1
10 Points

a) Make codes for Gaussian Elimination with and without pivoting. Also make codes for Cholesky factorization. Your code should be able to print L and U matrix for LU decomposition for the question (b) and (c).

b) Use your Gaussian Elimination with and without pivoting codes to solve the following linear system when $\epsilon = 10^{-k}$ for $k = 10, 30, 60$. At first, for each code compare the result of LU with and the given matrix. Then use backward substitution and forward substitution to solve the equation (To solve $LUx = b$). From your solutions, conclude which algorithm is more stable than the others.

$$\begin{pmatrix} \epsilon & 1 & 0 \\ 1 & 1 & \epsilon \\ 0 & 1 & 3\epsilon \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1+\epsilon \\ 2+\epsilon \\ 1+3\epsilon \end{pmatrix}$$

c) Let $m = 1000, Z = randn(m, m), A = Z' * Z$ and $b = randn(m, 1)$ Use your codes for Gaussian Elimination with pivoting and Cholesky factorization to solve the $Ax = b$. At first, decompose A as LU and use backward substitution and forward substitution to solve the equation. Do the same code for 3-times, respectively. Using tic, toc in matlab check the time and conclude which algorithm is better than the others.(If it takes too long times, you can reduce the m)