Data Structures
Homework #3

Due: Nov 7, 2023 (before class)

1. Show that the summation $\sum_{i=1}^{n} \lceil \log_2 i \rceil$ is $O(n \log n)$.

2. Let S be a set of n lines in the plane such that no two are parallel and no three meet in the same point. Show, by induction, that the lines in S determine $\Theta(n^2)$ intersection points. (**hint**: Consider the contribution made by one line.)

3. Suppose you have two nonempty stacks S and T and a deque D. Describe how to use D so that S stores all the elements of T below all of its original elements, with both sets of elements still in their original order.

4. Consider an implementation of an extendable array, but instead of copying the elements of the array list into an array of double the size (that is, from N to $2N$) when its capacity is reached, we copy the elements into an array with $\lceil N/2 \rceil$ additional cells, going from capacity N to $N + \lceil N/2 \rceil$. Show that performing a sequence of n push operations (that is, insertions at the end) still runs in $O(n)$ time in this case.

5. Suppose each row of an $n \times n$ array A consists of 1's and 0's such that, in any row i of A, all the 1's come before any 0's. We Assume that A is already in memory. Describe a method running in $O(n)$ time (not $O(n^2)$) for finding the row of A that contains the most 1's.

6. (**Programming**)
   A Gray code is an encoding of numbers so that two contiguous numbers have a single digit differing by 1. The term Gray code is often used to refer to a "reflected" code, or more specifically still, the binary reflected Gray code. For example, one-bit Gray code is $G_1 = (0, 1)$ and two-bit Gray code is $G_2 = (00, 01, 11, 10)$. Three-bit Gray code is as follows:

Dec	Gray
0	000
1	001
2	011
3	010
4	110
5	111
6	101
7	100

   This exercise asks you to list the n-bit Gray code using a stack. Please use a class for the stack. Your Python program should read a value n (the number of bits for the Gray

code) and use the stack to help you to generate the n-bit Gray codes as well as list them in order. Please note that your program **must use a *stack*** as an auxiliary tool. The execution of program should look like the sample in the given `HW3.ipynb` file.

About submitting this homework

1. For problem 1, 2, 3, 4 and 5, Please

   (1) write all of your solutions on the papers of size A4,

   (2) leave you name and student ID on the first page, and

   (3) hand in your solutions for problem 1, 2, 3, 4 and 5 to me in class

2. For problem 6, please upload the completed `.ipynb` file with the filename as `HW3_studentID.ipynb` to **i-school(Plus)** (`https://istudy.ntut.edu.tw/learn/index.php`).

3. The **deadline** is the **midnight of Nov 7**, 2023 and **Late work** is not acceptable.