The workflow of this solution:

1 Segment the input image into superpixels

2 Gauss-Seidel iterations to estimate dichromatic reflectance parameters.

Figure.1 Input image

For superpixel segmentation, Matlab function “superpixels”is used for computing.

https://www.mathworks.com/help/images/ref/superpixels.html

[L,NumLabels] = superpixels(A,N)

`[L,NumLabels] = superpixels(A,N)` computes superpixels of the 2-D grayscale or RGB image .

`A`. `N` specifies the number of superpixels you want to create.

The function returns:

`L`, a label matrix of type `double`, and

`NumLabels`, the actual number of superpixels that were computed.

The `superpixels` function uses the simple linear iterative clustering (SLIC) algorithm [1]. This algorithm groups pixels into regions with similar values. Using these regions in image processing operations, such as segmentation, can reduce the complexity of these operations.

Superpixels boundaries are fist computed out for the input image.

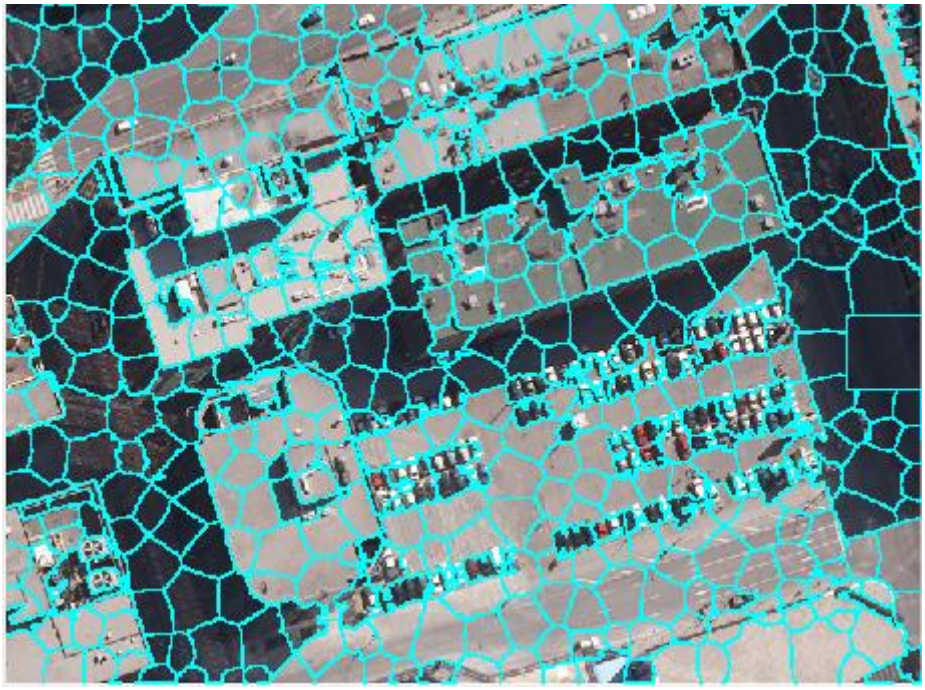

Figure.2 superpixel boundaries overlaid on the original image

For the superpixel image regions, set the color of each pixel in the output image to the mean RGB color of the superpixel region.

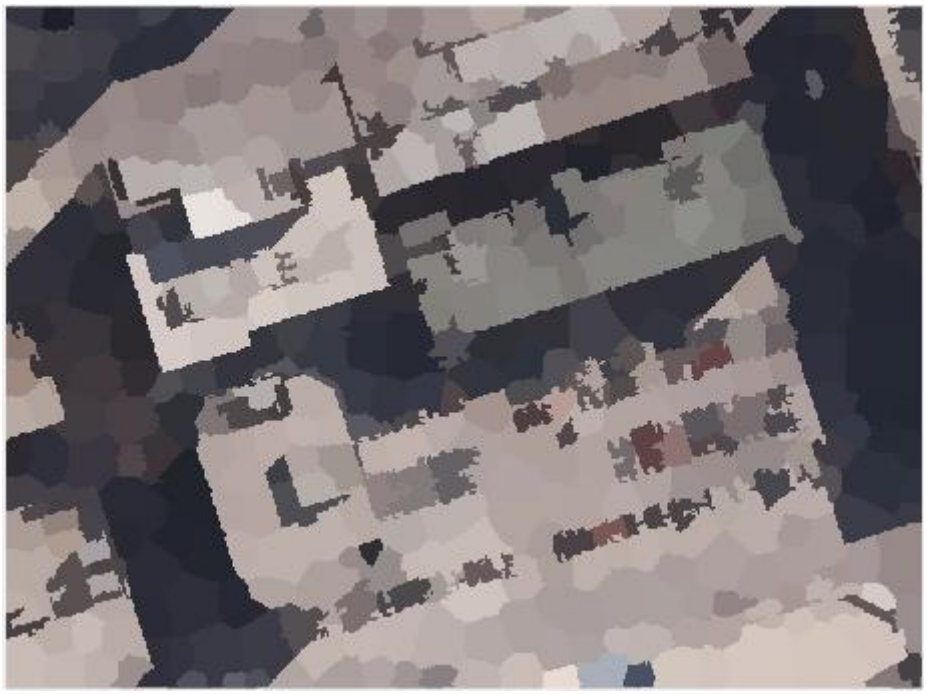

Figure.3 the color of each pixel set to the mean RGB value of the superpixel region

For the chromatic reflectance model, the following function in Figure.4 would be used.

$$L = L_D + L_S$$

$$L(\lambda, \theta) = L_D(\lambda, \theta) + L_S(\lambda, \theta)$$

$$L(\lambda, i, e, g) = L_D(\lambda, i, e, g) = L_S(\lambda, i, e, g)$$

$$C_L(x_{ij}) = m_D(x_{ij})C_D(X_k) + m_S(x_{ij})C_S$$

Figure.4 Function for the chromatic reflectance model

The cost will be minimized using

$$E(m_D, m_S, C_D, C_S) = \sum_{i=1}^{N} \sum_{j=1}^{M} r_{ij}^2$$

$$r_{ij} = m_D(x_{ij})\mathrm{C_D}(\mathrm{X_k}) + \mathrm{m_S}(x_{ij})\mathrm{C_S} - \mathrm{C_L}(\mathrm{x_{ij}})$$

A result of md, ms, Cd, md*Cd is shown in Figure. 5

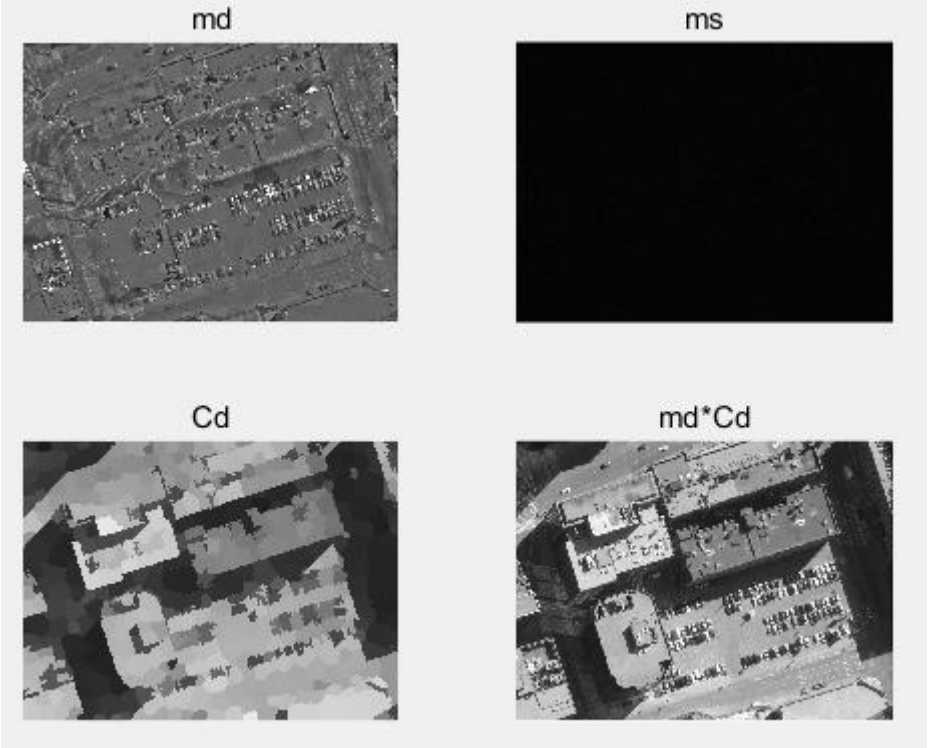

Figure.5 md, ms, Cd results.

Reference :

[1] Radhakrishna Achanta, Appu Shaji, Kevin Smith, Aurelien Lucchi, Pascal Fua, and Sabine Susstrunk, *SLIC Superpixels Compared to State-of-the-art Superpixel Methods*. IEEE Transactions on Pattern Analysis and Machine Intelligence, Volume 34, Issue 11, pp. 2274-2282, May 2012

[2] https://www5.cs.fau.de/research/groups/computer-vision/color-and-reflectance/

[3] https://www.mathworks.com/help/images/ref/superpixels.html