Supplement Information

We consider a population of size N where the number of mutant is i and the number of wilder is N － i. The probability to increase the number of mutant individuals from i to i + 1 is f_i^+. Similarly, f_i^- is the probability to decrease i by 1. Our goal is to calculate the fixation probabilities X_i , i.e. the probability that i mutant individuals succeed and take over the population. For the absorbing states, we have:

$$X_0 = 0 \quad \text{And} \quad X_N = 1 \tag{1}$$

For the intermediate states, the fixation probabilities are given by

$$X_i = f_i^- X_{i-1} + (1 - f_i^- - f_i^+) X_i + f_i^+ X_{i+1} \tag{2}$$

Rearrange this equation we can get:

$$\frac{X_{i+1} - X_i}{X_i - X_{i-1}} = \frac{f_i^-}{f_i^+} \tag{3}$$

Make i be 1,2,3,4 ….j-1, j and multiple this part together we can get below equation.

$$\frac{X_{j+1} - X_j}{X_j - X_{j-1}} \bullet \frac{X_j - X_{j-1}}{X_{j-1} - X_{j-2}} \cdots\cdots\cdots \frac{X_3 - X_2}{X_2 - X_1} \bullet \frac{X_2 - X_1}{X_1 - X_0} = \sum_{i=1}^{j} \frac{f_i^-}{f_i^+} \tag{4}$$

And obviously the denominator is the same with the numerator of the next part. So we can wipe them out together.Then we can get a simpler equation.

$$\frac{X_{j+1} - X_j}{X_1 - X_0} = \prod_{i=1}^{j} \frac{f_i^-}{f_i^+} \tag{5}$$

From (1) we know $X_0 = 0$, and then we can continue simplifying it to

$$X_{j+1} - X_j = (\prod_{i=1}^{j} \frac{f_i^-}{f_i^+}) X_1 \tag{6}$$

Let j be N-1,N-2,……3,2,1 we find

$$\begin{aligned}
X_N - X_{N-1} &= (\prod_{i=1}^{N-1} \frac{f_i^-}{f_i^+}) X_1 \\
X_{N-1} - X_{N-2} &= (\prod_{i=1}^{N-2} \frac{f_i^-}{f_i^+}) X_1 \\
&\cdots \\
&\cdots \\
X_2 - X_1 &= (\prod_{i=1}^{1} \frac{f_i^-}{f_i^+}) X_1
\end{aligned} \tag{7}$$

Add every row together we can get

$$X_N - X_1 = \sum_{j=1}^{N-1} (\prod_{i=1}^{j} \frac{f_i^-}{f_i^+}) X_1 \tag{8}$$

In this equation, $X_N = 1$ is known, so if we can get all $\frac{f_i^-}{f_i^+}$ we can get the answer for X_1 which is the fixation probability.

Randomly choose one mutant to death and randomly choose a wild node to reproduce proportion to the fitness. Then we get

$$f_i^- = \frac{i}{N} \bullet \frac{N - i}{(N - i) + (i - 1) r} \tag{9}$$

Similarly randomly choose one wild node to death and randomly choose a mutant node to reproduce proportion to the fitness. Then we get:

$$f_i^{+} = \frac{N-i}{N} \bullet \frac{ir}{(N-i-1)+ir} \tag{10}$$

Combine above two equations together we can easily get

$$\frac{f_i^{-}}{f_i^{+}} = \frac{(N-i-1)+ir}{(N-i)+(i-1)r} \bullet \frac{1}{r} \tag{11}$$

Then we find all the parameters we need to calculate X_1 through the equation (8) have been satisfied. But it' s really complicated and of course we will continue to simplify it.
For the equation (8) below,:

$$X_N - X_1 = \sum_{j=1}^{N-1} \left(\prod_{i=1}^{j} \frac{f_i^{-}}{f_i^{+}}\right) X_1$$

We simplify this $\prod_{i=1}^{j} \frac{f_i^{-}}{f_i^{+}}$ inner part first.

Unfold and list every part we get

$$\begin{aligned}
\frac{f_j^{-}}{f_j^{+}} &= \frac{(N-(j+1))+jr}{(N-j)+(j-1)r} \bullet \frac{1}{r} \\
\frac{f_{j-1}^{-}}{f_{j-1}^{+}} &= \frac{(N-j)+(j-1)r}{(N-(j-1))+(j-2)r} \bullet \frac{1}{r} \\
&\cdots \\
&\cdots \\
&\cdots \\
\frac{f_2^{-}}{f_2^{+}} &= \frac{(N-3)+2r}{(N-2)+r} \bullet \frac{1}{r} \\
\frac{f_1^{-}}{f_1^{+}} &= \frac{(N-2)+1r}{(N-1)} \bullet \frac{1}{r}
\end{aligned} \tag{12}$$

Multiply them all and it' s not too hard to find that every part' s numerator equal to the denominator of the next part. We wipe them out together and we can get:

$$\prod_{i=1}^{j}\frac{f_i^-}{f_i^+}=\frac{(N-j-1)+jr}{N-1}\bullet\frac{1}{r^j} \tag{13}$$

Use some math skills we can get the final result through below derivation

$$X_N - X_1 = \sum_{j=1}^{N-1}\left(\prod_{i=1}^{j}\frac{f_i^-}{f_i^+}\right)X_1$$

$$\Rightarrow X_N - X_1 = \sum_{j=1}^{N-1}\left(\frac{(N-j-1)+jr}{N-1}\bullet\frac{1}{r^j}\right)X_1$$

$$\Rightarrow X_N - X_1 = \frac{X_1}{N-1}\sum_{j=1}^{N-1}\left((N-j-1+jr)\bullet\frac{1}{r^j}\right)$$

$$\Rightarrow X_N - X_1 = \frac{X_1}{N-1}\sum_{j=1}^{N-1}\left(\frac{N}{r^j}-\frac{j+1}{r^j}+\frac{j}{r^{j-1}}\right)$$

$$\Rightarrow X_N - X_1 = \frac{X_1}{N-1}\left(\sum_{j=1}^{N-1}\left(\frac{N}{r^j}\right)+1-\frac{N}{r^{N-1}}\right)$$

$$\Rightarrow 1 = \frac{X_1}{N-1}\left(\sum_{j=1}^{N-1}\left(\frac{N}{r^j}\right)+1-\frac{N}{r^{N-1}}\right)+\frac{X_1}{N-1}\bullet(N-1)$$

$$\Rightarrow 1 = \frac{X_1}{N-1}\left(\sum_{j=1}^{N-1}\left(\frac{N}{r^j}\right)+1-\frac{N}{r^{N-1}}+\frac{N}{r^0}-1\right)$$

$$\Rightarrow 1 = \frac{X_1}{N-1}\left(\sum_{j=0}^{N-2}\left(\frac{N}{r^j}\right)\right)$$

$$\Rightarrow 1 = \frac{N\bullet X_1}{N-1}\left(\sum_{j=0}^{N-2}\left(\frac{1}{r^j}\right)\right)$$

$$\Rightarrow 1 = \frac{N\bullet X_1}{N-1}\left(\frac{1-\frac{1}{r^{N-1}}}{1-\frac{1}{r}}\right)$$

$$\Rightarrow X_1 = \frac{N-1}{N}\frac{1-1/r}{1-1/r^{N-1}}$$