X_i is the fixation probability when have i mutants.

f_i^- is the probability that i increase one mutant.

f_i^+ is the probability that i increase one mutant.

From the Markov process we can get this equation easily.

$$X_i = f_i^- X_{i-1} + (1 - f_i^- - f_i^+) X_i + f_i^+ X_{i+1} \tag{1}$$

Then we can get equation (2) from (1)

$$\frac{X_{i+1} - X_i}{X_i - X_{i-1}} = \frac{f_i^-}{f_i^+} \tag{2}$$

And we have two absorb states.

$$X_0 = 0 \qquad X_N = 1 \tag{3}$$

$$\frac{X_{j+1} - X_j}{X_1 - X_0} = \prod_{i=1}^{j} \frac{f_i^-}{f_i^+} \tag{4}$$

Then get (5) from (3)(4)

$$X_{j+1} - X_j = \left(\prod_{i=1}^{j} \frac{f_i^-}{f_i^+}\right) X_1 \tag{5}$$

Randomly choose one mutant to death and randomly choose a wild node to reproduce proportion to the fitness. Then we get (6) the formula used to calculate the transition probability.

$$f_i^- = \frac{i}{N} \bullet \frac{N - i}{(N - i) + (i - 1) r} \tag{6}$$

Randomly choose one wild node to death and randomly choose a mutant node to reproduce proportion to the fitness. Then we get (7) the formula used to calculate the transition probability.

$$f_i^+ = \frac{N-i}{N} \bullet \frac{ir}{(N-i-1)+ir} \tag{7}$$

Get (8) from (6) and (7)

$$\frac{f_i^-}{f_i^+} = \frac{(N-i-1)+ir}{(N-i)+(i-1)r} \bullet \frac{1}{r} \tag{8}$$

Multiple (8) Then we get (9)

$$\prod_{i=1}^{j} \frac{f_i^-}{f_i^+} = \frac{(N-j-1)+jr}{N-1} \bullet \frac{1}{r^j} \tag{9}$$

In (5) let the j be the N-1 then we get equation (10)

$$X_N - X_1 = \sum_{j=1}^{N-1} \left(\prod_{i=1}^{j} \frac{f_i^-}{f_i^+} \right) X_1 \tag{10}$$

Then the following process give the final result.

$$X_N - X_1 = \sum_{j=1}^{N-1} \left(\prod_{i=1}^{j} \frac{f_i^-}{f_i^+} \right) X_1$$

$$\Rightarrow X_N - X_1 = \sum_{j=1}^{N-1} \left(\frac{(N - j - 1) + jr}{N - 1} \bullet \frac{1}{r^j} \right) X_1$$

$$\Rightarrow X_N - X_1 = \frac{X_1}{N - 1} \sum_{j=1}^{N-1} \left((N - j - 1 + jr) \bullet \frac{1}{r^j} \right)$$

$$\Rightarrow X_N - X_1 = \frac{X_1}{N - 1} \sum_{j=1}^{N-1} \left(\frac{N}{r^j} - \frac{j+1}{r^j} + \frac{j}{r^{j-1}} \right)$$

$$\Rightarrow X_N - X_1 = \frac{X_1}{N - 1} \left(\sum_{j=1}^{N-1} \left(\frac{N}{r^j} \right) + 1 - \frac{N}{r^{N-1}} \right)$$

$$\Rightarrow 1 = \frac{X_1}{N - 1} \left(\sum_{j=1}^{N-1} \left(\frac{N}{r^j} \right) + 1 - \frac{N}{r^{N-1}} \right) + \frac{X_1}{N - 1} \bullet (N - 1)$$

$$\Rightarrow 1 = \frac{X_1}{N - 1} \left(\sum_{j=1}^{N-1} \left(\frac{N}{r^j} \right) + 1 - \frac{N}{r^{N-1}} + \frac{N}{r^0} - 1 \right)$$

$$\Rightarrow 1 = \frac{X_1}{N - 1} \left(\sum_{j=0}^{N-2} \left(\frac{N}{r^j} \right) \right.$$

$$\Rightarrow 1 = \frac{N \bullet X_1}{N - 1} \left(\sum_{j=0}^{N-2} \left(\frac{1}{r^j} \right) \right)$$

$$\Rightarrow 1 = \frac{N \bullet X_1}{N - 1} \left(\frac{1 - \frac{1}{r^{N-1}}}{1 - \frac{1}{r}} \right)$$

$$\Rightarrow X_1 = \frac{N - 1}{N} \frac{1 - 1/r}{1 - 1/r^{N-1}}$$