

# Finite-Time Information-Theoretic Bounds in Queueing Control

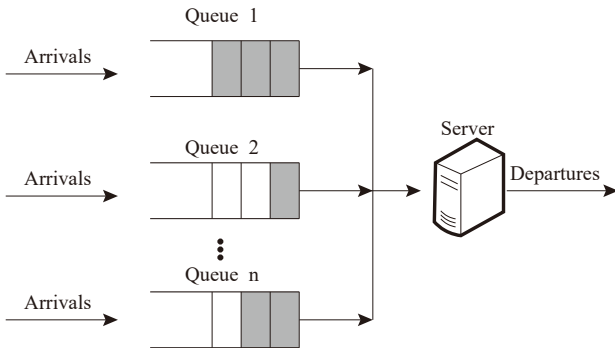
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# The Scheduling Problem in Queueing



Deciding which queues to serve at each time step to optimize performance metrics like delay, waiting time, or throughput

# Why Finite-Time Queue Scheduling is Critical



Figure: NVIDIA's open-sourced KAI Scheduler

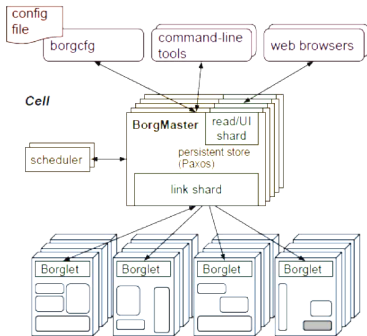


Figure: The high-level architecture of Google Borg

Bursty, non-stationary workload, millisecond-level lease granularity

**Question:** Can we speak about fundamental limits at time  $T$  (not just steady state), and design policies that hit those limits?

# MaxWeight Policy

## De facto policy: MaxWeight (Back-pressure) policy

- **Asymptotic (steady-state)** guarantees: throughput-optimality (stability for all rates in the interior of the capacity region), diffusion optimality in heavy traffic for many settings
  - [Tassiulas & Ephremides, 2002], [Stolyar, 2004], [Mandelbaum & Stolyar, 2004], [Dai & Lin, 2008]
- **Limitations** of MaxWeight in **finite-time** regime
  - MaxWeight tends to pick extreme schedules, and transient backlogs can grow large before averaging effects [Shah & Wischik, 2006], [Bramson, D'Auria, Walton 2021] (validated in our experiments)

**Gap:** little is known theoretically about its parameter-dependent performance and fundamental limitations in non-asymptotic settings

## Central Questions

- How can we formulate a finite-time language (minimax framework) for the scheduling problem in queueing systems?
- Within this framework, what is the minimum achievable queue length by time  $T$ ?
- Can MaxWeight attain this minimum?
- If not, what alternative scheduling policies can possibly achieve it, and under what conditions?

## Key Answers

- **Finite-time information-theoretic lower bound:** first minimax framework for the scheduling problem in queueing; fundamental limit for any scheduling policy
- **MaxWeight is not minimax-optimal:** In finite-time regime, its backlog exceeds the lower bound by a geometry-dependent factor
- **Introducing LyapOpt policy:** Minimizes the *full* Lyapunov drift (first- and second-order terms). LyapOpt matches the lower bound up to absolute constants
- **Extensive simulations:** LyapOpt consistently outperforms MaxWeight across a wide range of scenarios

# Bridging Queueing Control and Learning Theory

- **Not just regret.** Most learning results study *regret* when the model is unknown (RL). Here we ask: even if the model is known (oracle DP), what is the best achievable performance in finite time under randomness?
- **Queueing as structured DP.** Single-hop SPNs give a clean DP testbed. We build an **information-theoretic** toolkit for *finite-horizon* analysis (instead of only steady-state/asymptotic results).
- **Fundamental limits, which motivate algorithms.** We prove *minimax lower bounds* that hold for *any* policy, and design policies that match them—giving sharp benchmarks for short-horizon control.

## Related Works: Key Areas

- **Finite-horizon analyses in queueing.**
  - Challenging even for simple  $M/M/1$ . [Abate & Whitt, 1987]
  - Convergence-to-steady-state via coupling/spectral methods—not finite-time backlog with explicit scaling. [Robert, 2013; Gamarnik & Goldberg, 2013]
  - Results for specific policies/topologies, e.g., JSQ; do not cover general scheduling. [Luczak & McDiarmid, 2006; Ma & Maguluri, 2025]
- **Parameter learning in queueing (unknown rates, partial feedback).**
  - *Queueing regret*. [Krishnasamy, Sen, Johari, Shakkottai, 2021]; [Stahlbuhk, Shrade, Modiano, 2021]; [Freund, Lykouris, Weng, 2023]
  - *Time-averaged queue length*. [Yang, Srikant, Ying, 2023]
  - *Adversarial stability (AQT)*. Focus on universal stability/delay, not time- $T$  backlog scaling. [Borodin, Kleinberg, Raghavan, Sudan, Williamson, 2001]



## Related Works: Key Areas (Cont.)

- **Lower bounds for structured DP.**
  - Queueing control is computationally hard (curse of dimensionality). [Papadimitriou & Tsitsiklis, 1987]
  - One work on delay lower bound:  $G/D/1$  queue. [Gupta & Shroff, 2009]
- **Drift-method limitations and alternatives.**
  - Classical Lyapunov drift targets stability/steady-state performance. [Eryilmaz & Srikant, 2012; Maguluri & Srikant, 2016]
  - Drift-plus-penalty is a steady-state tradeoff framework. [Neely, 2010]
  - These are largely *asymptotic* and *first-order*; they do not directly yield *finite-time minimax* bounds with explicit parameter dependence.

# Outline

- **Part 1: Problem Setup & Minimax Framework**
- **Part 2: General Lower Bounds**
- **Part 3: Finite-Time Performance Guarantees**
- **Part 4: Experiments**

**Goal in mind:** Explain a **finite-time**, **parameter-explicit** theory for single-hop scheduling; show a gap for MaxWeight; present a policy (LyapOpt) that matches a minimax lower bound.

## **Part 1: Problem Setup & Minimax Framework**

## Problem Setup: Single-Hop SPN

Discrete-time single-hop SPN with  $n$  parallel queues.

- $Q(t)$ : Queue length vector at time  $t$ .
- $A(t)$ : Arrival vector at time  $t$ .  
 $\lambda(t) = \mathbb{E}[A(t)]$ : Mean arrival rate vector.
- Scheduling set  $\mathcal{D}_t \subseteq \mathbb{R}_+^n$ : Each  $D(t) \in \mathcal{D}_t$  is a "schedule" (jobs departing).

### Queueing Dynamics

$$Q(t+1) = \max\{Q(t) - D(t), \mathbf{0}\} + A(t+1), \quad t \geq 0$$

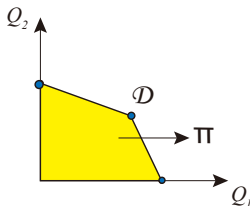
with  $Q(0) = \mathbf{0}$ .

## Problem Setup: Arrival Processes

- **Adversarial Arrivals and Departure Sets:**  $\{A(t), \mathcal{D}_t\}_{t \geq 0}$  chosen by an adversary, potentially with arbitrary dependencies.
- **Stochastic Arrivals and Fixed Departure Set (special case):**  $\{A(t)\}_{t \geq 0}$  is i.i.d. with mean  $\lambda$ .  $\mathcal{D}_t \equiv \mathcal{D}$  fixed.

Definition (Capacity region)

$$\Pi_t = \{\gamma \in \mathbb{R}_+^n : \gamma \leq d, \text{ for some } d \in \text{conv}(\mathcal{D}_t)\}.$$



Assumption

$$\lambda(t) \in \rho \Pi \text{ for all } t \geq 0, \rho \in (0, 1].$$

## Problem Setup: Policy

History:  $\mathcal{H}_t = \{(\mathcal{D}_0, D(0), A(1)), \dots, (\mathcal{D}_{t-1}, D(t-1), A(t)), \mathcal{D}_t\}$ .

### Policy

A *policy*  $\Phi = \{\phi_t\}_{t \geq 0}$ .  $\phi_t : \mathcal{H}_t \rightarrow$  Probability distribution over  $\mathcal{D}_t$

- $D(t) \in \mathcal{D}_t$  is chosen according to the distribution  $\phi_t(\mathcal{H}_t)$ .

**Goal:** Minimize cumulative queue length

# Minimax Criteria

- Standard approach in statistics [Wald, 1945], optimization [Nemirovsky & Yudin, 1978], and machine learning to study finite-sample (finite-horizon) difficulty.
- When no exact limit is known, quantify the best achievable performance via the  $\inf_{\Phi} \sup_{\mathcal{M}}$  criterion.
- Traditionally concerning regret due to model uncertainty and partial feedback; extends to queueing control and (stochastic) DP oracle here.
- First minimax formulation for finite-time fundamental limits of scheduling policies.

## Minimax Criteria: Performance Metrics

- **Total Queue Length:** captures the overall system backlog

$$\mathbb{E} \left[ \sum_{i=1}^n Q_i(T) \right]$$

This is a (stochastic) **Dynamic Programming** problem:

Objective at  $t$  depends on all past decisions.



# Minimax Criteria: Model Classes

## Model Classes: Arrival Process and Scheduling Set

General class  $\mathcal{M}^\rho(C, B)$ :

$$\left\{ (A(\cdot), \{\mathcal{D}_t\}) : \lambda(t) \in \rho \mathcal{D}_t, \frac{1}{n} \sum_{i=1}^n \text{Var}(A_i(t)) \leq C^2, \forall t \geq 0; \right. \\ \left. \frac{1}{n} \sum_{i=1}^n d_i^2 \leq B^2, \forall d \in \mathcal{D} \right\},$$

- $\rho \in (0, 1]$ : Traffic intensity.  $\rho \rightarrow 1$  is “heavy traffic”
- $C \geq 0$ : Arrival variability; can generalize to random departure as well.
- $B > 0$ : Scheduling set diameter.

# Minimax Criteria: Fundamental Lower Bound

**Goal:** find the fundamental minimax lower bound at time  $T$ :

$$\inf_{\Phi} \sup_{(A(\cdot), \{\mathcal{D}_t\}) \in \mathcal{M}^{\rho}(C, B)} \mathbb{E}_{\Phi, (A(\cdot), \mathcal{D})} \left[ \sum_{i=1}^n Q_i(T) \right].$$

- First-ever minimax formulation for finite-time fundamental limitations of queueing control
- Offers a principled approach to quantifying the hardness of structured dynamic scheduling problems

## **Part 2: General Lower Bounds**

# Theorem: Lower Bounds – Fundamental Limit

## Theorem (General Lower Bounds)

For any scheduling policy, and for arrival processes and scheduling sets within the model class  $\mathcal{M}^\rho(C, B)$ , the following lower bound holds:

For all  $T > c_0 \left( \frac{B^2}{nC^2} + \frac{nC^2}{B^2} + 1 \right)$ , there is a **unified lower bound** that covers both the heavy-traffic ( $\rho \rightarrow 1$ ) and interior ( $\rho \in (0, 1)$ ) regimes:

$$\inf_{\Phi} \sup_{\mathcal{M}^\rho(C, B)} \mathbb{E} \left[ \sum_{i=1}^n Q_i(T) \right] \geq c_1 \min \left\{ nC \sqrt{T-1}, \frac{nC^2}{B(1-\rho)} \right\} + \sqrt{n} \rho B,$$

where  $c_0$  and  $c_1$  are positive absolute constants.

# General Lower Bounds: Proof Blueprint

## Proof sketch (high level).

- 1 **Reduce DP to partial sums.** Lower bound the queueing recursion by a functional of the *supremum over time* of partial sums of random variables. In the *oracle DP* (known parameters), use *stochastic-process* lower bounds rather than statistical tools (Le Cam/Fano).
- 2 **Deviation via Gaussian / random walk.** Couple to a Gaussian or random walk, combine a sharp proxy bound with an approximation error. Heavy traffic ( $\rho \rightarrow 1$ ): mean-zero  $\sqrt{T}$ -type lower bound. Interior ( $\rho \in (0, 1)$ ): negatively drifted walk with  $(1 - \rho)^{-1}$  behavior.
- 3 **Gaussian-to-general approximation error.** Control the approximation error via strong-approximation techniques, e.g., the Komlós–Major–Tusnády (KMT) coupling, which provides uniform (in time) coupling with quantifiable error terms.

# Implications of Lower Bounds

- Bounds explicitly quantify **scaling with**:
  - Time horizon  $T$
  - Variance parameter of arrival  $C$  (no  $B$  for heavy-traffic  $\rho \rightarrow 1$ )
  - Number of queues  $n$
  - In interior cases, constant scaling with  $\frac{1}{1-\rho}$
- **Fundamental Benchmark:** No policy can guarantee better than  $nC\sqrt{T}$  (heavy-traffic) or  $\frac{nC^2}{B(1-\rho)}$  (interior) scaling in finite horizon.
- **Next Step:** Introduce a novel algorithm that matches this lower bound by explicitly optimizing both first- and second-order Lyapunov terms, addressing the identified gap.

## **Part 3: Finite-Time Performance Guarantees**

## Optimal Lyapunov Policy (LyapOpt)

Recall:  $Q(t+1) = \max\{Q(t) - D(t), 0\} + A(t+1)$

### LyapOpt policy

At each time  $t$ , select  $D(t)$  as the solution to:

$$D(t) \in \operatorname{argmin}_{d \in \mathcal{D}} \sum_{i=1}^n (\max\{Q_i(t) - d_i, 0\})^2$$

- Minimizes a surrogate of the Lyapunov ( $V(x) = \|x\|_2^2$ ) drift:  
 $\Delta V(t) = \mathbb{E}[V(Q(t+1) - A(t+1)) - V(Q(t) - A(t)) \mid \mathcal{H}_t]$ .
- **Novelty:** Optimizes the **full** one-step Lyapunov drift, including second-order terms.
- **Stability:** LyapOpt is **throughput optimal** (i.e., stable in interior regime).  
Smaller Lyapunov drift than MaxWeight  $\Rightarrow$  **positive recurrence** in one line.



# General Lyapunov Drift Analysis

- For any policy, arrival processes and scheduling sets in  $\mathcal{M}(C, B)$ :  
One-step Lyapunov drift:

$$\Delta V(t) \leq f(Q(t), D(t)) + r(Q(t), A(t+1))$$

$$\text{where } f(Q(t), d) = \mathbb{E} \left[ \underbrace{2 \sum_{i=1}^n Q_i(t) (\lambda_i(t) - d_i)}_{\text{first-order term}} + \underbrace{\sum_{i=1}^n (d_i^2 - \lambda_i(t)^2)}_{\text{second-order term}} \mid \mathcal{H}_t \right]$$

$$\text{and } \mathbb{E}[r(Q(t), A(t+1))] = \sum_{i=1}^n \text{Var}(A_i(t+1)).$$

- Summing over time and applying Jensen's and Cauchy-Schwarz inequalities:

$$\mathbb{E} \left[ \sum_{i=1}^n Q_i(T) \right] \leq n \sqrt{\sum_{t=1}^{T-1} \mathbb{E}[f(Q(t), D(t))]/n + (T-1)C^2} + \sum_{i=1}^n \mathbb{E}[A_i(T)].$$

## Theorem: Finite-Time Performance of LyapOpt

### Theorem (**Finite-Time Performance of the LyapOpt Policy**)

Within  $\mathcal{M}^\rho(C, B)$ , if  $\lambda(t) \in \mathcal{D}_t$  for all  $t \geq 0$ , the LyapOpt policy achieves:

$$\mathbb{E}\left[\sum_{i=1}^n Q_i(T)\right] \leq nC\sqrt{T-1} + \sum_{i=1}^n \mathbb{E}[A_i(T)]$$

- When  $\lambda(t) \in \mathcal{D}_t$ , LyapOpt perfectly matches the arrival rate and achieves the fundamental lower bound (up to a constant factor), establishing its **finite-time optimality**.

# MaxWeight Policy

## MaxWeight policy

Selects schedules  $D^{\text{MaxWeight}}(t)$  according to:

$$D^{\text{MaxWeight}}(t) \in \operatorname{argmax}_{d \in \mathcal{D}} \langle Q(t), d \rangle$$

- Optimizes the first-order Lyapunov term, prioritizing queues with larger backlogs.

## Theorem: Upper Bound of MaxWeight Policy

$$f(Q(t), d) = \mathbb{E} \left[ \underbrace{2 \sum_{i=1}^n Q_i(t) (\lambda_i(t) - d_i)}_{\text{first-order term}} + \underbrace{\sum_{i=1}^n (d_i^2 - \lambda_i(t)^2)}_{\text{second-order term}} \middle| \mathcal{H}_t \right]$$

### Theorem (Upper Bound of MaxWeight Policy)

Under  $\mathcal{M}^\rho(C, B)$ , the MaxWeight policy satisfies

$$\mathbb{E} \left[ \sum_{i=1}^n Q_i(T) \right] \leq n \sqrt{(B^2 + C^2)(T - 1)} + \sum_{i=1}^n \mathbb{E}[A_i(T)].$$

**Limitation** of MaxWeight (and drift-based methods)

- Ignores second-order effects
- Always selects **extreme points**, fails to adapt to arrival-rate geometry

## MaxWeight Lower Bound (Dimension 2)

### Proposition

There exists a family of instances with  $\rho \in (0, 1]$ ,  $B \geq 3\sqrt{2}$  and  $1 \leq C \leq B$ , for which the expected total queue lengths under MaxWeight satisfy:

$$\lim_{\rho \rightarrow 1} \sup_{\mathcal{M}^\rho(C, B)} \mathbb{E} \left[ \sum_{i=1}^2 Q_i^{\text{MaxWeight}}(T) \right] \geq \frac{C\sqrt{T}}{2\sqrt{2e\pi}} + \frac{BT^{\frac{1}{3}}}{4\sqrt{3}}, \quad T \geq \frac{B^2}{C^2} + \frac{C^2}{B^2} + 9.$$

In particular, there exists an instance in  $M^1(0, B)$  with  $B \geq 3\sqrt{2}$  and

$$\mathbb{E} \left[ \sum_{i=1}^2 Q_i^{\text{LyapOpt}}(T) \right] = 2 - \frac{1}{\sqrt{2}B}, \quad T \geq 1,$$

$$\mathbb{E} \left[ \sum_{i=1}^2 Q_i^{\text{MaxWeight}}(T) \right] \geq \frac{\sqrt{BT}}{2\sqrt{2e\pi}}, \quad \left\lceil \frac{2B^2}{\sqrt{2}B - 1} \right\rceil \leq T \leq \left\lceil \left( \frac{\sqrt{2}B}{2} - 1 \right)^3 \right\rceil.$$

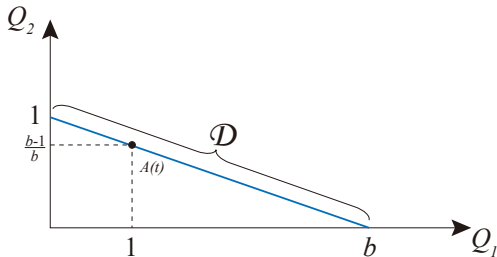
## MaxWeight Lower Bound (Dimension 2)

**Construction:** Consider the scheduling set and arrivals

$$\mathcal{D} = \{d \in \mathbb{R}^2 : d = x(b, 0) + (1 - x)(0, 1), 0 \leq x \leq 1\},$$

$$A(t) = (1, (b - 1)/b) \text{ for all } t \geq 1, \text{ and } A(0) = (1, (b - 1)/b - \varepsilon),$$

with  $b = \sqrt{2}B$



## MaxWeight Lower Bound (Dimension 2)

### Proof Sketch:

- **MaxWeight's Selection Rule:**

- MaxWeight always choose extreme points  $(0, 1)$  or  $(b, 0)$ ;
- At each time  $t$ , MaxWeight selects  $(b, 0)$  unless  $\frac{Q_2(t)}{Q_1(t)} \geq b$ .

- **Queue Dynamics under MaxWeight:**

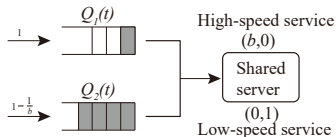
- $Q_2(t)$  accumulates to  $b$  before the first use of  $(0, 1)$ .
- Using  $(0, 1)$  increases  $Q_1(t)$  to 2.
- To use  $(0, 1)$  again,  $Q_2(t)$  must build up to  $2b$ .
- This alternating pattern causes  $Q_2(t)$  to grow at rate  $\sqrt{bT}$  over a finite horizon  $T$ .

- **Contrast with LyapOpt:**

- Always chooses the "true arrival" schedule  $(1, (b-1)/b)$ .
- Maintains constant queue lengths  $O(1)$ .

## MaxWeight Lower Bound (Dimension 2)

Common in wireless networks and data centers.



### MaxWeight Behavior:

- Over-prioritizes  $Q_1$  via extreme-point selection.
- $Q_2(t)$  builds up due to limited service.
- Highlights MaxWeight's finite-time inefficiency.

### LyapOpt Contrast:

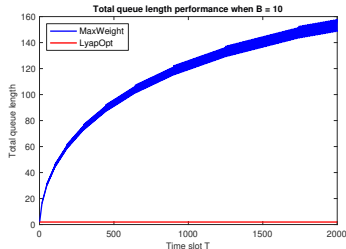
- Adapts to arrival asymmetry.
- Prevents backlog in this special case.



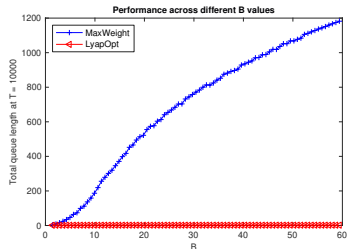
## **Part 4: Experiments**

## MaxWeight Lower Bound (Dimension 2)

**Empirical Validation:** The  $\sqrt{BT}$  gap is substantial for practical  $T$  and  $B$



(a) Total queue length when  $B = 10$



(b) Total queue length across different  $B$

**Figure:** Performance comparison of MaxWeight and LyapOpt policies versus  $B$

# Experiments with More Queues

## Experimental Setup

- **Scheduling Set  $\mathcal{D}$ :**  $10n$  integer vectors uniformly sampled from  $[1, 10]^n$ .
- **Arrival Rates:** 2000 vectors sampled from the boundary of the capacity region  $\Pi$ .
- **Arrival Distributions:** Binomial with variance 1, matching the sampled arrival rates.
- **Simulation:** 1000 time slots, averaged over 100 runs per scenario.

## Experiments with More Queues

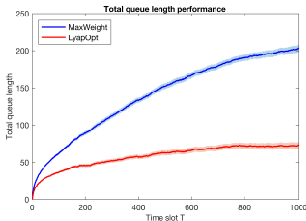
- ratio =  $\frac{\text{Total Queue Length (LyapOpt) at } t = 1000}{\text{Total Queue Length (MaxWeight) at } t = 1000}$

Table: Proportion of scenarios with ratio below 1, 0.9, and 0.5

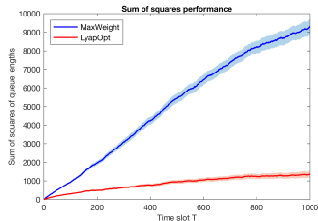
Number of Queues ( $n$ )	ratio $\leq 1$	ratio $\leq 0.9$	ratio $\leq 0.5$
2	84.7%	25.9%	0%
3	97.5%	54.1%	36.3%
4	99.9%	78.5%	46.1%
5	100%	67.0%	31.3%
6	97.4%	71.3%	26.5%
7	100%	90.0%	45.9%
8	100%	80.7%	35.9%

- LyapOpt achieves consistently better performance than MaxWeight for  $n = 2$  to 8.
- Significant improvements observed in many cases (ratio  $\leq 0.5$ ).

## Representative Case Study ( $n = 8$ Queues)



(a) Total queue length.



(b) Squared queue length.

Figure: Finite-time comparison of MaxWeight and LyapOpt policies ( $n = 8$ ).

- Both policies show  $\sqrt{T}$  growth in total queue length and linear  $T$  growth in squared queue length.
- LyapOpt yields lower total queue length and better balance.

# Summary

- Exposed a **finite-time gap** between MaxWeight and the minimax lower bound.
- Proposed LyapOpt, a **second-order** Lyapunov policy that *closes this gap*.
- Theory & simulations: LyapOpt yields **shorter queues** than MaxWeight over finite horizons.
- Clarifies the **limitations of drift-based (first-order) methods** in transient regimes.
- **Complements steady-state** analyses: revealing interesting finite-time, parameter-dependent phenomena (e.g., geometric structure and second-order effects).

## Future Directions

- **Multi-hop networks.** Extend lower bounds and LyapOpt-style policies beyond single hop.
- **Model geometry.** In  $G/G/1$ , the feasible set  $\mathcal{D}$  is a *weighted simplex*; refined analysis to common decision sets.
- **Unknown parameters.**
  - Unknown *arrival rates*: seamlessly covered by our work.
  - $n$ -queue,  $m$ -server systems with *unknown service rates*: requires **UCB-type exploration** with backlog-aware exploitation.
- **Computation.** MaxWeight solves a *linear* problem over  $\mathcal{D}$  (often LP / min-cut-max-flow); LyapOpt involves a *quadratic* objective over  $\mathcal{D}$ —design fast approximations and oracle reductions.