Image Analysis –Outline

- Introduction
- Image Segmentation
- Edge Detection
  - First-order derivative detector
  - Second-order derivative detector
- Hough Transform
  - Edge Linking
- Application

Image Analysis –Introduction

Image Analysis

Extract information from the image

Understand the content of the image
Recognize the meaning of image

Feature Extraction

- Edges and boundaries
- Spatial features
- Transform features
- Shape
- Texture
- Color

Segmentation

- Thresholding
- Region-based techniques
- Quad-trees
- Edge-based techniques

Classification

- Statistical classifiers
- Bayes Decision Theory
- Supervised learning
- Unsupervised learning

Image Analysis –Segmentation

- Segmentation is to subdivide an image into its constituent regions or objects.
- Segmentation should stop when the objects of interest in an application have been isolated.

➢ Segmentation algorithms generally are based on one of 2 basic properties of intensity values

similarity: to partition an image into regions that are similar according to a set of predefined criteria. (Thresholding)

discontinuity: to partition an image based on abrupt changes in intensity (Point, Line and Edge Detection)

Image Analysis –Segmentation

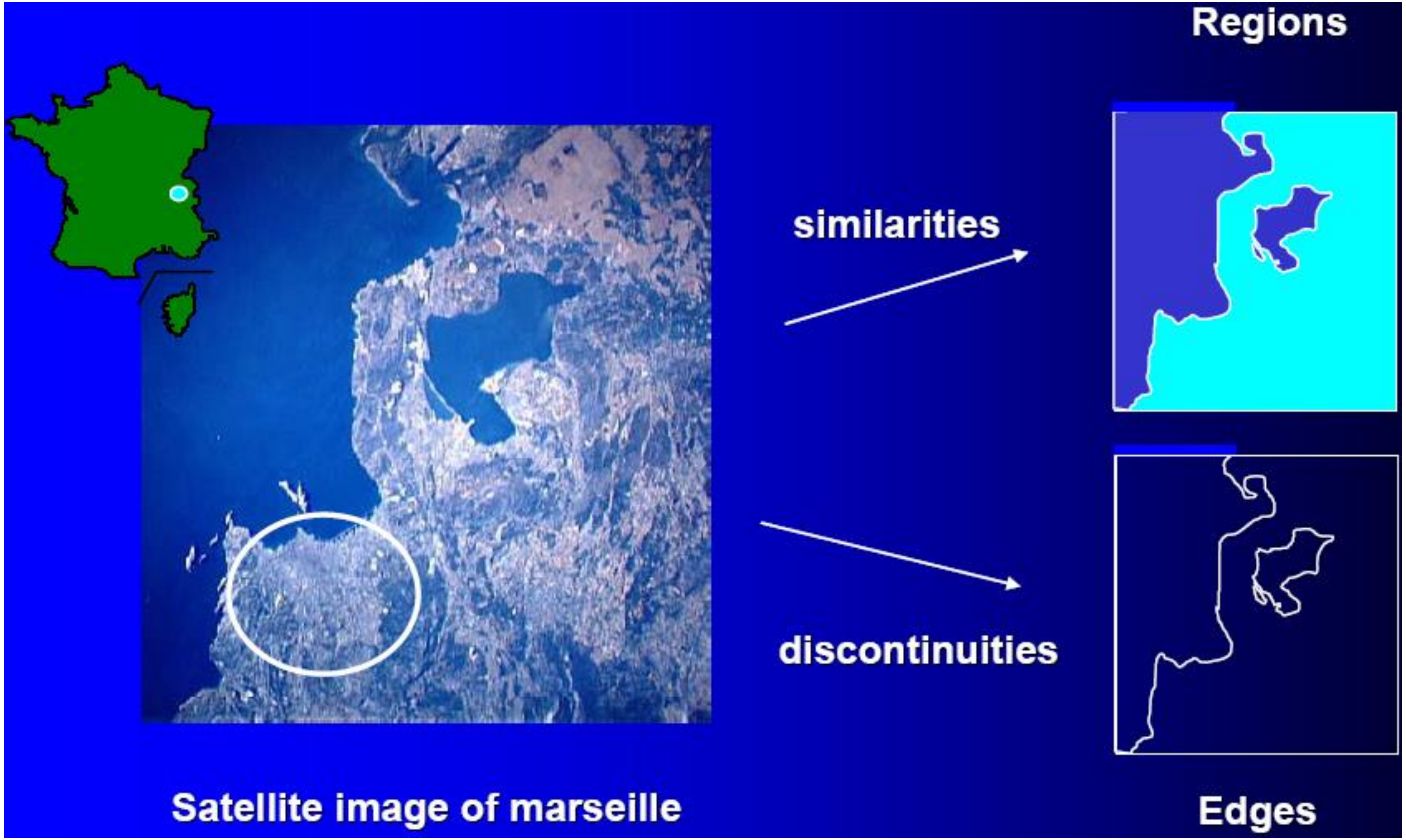


Image Analysis –Segmentation by Thresholding

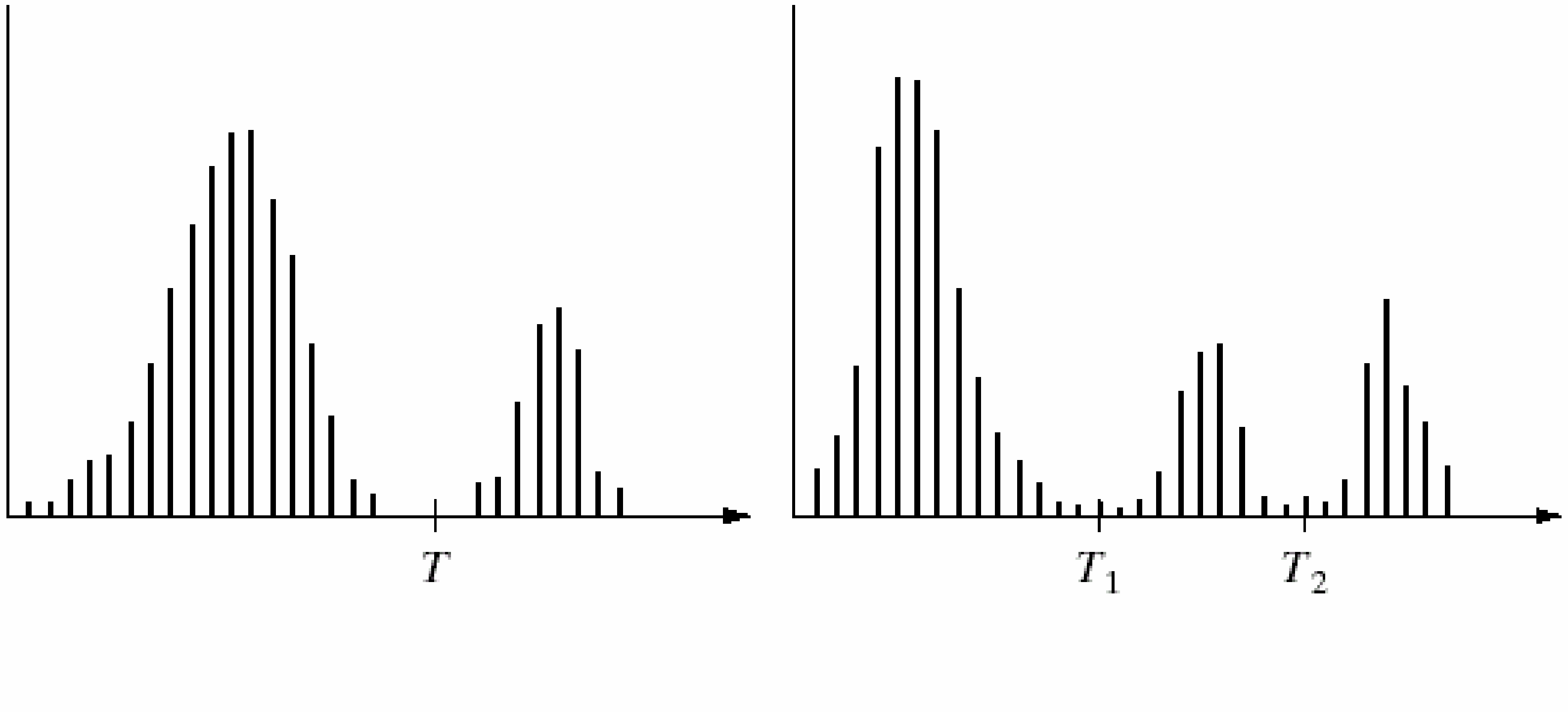
- Many images contain some objects of interest of uniform brightness placed against a background of differing brightness.

- Typical examples include handwritten and typewritten text, microscopic biomedical samples, fingerprints, and airplanes on a runway.

Image Analysis –Segmentation by Thresholding

image with dark background and a light object

image with dark background and two light objects



(a) Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.

Image Analysis –Segmentation by Thresholding

- A thresholded image g(x, y) is defined as

$$g(x,y)=\begin{cases}1 & \text{if } f(x,y)>T\\ 0 & \text{if } f(x,y)\le T\end{cases}$$

where T is the threshold given by

$$T=T[x,y,p(x,y),f(x,y)]$$

- Global threshold: T depends on gray-level values $f(x, y)$ of the whole image alone
- Local threshold: T depends on both $f(x, y)$ and its local neighbors property $p(x, y)$
- Adaptive threshold: T depends on x and y coordinates

Image Analysis –Segmentation by Thresholding

Basic Global Thresholding

Use T midway between the max and min gray levels generate binary image

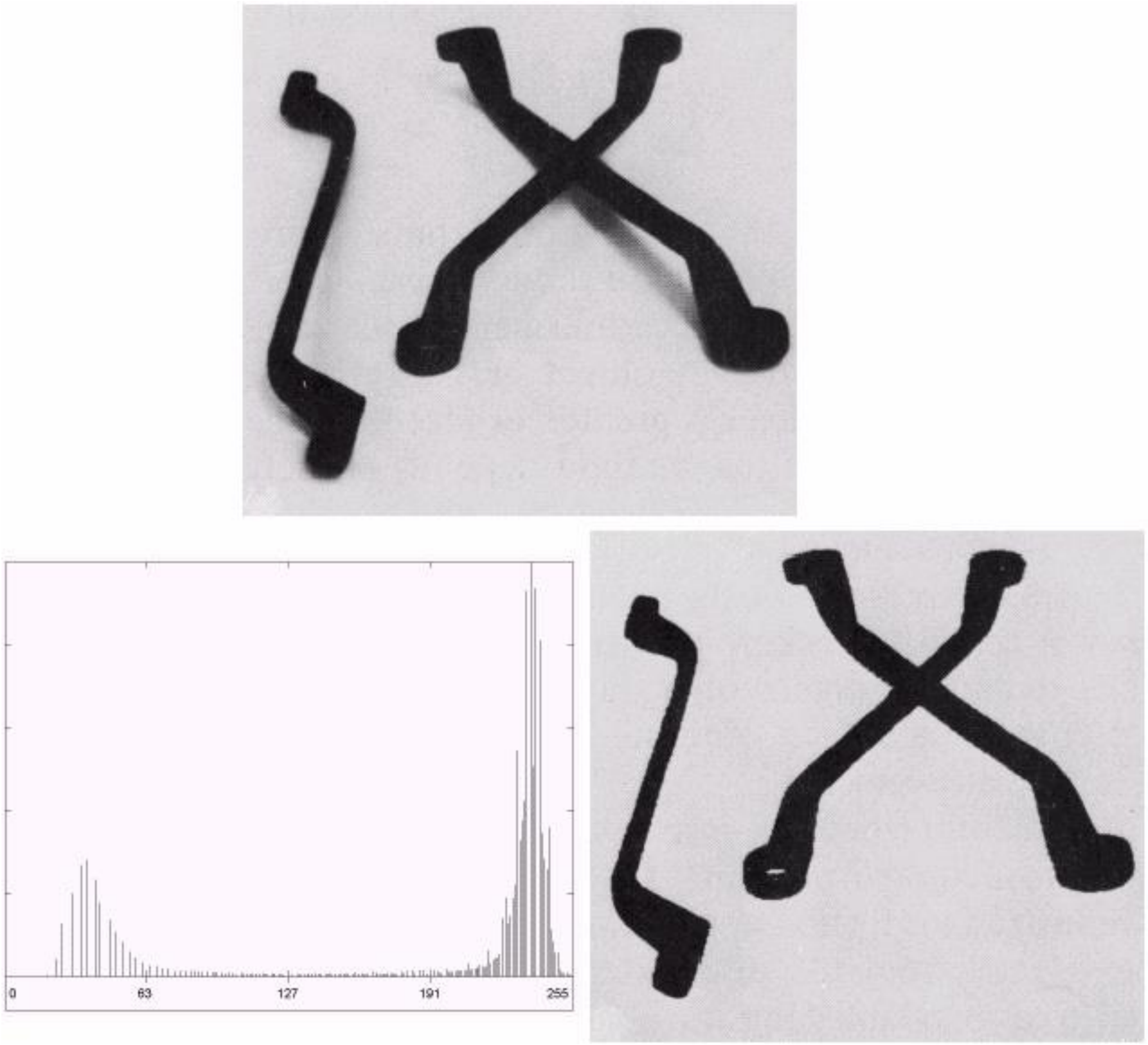


Image Analysis –Segmentation by Thresholding

- Heuristic approach to get global threshold T:

1. Select an initial estimate for T.
2. Segment the image using T. This will produce two groups of pixels: G_1 consisting of all pixels with gray level values > T and G_2 consisting of pixels with gray level values $\leq T$
3. Compute the average gray level values μ_1 and μ_2 for the pixels in regions G_1 and G_2
4. Compute a new threshold value $T = 0.5\ (\mu_1 + \mu_2)$
5. Repeat steps 2 through 4 until the difference in T in successive iterations is smaller than a predefined parameter T_o.

Image Analysis –Segmentation by Thresholding

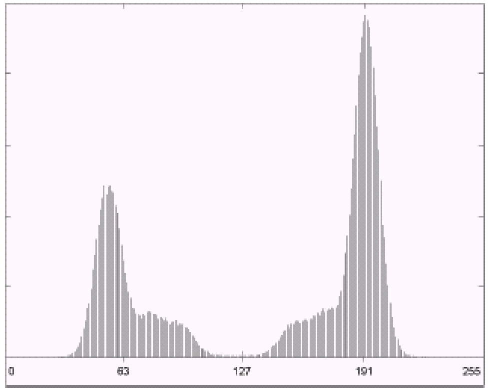
Example of

Heuristic approach:

T_0 = 0

3 iterations

with result T = 125



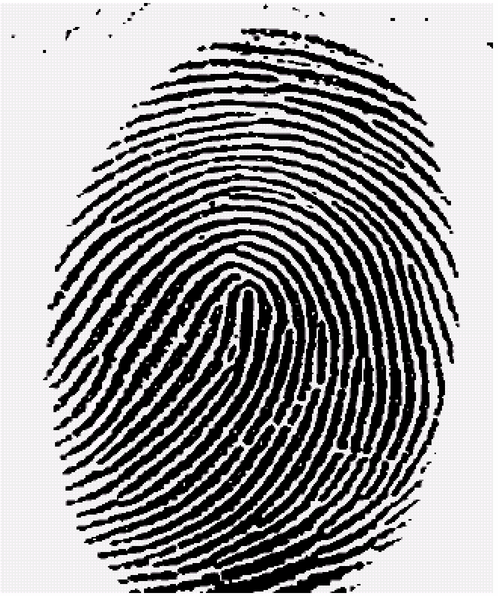


Image Analysis –Segmentation by Thresholding

If Object and background are separated in the grey value,

easily use global thresholding

If the grey value of object and background are overlapped, Difficult to segment using global thresholding

Solution: Adaptive local thresholding

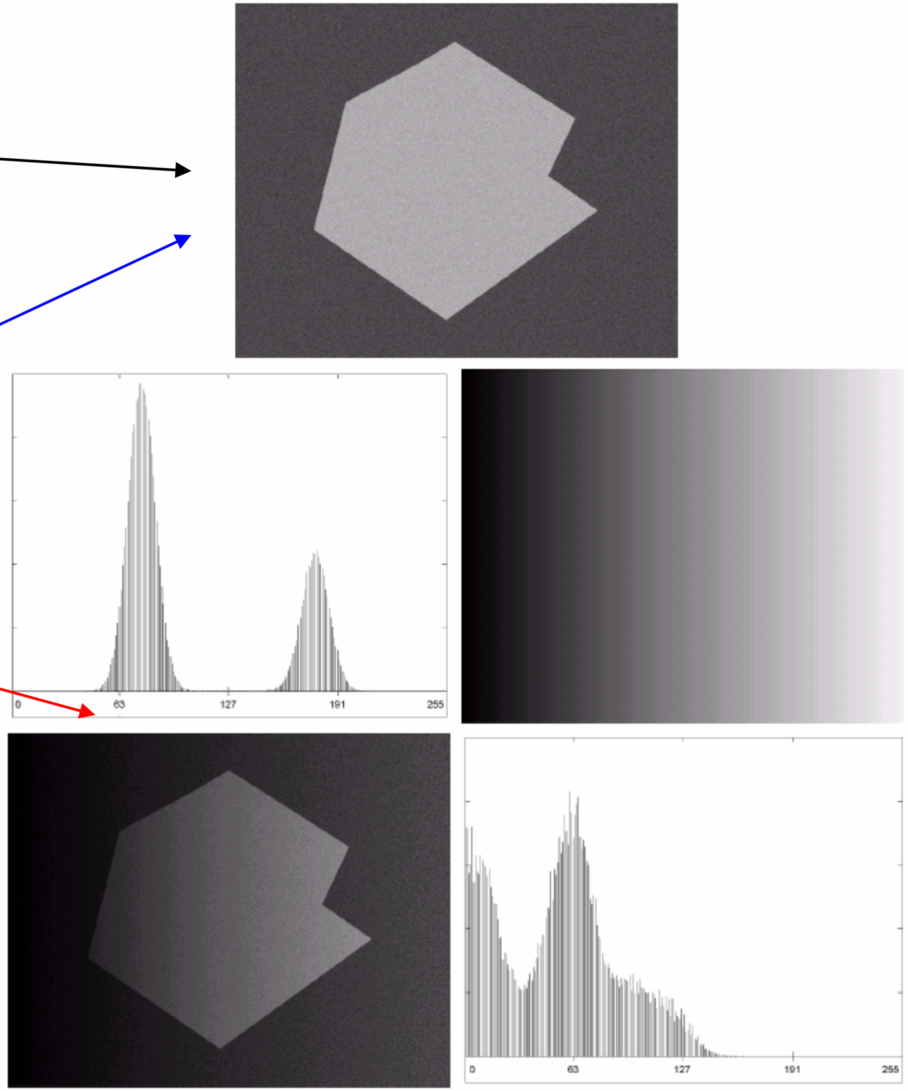


Image Analysis –Segmentation by Thresholding

- Adaptive local thresholding:

- Subdivide original image into small areas.
- Utilize a different threshold to segment each subimages.
- Since the threshold used for each pixel depends on the location of the pixel in terms of the subimages, this type of thresholding is adaptive.

Image Analysis –Segmentation by Thresholding

a b
c d

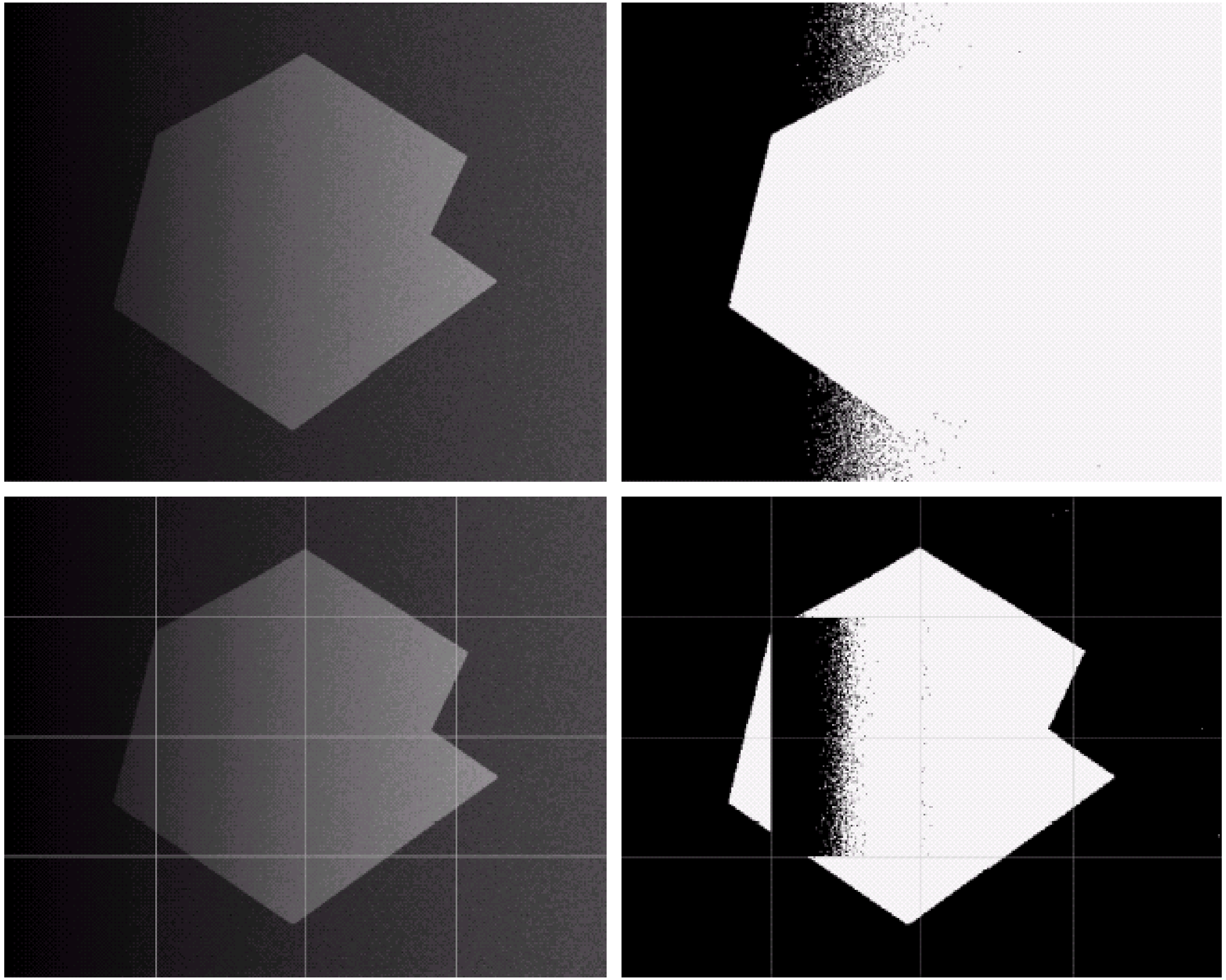


FIGURE 10.30 (a) Original image. (b) Result of global thresholding. (c) Image subdivided into individual subimages. (d) Result of adaptive thresholding.

Image Analysis –Segmentation by Thresholding

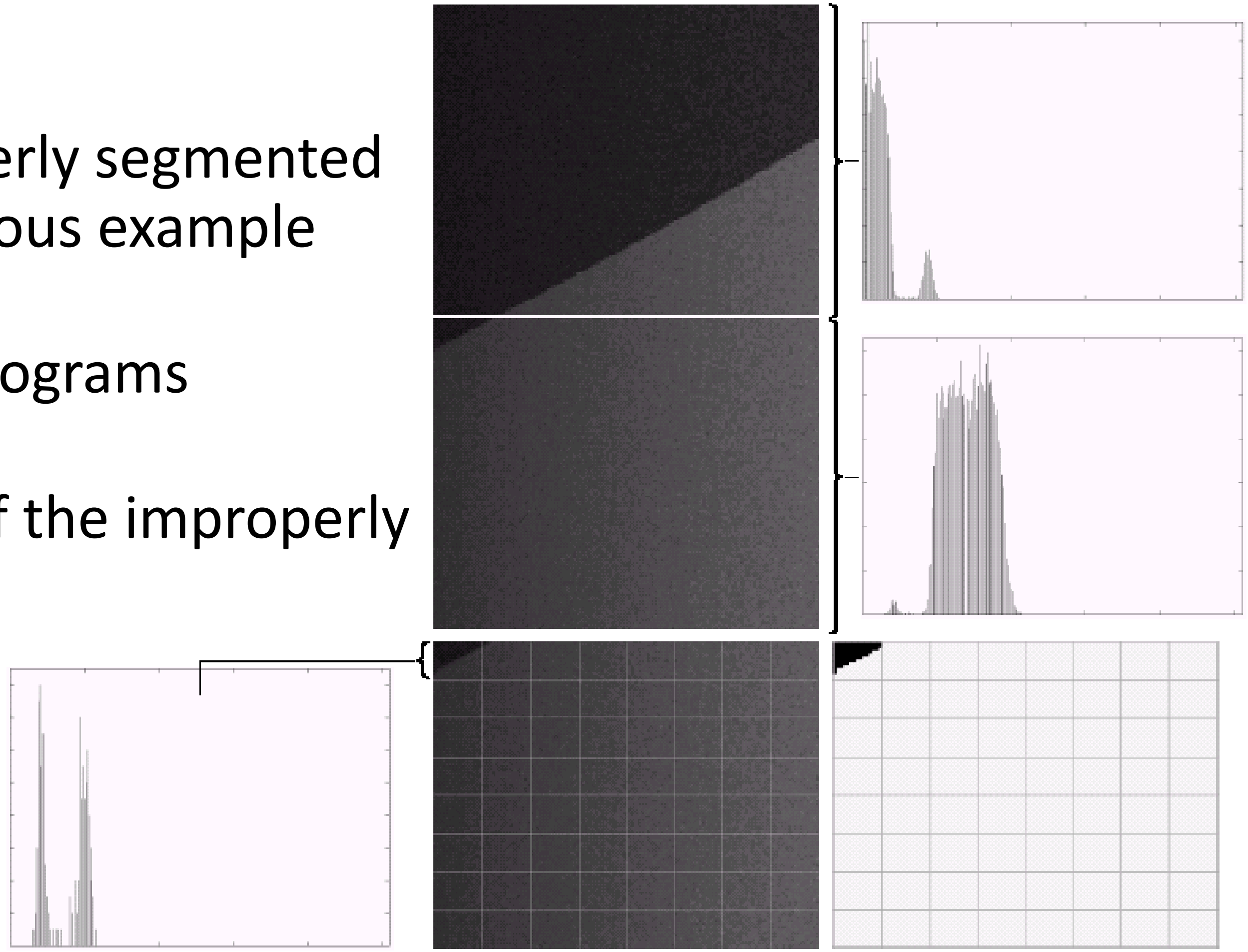
Further subdivision:

a) properly and improperly segmented subimages from previous example

b)-c) corresponding histograms

d) further subdivision of the improperly segmented subimage.

e) histogram of small subimage at top



f) result of adaptively segmenting d)

Image Analysis –Optimal Thresholding

- Objective:

  Minimize the average error in making decisions that a given pixel belongs to an object or the background

  - Assumptions:

    Image contains only 2 gray-level regions.

    $p_1(z)$ and $p_2(z)$ are the probability density functions of grey level z for region 1 (object) and 2 (background) respectively

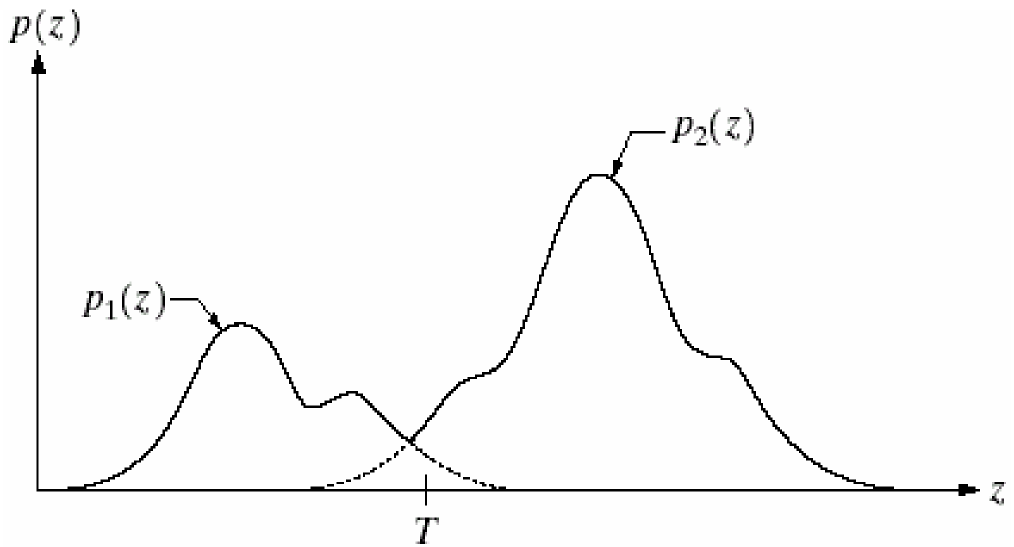
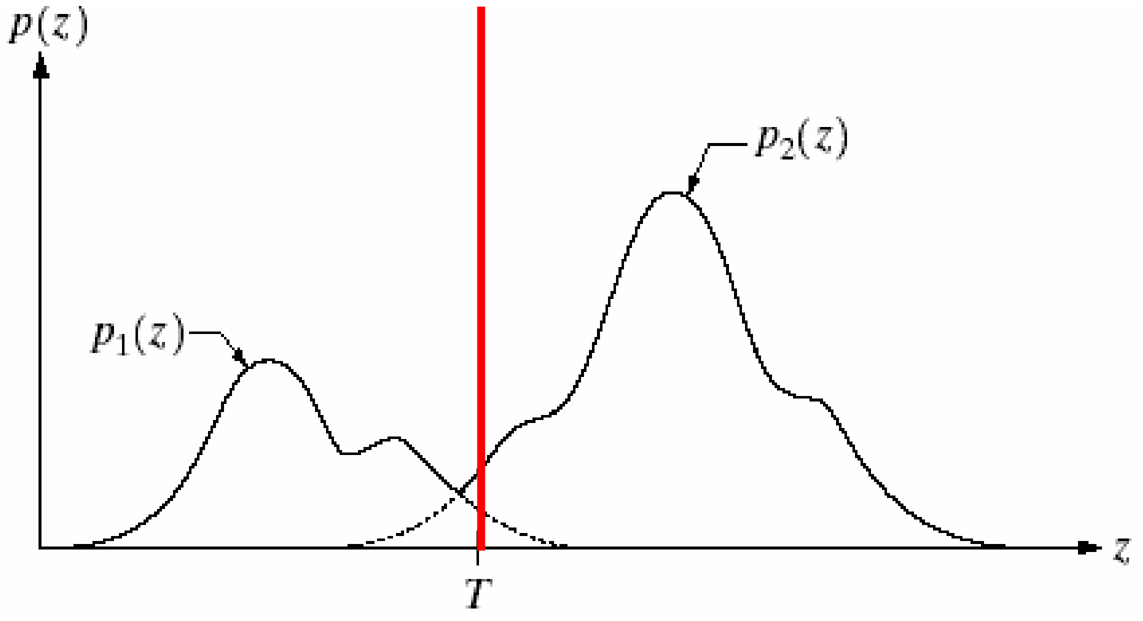


Image Analysis –Optimal Thresholding

- Probability of error in classifying a background point as an object:

$$E_1(T) = \int_{-\infty}^{T} p_2(z)dz$$



- Probability of error in classifying an object point as background:

$$E_2(T) = \int_{T}^{\infty} p_1(z)dz$$

Image Analysis –Optimal Thresholding

Mixture pdf of the overall image:

$$p(z) = P_1 p_1(z) + P_2 p_2(z)$$

- Assume any pixel belongs to either object or background:

$$P_1 + P_2 = 1$$

- Overall probability of error:

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$

Image Analysis –Optimal Thresholding

- To minimize the error, differentiate $E(T)$ with respect to T and let the result equal to 0

$$\frac{dE(T)}{dT} = \frac{d(P_2 E_1(T) + P_1 E_2(T))}{dT} = 0$$

$$\Downarrow$$

- Find T which makes $P_1 p_1(T) = P_2 p_2(T)$

- If $P_1 = P_2$, the optimum threshold is where the curve $p_1(z)$ and $p_2(z)$ intersect.

Image Analysis –Optimal Thresholding

- Assuming both $p_1(z)$ and $p_2(z)$ follow Gaussian distribution:

$$p(z) = \frac{P_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(z-\mu_1)^2}{2\sigma_1^2}} + \frac{P_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{(z-\mu_2)^2}{2\sigma_2^2}}$$

where

μ_1 and σ_1^2 are the mean and variance of the Gaussian density of one object.

μ_2 and σ_2^2 are the mean and variance of the Gaussian density of the other object.

Image Analysis –Optimal Thresholding

- The optimum T is obtained by solve:

$$P_1 p_1(T) = P_2 p_2(T) \Rightarrow \frac{P_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(T-\mu_1)^2}{2\sigma_1^2}} = \frac{P_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{(T-\mu_2)^2}{2\sigma_2^2}}$$

- This results in a quadratic equation:

$$AT^2 + BT + C = 0$$

$$\text{where } A = \sigma_1^2 - \sigma_2^2, \quad B = 2(\mu_1\sigma_2^2 - \mu_2\sigma_1^2)$$

$$C = \sigma_1^2\mu_2^2 - \sigma_2^2\mu_1^2 + 2\sigma_1^2\sigma_2^2 \ln(\sigma_2 P_1 / \sigma_1 P_2)$$

Image Analysis –Optimal Thresholding

- If $\sigma_1 = \sigma_2 = \sigma$, the optimum threshold is simply obtained by

$$T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln\left(\frac{P_2}{P_1}\right)$$

- if $P_1 = P_2$, then the optimal threshold is the average of the two means

$$T = \frac{\mu_1 + \mu_2}{2}$$

Image Analysis –Optimal Thresholding

- Example:

cardioangiogram before and after preprocessing.

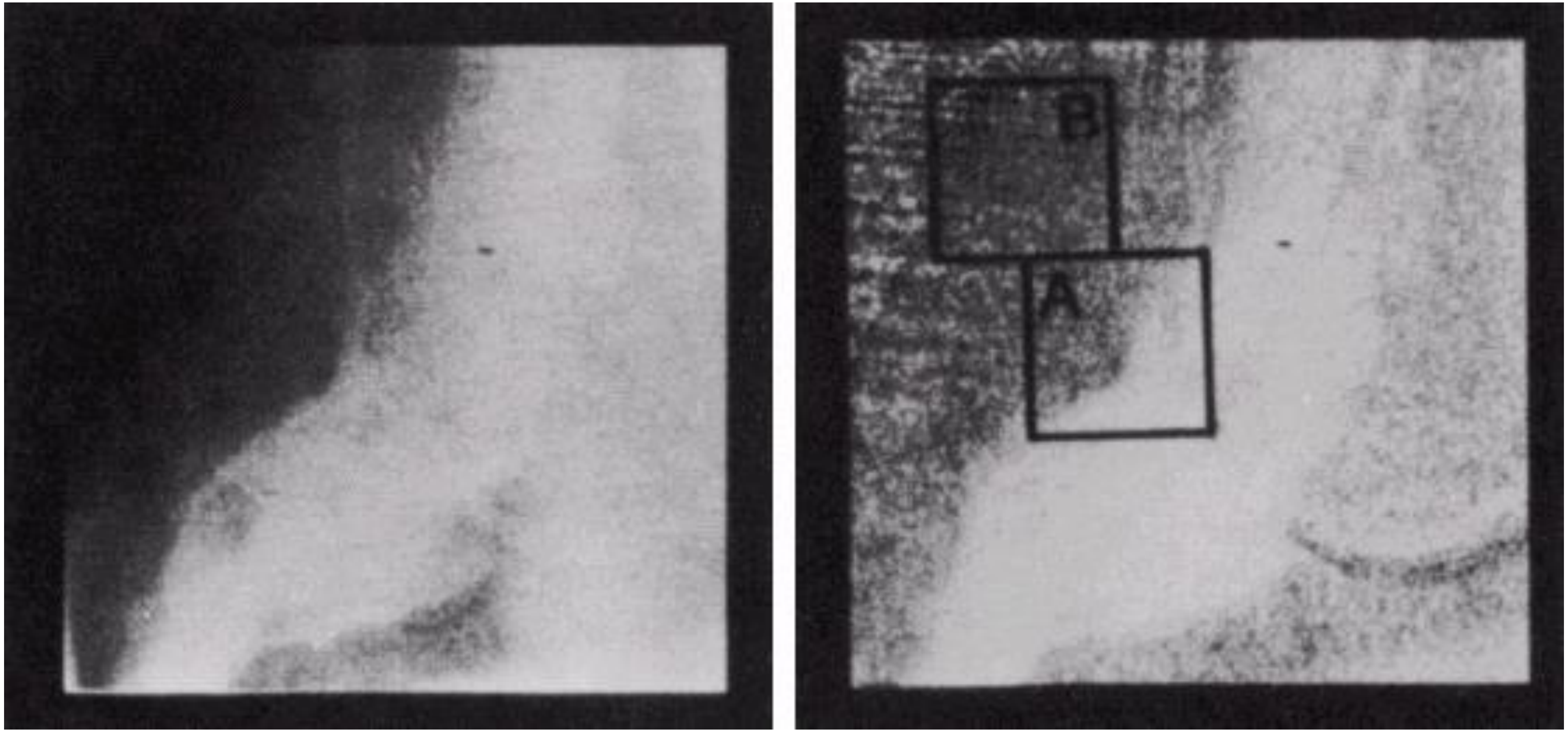
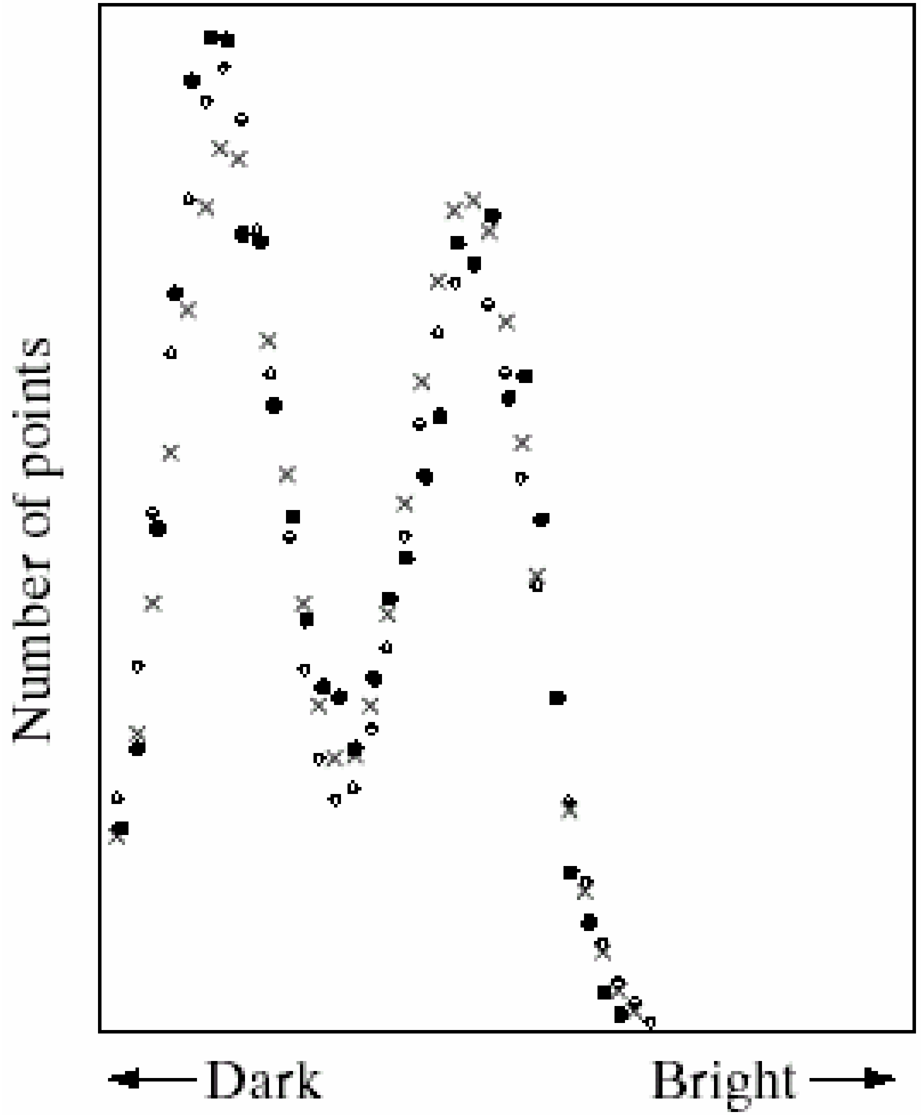
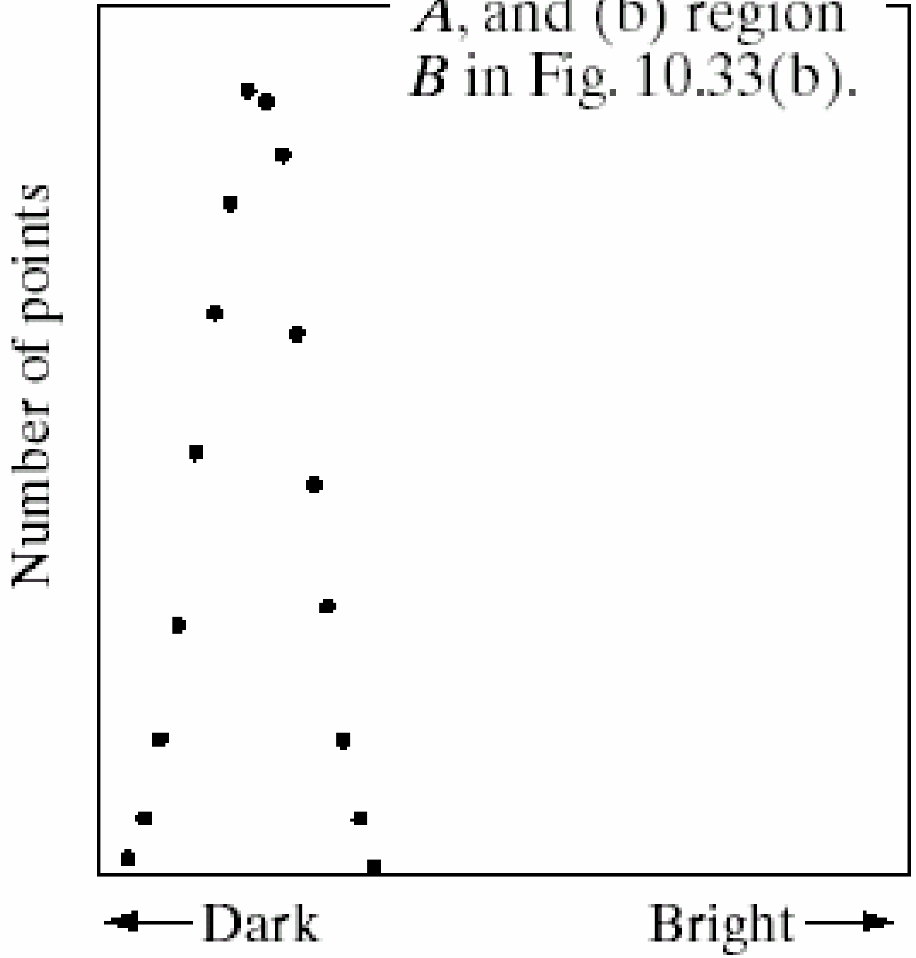


Image Analysis –Optimal Thresholding

Example:





Histograms (black dots) of (a) region A, and (b) region B in Fig. 10.33(b).

Image Analysis –Optimal Thresholding

Example:

Cardioangiogram showing superimposed boundaries.

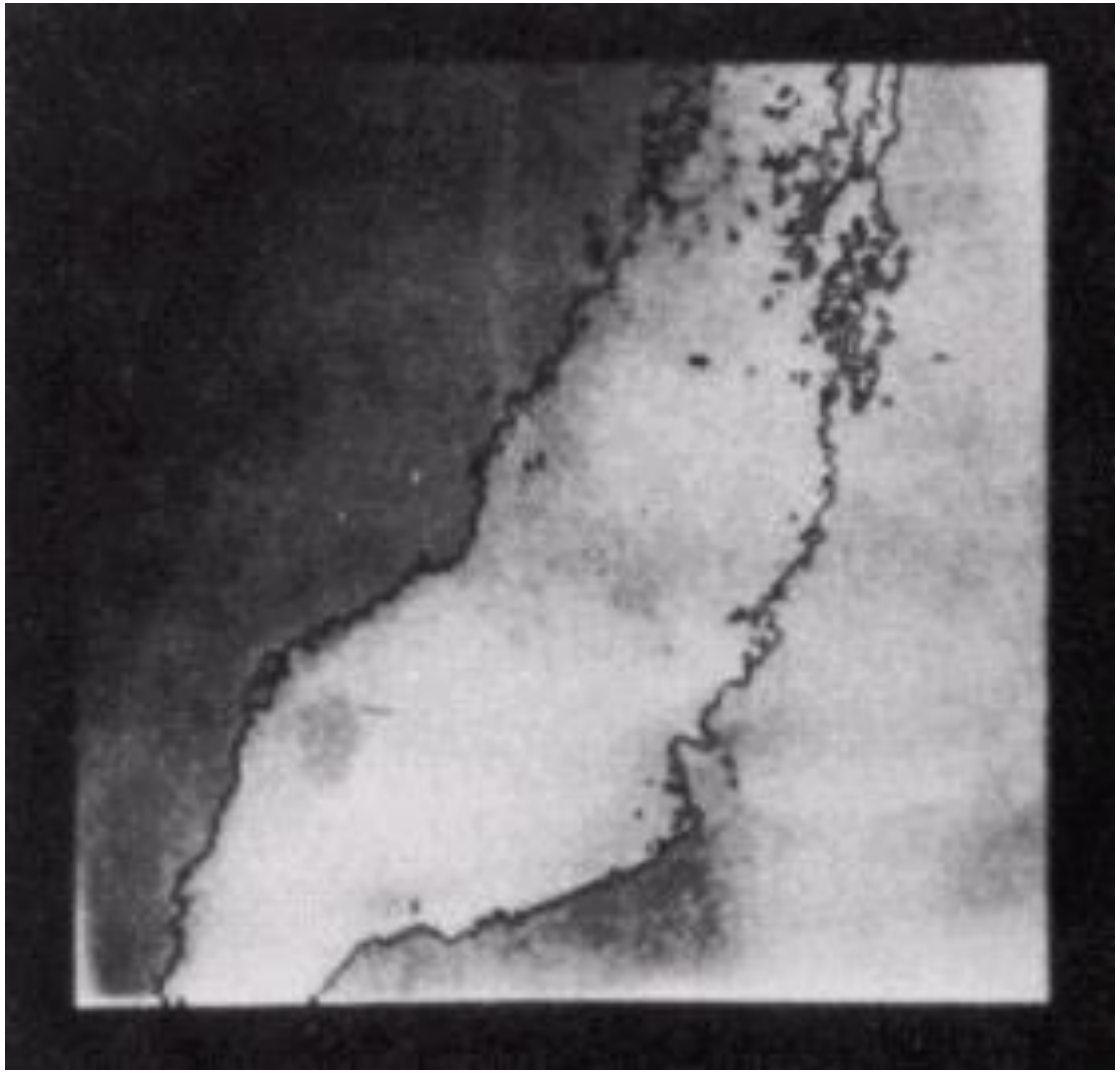


Image Analysis –Detection of Discontinuities

- Detect the three basic types of gray-level discontinuities (abrupt changes in intensity)
  - Point detection
  - Line detection
  - Edge detection

- The common way is to run a mask through the image

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Image Analysis –Point Detection

$$R = \sum_{i=1}^{9} w_i z_i$$

−1	−1	−1
−1	8	−1
−1	−1	−1

- The formulation measures the weighted difference between the center point and its neighbors.
- A point has been detected at the location on which the mask is centered if

$$|R| \geq T$$

where

- T is a nonnegative threshold
- R is the sum of products of the coefficients with the gray levels contained in the region encompassed by the mark.

Image Analysis –Point Detection example

(b) X-ray image of a turbine blade with a porosity.

Result of mask output and point detection

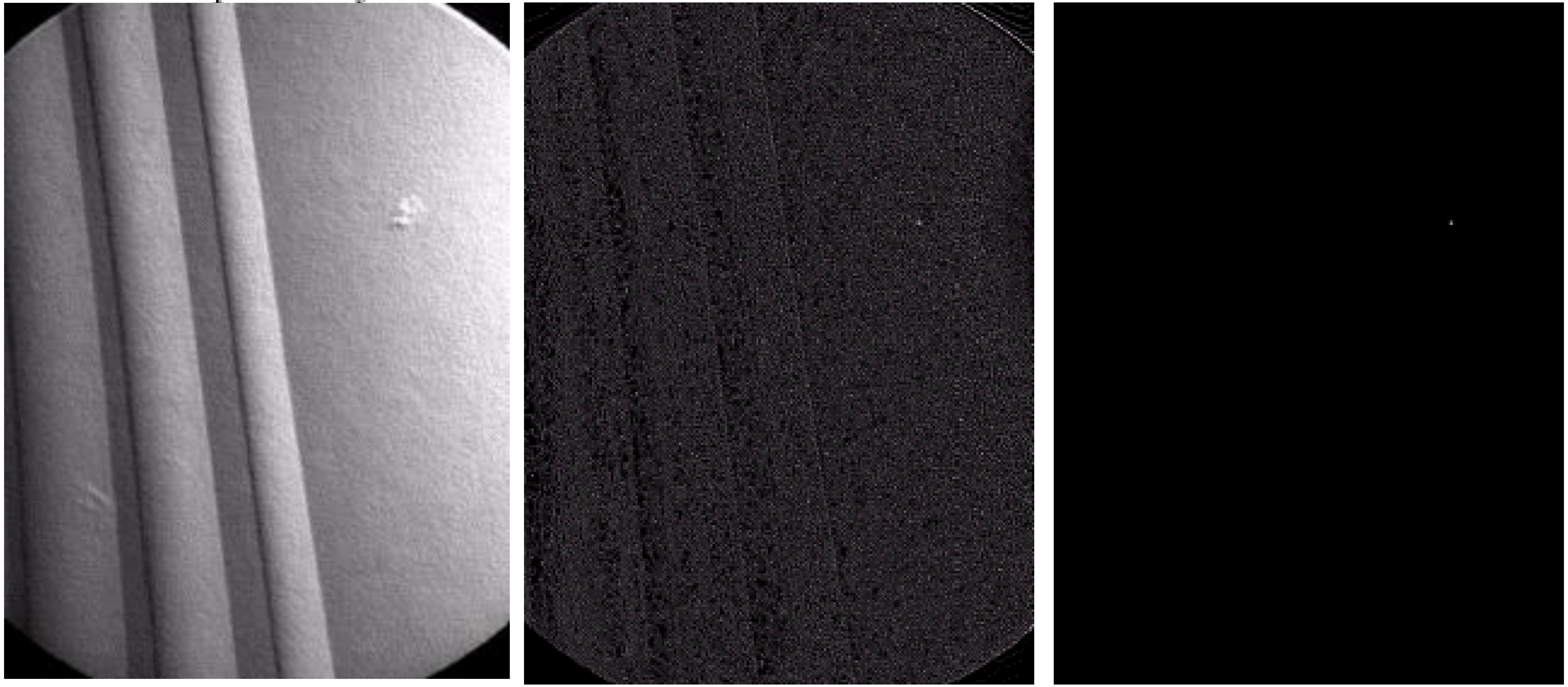


Image Analysis –Line Detection

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

−1	−1	2
−1	2	−1
2	−1	−1

+45°

−1	2	−1
−1	2	−1
−1	2	−1

Vertical

2	−1	−1
−1	2	−1
−1	−1	2

−45°

- Horizontal mask will result in maximum response when a line passed through the middle row of the mask with a constant background.
- The similar idea is used with other masks.
- note: The preferred direction of each mask is weighted with a larger coefficient (i.e.,2) than other possible directions.

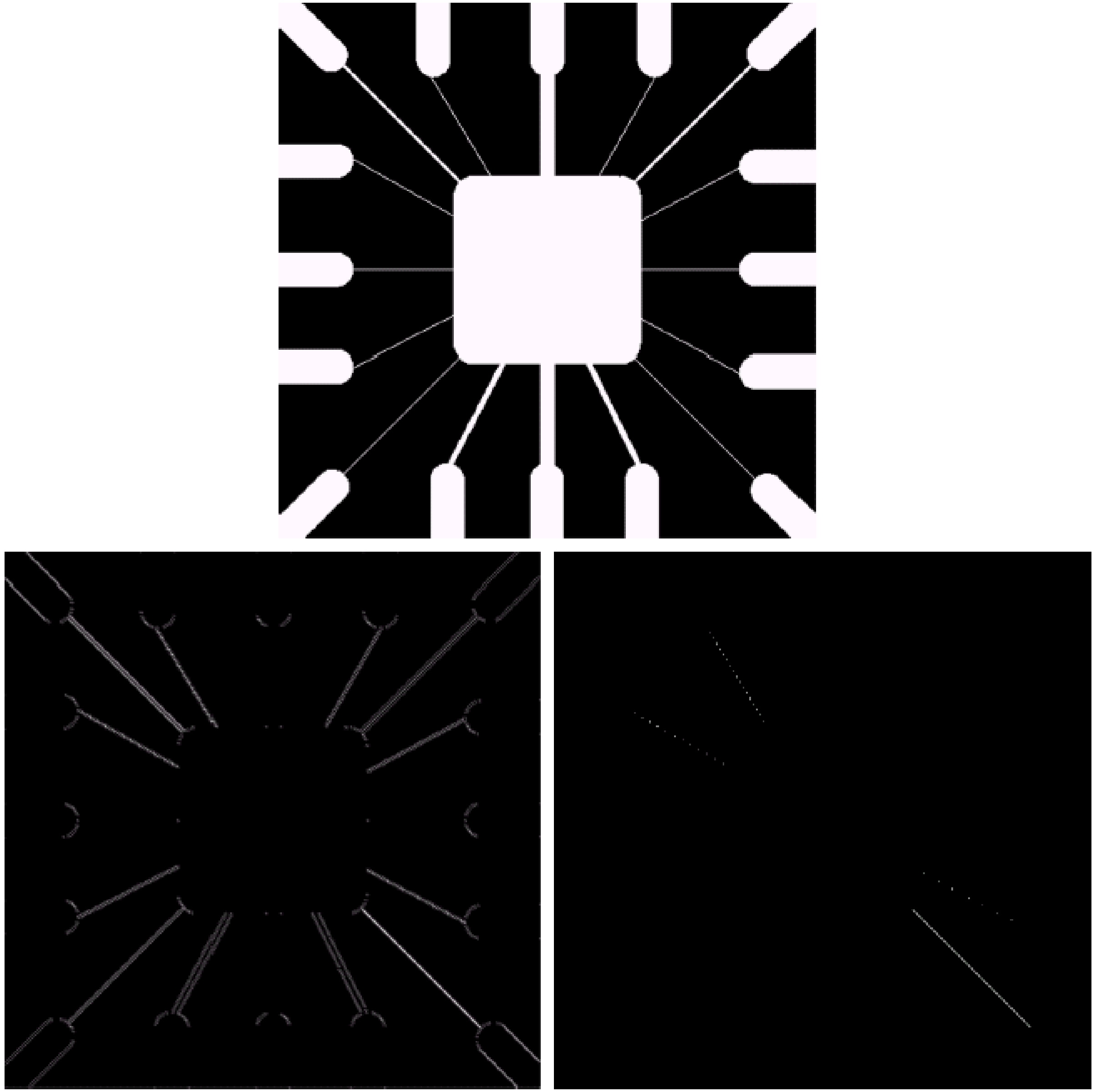
Image Analysis –Line Detection

- Apply every mask on the image.
- let R_1, R_2, R_3, R_4 denotes the response of the horizontal, +45 degree, vertical and -45 degree masks, respectively.
- if, at a certain point in the image

$$|R_i| > |R_j|,$$

- for all $j \neq i$, that point is said to be more likely associated with a line in the direction of mask i.
- To detect all lines in an image in the direction defined by a given mask, we simply run the mask through the image and threshold the absolute value of the result.
- Points left are the strongest responses, which, correspond closest to the direction defined by the mask.

Image Analysis –Line Detection Example



a
b c

FIGURE 10.4 Illustration of line detection. (a) Binary wire-bond mask. (b) Absolute value of result after processing with $-45°$ line detector. (c) Result of thresholding image (b).

Image Analysis –Edge Detection

- How can an algorithm extract relevant information from an image to recognize objects?

- Most important information for the interpretation of an image (for both technical and biological systems) is the contour of objects.

- Contours are indicated by abrupt changes in brightness.

- We can use edge detection filters to extract contour information from an image.

Image Analysis –Edge Detection

- Edge detection is the most common approach for detecting meaningful discontinuities in gray level.
- We will discuss approaches of
  - first-order derivative (Gradient operator)
  - second-order derivative (Laplacian operator)

- Intuitively, an edge is a set of connected pixels that lie on the boundary between two regions.
- Changes or discontinuities in image amplitude provide an indication of physical extent of object

Image Analysis –Edge Detection

- Edge

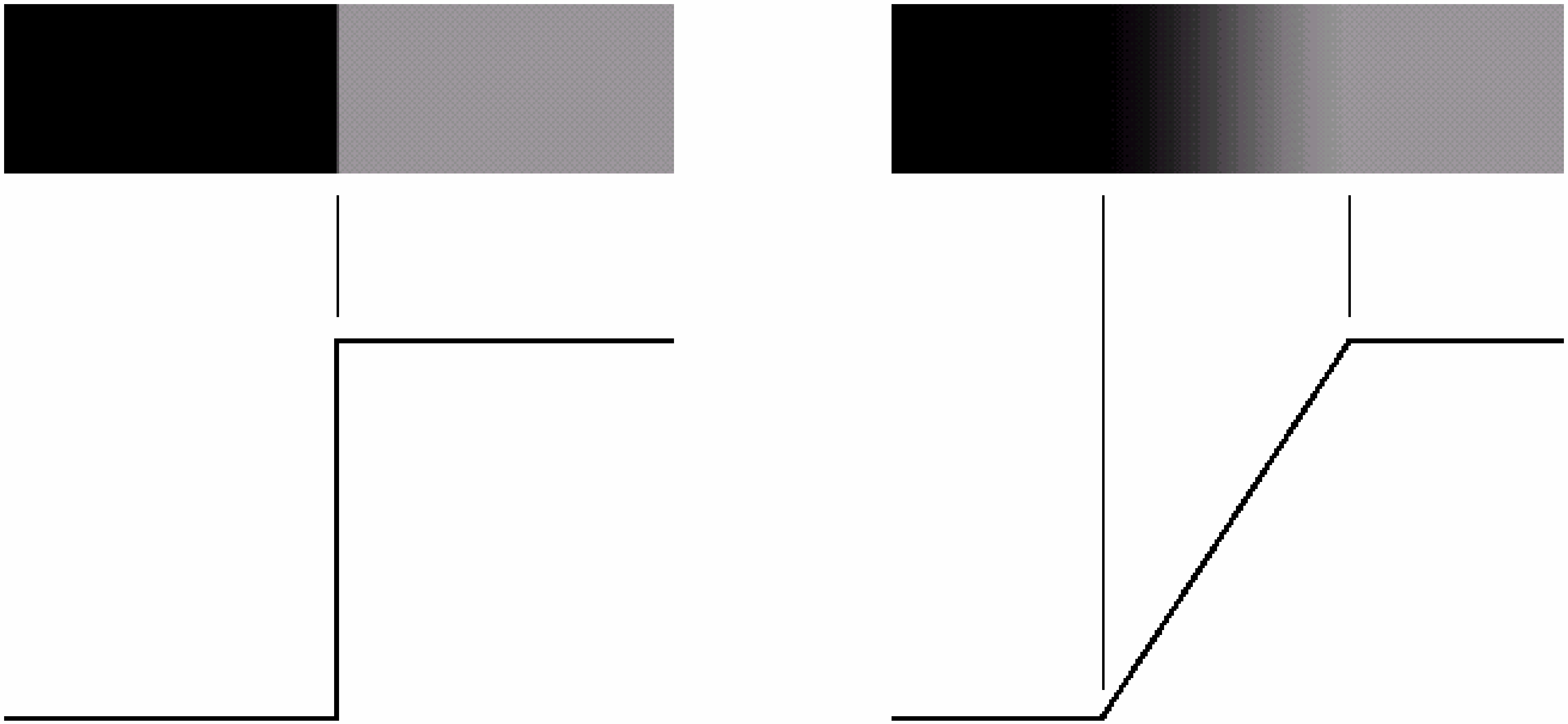
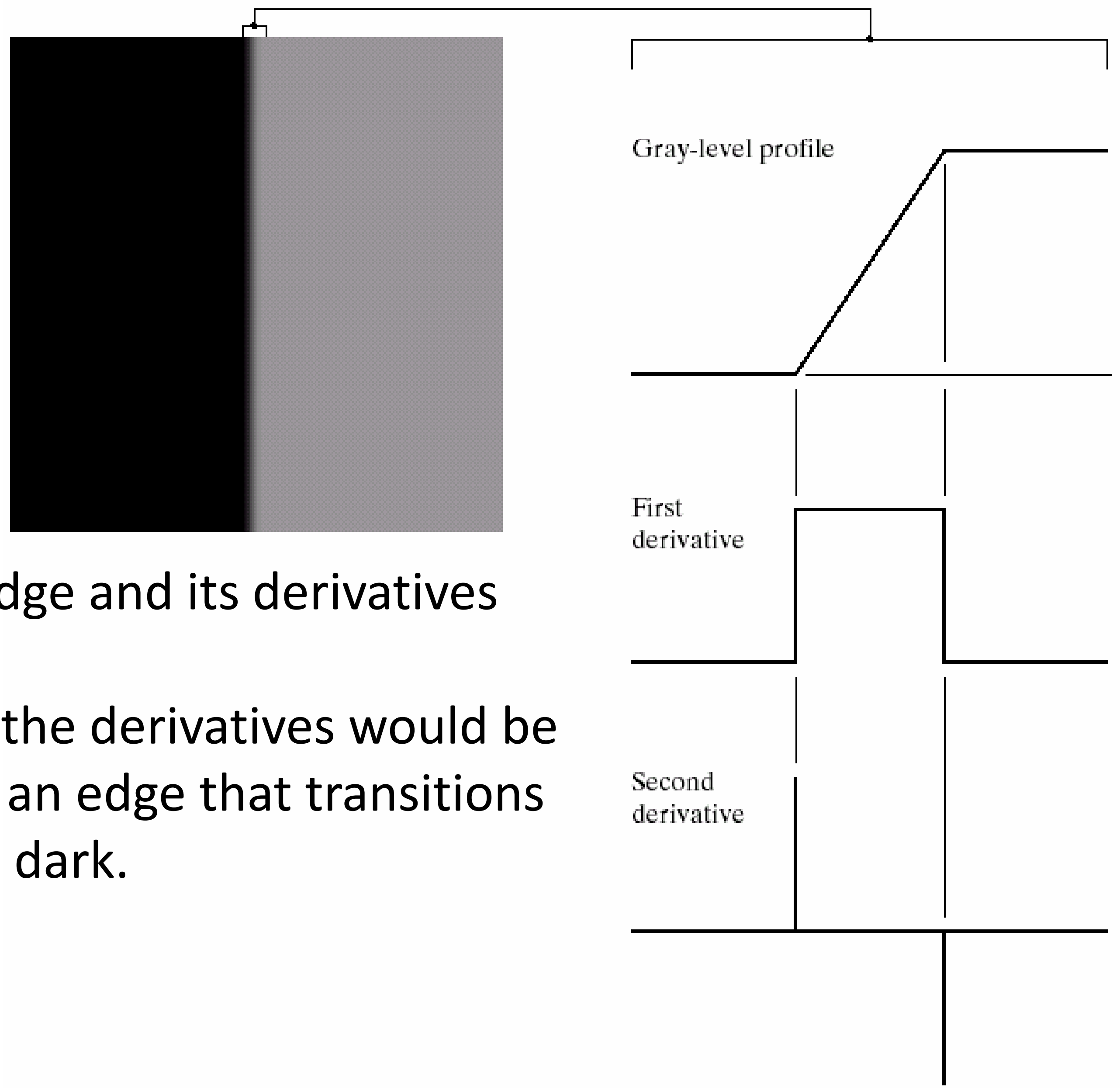


Image Analysis –Edge First & Second Derivatives



- Noise free edge and its derivatives
- The signs of the derivatives would be reversed for an edge that transitions from light to dark.

Image Analysis –Noisy Edge Derivatives

- First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and σ = 0.0, 0.1, 1.0 and 10.0, respectively.
- Second column: first-derivative images and gray-level profiles.
- Third column : second-derivative images and gray-level profiles..

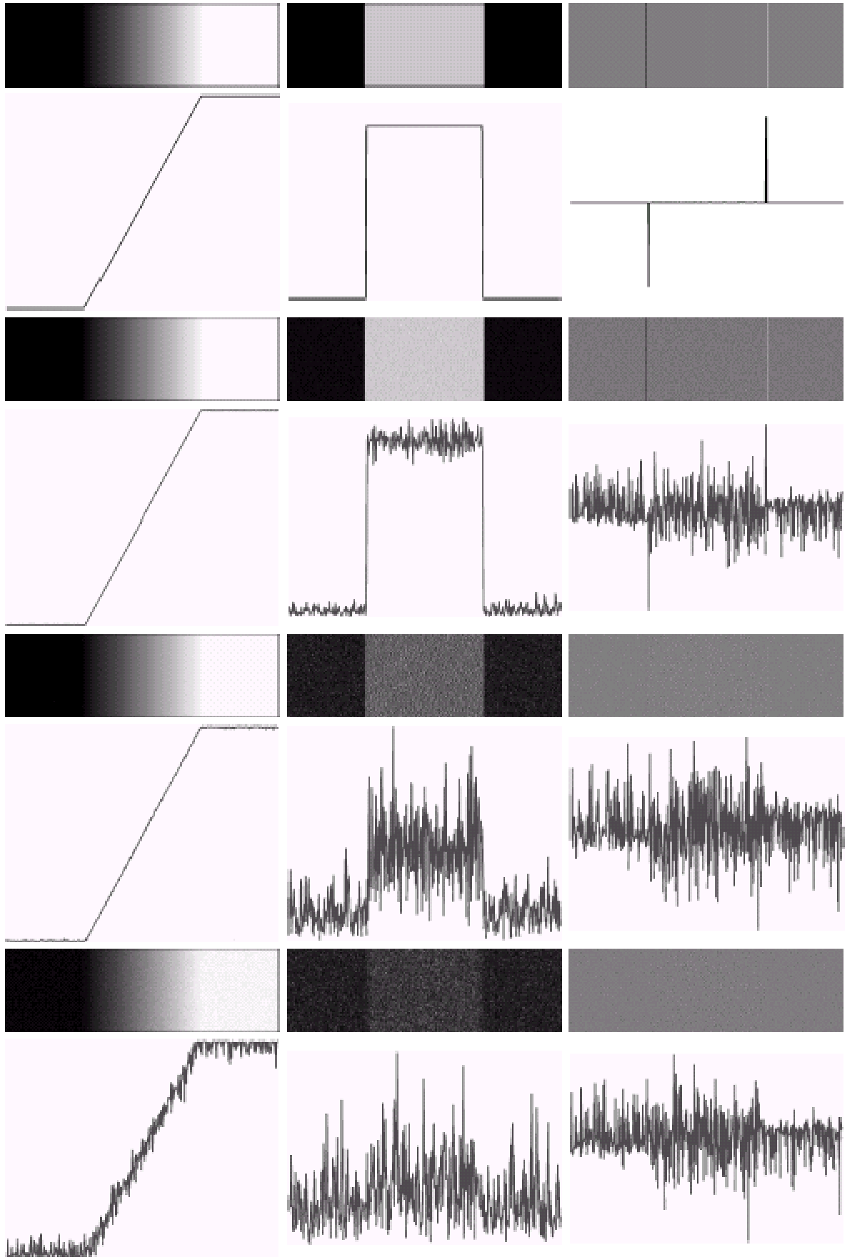


Image Analysis –Noisy Edge Derivatives

- Fairly little noise can have a significant impact on the two key derivatives used for edge detection in images.
- Image smoothing should be serious consideration prior to the use of derivatives in applications where noise is likely to be present.

- To determine a point as an edge point
  The transition in grey level associated with the point has to be significantly stronger than the background at that point.
  Use threshold to determine whether a value is “significant” or not.
  The point’s two-dimensional first-order derivative must be greater than a specified threshold.

Image Analysis –Image First Derivative: Gradient

- The first-order derivative of image called gradient is a two-dimensional vector, which consists of x- and y-differentials.

$$\nabla f(x,y) = \begin{bmatrix} G_x(x,y) \\ G_y(x,y) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f(x,y)}{\partial x} \\ \dfrac{\partial f(x,y)}{\partial y} \end{bmatrix}$$

- The strength of the differentials is proportional to the degree of discontinuity of the image.
- Thus, image differentiation
  enhances edges and other discontinuities (noise)
  deemphasizes area with slowly varying gray-level values.

Image Analysis –Image First Derivative: Gradient

- A image gradient vector has magnitude

$$\left|\nabla f(x,y)\right| = [G_x^2(x,y) + G_y^2(x,y)]^{1/2}$$

And direction

$$\theta(x,y) = \tan^{-1}\left(\frac{G_y(x,y)}{G_x(x,y)}\right)$$

- Gradient vector points in the direction of maximum rate of change of f at coordinate (x, y).

- The direction of an edge at (x, y) is perpendicular to the direction of the gradient vector at that point.

Image Analysis –Image First Derivative: Gradient

- A simple approximation of the x- and y-differentials for digital image.

$$G_x(x,y) = f(x+1,y) - f(x-1,y)$$
$$G_y(x,y) = f(x,y+1) - f(x,y-1)$$

- This is equivalent to run the two simple 3X3masks through the image:

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

- However, it is very sensitive to noise.

Image Analysis –Image First Derivative: Gradient

- Therefore, certain smoothing is desirable prior to application of differentiation.

$$\nabla[h(x,y) * f(x,y)] = [\nabla h(x,y)] * f(x,y)$$

- Due to linearity of differentiation, differentiate the image convolved (smoothed) with h is same as convolving an image with $\nabla h(x,y)$. So we have gradient operator (Mask) $\nabla h(x,y)$.

- Different design of the smooth filter $h(x,y)$ leads to various different gradient operators (Masks) $\nabla h(x,y)$.

Image Analysis –Image First Derivative: Gradient

- Some gradient masks

$$G_y = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$G_x = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

Image Analysis –Image First Derivative: Gradient

Gradient operator in diagonal form

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

Image Analysis –Gradient Examples

original, x- and y-differentials and gradient images with 3×3 masks

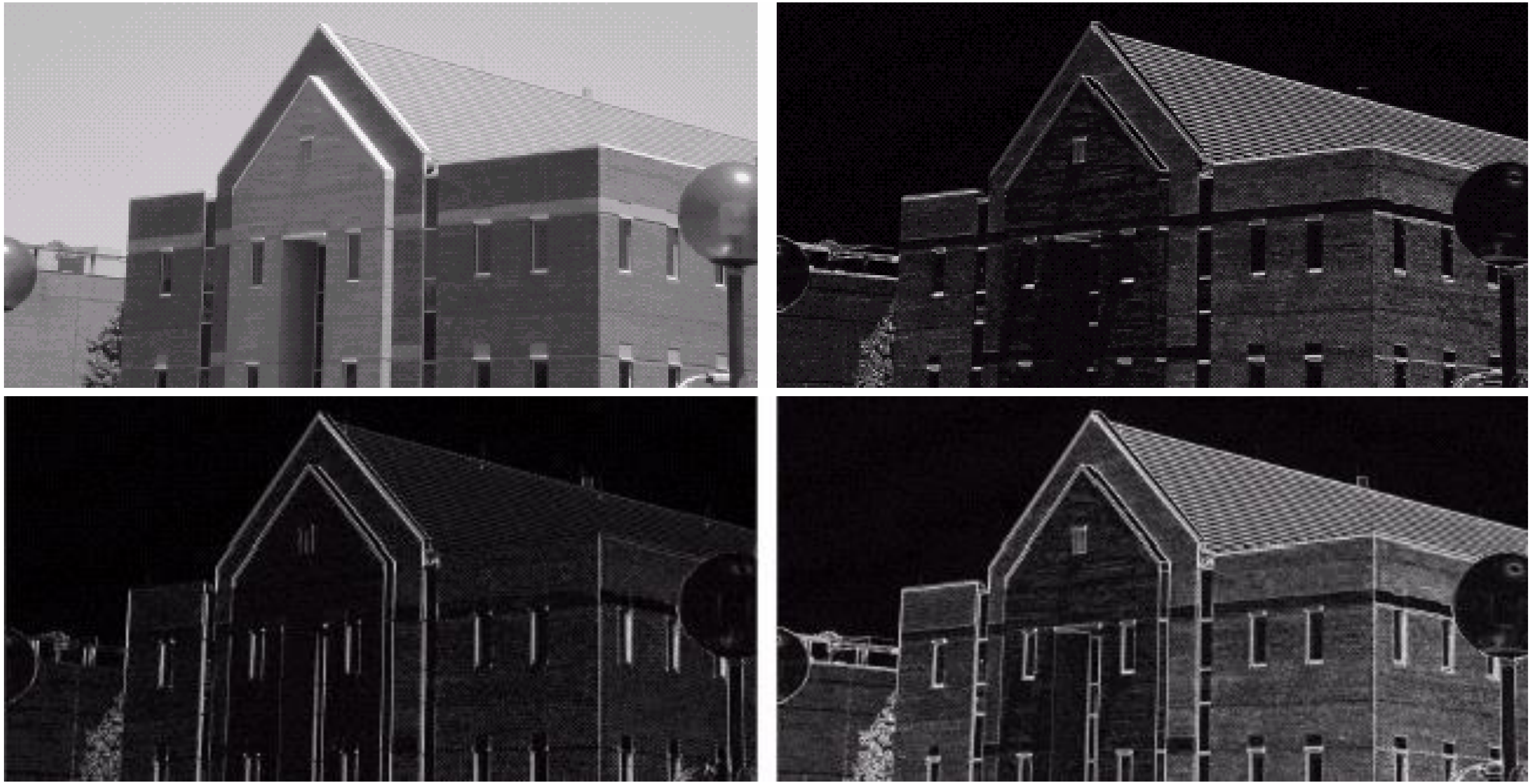


Image Analysis –Gradient Examples

original, x- and y-differentials and gradient images with 5×5 masks

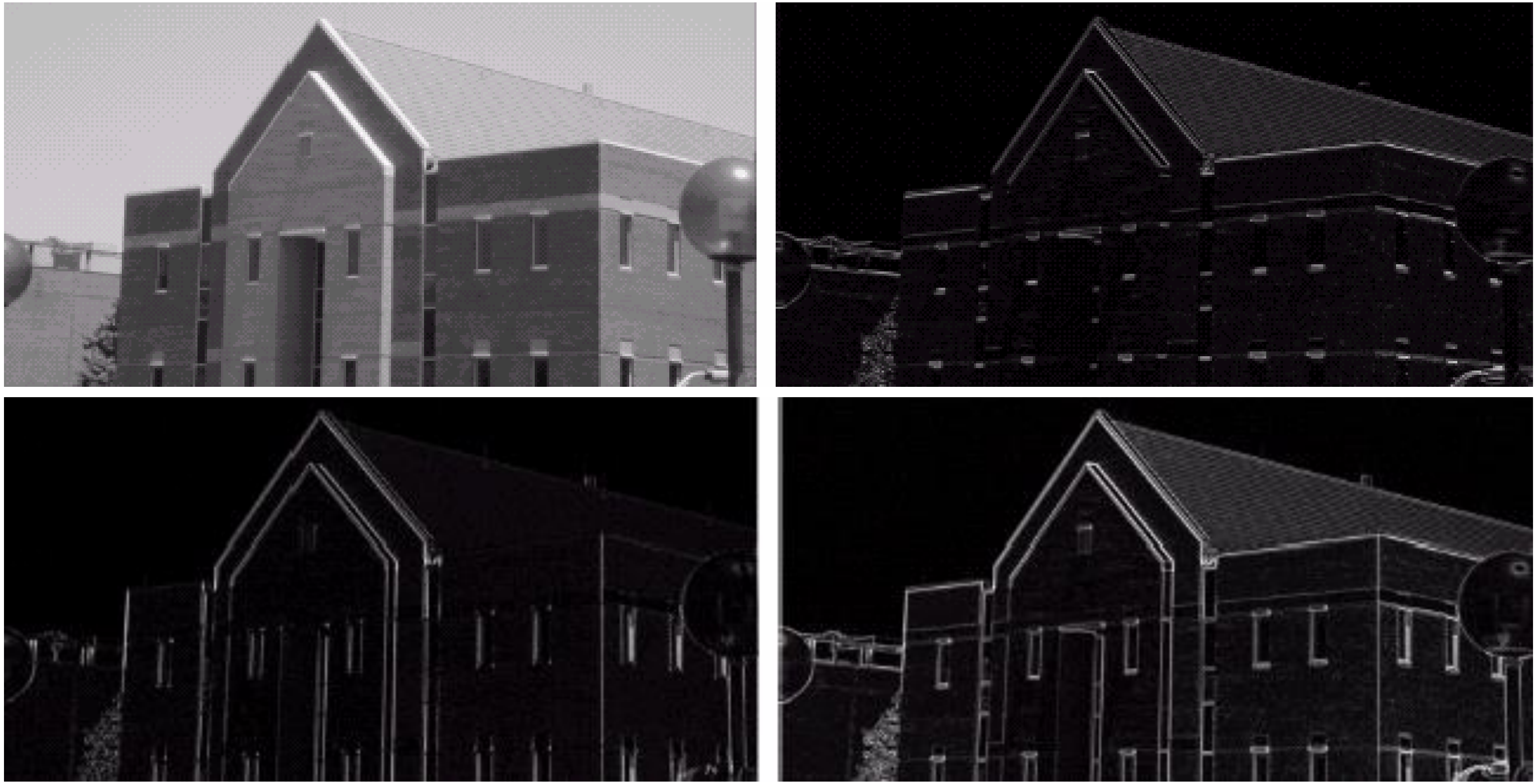


Image Analysis –Gradient Examples

Example

Image Analysis –Gradient Examples

Example

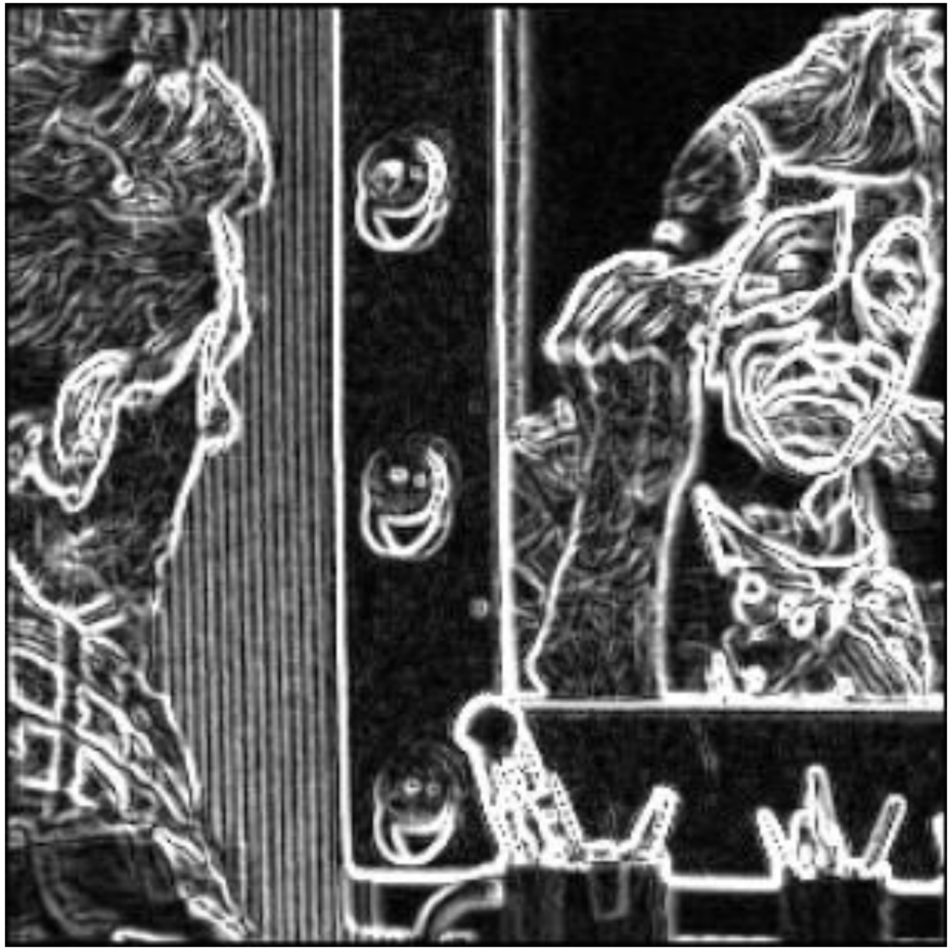


Image Analysis –Second-Order Derivative

- Second-order derivative is also called Laplacian operator defined by

$$\nabla^2 f = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

- Approximation in discrete domain:

$$\frac{\partial^2 f}{\partial x^2} = [f(x+1,y) - f(x,y)] - [f(x,y) - f(x-1,y)]$$

$$= f(x+1,y) + f(x-1,y) - 2f(x,y)$$

- Similarly: $$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Image Analysis –Second-Order Derivative

- This yield

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

- This can be implemented by a four-neighbor Laplacian mask or an eight-neighbor Laplacian mask

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Image Analysis –Laplacian of Gaussian (LoG)

- Laplacian is sensitive to noise
- Therefore, certain smoothing is desirable prior to application of Laplacian
- Solution: Employ Gaussian-shaped smoothing

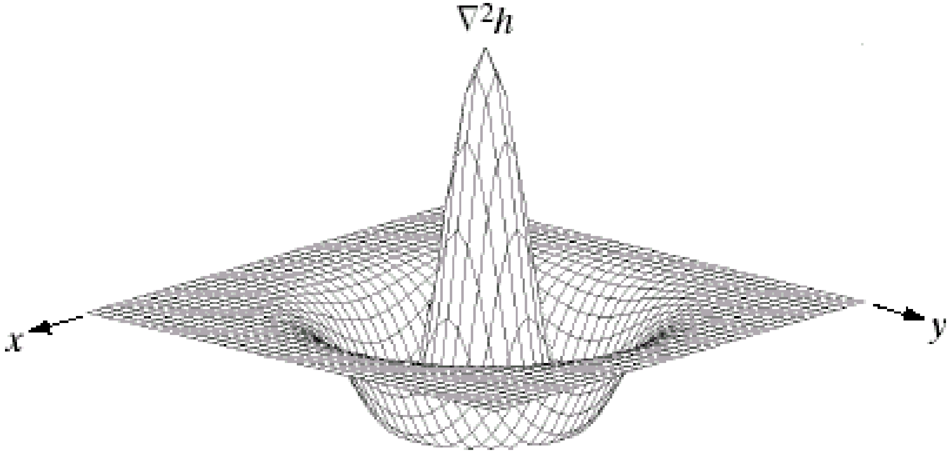
$$h(x,y) = -e^{-\frac{x^2+y^2}{2\sigma^2}} = -e^{-\frac{r^2}{2\sigma^2}}$$

- Due to linearity of second derivative, taking the Laplacian of the image convolved (smoothed) with *h* is same as convolving an image with $\nabla^2 h$.

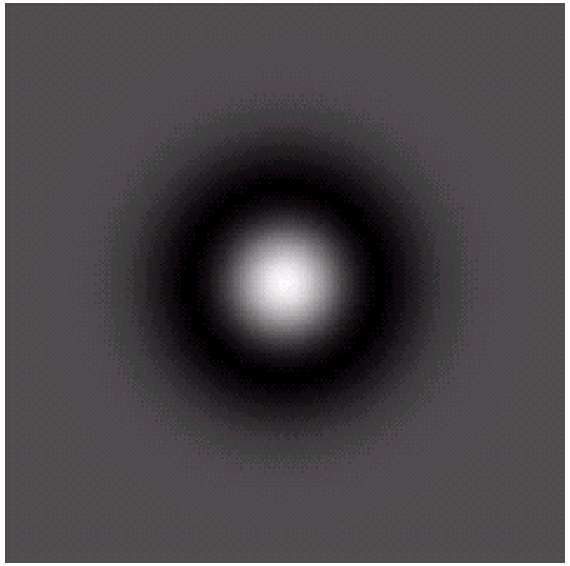
$$\nabla^2(h * f) = (\nabla^2 h) * f$$

Image Analysis –Laplacian of Gaussian (LoG)

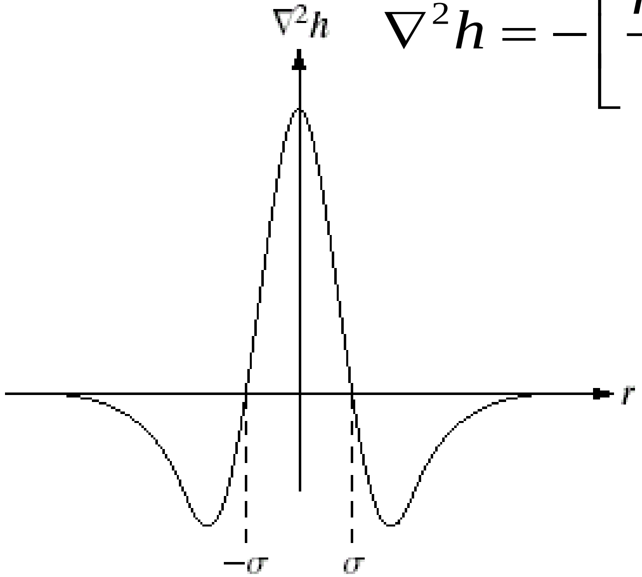
- It is easy to have



Mexican hat



$$\nabla^2 h = -\left[\frac{r^2 - \sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$



0	0	–1	0	0
0	–1	–2	–1	0
–1	–2	16	–2	–1
0	–1	–2	–1	0
0	0	–1	0	0

a b
c d

FIGURE 10.14 Laplacian of a Gaussian (LoG). (a) 3-D plot. (b) Image (black is negative, gray is the zero plane, and white is positive). (c) Cross section showing zero crossings. (d) 5 × 5 mask approximation to the shape of (a).

the coefficient must be sum to zero

Image Analysis –Example of LoG

a) original image
b) Sobel gradient
c) spatial Gaussian
  smoothing function
d) Laplacian mask
e) LoG
f) threshold LoG
g) zero crossing

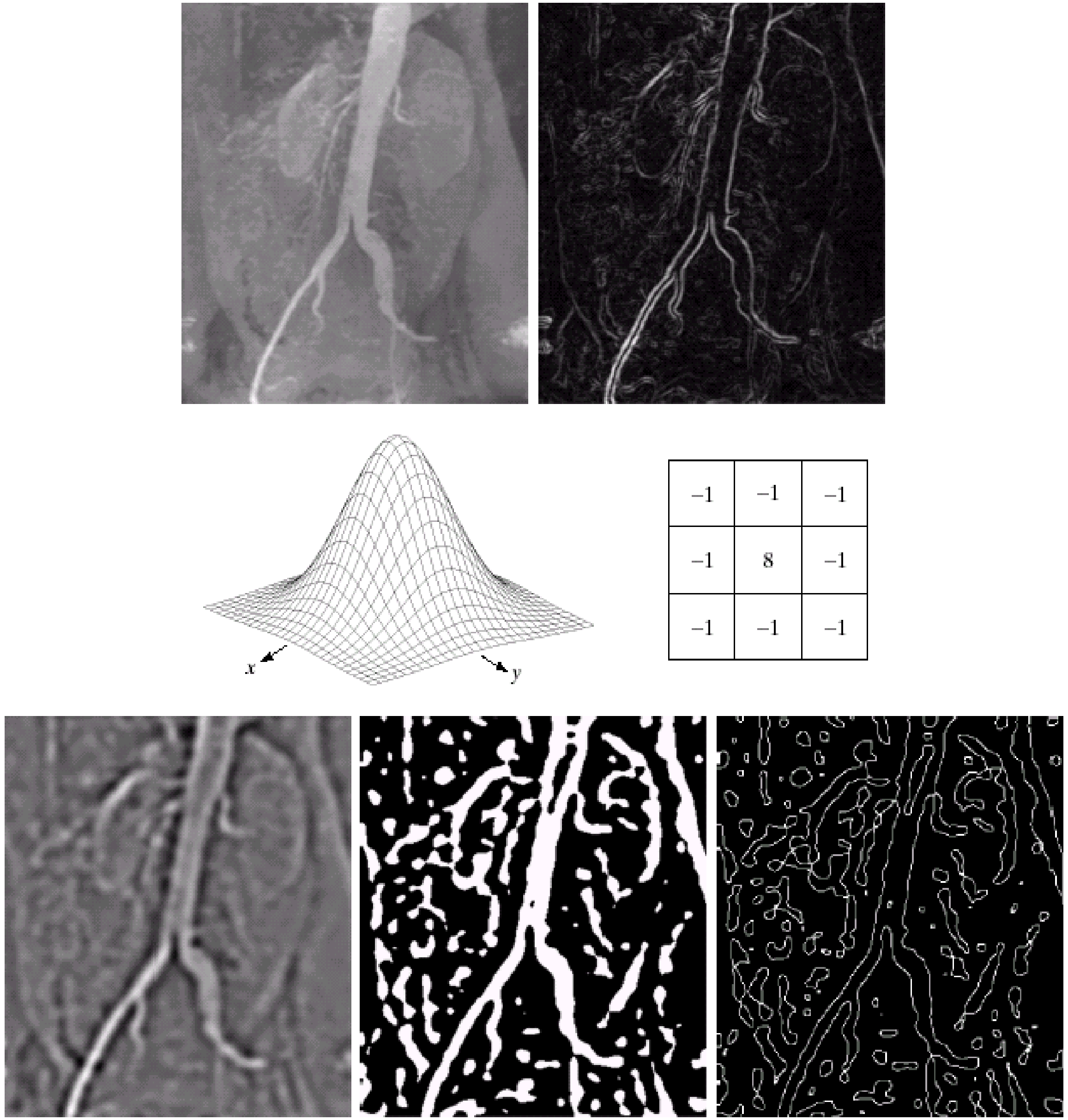


Image Analysis –Example of LoG

Example

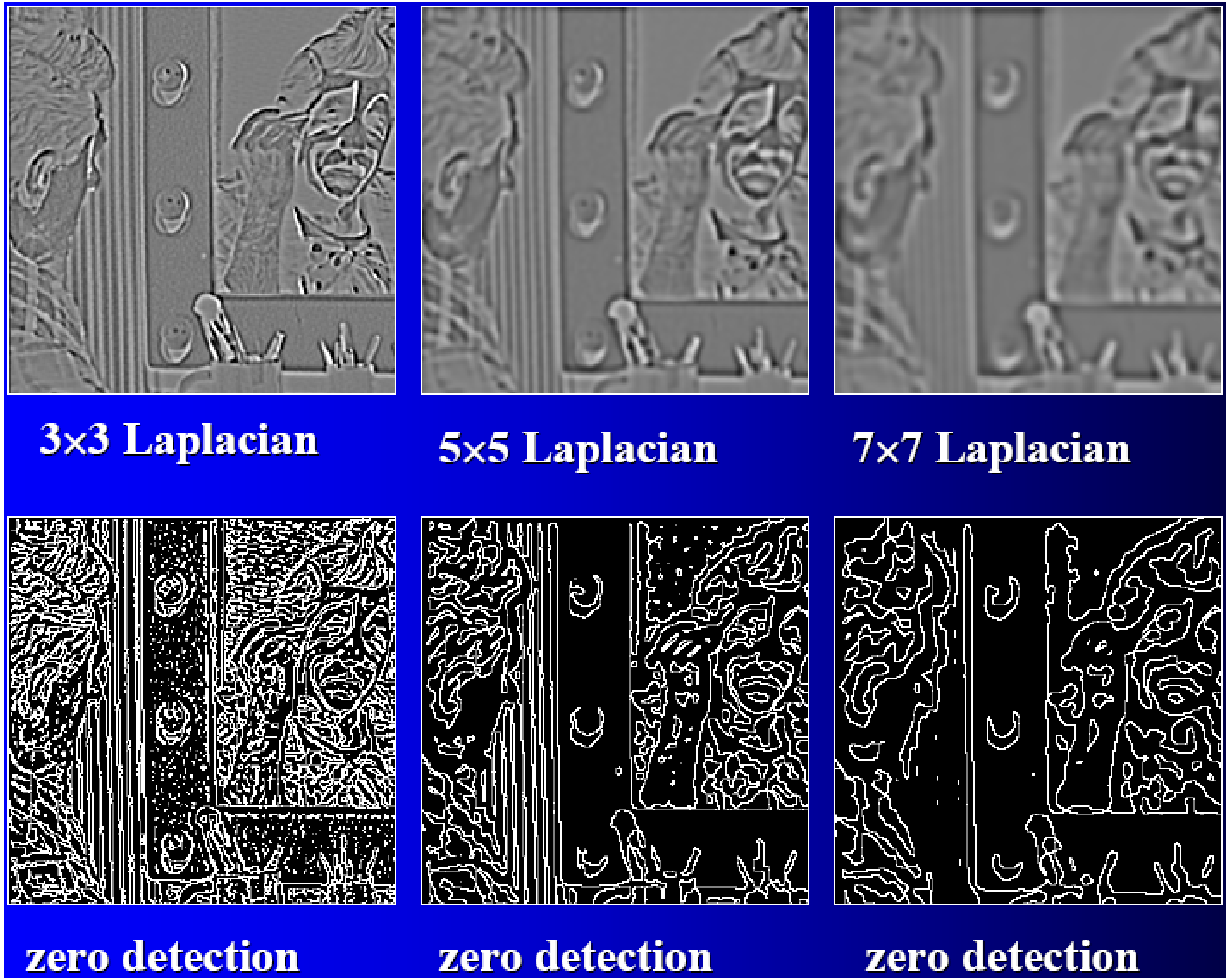


Image Analysis –Edge Linking & Boundary Detection

- Edge detection algorithm are followed by linking procedures to assemble edge pixels into meaningful edges.

- Basic approaches

  Local Processing

  Global Processing via the Hough Transform

Image Analysis –Local Processing

- Analyze the characteristics of pixels in a small neighborhood (say, 3x3, 5x5) about every edge pixels (x,y) in an image.

- All points that are similar according to a set of predefined criteria are linked, forming an edge of pixels that share those criteria.

Image Analysis –Local Processing

1. The strength of the gradient vector
   An edge pixel with coordinates (x_0,y_0) in a predefined neighborhood of (x,y) is similar in magnitude to the pixel at (x,y) if $|\nabla f(x,y) - \nabla f(x_0,y_0)| \leq E$
2. The direction of the gradient vector
   An edge pixel with coordinates (x_0,y_0) in a predefined neighborhood of (x,y) is similar in angle to the pixel at (x,y) if $|\theta(x,y) - \theta(x_0,y_0)| < A$

A point in the predefined neighborhood of (x_0,y_0) is linked to the pixel at (x,y) if both magnitude and direction criteria are satisfied.

Image Analysis –Local Processing

Example

use horizontal and
vertical Sobel operators

eliminate isolated
short segments

link conditions:
gradient value > 25
gradient direction difference < 15°

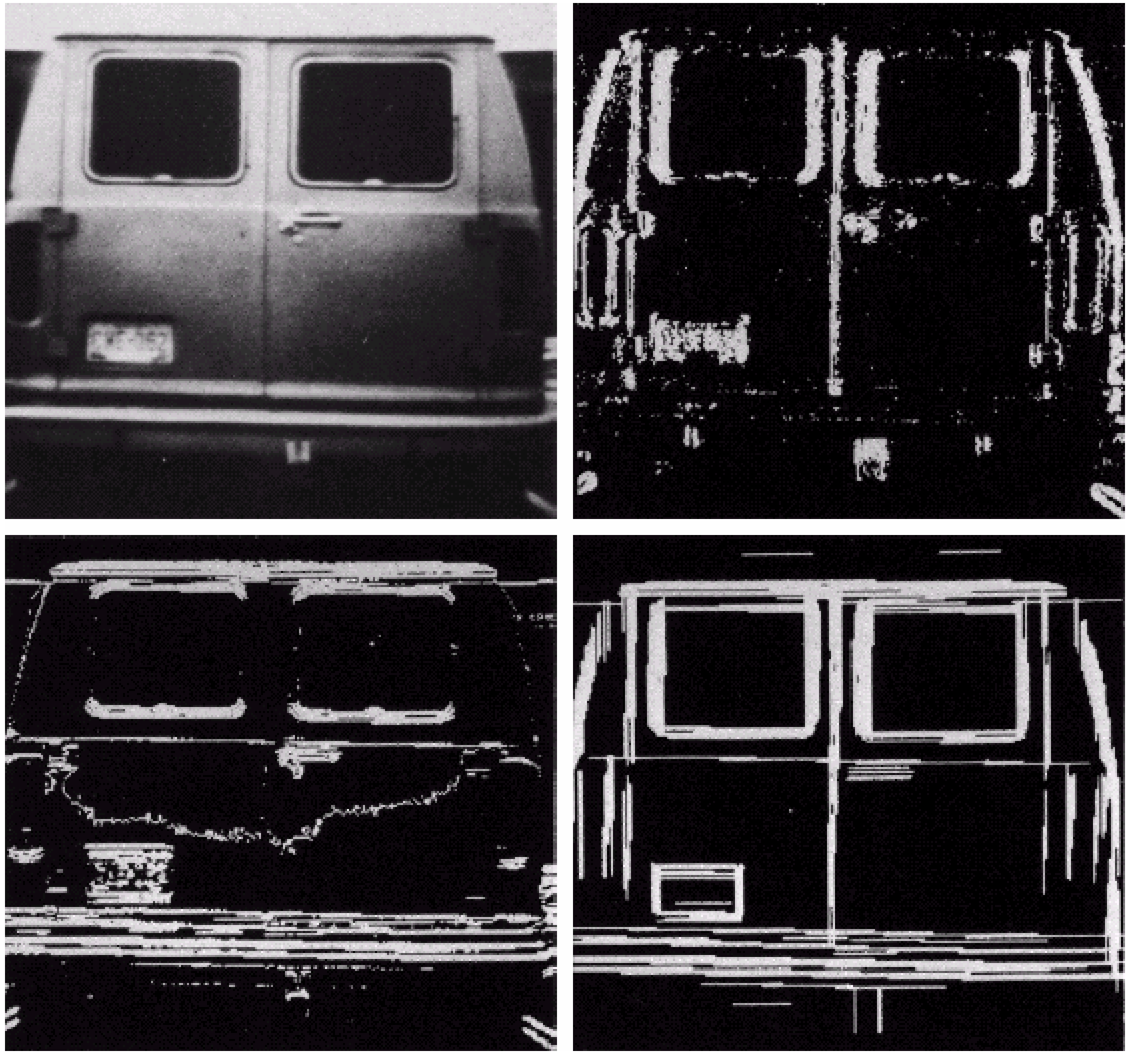


Image Analysis –Hough Transform

- Hough transform is a technique that can be used to detect (link) regular curves such as lines, circles, and ellipses in an image.

  Line segment in spatial space:

  If the line passes through a point (pixel) (x_i, y_i), we obtain:

$$y = ax + b$$

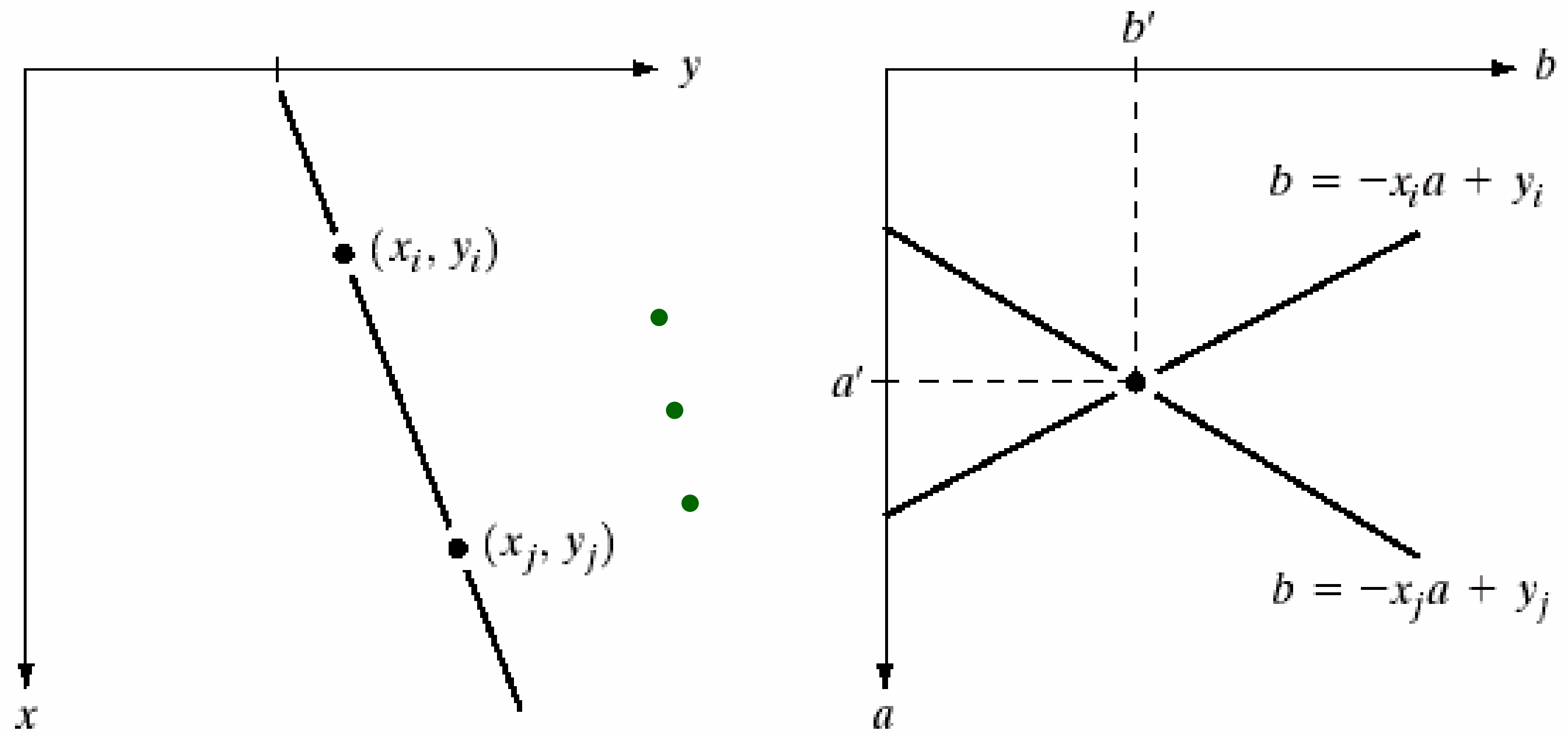
- Rewrite it in ab-plane or parametric space:

$$y_i = ax_i + b$$

$$b = y_i - ax_i$$

Image Analysis –Hough Transform

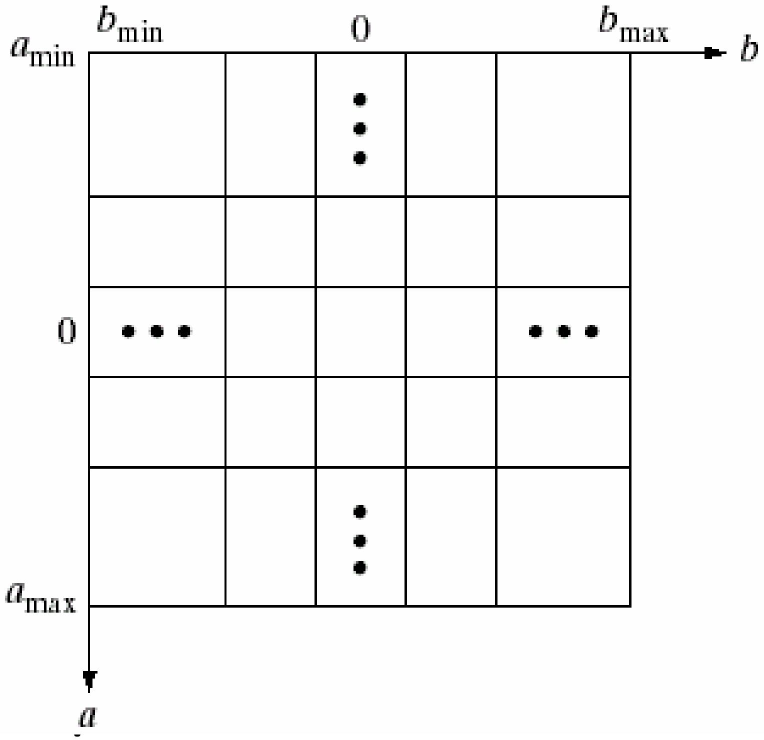
- A line in *xy*-plane is a set of points (*x*, *y*) that satisfy equation: $y = a'x + b'$ which is mapped into a point in ab-plane (a', b')



- A point in *xy*-plane may have many lines go through it, which is mapped into a line in *ab*-plane. $(x_j, y_j) \Leftrightarrow b = y_j - ax_j$

Image Analysis –Hough Transform

- All points (x_i, y_i) on a same line in the image must fall into a same point (a_i, b_i) in the parametric space.



- Hough transform:
  1. Ddivision of parameter space into a cells (a, b).
  2. All cells are initialized to zero,

$$A(a,b) = 0$$

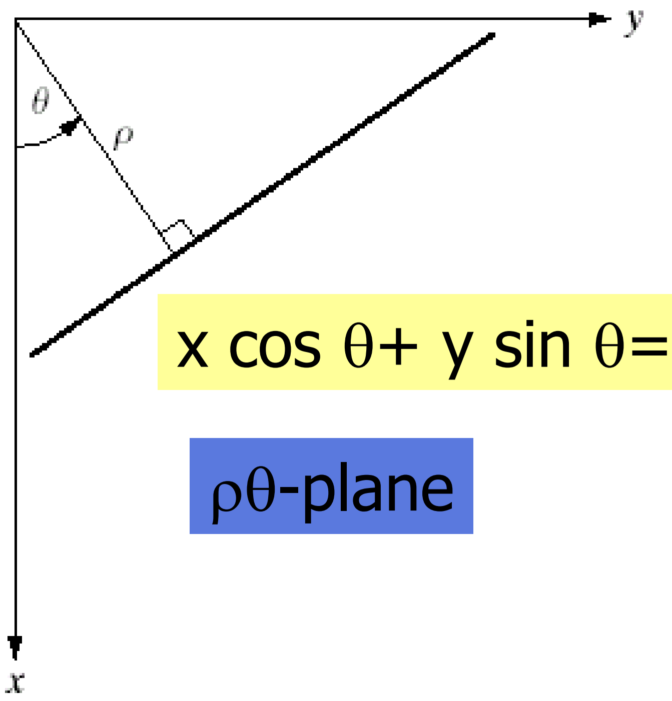
  3. For each detected point (x_i, y_i) in the image:

$$A(a,b)+1 \Rightarrow A(a,b) \text{ for all } a \text{ and } b \text{ satisfying } b = y_i - ax_i$$

- At the end of the procedure, value $A(a, b)$ corresponds to the number of points in image lying on the line $y = ax+b$

Image Analysis –Hough Transform

- Problem of using $y=ax+b$ is that a is infinite for a vertical line.
- To avoid the problem, use equation $x\cos\theta + y\sin\theta = \rho$ to represent a line instead.
- Vertical line has $\theta = 90°$ with ρ equals to the positive y-intercept or $\theta = -90°$ with ρ equals to the negative y-intercept.



$x \cos \theta + y \sin \theta = \rho$

$\rho\theta$-plane

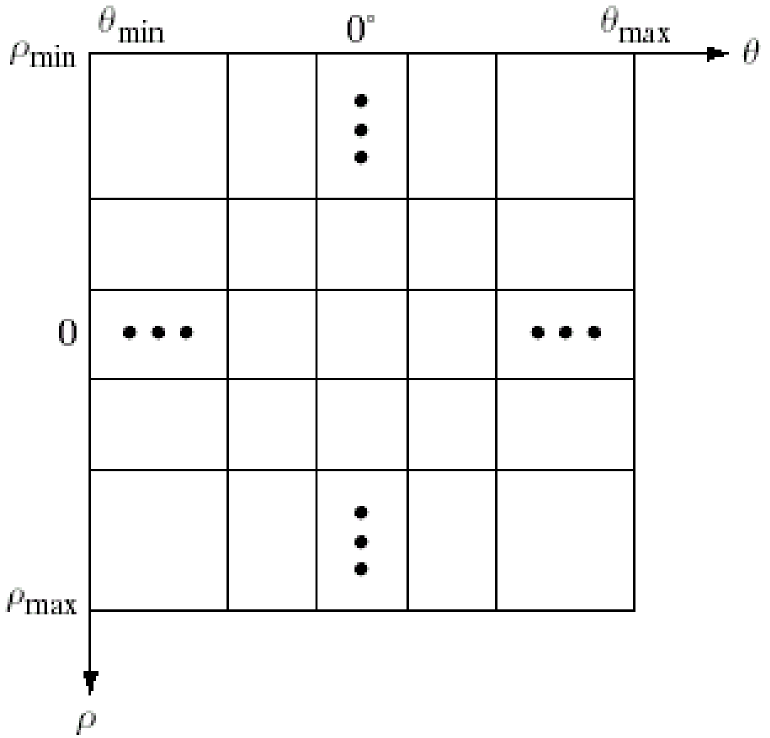
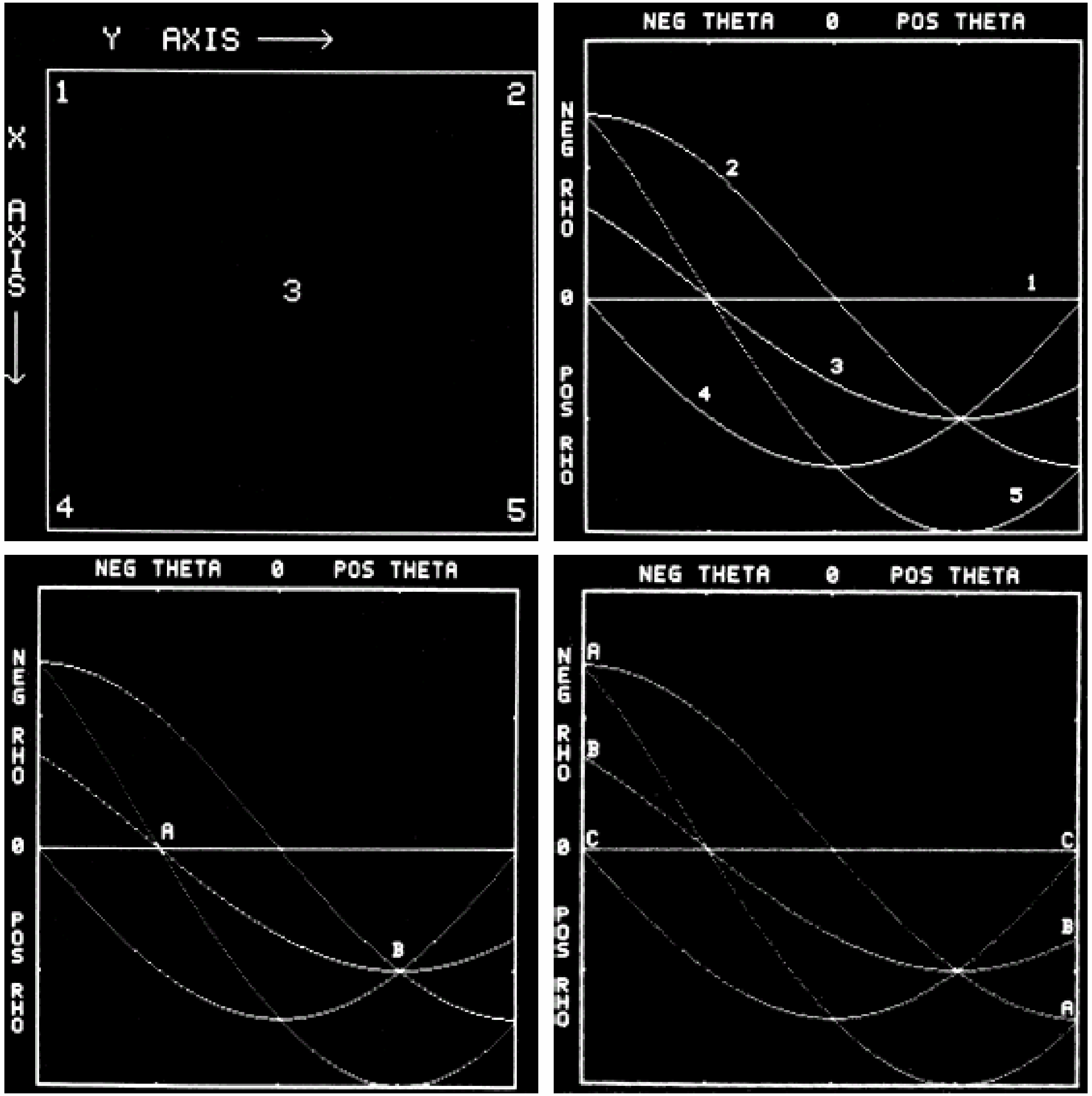


Image Analysis –Hough Transform

- Example

5 points in the image



ρθ-plane

$x\cos\theta + y\sin\theta = \rho$

Image Analysis –Hough Transform

- Generalized Hough transform can be used for any function of the form

$$g(v,c) = 0$$

v is a vector of coordinates, c is a vector of coefficients

- For example a circle is represented by equation:

$$(x-c_1)^2 + (y-c_2)^2 = c_3^2$$

- three parameters (c_1, c_2, c_3)
- cube like cells, accumulators of the form $A(c_1, c_2, c_3)$
- For each point in the image, update the value of $A(c_1, c_2, c_3)$ $\{A(c_1, c_2, c_3) + 1 \rightarrow A(c_1, c_2, c_3)\}$ that satisfies the equation $(x-c_1)^2 + (y-c_2)^2 = c_3^2$.

Image Analysis –Edge Detection by Hough Transform

1. Compute the gradient of an image and threshold it to obtain a binary image.

2. Specify subdivisions in the $\rho\theta$-plane.

3. Examine the counts of the accumulator cells for high pixel concentrations.

4. Examine the relation (principally for continuity) between pixels in a chosen cell.

5. A gap at any point is linked if the distance between that point and its closet neighbor below a certain threshold.

Image Analysis –Edge Detection by Hough Transform

Example

link criteria:
pixels belonged to a set is linked according to the highest count.

no gaps were longer than 5 pixels

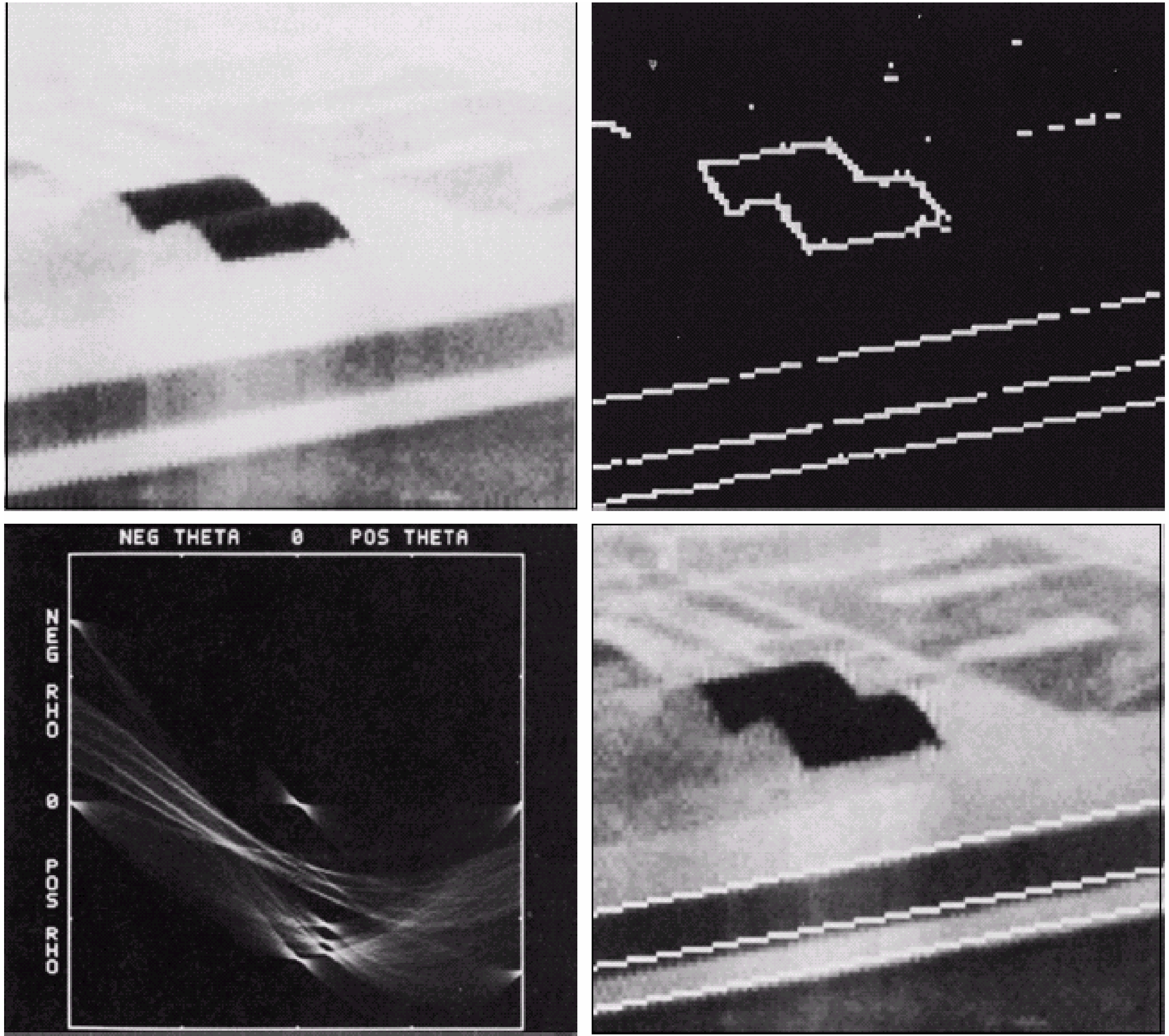


Image Analysis –Edge Detection by Hough Transform

Further example

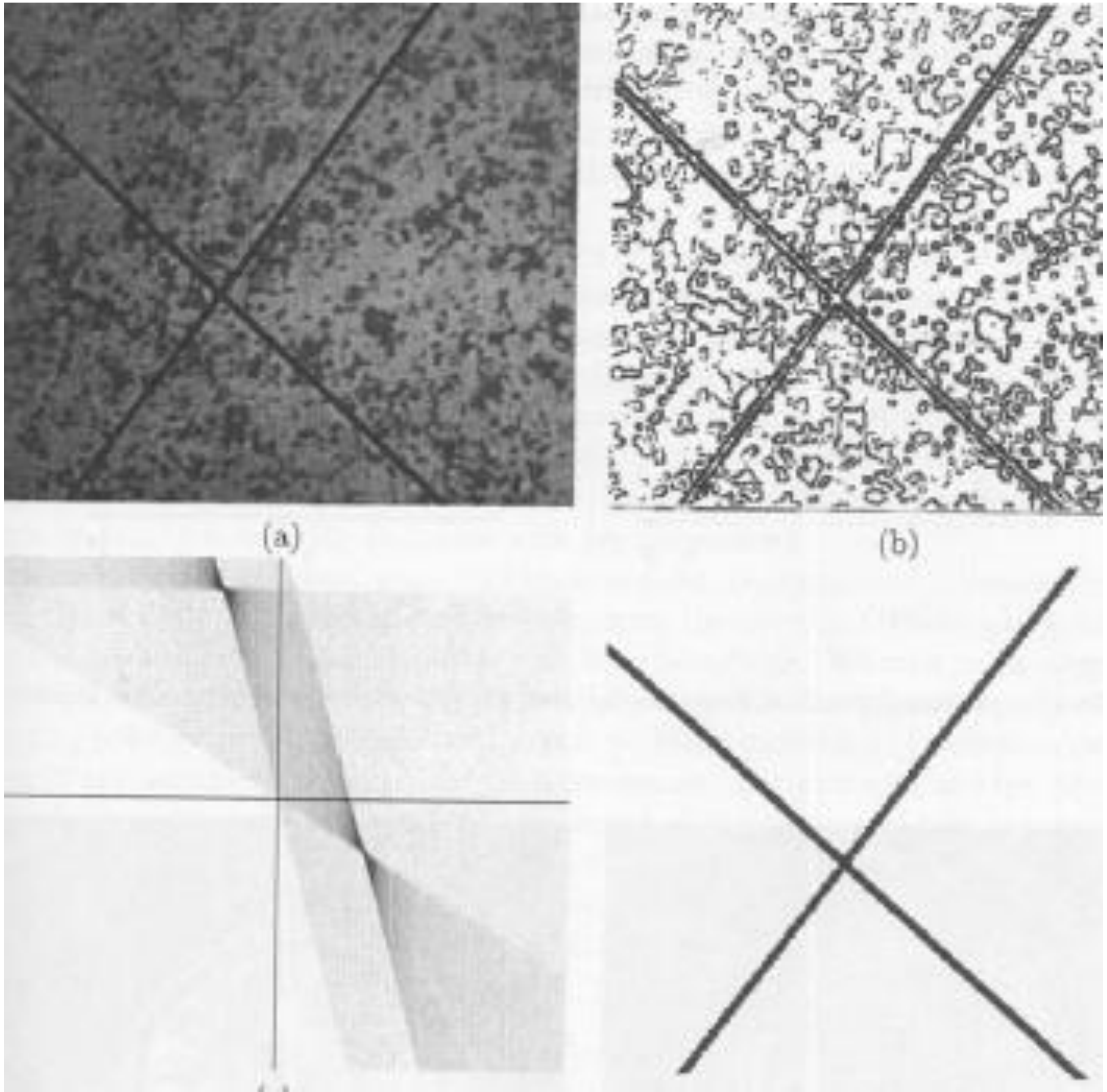


Image Analysis –Application

Features useful in the Automatic Fingerprint Identification

Image Analysis –Application

What features are effective for Content Based Image Retrieval?

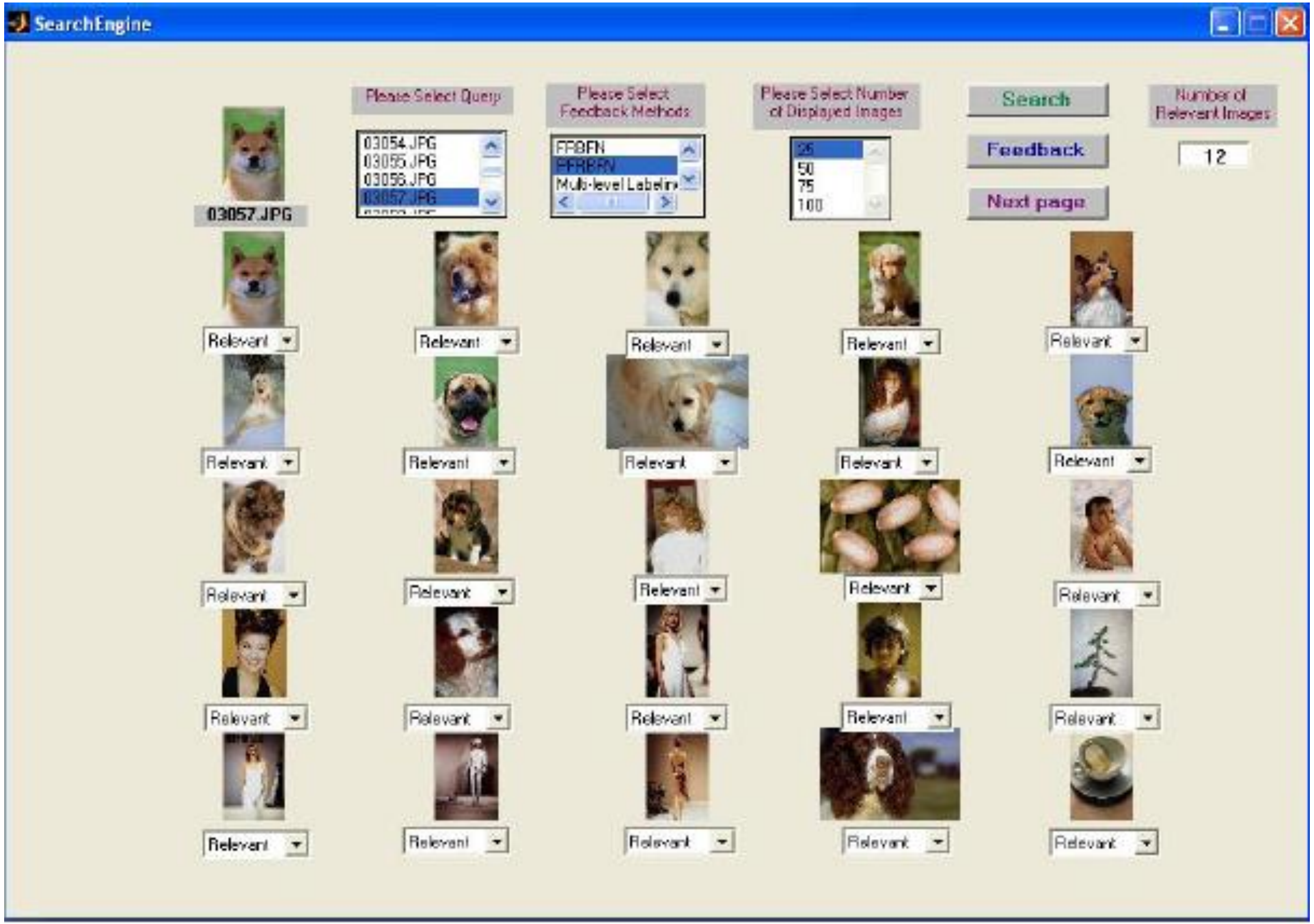
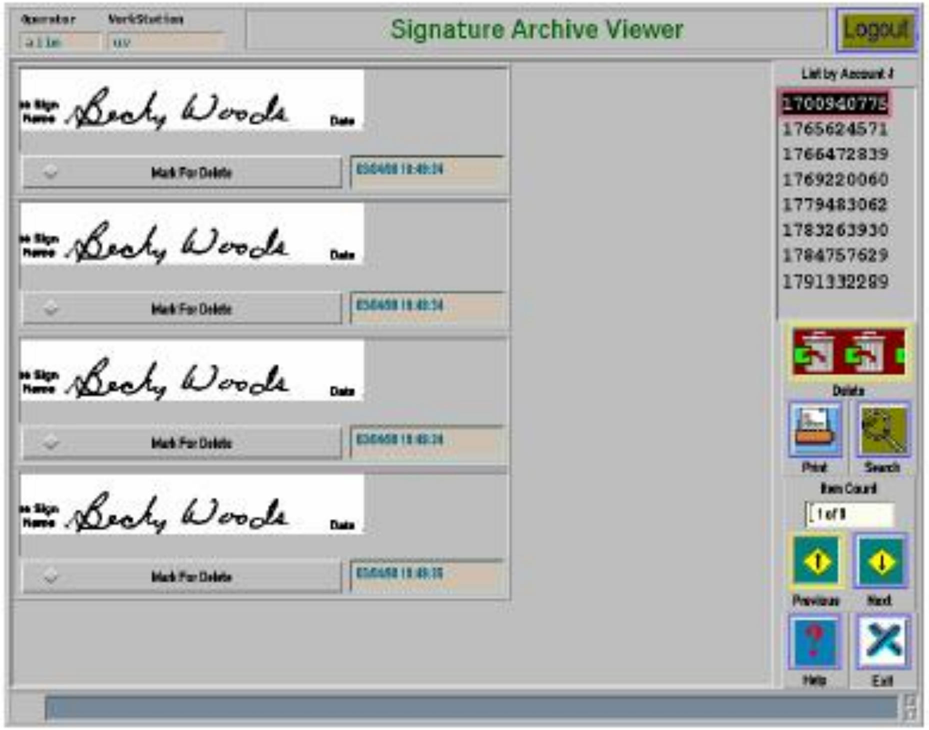


Image Analysis –Application

Automatic signature verification



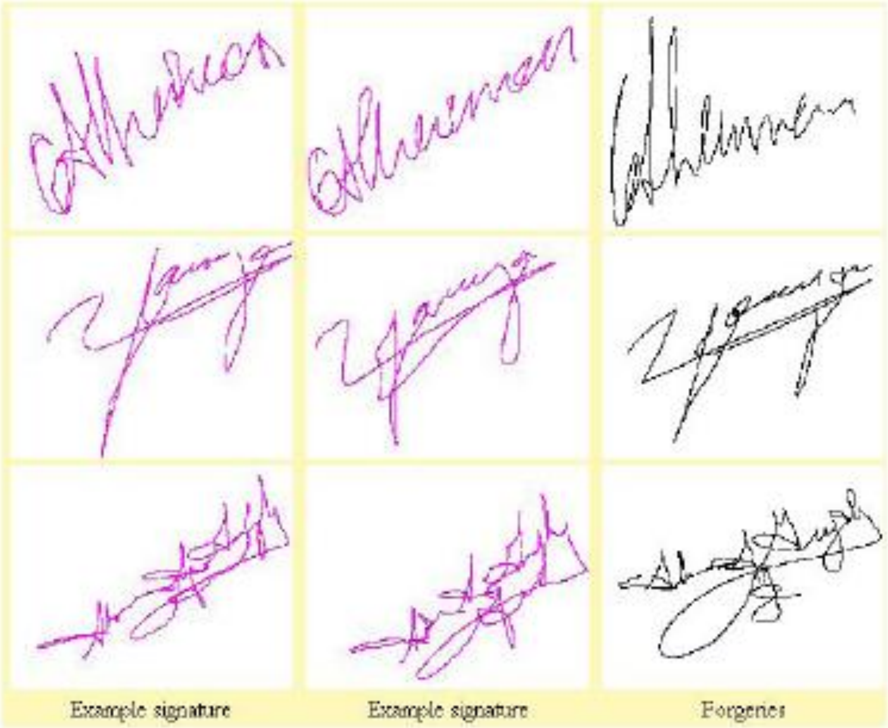
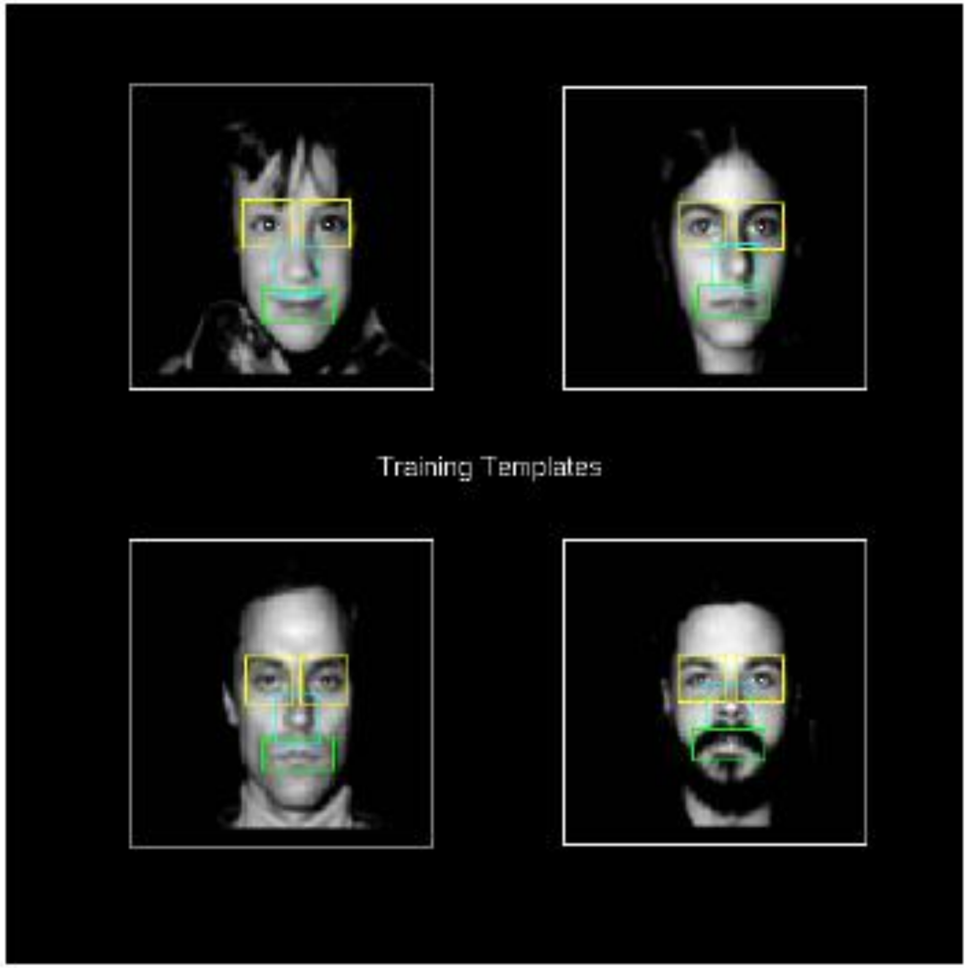


Image Analysis –Application

Are these feature effective for Automatic Face Recognition?



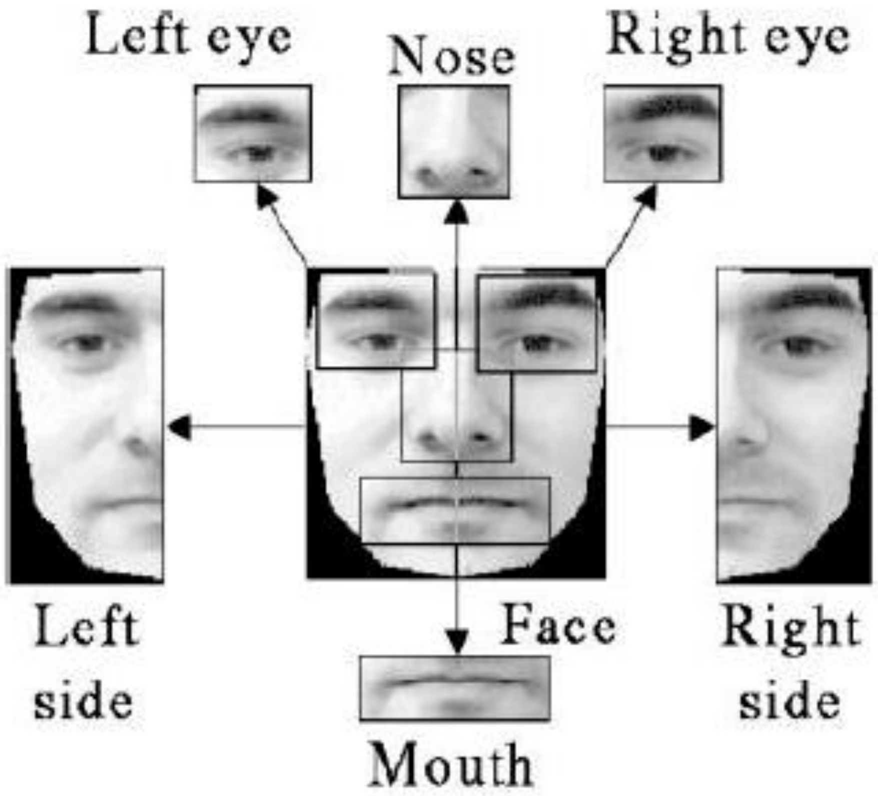


Image Analysis –Application

- Digital Watermarking

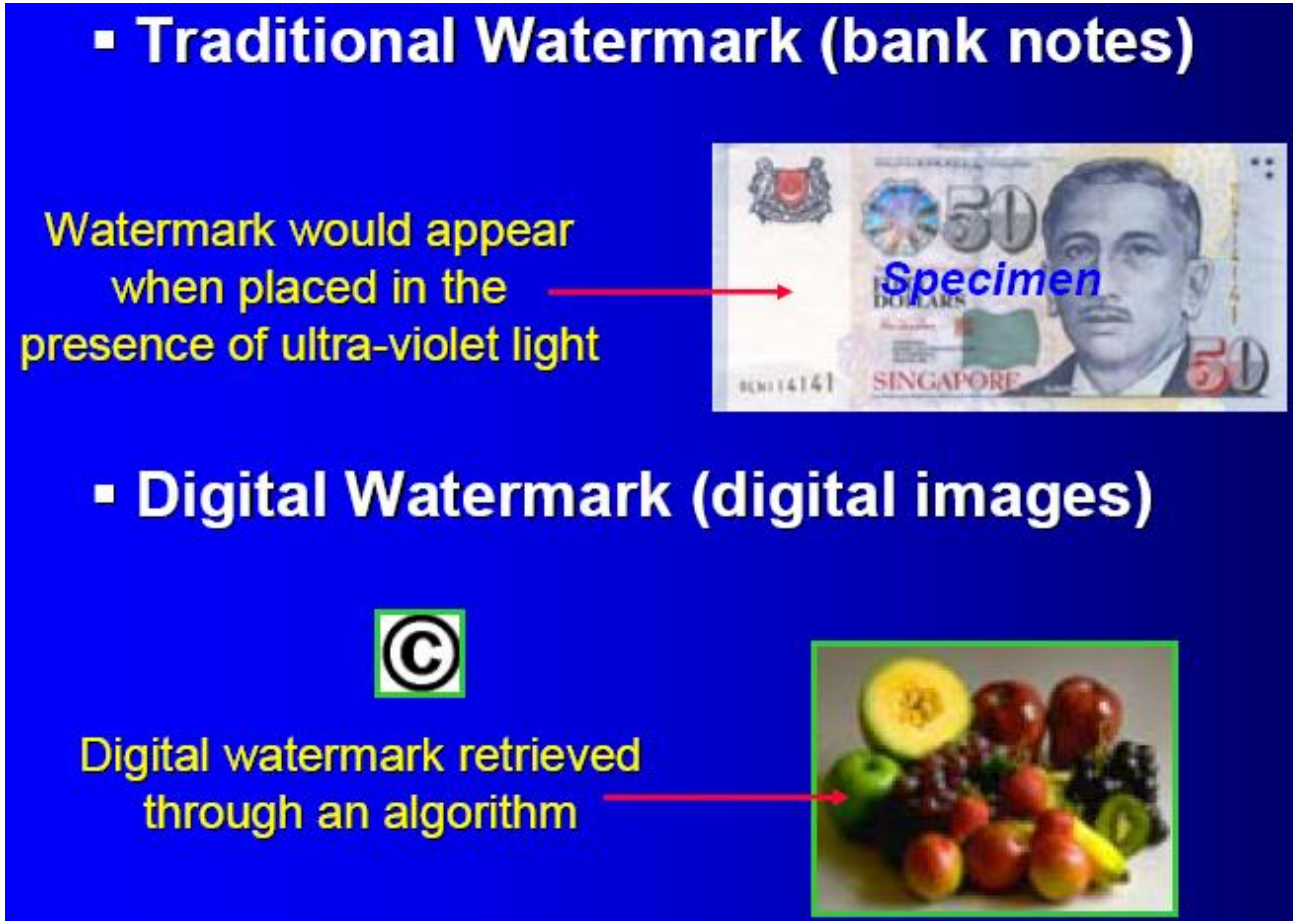


Image Analysis –Application

Digital content tampering detection

The photograph is a composite created by ex-composite LA Times photographer Brian Walski.

He was dismissed when the photograph was found be altered.