Finite-Time Information-Theoretic Bounds in Queueing Control

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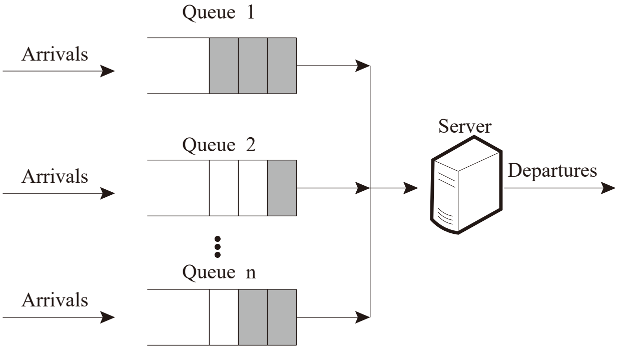
The Scheduling Problem in Queueing



Deciding which queues to serve at each time step to optimize performance metrics like delay, waiting time, or throughput

Why Finite-Time Queue Scheduling is Critical

Figure: NVIDIA's open-sourced KAI Scheduler

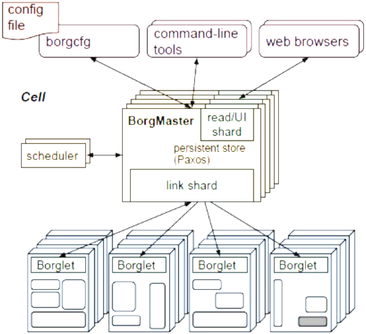


Figure: The high-level architecture of Google Borg

Bursty, non-stationary workload, millisecond-level lease granularity

Question: Can we speak about fundamental limits at time T (not just steady state), and design policies that hit those limits?

MaxWeight Policy

De facto policy: MaxWeight (Back-pressure) policy

- **Asymptotic (steady-state)** guarantees: throughput-optimality (stability for all rates in the interior of the capacity region), diffusion optimality in heavy traffic for many settings
  - [Tassiulas & Ephremides, 2002], [Stolyar, 2004], [Mandelbaum & Stolyar, 2004], [Dai & Lin, 2008]
- **Limitations** of MaxWeight in **finite-time** regime
  - MaxWeight tends to pick extreme schedules, and transient backlogs can grow large before averaging effects [Shah & Wischik, 2006], [Bramson, D'Auria, Walton 2021] (validated in our experiments)

Gap: little is known theoretically about its parameter-dependent performance and fundamental limitations in non-asymptotic settings

Central Questions

- How can we formulate a finite-time language (minimax framework) for the scheduling problem in queueing systems?
- Within this framework, what is the minimum achievable queue length by time T?
- Can MaxWeight attain this minimum?
- If not, what alternative scheduling policies can possibly achieve it, and under what conditions?

Key Answers

- **Finite-time information-theoretic lower bound:** first minimax framework for the scheduling problem in queueing; fundamental limit for any scheduling policy
- **MaxWeight is not minimax-optimal:** In finite-time regime, its backlog exceeds the lower bound by a geometry-dependent factor
- **Introducing** $\mathrm{LyapOpt}$ **policy:** Minimizes the *full* Lyapunov drift (first- and second-order terms). $\mathrm{LyapOpt}$ matches the lower bound up to absolute constants
- **Extensive simulations:** $\mathrm{LyapOpt}$ consistently outperforms MaxWeight across a wide range of scenarios

Bridging Queueing Control and Learning Theory

- **Not just regret.** Most learning results study *regret* when the model is unknown (RL). Here we ask: even if the model is known (oracle DP), what is the best achievable performance in finite time under randomness?
- **Queueing as structured DP.** Single-hop SPNs give a clean DP testbed. We build an **information-theoretic** toolkit for *finite-horizon* analysis (instead of only steady-state/asymptotic results).
- **Fundamental limits, which motivate algorithms.** We prove *minimax lower bounds* that hold for *any* policy, and design policies that match them—giving sharp benchmarks for short-horizon control.

Related Works: Key Areas

- **Finite-horizon analyses in queueing.**
  - Challenging even for simple $M/M/1$. [Abate & Whitt, 1987]
  - Convergence-to-steady-state via coupling/spectral methods—not finite-time backlog with explicit scaling. [Robert, 2013; Gamarnik & Goldberg, 2013]
  - Results for specific policies/topologies, e.g., JSQ; do not cover general scheduling. [Luczak & McDiarmid, 2006; Ma & Maguluri, 2025]
- **Parameter learning in queueing (unknown rates, partial feedback).**
  - *Queueing regret.* [Krishnasamy, Sen, Johari, Shakkottai, 2021]; [Stahlbuhk, Shrade, Modiano, 2021]; [Freund, Lykouris, Weng, 2023]
  - *Time-averaged queue length.* [Yang, Srikant, Ying, 2023]
  - *Adversarial stability (AQT).* Focus on universal stability/delay, not time-T backlog scaling. [Borodin, Kleinberg, Raghavan, Sudan, Williamson, 2001]

Related Works: Key Areas (Cont.)

- **Lower bounds for structured DP.**
  - Queueing control is computationally hard (curse of dimensionality). [Papadimitriou & Tsitsiklis, 1987]
  - One work on delay lower bound: $G/D/1$ queue. [Gupta & Shroff, 2009]
- **Drift-method limitations and alternatives.**
  - Classical Lyapunov drift targets stability/steady-state performance. [Eryilmaz & Srikant, 2012; Maguluri & Srikant, 2016]
  - Drift-plus-penalty is a steady-state tradeoff framework. [Neely, 2010]
  - These are largely *asymptotic* and *first-order*; they do not directly yield *finite-time minimax* bounds with explicit parameter dependence.

Outline

- **Part 1: Problem Setup & Minimax Framework**
- **Part 2: General Lower Bounds**
- **Part 3: Finite-Time Performance Guarantees**
- **Part 4: Experiments**

Goal in mind: Explain a finite-time, parameter-explicit theory for single-hop scheduling; show a gap for MaxWeight; present a policy (LyapOpt) that matches a minimax lower bound.

Part 1: Problem Setup & Minimax Framework

Problem Setup: Single-Hop SPN

Discrete-time single-hop SPN with n parallel queues.

- $Q(t)$: Queue length vector at time t.
- $A(t)$: Arrival vector at time t.
  $\lambda(t) = \mathbb{E}[A(t)]$: Mean arrival rate vector.
- Scheduling set $\mathcal{D}_t \subseteq \mathbb{R}_+^n$: Each $D(t) \in \mathcal{D}_t$ is a "schedule" (jobs departing).

Queueing Dynamics

$$Q(t+1) = \max\{Q(t) - D(t), \mathbf{0}\} + A(t+1), \quad t \geq 0$$

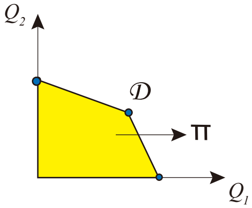
with $Q(0) = \mathbf{0}$.

Problem Setup: Arrival Processes

- **Adversarial Arrivals and Departure Sets:** $\{A(t), \mathcal{D}_t\}_{t \geq 0}$ chosen by an adversary, potentially with arbitrary dependencies.
- **Stochastic Arrivals and Fixed Departure Set (special case):** $\{A(t)\}_{t \geq 0}$ is i.i.d. with mean λ. $\mathcal{D}_t \equiv \mathcal{D}$ fixed.

Definition (Capacity region)

$\Pi_t = \{\gamma \in \mathbb{R}_+^n : \gamma \leq d, \text{ for some } d \in \text{conv}(\mathcal{D}_t)\}$.



Assumption

$\lambda(t) \in \rho\Pi$ for all $t \geq 0$, $\rho \in (0, 1]$.

Problem Setup: Policy

History: $\mathcal{H}_t = \{(\mathcal{D}_0, D(0), A(1)), \ldots, (\mathcal{D}_{t-1}, D(t-1), A(t)), \mathcal{D}_t\}$.

Policy

A *policy* $\Phi = \{\phi_t\}_{t \geq 0}$. $\phi_t : \mathcal{H}_t \rightarrow$ Probability distribution over $\mathcal{D}_t$

- $D(t) \in \mathcal{D}_t$ is chosen according to the distribution $\phi_t(\mathcal{H}_t)$.

Goal: Minimize cumulative queue length

Minimax Criteria

- Standard approach in statistics [Wald, 1945], optimization [Nemirovsky & Yudin, 1978], and machine learning to study finite-sample (finite-horizon) difficulty.
- When no exact limit is known, quantify the best achievable performance via the $\inf_{\Phi} \sup_{\mathcal{M}}$ criterion.
- Traditionally concerning regret due to model uncertainty and partial feedback; extends to queueing control and (stochastic) DP oracle here.
- First minimax formulation for finite-time fundamental limits of scheduling policies.

Minimax Criteria: Performance Metrics

- **Total Queue Length:** captures the overall system backlog

$$\mathbb{E}\left[\sum_{i=1}^{n} Q_i(T)\right]$$

This is a (stochastic) Dynamic Programming problem:

Objective at t depends on all past decisions.

Minimax Criteria: Model Classes

Model Classes: Arrival Process and Scheduling Set

General class $\mathcal{M}^{\rho}(C, B)$:

$$\Big\{ (A(\cdot), \{\mathcal{D}_t\}) : \lambda(t) \in \rho \mathcal{D}_t, \frac{1}{n}\sum_{i=1}^{n} \mathsf{Var}(A_i(t)) \le C^2, \ \forall t \ge 0; \quad \frac{1}{n}\sum_{i=1}^{n} d_i^2 \le B^2, \ \forall d \in \mathcal{D} \Big\},$$

- $\rho \in (0, 1]$: Traffic intensity. $\rho \to 1$ is "heavy traffic"
- $C \ge 0$: Arrival variability; can generalize to random departure as well.
- $B > 0$: Scheduling set diameter.

Minimax Criteria: Fundamental Lower Bound

Goal: find the fundamental minimax lower bound at time T:

$$\inf_{\Phi} \sup_{(A(\cdot),\{\mathcal{D}_t\})\in\mathcal{M}^{\rho}(C,B)} \mathbb{E}_{\Phi,(A(\cdot),\mathcal{D})}\left[\sum_{i=1}^{n} Q_i(T)\right].$$

- First-ever minimax formulation for finite-time fundamental limitations of queueing control
- Offers a principled approach to quantifying the hardness of structured dynamic scheduling problems

Part 2: General Lower Bounds

Theorem: Lower Bounds – Fundamental Limit

Theorem (**General Lower Bounds**)

For any scheduling policy, and for arrival processes and scheduling sets within the model class $\mathcal{M}^{\rho}(C, B)$, the following lower bound holds:
*For all $T > c_0 \left(\frac{B^2}{nC^2} + \frac{nC^2}{B^2} + 1 \right)$, there is a **unified lower bound** that covers both the heavy-traffic ($\rho \to 1$) and interior ($\rho \in (0, 1)$) regimes:*

$$\inf_{\Phi} \sup_{\mathcal{M}^{\rho}(C,B)} \mathbb{E}\left[\sum_{i=1}^{n} Q_i(T)\right] \geq c_1 \min\left\{ nC\sqrt{T-1}, \frac{nC^2}{B(1-\rho)} \right\} + \sqrt{n}\,\rho B,$$

where c_0 and c_1 are positive absolute constants.

General Lower Bounds: Proof Blueprint

Proof sketch (high level).

1. **Reduce DP to partial sums.** Lower bound the queueing recursion by a functional of partial sums (i.e., finite sums of random variables). In the *oracle DP* (known parameters), use *stochastic-process* lower bounds rather than statistical tools (Le Cam/Fano).
2. **Deviation via Gaussian / random walk.** Couple to a Gaussian or random walk, combine a sharp proxy bound with an approximation error. Heavy traffic ($\rho \to 1$): mean-zero $\sqrt{T}$-type lower bound. Interior ($\rho \in (0,1)$): negatively drifted walk with $(1-\rho)^{-1}$ behavior.
3. **Gaussian-to-general approximation error.** Control the approximation error via strong-approximation techniques, e.g., the Komlós–Major–Tusnády (KMT) coupling, which provides uniform (in time) coupling with quantifiable error terms.

Implications of Lower Bounds

- Bounds explicitly quantify **scaling with**:
  - Time horizon T
  - Variance parameter of arrival C (no B for heavy-traffic $\rho \to 1$)
  - Number of queues n
  - In interior cases, constant scaling with $\frac{1}{1-\rho}$
- **Fundamental Benchmark:** No policy can guarantee better than $nC\sqrt{T}$ (heavy-traffic) or $\frac{nC^2}{B(1-\rho)}$ (interior) scaling in finite horizon.
- **Next Step:** Introduce a novel algorithm that matches this lower bound by explicitly optimizing both first- and second-order Lyapunov terms, addressing the identified gap.

Part 3: Finite-Time Performance Guarantees

Optimal Lyapunov Policy ($\mathrm{LyapOpt}$)

Recall: $Q(t+1) = \max\{Q(t) - D(t), \mathbf{0}\} + A(t+1)$

$\mathrm{LyapOpt}$ policy

At each time t, select $D(t)$ as the solution to:

$$D(t) \in \operatorname*{argmin}_{d \in \mathcal{D}} \sum_{i=1}^{n} \left(\max\{Q_i(t) - d_i, 0\}\right)^2$$

- Minimizes a surrogate of the Lyapunov ($V(x) = \|x\|_2^2$) drift: $\Delta V(t) = \mathbb{E}[V(Q(t+1) - A(t+1)) - V(Q(t) - A(t)) \mid \mathcal{H}_t]$.
- **Novelty:** Optimizes the full one-step Lyapunov drift, including second-order terms.
- **Stability:** $\mathrm{LyapOpt}$ is throughput optimal (i.e., stable in interior regime). Smaller Lyapunov drift than MaxWeight $\Rightarrow$ positive recurrence in one line.

General Lyapunov Drift Analysis

- For any policy, arrival processes and scheduling sets in $\mathcal{M}(C, B)$:
  One-step Lyapunov drift:

$$\Delta V(t) \le f(Q(t), D(t)) + r(Q(t), A(t+1))$$

$$\text{where } f(Q(t), d) = \mathbb{E}\left[\underbrace{2\sum_{i=1}^{n} Q_i(t)\big(\lambda_i(t) - d_i\big)}_{\text{first-order term}} + \underbrace{\sum_{i=1}^{n} \big(d_i^2 - \lambda_i(t)^2\big)}_{\text{second-order term}} \,\Bigg|\, \mathcal{H}_t \right]$$

and $\mathbb{E}[r(Q(t), A(t+1))] = \sum_{i=1}^{n} \mathrm{Var}(A_i(t+1))$.

- Summing over time and applying Jensen's and Cauchy–Schwarz inequalities:

$$\mathbb{E}\left[\sum_{i=1}^{n} Q_i(T) \right] \le n \sqrt{ \sum_{t=1}^{T-1} \mathbb{E}[f(Q(t), D(t))]/n + (T-1)C^2 + \sum_{i=1}^{n} \mathbb{E}[A_i(T)]}.$$

Theorem: Finite-Time Performance of LyapOpt

Theorem (Finite-Time Performance of the LyapOpt Policy)

Within $\mathcal{M}^\rho(C, B)$, if $\lambda(t) \in \mathcal{D}_t$ for all $t \geq 0$, the LyapOpt *policy achieves:*

$$\mathbb{E}\Big[\sum_{i=1}^{n} Q_i(T)\Big] \leq nC\sqrt{T-1} + \sum_{i=1}^{n} \mathbb{E}[A_i(T)]$$

- When $\lambda(t) \in \mathcal{D}_t$, LyapOpt perfectly matches the arrival rate and achieves the fundamental lower bound (up to a constant factor), establishing its finite-time optimality.

MaxWeight Policy

MaxWeight policy

Selects schedules $D^{\mathsf{MaxWeight}}(t)$ according to:

$$D^{\mathsf{MaxWeight}}(t) \in \underset{d \in \mathcal{D}}{\operatorname{argmax}} \langle Q(t), d \rangle$$

- Optimizes the first-order Lyapunov term, prioritizing queues with larger backlogs.

Theorem: Upper Bound of MaxWeight Policy

$$f(Q(t), d) = \mathbb{E}\left[\underbrace{2\sum_{i=1}^{n} Q_i(t)\big(\lambda_i(t) - d_i\big)}_{\text{first-order term}} + \underbrace{\sum_{i=1}^{n}\big(d_i^2 - \lambda_i(t)^2\big)}_{\text{second-order term}} \;\middle|\; \mathcal{H}_t \right]$$

Theorem (Upper Bound of MaxWeight Policy)

Under $\mathcal{M}^\rho(C, B)$, the MaxWeight policy satisfies

$$\mathbb{E}\left[\sum_{i=1}^{n} Q_i(T)\right] \le n\sqrt{(B^2 + C^2)(T-1)} + \sum_{i=1}^{n} \mathbb{E}[A_i(T)].$$

Limitation of MaxWeight (and drift-based methods)

- Ignores second-order effects
- Always selects extreme points, fails to adapt to arrival-rate geometry

MaxWeight Lower Bound (Dimension 2)

Proposition

There exists a family of instances with $\rho \in (0,1]$, $B \geq 3\sqrt{2}$ and $1 \leq C \leq B$, for which the expected total queue lengths under MaxWeight satisfy:

$$\lim_{\rho \to 1} \sup_{\mathcal{M}^{\rho}(C,B)} \mathbb{E}\Big[\sum_{i=1}^{2} Q_i^{\mathrm{MaxWeight}}(T)\Big] \geq \frac{C\sqrt{T}}{2\sqrt{2e\pi}} + \frac{BT^{\frac{1}{3}}}{4\sqrt{3}}, \quad T \geq \frac{B^2}{C^2} + \frac{C^2}{B^2} + 9.$$

In particular, there exists an instance in $M^1(0,B)$ with $B \geq 3\sqrt{2}$ and

$$\mathbb{E}\Big[\sum_{i=1}^{2} Q_i^{\mathsf{LyapOpt}}(T)\Big] = 2 - \frac{1}{\sqrt{2}\,B}, \quad T \geq 1,$$

$$\mathbb{E}\Big[\sum_{i=1}^{2} Q_i^{\mathrm{MaxWeight}}(T)\Big] \geq \frac{\sqrt{BT}}{2\sqrt{2e\pi}}, \quad \left\lceil \frac{2B^2}{\sqrt{2}B - 1} \right\rceil \leq T \leq \left\lceil (\frac{\sqrt{2}B}{2} - 1)^3 \right\rceil.$$

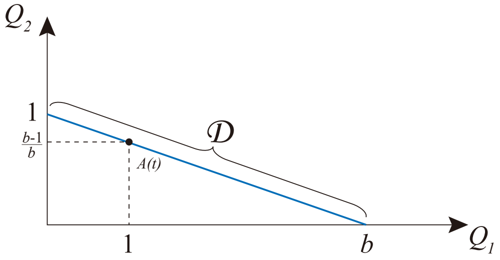
MaxWeight Lower Bound (Dimension 2)

Construction: Consider the scheduling set and arrivals

$$\mathcal{D} = \{d \in \mathbb{R}^2 : d = x(b,0) + (1-x)(0,1), 0 \le x \le 1\},$$

$$A(t) = (1, (b-1)/b) \text{ for all } t \ge 1, \text{ and } A(0) = (1, (b-1)/b - \varepsilon),$$

with $b = \sqrt{2}B$



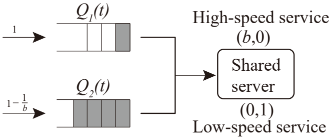
MaxWeight Lower Bound (Dimension 2)

Proof Sketch:

- **MaxWeight's Selection Rule:**
  - MaxWeight always choose extreme points $(0, 1)$ or $(b, 0)$;
  - At each time t, MaxWeight selects $(b, 0)$ unless $\frac{Q_2(t)}{Q_1(t)} \geq b$.
- **Queue Dynamics under MaxWeight:**
  - $Q_2(t)$ accumulates to b before the first use of $(0, 1)$.
  - Using $(0, 1)$ increases $Q_1(t)$ to 2.
  - To use $(0, 1)$ again, $Q_2(t)$ must build up to $2b$.
  - This alternating pattern causes $Q_2(t)$ to grow at rate $\sqrt{bT}$ over a finite horizon T.
- **Contrast with $\mathrm{LyapOpt}$:**
  - Always chooses the "true arrival" schedule $(1, (b-1)/b)$.
  - Maintains constant queue lengths $O(1)$.

MaxWeight Lower Bound (Dimension 2)

Common in wireless networks and data centers.



MaxWeight Behavior:

- Over-prioritizes Q_1 via extreme-point selection.
- $Q_2(t)$ builds up due to limited service.
- Highlights MaxWeight's finite-time inefficiency.

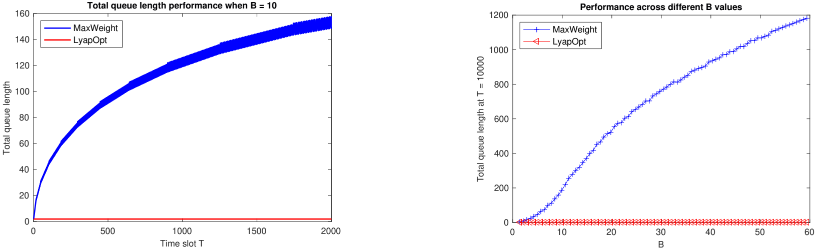
LyapOpt Contrast:

- Adapts to arrival asymmetry.
- Prevents backlog in this special case.

Part 4: Experiments

MaxWeight Lower Bound (Dimension 2)

Empirical Validation: The $\sqrt{BT}$ gap is substantial for practical T and B



(a) Total queue length when $B = 10$

(b) Total queue length across different B

Figure: Performance comparison of MaxWeight and LyapOpt policies versus B

Experiments with More Queues

Experimental Setup

- **Scheduling Set $\mathcal{D}$:** $10n$ integer vectors uniformly sampled from $[1, 10]^n$.
- **Arrival Rates:** 2000 vectors sampled from the boundary of the capacity region Π.
- **Arrival Distributions:** Binomial with variance 1, matching the sampled arrival rates.
- **Simulation:** 1000 time slots, averaged over 100 runs per scenario.

Experiments with More Queues

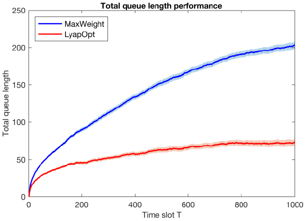
- ratio $= \frac{\text{Total Queue Length } (\mathrm{LyapOpt}) \text{ at } t = 1000}{\text{Total Queue Length (MaxWeight) at } t = 1000}$

Table: Proportion of scenarios with ratio below 1, 0.9, and 0.5

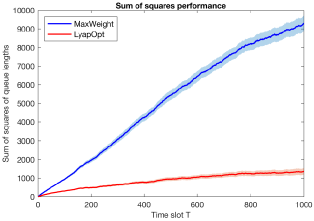
Number of Queues (n)	ratio ≤ 1	ratio ≤ 0.9	ratio ≤ 0.5
2	84.7%	25.9%	0%
3	97.5%	54.1%	36.3%
4	99.9%	78.5%	46.1%
5	100%	67.0%	31.3%
6	97.4%	71.3%	26.5%
7	100%	90.0%	45.9%
8	100%	80.7%	35.9%

- $\mathrm{LyapOpt}$ achieves consistently better performance than MaxWeight for $n = 2$ to 8.
- Significant improvements observed in many cases (ratio ≤ 0.5).

Representative Case Study ($n = 8$ Queues)



(a) Total queue length.



(b) Squared queue length.

Figure: Finite-time comparison of MaxWeight and LyapOpt policies ($n = 8$).

- Both policies show $\sqrt{T}$ growth in total queue length and linear T growth in squared queue length.
- $\mathrm{LyapOpt}$ yields lower total queue length and better balance.

Summary

- Exposed a **finite-time gap** between MaxWeight and the minimax lower bound.
- Proposed $\mathrm{LyapOpt}$, a **second-order** Lyapunov policy that *closes this gap*.
- Theory & simulations: $\mathrm{LyapOpt}$ yields **shorter queues** than MaxWeight over finite horizons.
- Clarifies the **limitations of drift-based (first-order) methods** in transient regimes.
- **Complements steady-state** analyses: revealing interesting finite-time, parameter-dependent phenomena (e.g., geometric structure and second-order effects).

Future Directions

- **Multi-hop networks.** Extend lower bounds and $\mathrm{LyapOpt}$-style policies beyond single hop.
- **Model geometry.** In $G/G/1$, the feasible set $\mathcal{D}$ is a *weighted simplex*; refined analysis to common decision sets.
- **Unknown parameters.**
    - Unknown *arrival rates*: seamlessly covered by our work.
    - n-queue, m-server systems with *unknown service rates*: requires **UCB-type exploration** with backlog-aware exploitation.
- **Computation.** MaxWeight solves a *linear* problem over $\mathcal{D}$ (often LP / min-cut–max-flow); $\mathrm{LyapOpt}$ involves a *quadratic* objective over $\mathcal{D}$—design fast approximations and oracle reductions.