble, and learning control is the most powerful candidate to achieve robust performance in the problem of non-uniform quantum ensemble control [76]. The object controlled by the inhomogeneous quantum ensemble is composed of many different individual quantum systems (for example, atom, molecule, or spin system), and the dynamic parameters describing the inhomogeneous quantum ensemble have significant uncertainties [77,78], and each independent quantum system in the ensemble has a different initial state. The goal of robust control is to control all independent quantum systems in the ensemble to the same target state under the influence of various uncertainties. Aiming at the problem of non-uniform quantum ensemble control, a sampling-based learning control (SLC) method has been proposed to achieve high-fidelity control of non-uniform quantum ensemble [78]. The SLC control method includes several steps of training, testing, and evaluation. In addition to the non-uniform quantum ensemble, the SLC method is also useful for robust control of single quantum systems with various uncertainties. In order to achieve robust control

of various uncertain quantum systems, the SLC method can be used to search for robust control pulses [79-81]. By exploring the richness and diversity of samples, the performance of the SLC method can be further improved.

Inspired by deep learning, a batch-based gradient algorithm (b-GRAPE) has been proposed, which can effectively search for robust quantum control. The numerical results show that b-GRAPE can achieve higher performance than the SLC method while maintaining high fidelity, and significantly enhancing the robustness of the control [82]. In other applications that need to enhance the robustness of closed-loop learning control, Hessian matrix information can be used [83] or the idea of SLC can be integrated into the learning algorithm to search for the field of robust control. For example, an improved DE algorithm (called msMS\_DE) has been proposed to find powerful femtosecond laser pulses to control the fragmentation of CH<sub>2</sub>BrI molecules [76]. The msMS\_DE method uses multiple samples for fitness evaluation and uses a hybrid strategy for mutation operation.

The future development directions of quantum learning control include: (1) Further develop or improve existing machine learning algorithms to effectively solve complex quantum control problems derived from new quantum technologies (especially multi-dimensional quantum systems); (2) Explore various Applications of cutting-edge machine learning technologies (such as deep learning) in the field of quantum control; (3) Integrating machine learning algorithms into the design of quantum control experiments to save the use of quantum resources (such as quantum entanglement and quantum coherence); (4) Development New quantum machine learning algorithms for simulating complex quantum systems.

### **3. Quantum measurement, quantum filtering, and quantum system identification**

Since any measurement of the quantum state will destroy its existing state to some extent, the development of quantum feedback control is challenged. At first, feedback control can only be cut in from the perspective of learning control. Although the quantum-limited feedback theory was first proposed by Wisemen [8] in 1994, there has been no research on the observability of quantum systems. Great progress. It was not until the late 1990s, with breakthroughs in measurement theory and technology, that the measurement of the state of quantum systems and the study of the combination of classical system control theory began one after another.

- There are two quantum measurement methods. One is the 3D quantum state tomography, which is based on multiple projective measurements, and then combines these projections from different angles to reconstruct the original 3D appearance of the quantum state. The other method is the quantum non-demolition measurement [84], which is a continuous weak measurement because the strength of the measurement is weak enough to not cause the collapse of the quantum state. The measured quantum state can retain its original nature of continuous change. For the research of using continuous weak measurement for parameter estimation, please refer to Mabuchi [85], Gambetta [86]. On the other hand, weak continuity measurement can provide

continuity feedback signals, and thus becomes an indispensable part of quantum feedback control, see Belavkin [87], Lloyd [88], James [89].

- Quantum filtering: In the case of weak continuity measurement, the weaker the measurement intensity, the stronger the proportion of noise. In order to filter out this noise, which inspired the establishment of quantum filtering theory, see Bouten [90] and Bouten [91] (this is a long review paper on quantum filtering theory).
- In 2003, Geremia [92] applied the classical Kalman filtering theory to quantum systems, which resulted in the quantum Kalman filtering theory (quantum Kalman filtering). Quantum filtering theory later added the concept of robustness to resist quantum uncertainty and external disturbances, see Stockton [93], Yamamoto [94], Yamamoto [95].

The purpose of quantum system identification is to estimate the components in the black box by observing the input and output behavior of the black box. When a single quantum mechanical process occurs in the black box. Hsu [97] established a basic and general framework for quantum system identification, which allows us to classify how much knowledge about quantum systems can be obtained in principle from a given experimental setting. When the topology of the system is known, the framework enables us to establish a general standard for the estimability of the coupling constant of its Hamiltonian.

Inspired by the classical system identification theory, the research of quantum parameter identification is booming, among which the Hamiltonian identification has been extensively studied. Since the Hamiltonian of a quantum system determines the evolution of its quantum state [96], the identification of the quantum Hamiltonian has become an important research field. The core problem is to verify the uniqueness of the existence of the quantum Hamiltonian identification solution and the Hamiltonian identifiability of pauses. Le Bris et al. first established the recognizability condition of a closed quantum system driven by a laser field in 2007 and measured the population of all quantum states [96]. Hsu can still find the condition of recognizability by relaxing the above measurement conditions, that is, measuring the limited observable quantity [97]. Sone and Cappellaro [98] in 2017 and Wang et al. [99] in 2020 discussed the identification of the Hamiltonian under a single quantum probe and tested the estimation performance in the presence of Gaussian noise. Guta and Yamamoto discussed the identification of linear passive quantum systems coupled to quantum boson fields in 2016 [100].

The research on the identification of quantum Hamiltonian provides a theoretical analysis of the identification algorithm, which can be divided into two methods: time domain and frequency domain. The time-domain method combines the design of the observer. For example, Kosut and Rabitz [101] use an invariant asymptotic state observer to estimate the parameters in the Hamiltonian and use the gradient algorithm. Bonnabel and Mirrahimi [102] proposed an exponentially convergent adaptive observer for the direct estimation of parameters with Gaussian measurement noise and control. In addition, quantum tomography technology has also been applied to quantum Hamiltonian identification [103]. Literature [104] proposed a two-step identification algorithm for a closed quantum system based on a quantum tomography framework. The algorithm proposed by Jagadish

and Shaji [105] uses measurement data from quantum tomography to identify the coupling Hamiltonian between the qubit and its environment. In addition, a method based on system implementation has also been proposed to identify unknown parameters in the Hamiltonian of linear passive quantum systems [100].

On the other hand, the frequency domain-based quantum Hamiltonian identification method also launched another research route at the same time. Zhang and Sarovar [106] used an equivalent transfer function to estimate the parameters in the Hamiltonian of the spin system, in which the observable value of the time trajectory was measured. This method was later extended to open quantum systems [107]. Fourier analysis has also been applied to quantum Hamiltonian identification. Cole [108] and Schirmer [109] used Fourier transform to identify the Hamiltonian of a two-level closed quantum system for an observable measurement. Burgarth [110] provides a solution for quantum Hamiltonian identification of N-level quantum systems. If classic equipment is used in the measurement process, it may cause the generation of classic colored noise [111-112]. In response to this problem, Dong [113] introduced an enhanced system model in 2020 to describe the total dynamics, parameterized the classical colored noise, and realized the algorithm by using eigenstates to identify quantum systems with unknown parameters.

## 4. Quantum feedback control

There are currently three types of quantum feedback controllers: classical controllers based on Markovian feedback, classical controllers based on Bayesian feedback, and quantum controllers based on quantum coherence. The measurement method used by the Markovian feedback controller [114] is projective measurement. Because the measurement is completed in an instant, the measurement dynamics can be ignored, so the Markovian master equation is used to describe the control process. The measurement method used in Bayesian quantum feedback control is a continuous measurement, so the controlled quantum system is described by the stochastic master equation. Nowadays, Bayesian feedback control has been widely used in the preparation of quantum states [115] and quantum error correction [116, 117].

In quantum coherent feedback control, the controller itself is a quantum system, and the feedback signal processing is also quantum information. That is, quantum coherent feedback control uses one quantum system to control another quantum system. Since the intervention of a quantum controller will not destroy the coherence of the original quantum system, it is called coherent control or coherent control. The three main papers discussing quantum coherent feedback control are Lloyd [17] in 2000, Mabuchi [118] in 2008, and Nurdin [2] in 2009. The commonly used LQG control and robust control in classical systems have been successfully introduced into quantum coherent control, such as the LQG quantum coherent control proposed by Nurdin [2] and the  $H_\infty$  quantum coherent control proposed by James [20]. However, so far, only some non-convex optimization methods have been applied to LQG quantum coherent control [2, 119], and there are still some open problems to be solved for the general solution of LQG quantum coherent control.



Robustness has been recognized as a key issue in the development of quantum control theory and practical quantum technology because most practical quantum systems inevitably have various types of uncertainty and interference. These uncertainties may come from decoherence, systematic errors, environmental noise, or inaccuracy in the identification of Hamiltonian [120-124]. Therefore, in the face of different types of uncertainties in quantum systems, the development of quantum robust control has become the most important topic in the field of quantum control [9,26, 78, 81,125-135,136]. The main research results in recent years are listed below:

- Chen proposed the sampled learning control (SLC), which can control quantum systems with large fluctuations in uncertain parameters [78].
- Dong proposed a sliding mode control for a two-level system with bounded uncertainty in the Hamiltonian, which can ensure the required robustness under the effect of uncertainty [130].
- Qi proposed a two-step strategy combining the concepts of feedback and open-loop control, which can achieve robust control of quantum systems with uncertainties [131].
- Kosut proposed a sequential convex programming method to design robust quantum logic gates [132].
- Soare applied a noise filtering method to enhance the robustness of quantum control [133].
- Quantum coherent control (quantum controller to control the quantum system) is given a strong function, which forms a strong quantum coherent control [9,26, 134-137].
- James [22] solved the problem of quantum controller design for a class of linear random systems, which can limit the adverse effects of external disturbances on quantum performance.
- Maalouf and Petersen [135] designed the  $H_\infty$  quantum coherence controller for a group of linear quantum systems described by annihilation operators.
- Different from only considering interference input [134,135], Xiang [26] further studied the design of robust quantum coherent controllers under the simultaneous action of interference input and parameter uncertainty and proposed a robust control and quantum system scale. Relevance between. Lu [137] further extended the results of [26] to a class of quantum passive systems.
- Most research on quantum coherent control only considers indirect coupling [26,134,135,137], Zhang [9] discussed the coherent feedback control of linear quantum systems with direct and indirect coupling. Xiang [139] further combined the advantages of direct and indirect coupling quantum coherent control design, first designing a static quantum controller with indirect coupling, and then combined with a dynamic quantum controller designed with direct coupling.

## 5. Prospects and challenges of quantum control

In summary, quantum incoherent control, quantum feedback control, robust quantum control, decoherence control, quantum entanglement control, Quantum risk-sensitive control (quantum risk-sensitive control), quantum ensemble control, etc., will still be the main research topics of quantum control in the next few years. However, in terms of future development trends, quantum risk-sensitive control is particularly important. Regardless of the application of quantum field theory [140], or the

field of robust quantum control or quantum estimation [141], risk-sensitive performance indicators will gradually gain a dominant position.

As far as the controlled quantum system is concerned, the development trend of quantum control will be from the previous single control to the current group control, and then to the future many-body control. The physics corresponding to quantum control has gradually expanded from microscopic atomic physics to macroscopic solid-state physics. Compared with ensemble control, which refers to a group of particles that satisfy the Bloch equation. Many-body control directly discusses the quantum emergence of the entire group in the form of quantum field theory. Quantum many-particle systems can exhibit collective phenomena that cannot be imagined in classical systems, such as the spontaneous formation of macroscopic quantum phase coherence in superfluids and superconductors. If collective quantum phenomena can be stably presented under controllable environmental conditions, then revolutionary applications can be imagined. Unfortunately, almost all quantum multiparticle states known to date are discovered haphazardly, and it is difficult to manipulate their properties in a controlled manner. Can quantum control surpass this accidental discovery process and enter a mode of quantum many-body control on purpose? Can we achieve new quantum many-body states and optimize their functions through controlled methods? Several current parallel development research routes show that decisive progress will be made on these issues in the next ten years [142-144].

## Reference

- [1] H.M. Wiseman, G.J. Milburn, *Quantum Measurement and Control*, Cambridge University Press, Cambridge, U.K., 2010.
- [2] H. I. Nurdin, M. R. James, and I. R. Petersen, "Coherent quantum LQG control," *Automatica*, vol. 45, pp. 1837-1846, 2009.
- [3] M. R. James and J. E. Gough, "Quantum Dissipative Systems and Feedback Control Design by Interconnection," *IEEE Transactions on Automatic Control*, vol. 55, no. 8, pp. 1806-1821, Aug. 2010.
- [4] Y. Pan, D. Dong, and I. R. Petersen, "Dark Modes of Quantum Linear Systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 4180-4186, Aug. 2017.
- [5] S. Ma, M. J. Woolley, I. R. Petersen, and N. Yamamoto, "Cascade and locally dissipative realizations of linear quantum systems for pure Gaussian state covariance assignment," *Automatica*, vol. 90, pp. 263-270, Apr. 2018.
- [6] D. Dong and I. R. Petersen, "Sliding mode control of two-level quantum systems," *Automatica*, vol. 48, no. 5, pp. 725 - 735, 2012.
- [7] M. Yanagisawa, H. Kimura, "Transfer function approach to quantum control - part I: Dynamics of Quantum Feedback System", *IEEE Transactions on Automatic Control*, vol. 48, pp. 2107-2120, 2003; part II: Control concepts and applications', *IEEE Trans. Autom. Control*, vol. 48, pp. 2121-2132, 2003.
- [8] H. M. Wiseman and G. J. Milburn, "All-optical versus electro-optical quantum-limited feedback," *Physical Review A*, vol. 49, no. 5, pp. 4110-4125, May 1994.
- [9] G. Zhang and M. R. James, "Direct and Indirect Couplings in Coherent Feedback Control of Linear Quantum Systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 7, pp. 1535-1550, Jul. 2011.
- [10] A. I. Maalouf and I. R. Petersen, "Sampled-data LQG control for a class of linear quantum systems," *Systems & Control Letters*, vol. 61, no. 2, pp. 369-374, Feb. 2012.
- [11] C.-C. Shu, Y. Guo, K.-J. Yuan, D. Dong, and A. D. Bandrauk, "Attosecond all-optical control and visualization of quantum interference between degenerate magnetic states by circularly polarized pulses," *Optics Letters*, vol. 45, no. 4, p. 960, Feb. 2020.

- [12] Y. Guo, C.-C. Shu, D. Dong, and F. Nori, “Vanishing and Revival of Resonance Raman Scattering,” *Physical Review Letters*, vol. 123, no. 22, Nov. 2019.
- [13] B. Qi and L. Guo, “Is measurement-based feedback still better for quantum control systems?,” *Systems & Control Letters*, vol. 59, no. 6, pp. 333–339, Jun. 2010.
- [14] C. Altafini and F. Ticozzi, “Modeling and Control of Quantum Systems: An Introduction,” *IEEE Transactions on Automatic Control*, vol. 57, no. 8, pp. 1898–1917, Aug. 2012.
- [15] F. Ticozzi, K. Nishio, and C. Altafini, “Stabilization of Stochastic Quantum Dynamics via Open- and Closed-Loop Control,” *IEEE Transactions on Automatic Control*, vol. 58, no. 1, pp. 74–85, Jan. 2013.
- [16] N. Yamamoto, “Coherent versus Measurement Feedback: Linear Systems Theory for Quantum Information,” *Physical Review X*, vol. 4, no. 4, Nov. 2014.
- [17] S. Lloyd, “Coherent quantum feedback,” *Phys. Rev. A*, vol. 62, p. 022108, 2000.
- [18] D. D’Alessandro, *Introduction to Quantum Control and Dynamics*, Chapman & Hall/CRC, 2007.
- [19] C. D’Helon, and M. R. James, “Stability, gain, and robustness in quantum feedback networks,” *Phys. Rev. A*, vol. 73, p.053803, 2006.
- [20] M. R. James, H. I. Nurdin, and I. R., Petersen, “ $H_\infty$  control of linear quantum stochastic systems,” *IEEE Trans. Autom. Control*, vol. 53, pp.1787-1803, 2008.
- [21] I. R. Petersen, V. Ugrinovskii, and M. R. James, “Robust stability of uncertain linear quantum systems,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 370, no. 1799, pp. 5354–5363, Nov. 2012.
- [22] M.R. James, I.R. Petersen, V. Ugrinovskii, A Popov stability condition for uncertain linear quantum systems, in: *Proceedings of the 2013 American Control Conference*, Washington, DC, USA, 2013.
- [23] H.-T. Lim, J.-C. Lee, K.-H. Hong, and Y.-H. Kim, “Avoiding entanglement sudden death using single-qubit quantum measurement reversal,” *Optics Express*, vol. 22, no. 16, p. 19055, Jul. 2014.
- [24] C. Xiang, I. R. Petersen, and D. Dong, “Performance Analysis and Coherent Guaranteed Cost Control for Uncertain Quantum Systems Using Small Gain and Popov Methods,” *IEEE Transactions on Automatic Control*, vol. 62, no. 3, pp. 1524–1529, Mar. 2017.
- [25] C. Xiang, I.R. Petersen, D. Dong, Guaranteed cost dynamic coherent control for a class of uncertain linear quantum systems, in: *Proceedings of the 2015 IEEE Multi-Conference on Systems and Control Conference*, Sydney, Australia, 2015.
- [26] C. Xiang, I. R. Petersen, and D. Dong, “Coherent robust  $H_\infty$  control of linear quantum systems with uncertainties in the Hamiltonian and coupling operators,” *Automatica*, vol. 81, pp. 8–21, Jul. 2017.
- [27] A. P. Peirce, and M. A. Dahleh, “Optimal control of quantum-mechanical systems: Existence, numerical approximation and applications,” *Physical Review*, vol. 37, no. 12, pp. 4950-4956, 1988.
- [28] S. Shi, and H. Rabitz, “Selective excitation in harmonic molecular systems by optimally designed fields,” *Chemical Physics*, vol. 139, pp. 185-199, 1989.
- [29] R. Kosloff, S. A. Rice, P. Gaspard, S. Tersigni, and D. J. Tannor, “Wavepacket Dancing: Achieving chemical selectivity by shaping light pulses,” *Chemical Physics*, vol. 139, pp. 201-220, 1989.
- [30] M. Dahleh, A. P. Peirce, and H. Rabitz, “Optimal control of uncertain quantum systems,” *Phys. Rev. A*, vol. 42, pp.1065-1079, 1990.
- [31] W. S. Warren, H. Rabitz, and M. A. Dahleh, “Coherent Control of Quantum Dynamics: The Dream Is Alive,” *Science*, vol. 259, no. 12, pp. 1581-1585, 1993.
- [32] W. S. Zhu, and H. Rabitz, “A rapid monotonically convergent iteration algorithm for quantum optimal control over the expectation value of a positive definite operator,” *J. Chem. Phys.*, vol. 109, pp.385-391, 1998.
- [33] N. Khaneja, R. rockett, and S. J. Glaser, “Time optimal control in spin systems,” *Phys. Rev. A*, vol. 63, p. 032308, 2001.
- [34] U. Boscain, G. Charlot, J. P. Gauthier, S. Gu´erin, and H. R. Jauslin, “Optimal control in laser-induced population transfer for two- and three-level quantum systems,” *J. Math. Phys.*, vol. 43, pp.2107-2132, 2002.
- [35] U. Boscain, and P. Mason, “Time minimal trajectories for a spin 1/2 particle in a magnetic field,” *J. Math. Phys.*, vol. 47, p.062101, 2006.
- [36] N. Khaneja, T. Reiss, B. Luy, and S. J. Glaser, “Optimal control of spin dynamics in the presence of relaxation,” *J.*

- Magn. Reson.*, vol. 162, pp.311-319, 2003.
- [37] D. Dong, J. Lam, and I. R. Petersen, "Robust incoherent control of qubit systems via switching and optimization," *Int. J. Control*, vol. 83, no. 1, pp. 206 - 217, 2010.
  - [38] U. Boscain, and P. Mason, "Time minimal trajectories for a spin 1/2 particle in a magnetic field," *J. Math. Phys.*, vol. 47, p.062101, 2006.
  - [39] D. D'Alessandro, "Optimal evaluation of generalized Euler angles with applications to control," *Automatica*, vol. 40, pp.1997-2002, 2004.
  - [40] L. Viola, and S. Lloyd, "Dynamical suppression of decoherence in two-state quantum systems," *Phys. Rev. A*, 1998, 58, pp.2733-2744.
  - [41] D'Alessandro, and M. Dahleh, "Optimal control of two-level quantum systems," *IEEE Trans. Autom. Control*, vol. 46, pp.866-876, 2001.
  - [42] S. Grivopoulos, and B. Bamieh, "Optimal population transfers in a quantum system for large transfer time," *IEEE Trans. Autom. Control*, vol. 53, pp.980-992, 2008.
  - [43] M. Mirrahimi, P. Rouchon, and G. Turinici, "Lyapunov control of bilinear Schrodinger equations," *Automatica*, vol. 41, pp.1987-1994, 2005.
  - [44] S. Kuang, and S. Cong, "Lyapunov control methods of closed quantum systems," *Automatica*, vol. 44, pp. 98-108, 2008.
  - [45] C. Altafini, "Feedback stabilization of isospectral control systems on complex flag manifolds: application to quantum ensembles," *IEEE Trans. Autom. Control*, vol. 52, pp. 2019-2028, 2007.
  - [46] P. Vettori, "On the convergence of a feedback control strategy for multilevel quantum systems," Sep. ,2002.
  - [47] M. Mirrahimi, P. Rouchon, and G. Turinici, "Lyapunov control of bilinear Schrodinger equations," *Automatica*, vol. 41, pp.1987-1994, 2005.
  - [48] S. Grivopoulos, and B. Bamieh, "Lyapunov-based control of quantum systems," *Proc. of the 42nd IEEE Conf. on Decision and Control*, pp.434-438, Maui, Hawaii USA, 2003.
  - [49] X. Wang and S. G. Schirmer, "Analysis of Lyapunov method for control of quantum states," *IEEE Transactions on Automatic Control*, vol. 55, no. 10, pp. 2259–2270, 2010.
  - [50] S. C. Hou, M. A. Khan, X. Yi, D. Dong, and I. R. Petersen, "Optimal Lyapunov-based quantum control for quantum systems," *Physical Review A*, vol. 86, no. 2, p. 022321, 2012.
  - [51] H. I. Nurdin and N. Yamamoto, *Linear Dynamical Quantum Systems: Analysis, Synthesis, and Control*. Springer International Publishing, 2017.
  - [52] M. Shapiro, and P. Brumer, "Principles of the Quantum Control of Molecular Processes," John Wiley & Sons, Inc., 2003.
  - [53] R. Romano, R., and D. D'Alessandro, "Incoherent control and entanglement for two-dimensional coupled Systems," *Phys. Rev. A*, vol. 73, p.022323, 2006.
  - [54] D. Dong, C. Zhang, H. Rabitz, A. Pechen, and T. J. Tarn, "Incoherent control of locally controllable quantum systems," *J. Chem. Phys.*, vol. 129, p.154103, 2008.
  - [55] D. Dong, J. Lam, and T. J. Tarn, "Rapid incoherent control of quantum systems based on continuous measurements and reference model," *IET Control Theory Appl.*, vol. 3, pp.161-169, 2009.
  - [56] M. A. Nielsen, and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, 2000.
  - [57] J. Kerckhoff, H. I. Nurdin, D. S. Pavlichin, and H. Mabuchi, "Designing Quantum Memories with Embedded Control: Photonic Circuits for Autonomous Quantum Error Correction," *Physical Review Letters*, vol. 105, no. 4, Jul. 2010.
  - [58] K. Jacobs, D. A. Steck, "A straightforward introduction to continuous quantum measurement," *Contemp. Phys.*, vol. 47, pp.279-303, 2006.
  - [59] B. Qi, "On the quantum master equation under feedback control," *Science in China Series F: Information sciences*, vol. 52, pp 2133-2139, 2009.
  - [60] A. J. Shaiju and I. R. Petersen, "A Frequency Domain Condition for the Physical Realizability of Linear Quantum Systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 8, pp. 2033–2044, Aug. 2012.
  - [61] I. R. Petersen, "Quantum Linear Systems Theory," *The Open Automation and Control Systems Journal*, vol. 8, no.

- 1, pp. 67–93, Oct. 2016.
- [62] I. R. Petersen, “Networked Quantum Systems,” *Systems & Control: Foundations & Applications*, pp. 583–618, 2018.
- [63] Bertet P, Ong FR, Boissonneault M, Bolduc A, Mallet F, Doherty AC, Blais A, Vion D, Esteve D, “Circuit quantum electrodynamics with a nonlinear resonator”. In: Dykman M (ed) *Fluctuating nonlinear oscillators: from nanomechanics to quantum superconducting circuits*. Oxford University Press, Oxford, 2012
- [64] G. Zhang, S. Grivopoulos, I. R. Petersen, and J. E. Gough, “The Kalman Decomposition for Linear Quantum Systems,” *IEEE Transactions on Automatic Control*, vol. 63, no. 2, pp. 331–346, Feb. 2018.
- [65] J. Gough, and M. R. James, “The series product and its application to quantum feedforward and feedback networks,” *IEEE Trans. Autom. Control*, vol. 54, pp. 2530–2544, 2009.
- [66] L. Bouten, R. van Handel, and M. R. James, “An introduction to quantum filtering,” *SIAM J. Control Optim.*, vol. 46, pp. 2199–2241, 2007.
- [67] C. Brif, R. Chakrabarti, and H. Rabitz, “Control of quantum phenomena: past, present and future,” *New Journal of Physics*, vol. 12, p. 075008, 2010.
- [68] R. S. Sutton and A. G. Barto, *Reinforcement Learning, second edition: An Introduction*. MIT Press, 2018.
- [69] C. Chen, D. Dong, H.-X. Li, and T.-J. Tarn, “HybridMDP based integrated hierarchical Q-learning,” *Science China Information Sciences*, vol. 54, no. 11, pp. 2279–2294, 2011.
- [70] Chunlin Chen, Daoyi Dong, Han-Xiong Li, Jian Chu, and Tzyh-Jong Tarn, “Fidelity-Based Probabilistic Q-Learning for Control of Quantum Systems,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 25, no. 5, pp. 920–933, May 2014.
- [71] D. Dong, C. Chen, H. Li, and T.-J. Tarn, “Quantum reinforcement learning,” *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 38, no. 5, pp. 1207–1220, 2008.
- [72] D. Y. Dong, C. Chen, T.-J. Tarn, A. Pechen, and H. Rabitz, “Incoherent control of quantum systems with wavefunction controllable subspaces via quantum reinforcement learning,” *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 38, no. 4, pp. 957–962, 2008.
- [73] M. Bukov, A. G. R. Day, D. Sels, P. Weinberg, A. Polkovnikov, and P. Mehta, “Reinforcement Learning in Different Phases of Quantum Control,” *Physical Review X*, vol. 8, no. 3, Sep. 2018.
- [74] M. Y. Niu, S. Boixo, V. N. Smelyanskiy, and H. Neven, “Universal quantum control through deep reinforcement learning,” *npj Quantum Information*, vol. 5, no. 1, Apr. 2019.
- [75] T. Fösel, P. Tighineanu, T. Weiss, and F. Marquardt, “Reinforcement Learning with Neural Networks for Quantum Feedback,” *Physical Review X*, vol. 8, no. 3, Sep. 2018.
- [76] D. Dong, X. Xing, H. Ma, C. Chen, Z. Liu, and H. Rabitz, “Learning-Based Quantum Robust Control: Algorithm, Applications, and Experiments,” *IEEE Transactions on Cybernetics*, vol. 50, no. 8, pp. 3581–3593, Aug. 2020.
- [77] J.-S. Li and N. Khaneja, “Control of inhomogeneous quantum ensembles,” *Physical Review A*, vol. 73, no. 3, Mar. 2006.
- [78] C. Chen, D. Dong, R. Long, I. R. Petersen, and H. A. Rabitz, “Sampling-based learning control of inhomogeneous quantum ensembles,” *Physical Review A*, vol. 89, no. 2, Feb. 2014.
- [79] D. Dong, M. A. Mabrok, I. R. Petersen, B. Qi, C. Chen, and H. Rabitz, “Sampling-Based Learning Control for Quantum Systems With Uncertainties,” *IEEE Transactions on Control Systems Technology*, vol. 23, no. 6, pp. 2155–2166, Nov. 2015.
- [80] D. Dong, C. Chen, B. Qi, I. R. Petersen, and F. Nori, “Robust manipulation of superconducting qubits in the presence of fluctuations,” *Scientific Reports*, vol. 5, no. 1, Jan. 2015.
- [81] C. Wu, B. Qi, C. Chen, and D. Dong, “Robust Learning Control Design for Quantum Unitary Transformations,” *IEEE Transactions on Cybernetics*, vol. 47, no. 12, pp. 4405–4417, Dec. 2017.
- [82] R.-B. Wu, H. Ding, D. Dong, and X. Wang, “Learning robust and high-precision quantum controls,” *Physical Review A*, vol. 99, no. 4, Apr. 2019.
- [83] X. Xing, R. Rey-de-Castro, and H. Rabitz, “Assessment of optimal control mechanism complexity by experimental landscape Hessian analysis: fragmentation of CH<sub>2</sub>BrI,” *New Journal of Physics*, vol. 16, no. 12, p. 125004, Dec. 2014.
- [84] V. B. Braginsky, and F. Y. Khalili, “Quantum non-demolition measurements: the route from toys to tools,” *Rev.*

- Mod. Phys.*, vol. 68, pp. 1-11, 1996.
- [85] H. Mabuchi, "Dynamical identification of quantum open quantum systems," *Quantum Semiclass. Opt.*, vol. 8, pp. 1103-1108, 1996.
  - [86] J. Gambetta, and H. M. Wiseman, "State and dynamical parameter estimation for open quantum systems," *Phys. Rev. A*, vol. 64, p. 042105, 2001.
  - [87] V. P. Belavkin, "Measurement, filtering and control in quantum open dynamical systems," *Rep. Math. Phys.*, vol. 43, pp. 405-425, 1999.
  - [88] S. Lloyd, and J. J. E. Slotine, "Quantum feedback with weak measurements," *Phys. Rev. A*, vol. 62, p. 012307, 2000.
  - [89] M. R. James, "Risk-sensitive optimal control of quantum systems," *Phys. Rev. A*, vol. 69, p. 032108, 2004.
  - [90] L. Bouten, R. van Handel, and M. R. James, "An introduction to quantum filtering," *SIAM J. Control Optim.*, vol. 46, pp. 2199-2241, 2007.
  - [91] L. Bouten, R. van Handel, and M. R. James, "A discrete invitation to quantum filtering and feedback control," *SIAM Review*, vol. 51, pp. 239-316, 2009.
  - [92] J. M. Geremia, J. K. Stockton, A. C. Doherty, and H. Mabuchi, "Quantum Kalman filtering and the Heisenberg limit in atomic magnetometry," *Phys. Rev. Lett.*, vol. 91, p. 250801, 2003.
  - [93] J. K. Stockton, J. M. Geremia, A. C. Doherty, and H. Mabuchi, "Robust quantum parameter estimation: Coherent magnetometry with feedback," *Phys. Rev. A*, vol. 69, p. 032109, 2004.
  - [94] N. Yamamoto, "Robust observer for uncertain linear quantum systems," *Phys. Rev. A*, vol. 74, p. 032107, 2006.
  - [95] N. Yamamoto, and L. Bouten, "Quantum risk-sensitive estimation and robustness," *IEEE Trans. Autom. Control*, vol. 54, pp. 92-107, 2009.
  - [96] C. Le Bris, M. Mirrahimi, H. Rabitz, and G. Turinici, "Hamiltonian identification for quantum systems: Well-posedness and numerical approaches," *ESAIM Control Optim. Calculus Variations*, vol. 13, no. 2, pp. 378-395, 2007.
  - [97] L.-Y. Hsu, "Quantum system identification," *Int. J. Quantum Inf.*, vol. 3, no. supp01, pp. 215-222, Nov. 2005.
  - [98] A. Sone and P. Cappellaro, "Hamiltonian identifiability assisted by a single-probe measurement," *Phys. Rev. A, Gen. Phys.*, vol. 95, no. 2, Feb. 2017.
  - [99] Y. Wang, D. Dong, A. Sone, I. R. Petersen, H. Yonezawa, and P. Cappellaro, "Quantum Hamiltonian identifiability via a similarity transformation approach and beyond," *IEEE Trans. Autom. Control*, pp. 1-1, 2020.
  - [100] M. Guta and N. Yamamoto, "System identification for passive linear quantum systems," *IEEE Trans. Autom. Control*, vol. 61, no. 4, pp. 921-936, Apr. 2016.
  - [101] R. L. Kosut and H. Rabitz, "Identification of quantum systems," *IFAC Proc. Vols.*, vol. 35, no. 1, pp. 397-402, 2002.
  - [102] S. Bonnabel, M. Mirrahimi, and P. Rouchon, "Observer-based Hamiltonian identification for quantum systems," *Automatica*, vol. 45, pp. 1144-1155, May 2009.
  - [103] Y. Wang et al., "Quantum gate identification: Error analysis, numerical results and optical experiment," *Automatica*, vol. 101, pp. 269-279, Mar. 2019.
  - [104] Y. Wang, D. Dong, B. Qi, J. Zhang, I. R. Petersen, and H. Yonezawa, "A quantum Hamiltonian identification algorithm: Computational complexity and error analysis," *IEEE Trans. Autom. Control*, vol. 63, no. 5, pp. 1388-1403, May 2018.
  - [105] V. Jagadish and A. Shaji, "The dynamics of a qubit reveals its coupling to a N-level system," *Ann. Phys.*, vol. 362, pp. 287-297, Nov. 2015.
  - [106] J. Zhang and M. Sarovar, "Quantum Hamiltonian identification from measurement time traces," *Phys. Rev. Lett.*, vol. 113, no. 8, Aug. 2014.
  - [107] J. Zhang and M. Sarovar, "Identification of open quantum systems from observable time traces," *Phys. Rev. A, Gen. Phys.*, vol. 91, no. 5, May 2015.
  - [108] J. H. Cole, S. G. Schirmer, A. D. Greentree, C. J. Wellard, D. K. L. Oi, and L. C. L. Hollenberg, "Identifying an experimental two-state Hamiltonian to arbitrary accuracy," *Phys. Rev. A, Gen. Phys.*, vol. 71, no. 6, Jun. 2005.
  - [109] S. G. Schirmer, D. K. L. Oi, and S. J. Devitt, "Physics-based mathematical models for quantum devices via experimental system identification," *J. Phys., Conf. Ser.*, vol. 107, Mar. 2008.
  - [110] D. Burgarth, K. Maruyama, and F. Nori, "Coupling strength estimation for spin chains despite restricted access,"

- Phys. Rev. A, Gen. Phys., vol. 79, no. 2, Feb. 2009.
- [111] W.-X. Zheng, “Estimation of the parameters of autoregressive signals from colored noise-corrupted measurements,” *IEEE Signal Process. Lett.*, vol. 7, no. 7, pp. 201–204, Jul. 2000.
  - [112] F. Ding, P. X. Liu, and G. Liu, “Auxiliary model based multi-innovation extended stochastic gradient parameter estimation with colored measurement noises,” *Signal Process.*, vol. 89, no. 10, pp. 1883–1890, Oct. 2009
  - [113] L. Tan, D. Dong, D. Li, and S. Xue, “Quantum Hamiltonian Identification With Classical Colored Measurement Noise,” *IEEE Transactions on Control Systems Technology*, vol. 29, no. 3, pp. 1356–1363, May 2021.
  - [114] H. M. Wiseman, and G. J. Milburn, “Quantum theory of optical feedback via homodyne detection,” *Phys. Rev. Lett.*, vol. 70, pp. 548–551, 1993.
  - [115] R. Ruskov, and A. N. Korotkov, “Quantum feedback control of a solid-state qubit,” *Phys. Rev. B*, vol. 66, p.041401, 2002.
  - [116] C. Ahn C., A. C. Doherty, A. J. Landahl, “Continuous quantum error correction via quantum feedback control,” *Phys. Rev. A*, vol. 65, p. 042301, 2002.
  - [117] M. Sarovar, C. Ahn, K. Jacobs, and G. J. Milburn, “Practical scheme for error control using feedback,” *Phys. Rev. A*, vol. 69, p. 052324, 2004.
  - [118] H. Mabuchi, “Coherent-feedback quantum control with a dynamic compensator,” *Phys. Rev. A*, vol. 78, p. 032323, 2008.
  - [119] A. Kh. Sichani, I. G. Vladimirov, and I. R. Petersen, “A numerical approach to optimal coherent quantum LQG controller design using gradient descent,” *Automatica*, vol. 85, pp. 314–326, Nov. 2017
  - [120] Q. Gao, D. Dong, and I. R. Petersen, “Fault tolerant filtering and fault detection for quantum systems,” *Automatica*, vol. 71, pp.125–134, Sept. 2016.
  - [121] R. B. Wu, H. Ding, D. Dong, and X. Wang, “Learning robust and high precision quantum controls,” *Phys. Rev. A*, vol.99, no. 4, p.042327, Apr. 2019.
  - [122] Y. Wang, D. Dong, A. Sone, I. R. Petersen, H. Yonezawa, and P. Cappellaro, “Quantum Hamiltonian identifiability via a similarity transformation approach and beyond,” *IEEE Trans. Autom. Control*, vol.65, no.12, pp.4632–4647, Nov. 2020.
  - [123] Y. Wang, D. Dong, B. Qi, J. Zhang, I. R. Petersen, and H. Yonezawa, “A quantum Hamiltonian identification algorithm: Computational complexity and error analysis,” *IEEE Trans. Autom. Control*, vol. 63, no.5, pp.1388–1403, May 2018.
  - [124] Y. Guo, D. Dong, and C. C. Shu, “Optimal and robust control of quantum state transfer by shaping the spectral phase of ultrafast laser pulses,” *Phys. Chem. Chem. Phys.*, vol. 20, no.14, pp. 9498–9506, Mar. 2018.
  - [125] S. Wang and D. Dong, “Fault-tolerant control of linear quantum stochastic systems,” *IEEE Trans. Autom. Control*, vol. 62, no.6, pp.2929–2935, Jun. 2017.
  - [126] C. Xiang, I. R. Petersen, and D. Dong, “Performance analysis and coherent guaranteed cost control for uncertain quantum systems using small gain and Popov methods,” *IEEE Trans. Autom. Control*, vol. 62, no.3, pp.1524–1529, Mar. 2017.
  - [127] H. J. Ding and R. B. Wu, “Robust quantum control against clock noises in multiqubit systems,” *Phys. Rev. A*, vol.100, no.2, p.022302, Aug. 2019.
  - [128] D. Dong, X. Xing, H. Ma, C. Chen, Z. Liu, and H. Rabitz, “Learning based quantum robust control: Algorithm, applications, and experiments,” *IEEE Trans. Cybern.*, vol. 50, no.8, pp. 3581–3593, Aug. 2020.
  - [129] D. Dong, M. A. Mabrok, I. R. Petersen, B. Qi, C. Chen, and H. Rabitz, “Sampling-based learning control for quantum systems with uncertainties,” *IEEE Trans. Control Syst. Technol.*, vol. 23, no.6, pp. 2155–2166, Nov. 2015.
  - [130] D. Dong and I. R. Petersen, “Notes on sliding mode control of two-level quantum systems,” *Automatica*, vol. 48, no. 12, pp. 3089–3097, Dec. 2012.
  - [131] B. Qi, “A two-step strategy for stabilizing control of quantum systems with uncertainties,” *Automatica*, vol.49, no.3, pp.834–839, Mar. 2013.
  - [132] R. L. Kosut, M. D. Grace, and C. Brif, “Robust control of quantum gates via sequential convex programming,” *Phys. Rev. A*, vol.88, no. 5, p. 052326, Nov. 2013.
  - [133] A. Soare, H. Ball, D. Hayes, J. Sastrawan, M. C. Jarratt, J. J. McLoughlin, Z. Zhen, T. J. Green, and M. J. Biercuk, “Experimental noise filtering by quantum control,” *Nat. Phys.*, vol. 10, pp.825–829, Oct. 2014.

- [134] M. R. James, H. I. Nurdin, and I. R. Petersen, “ $H_\infty$  control of linear quantum stochastic systems,” *IEEE Trans. Autom. Control*, vol.53, no. 8, pp.1787–1803, Sept. 2008.
- [135] A. I. Maalouf and I. R. Petersen, “Coherent  $H_\infty$  control for a class of annihilation operator linear quantum systems,” *IEEE Trans. Autom. Control*, vol. 56, no.2, pp.309–319, Feb. 2011.
- [136] X. Lu and S. Kuang, “Coherent  $H_\infty$  control for linear quantum passive systems with model uncertainties,” *IET Control Theory Appl.*, vol.13, no. 5, pp.711–720, Apr. 2019.
- [137] A. Elahi, A. Alfi, and H. Modares, “ $H_\infty$  consensus control of discretetime multi-agent systems under network imperfections and external disturbance,” *IEEE/CAA J. Autom. Sinica*, vol.6, no. 3, pp. 667–675, May 2019.
- [138] M. Zhou, Z. Cao, and Y. Wang, “Robust fault detection and isolation based on finite-frequency  $H_-/H_\infty$  unknown input observers and zonotopic threshold analysis,” *IEEE/CAA J. Autom. Sinica*, vol.6, no. 3, pp. 750–759, May 2019.
- [139] C. Xiang, I. R. Petersen, and D. Dong, “Static and dynamic coherent robust control for a class of uncertain quantum systems,” *Systems & Control Letters*, vol. 141, p. 104702, Jul. 2020
- [140] C. D’Helon, A. C. Doherty, M. R. James, and S. D. Wilson, “Quantum risk-sensitive control,” in *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 3132–3137, December 2006.
- [141] N. Yamamoto and L. Bouten, “Quantum risk-sensitive estimation and robustness,” *IEEE Transactions on Automatic Control*, vol. 54, no. 1, pp. 92–107, 2009.
- [142] R. S. Bliss and D. Burgarth, “Quantum control of infinite-dimensional many-body systems,” *Physical Review A*, vol. 89, no. 3, Mar. 2014.
- [143] S. E. B. Nielsen, M. Ruggenthaler, and R. van Leeuwen, “Quantum Control of Many-Body Systems by the Density,” arXiv:1412.3794 [quant-ph], Dec. 2014, Accessed: Sep. 16, 2021.
- [144] S. Choi, N. Y. Yao, S. Gopalakrishnan, and M. D. Lukin, “Quantum Control of Many-body Localized States,” arXiv:1508.06992 [cond-mat, physics:quant-ph], Aug. 2015