Quantum dot

In this notebook we consider one particular example of nc rational functions: the one that appears in the paper of Beenakker

In[391]:=

```
SetDirectory[NotebookDirectory[]];
```

Add the folder(s) containing the current package as well as NC package to $Path

In[287]:=

```
AppendTo[$Path,
  "/Users/yumish/Dropbox/a_recherche/mathematica_all/mathematica_packages/"];
<< NC` (*load NCAlgebra*)
<< NCAlgebra`
<< PolynomialsOfRandomMatrices`
```

NC : You are using the version of NCAlgebra which is found in:
"/Users /yumish /Library /CloudStorage /Dropbox /a_recherche /mathematica _all /mathematica _packages /NC/".

NCAlgebra : All lower cap single letter symbols (e.g. a,b,c,...) were set as noncommutative.

```
DoSplot[N[LLP /. En → -1], {-.5, 1.5}, SolutionAccuracy → 0.0001,
 Iterations → 50 000, ImaginaryPart → 0.00001, GridNumber → 5000,
 PlotRange → {{-0.5, 1.5}, {0, 100}}, Ticks → Automatic, PlotLegends → Automatic]
```

In[322]:=

```
Options[DoSplot] = Join[{SolutionAccuracy → 0.001, Iterations → 5000,
    ImaginaryPart → 0.001, GridNumber → 1000, Rescaling → 1}, Options[Plot]];
DoSplot[LLinearization_, Interval_, opts : OptionsPattern[]] :=
  Block[{VVariableList, LL0, LL, JJ, SSOperator, MM},

   MM = IterativelySolveMDEonInterval2[LLinearization, z , Interval⟦1⟧,
     Interval⟦2⟧, OptionValue[ImaginaryPart], OptionValue[GridNumber],
     OptionValue[Iterations], OptionValue[SolutionAccuracy]];

   Return[ListLinePlot[
     Table[{MM⟦i, 1⟧, Im[MM⟦i, 2⟧⟦1, 1⟧] / Pi * OptionValue[Rescaling]},
      {i, Length[MM]}], FilterRules[{opts}, Options[Plot]],
     PlotRange → {0, 1.02 Max[Table[Im[MM⟦i, 2⟧⟦1, 1⟧] / Pi, {i, Length[MM]}]]},
     Ticks → Automatic, PlotLegends → Automatic, ImageSize → 270,
     PlotStyle → {Blue, Thick}, Filling → Bottom]];
  ];
```

Computing Fano factor

*Function ***FanoFactor[evlist]***

In[291]:=

```
FanoFactor[evlist_] := Block[{list01},
   (*list01=Select[evlist,(#>0 && #<1)&];*)
   list01 = evlist;
   Return[Sum[list01[[i]] * (1 - list01[[i]]), {i, Length[list01]}] / Total[list01]];
  ];
```

Linearization (run)

We define the semicircular elements x, s, t, u, v, so that $y_1 = (s + i\,t) \big/ \sqrt{2}$, $y_2 = (u + i\,v) \big/ \sqrt{2}$.

The initial linearization

In[292]:=

```
Clear[En];
LL = {{-z, 0, 0, 0, 0, (u - ⅈ * v) / Sqrt[2], 0, 0},
    {0, 0, 0, 0, (s + ⅈ * t) / Sqrt[2], En - x, (s + ⅈ * t) / Sqrt[2], (u + ⅈ * v) / Sqrt[2]},
    {0, 0, 0, 0, 0, (s - ⅈ * t) / Sqrt[2], ⅈ / Pi, 0}, {0, 0, 0, 0, 0, (u - ⅈ * v) / Sqrt[2],
     0, ⅈ / Pi}, {0, (s - ⅈ * t) / Sqrt[2], 0, 0, -1 / (4 * Pi ^ 2), 0, 0, 0},
    {(u + ⅈ * v) / Sqrt[2], En - x, (s + ⅈ * t) / Sqrt[2], (u + ⅈ * v) / Sqrt[2], 0, 0, 0, 0},
    {0, (s - ⅈ * t) / Sqrt[2], -ⅈ / Pi, 0, 0, 0, 0, 0},
    {0, (u - ⅈ * v) / Sqrt[2], 0, -ⅈ / Pi, 0, 0, 0, 0}};
MatrixForm@LL
```

Out[294]//MatrixForm=

$$\begin{pmatrix} -z & 0 & 0 & 0 & 0 & \frac{u-i\,v}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{s+i\,t}{\sqrt{2}} & \text{En}-x & \frac{s+i\,t}{\sqrt{2}} & \frac{u+i\,v}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & \frac{s-i\,t}{\sqrt{2}} & \frac{i}{\pi} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{u-i\,v}{\sqrt{2}} & 0 & \frac{i}{\pi} \\ 0 & \frac{s-i\,t}{\sqrt{2}} & 0 & 0 & -\frac{1}{4\,\pi^2} & 0 & 0 & 0 \\ \frac{u+i\,v}{\sqrt{2}} & \text{En}-x & \frac{s+i\,t}{\sqrt{2}} & \frac{u+i\,v}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & \frac{s-i\,t}{\sqrt{2}} & -\frac{i}{\pi} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{u-i\,v}{\sqrt{2}} & 0 & -\frac{i}{\pi} & 0 & 0 & 0 & 0 \end{pmatrix}$$

Now we obtain the linearization leading to the block structure

In[295]:=

```
P23 = SparseArray[{{1, 1} → 1, {2, 3} → 1, {3, 2} → 1,
    {4, 4} → 1, {5, 5} → 1, {6, 6} → 1, {7, 7} → 1, {8, 8} → 1}];
P28 = SparseArray[{{1, 1} → 1, {2, 8} → 1, {3, 3} → 1,
    {4, 4} → 1, {5, 5} → 1, {6, 6} → 1, {7, 7} → 1, {8, 2} → 1}];
P67 = SparseArray[{{1, 1} → 1, {2, 2} → 1, {3, 3} → 1,
    {4, 4} → 1, {5, 5} → 1, {6, 7} → 1, {7, 6} → 1, {8, 8} → 1}];
P34 = SparseArray[{{1, 1} → 1, {2, 2} → 1, {3, 4} → 1,
    {4, 3} → 1, {5, 5} → 1, {6, 6} → 1, {7, 7} → 1, {8, 8} → 1}];
P78 = SparseArray[{{1, 1} → 1, {2, 2} → 1, {3, 3} → 1,
    {4, 4} → 1, {5, 5} → 1, {6, 6} → 1, {7, 8} → 1, {8, 7} → 1}];
P47 = SparseArray[{{1, 1} → 1, {2, 2} → 1, {3, 3} → 1,
    {4, 7} → 1, {5, 5} → 1, {6, 6} → 1, {7, 4} → 1, {8, 8} → 1}];
P25 = SparseArray[{{1, 1} → 1, {2, 5} → 1, {3, 3} → 1,
   {4, 4} → 1, {5, 2} → 1, {6, 6} → 1, {7, 7} → 1, {8, 8} → 1}];
P35 = SparseArray[{{1, 1} → 1, {2, 2} → 1, {3, 5} → 1, {4, 4} → 1,
   {5, 3} → 1, {6, 6} → 1, {7, 7} → 1, {8, 8} → 1}];
P46 = SparseArray[{{1, 1} → 1, {2, 2} → 1, {3, 3} → 1,
    {4, 6} → 1, {5, 5} → 1, {6, 4} → 1, {7, 7} → 1, {8, 8} → 1}];
LLP = P78.P23.P34.P67.P28.LL.P28.P67.P34.P23.P78;
LLP2 = P46.P35.P25.P47.LL.P47.P25.P35.P46;
MatrixForm@LLP
MatrixForm@LLP2
```

Out[305]//MatrixForm=

$$\begin{pmatrix}
-z & 0 & 0 & 0 & 0 & 0 & 0 & \frac{u-i\,v}{\sqrt{2}} \\
0 & 0 & \frac{i}{\pi} & 0 & 0 & 0 & 0 & \frac{u-i\,v}{\sqrt{2}} \\
0 & -\frac{i}{\pi} & 0 & 0 & 0 & 0 & \frac{u-i\,v}{\sqrt{2}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{\pi} & 0 & \frac{s-i\,t}{\sqrt{2}} \\
0 & 0 & 0 & 0 & -\frac{1}{4\pi^2} & 0 & \frac{s-i\,t}{\sqrt{2}} & 0 \\
0 & 0 & 0 & -\frac{i}{\pi} & 0 & 0 & \frac{s-i\,t}{\sqrt{2}} & 0 \\
0 & 0 & \frac{u+i\,v}{\sqrt{2}} & 0 & \frac{s+i\,t}{\sqrt{2}} & \frac{s+i\,t}{\sqrt{2}} & 0 & En-x \\
\frac{u+i\,v}{\sqrt{2}} & \frac{u+i\,v}{\sqrt{2}} & 0 & \frac{s+i\,t}{\sqrt{2}} & 0 & 0 & En-x & 0
\end{pmatrix}$$

Out[306]//MatrixForm=

$$\begin{pmatrix} -z & 0 & 0 & \frac{u-i v}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{4\pi^2} & \frac{s-i t}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{s+i t}{\sqrt{2}} & 0 & En-x & 0 & \frac{s+i t}{\sqrt{2}} & 0 & \frac{u+i v}{\sqrt{2}} \\ \frac{u+i v}{\sqrt{2}} & 0 & En-x & 0 & \frac{s+i t}{\sqrt{2}} & 0 & \frac{u+i v}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{s-i t}{\sqrt{2}} & 0 & \frac{i}{\pi} & 0 & 0 \\ 0 & 0 & \frac{s-i t}{\sqrt{2}} & 0 & -\frac{i}{\pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{u-i v}{\sqrt{2}} & 0 & 0 & 0 & \frac{i}{\pi} \\ 0 & 0 & \frac{u-i v}{\sqrt{2}} & 0 & 0 & 0 & -\frac{i}{\pi} & 0 \end{pmatrix}$$

Superoperator Γ and related matrices (run)

Definitions

In[307]:=

```
Clear[En];
```

In[308]:=

```
K = SparseArray[{{7, 8} → -1, {8, 7} → -1}, {8, 8}]; MatrixForm@K
```

Out[308]//MatrixForm=

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&-1 \\ 0&0&0&0&0&0&-1&0 \end{pmatrix}$$

In[309]:=

```
L1 = SparseArray[{{7, 5} → 1, {7, 6} → 1, {8, 4} → 1}, {8, 8}]; MatrixForm@L1
```

Out[309]//MatrixForm=

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0 \\ 0&0&0&1&0&0&0&0 \end{pmatrix}$$

In[310]:=

```
L2 = SparseArray[{{7, 3} → 1, {8, 1} → 1, {8, 2} → 1}, {8, 8}]; MatrixForm@L2
```

Out[310]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[311]:=

```
κ1 = SparseArray[{{2, 3} → i / Pi, {3, 2} → -i / Pi}, {3, 3}]; MatrixForm@κ1
```

Out[311]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{i}{\pi} \\ 0 & -\frac{i}{\pi} & 0 \end{pmatrix}$$

In[312]:=

```
κ2 = SparseArray[{{1, 3} → i / Pi, {3, 1} → -i / Pi, {2, 2} → -1 / (4 * Pi ^ 2)}, {3, 3}];
MatrixForm@κ2
```

Out[312]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{i}{\pi} \\ 0 & -\frac{1}{4\pi^2} & 0 \\ -\frac{i}{\pi} & 0 & 0 \end{pmatrix}$$

In[313]:=

```
κ3 = {{0, 1}, {1, 0}}; MatrixForm@κ3
```

Out[313]//MatrixForm=

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

In[314]:=

```
κ4 = {{0, 1, 1}, {1, 0, 0}}; MatrixForm@κ4
```

Out[314]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

In[315]:=

```
κ5 = {{0, 0, 1}, {1, 1, 0}}; MatrixForm@κ5
```

Out[315]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

In[316]:=

```
K0 = {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, ⅈ / Pi, 0, 0, 0, 0, 0}, {0, -ⅈ / Pi, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, ⅈ / Pi, 0, 0}, {0, 0, 0, 0, -1 / (4 * Pi ^ 2), 0, 0, 0},
   {0, 0, 0, -ⅈ / Pi, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, En}, {0, 0, 0, 0, 0, 0, En, 0}};
MatrixForm@(K0 /. En → 1)
```

Out[316]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{i}{\pi} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{4\pi^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{\pi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[317]:=

```
Γ[R_] := K.R.K + L1.R.ConjugateTranspose[L1] + ConjugateTranspose[L1].R.L1 +
    L2.R.ConjugateTranspose[L2] + ConjugateTranspose[L2].R.L2;
```

Numerical solution of the MDE

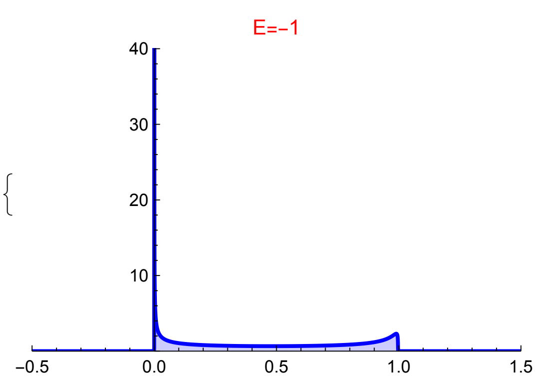
Plots of the DoS for different values of E

In[332]:=

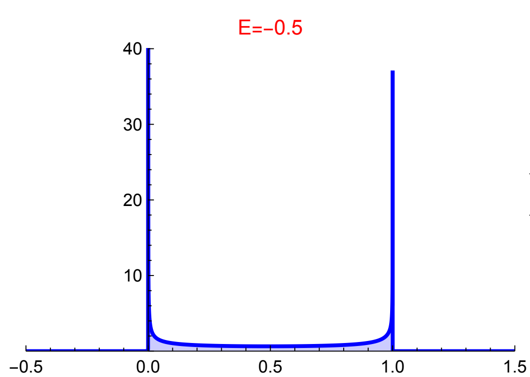
```
p = {};
EnList = {-1, -0.5 , -0.1, 0,  0.05,  0.1, 0.15, 0.4, 0.5, 0.6, 0.7, 1, 2, 5};
Do[plotLabel = "E=" <> ToString[EnList⟦ind⟧];
  p = Append[p, DoSplot[N[LLP /. En → EnList⟦ind⟧], {-.5, 1.5},
      SolutionAccuracy → 0.0001, Iterations → 50 000, ImaginaryPart → 0.00001,
      GridNumber → 5000, PlotRange → {{-0.5, 1.5}, {0, 40}}, Ticks → Automatic,
      PlotLegends → Automatic, PlotLabel → Style[plotLabel, FontColor → Red]]]
  , {ind, Length[EnList]}];
Table[Table[p⟦ind * 2 + j⟧, {j, 2}], {ind, 0, Length[EnList] / 2 - 1}]
```

Out[334]=

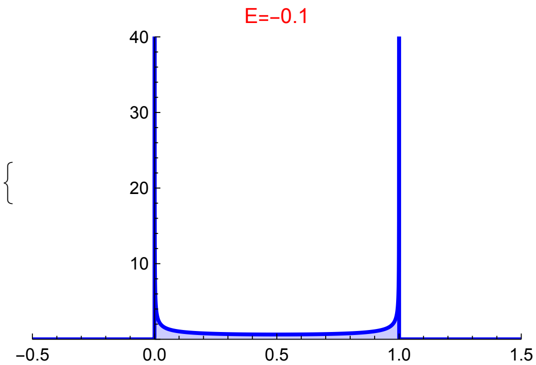
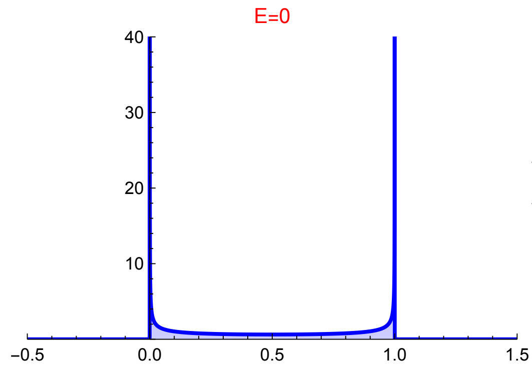
{{

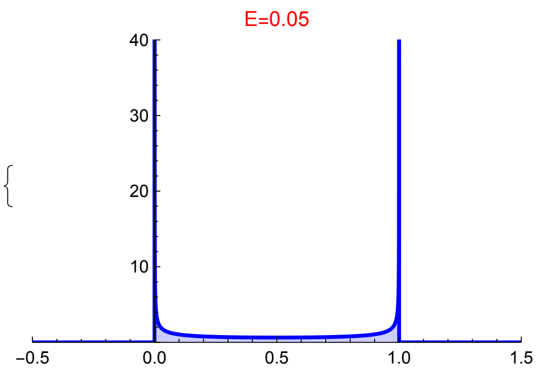
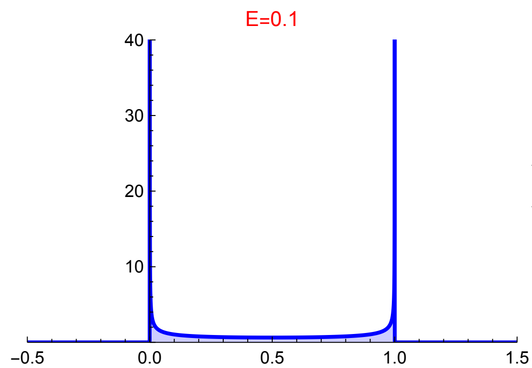


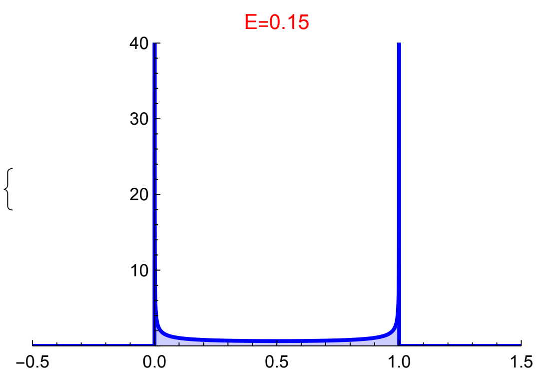
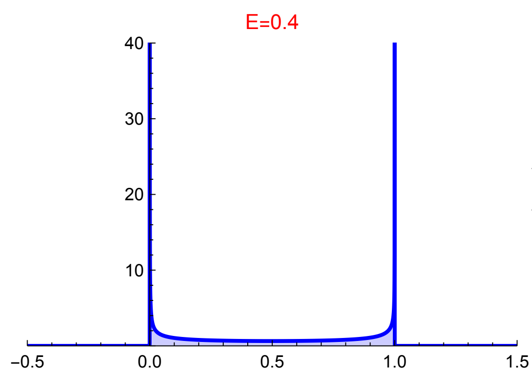
,

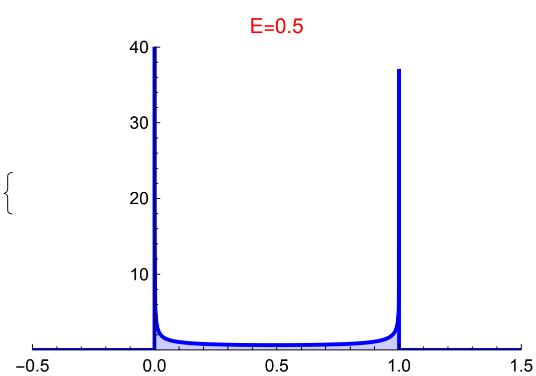
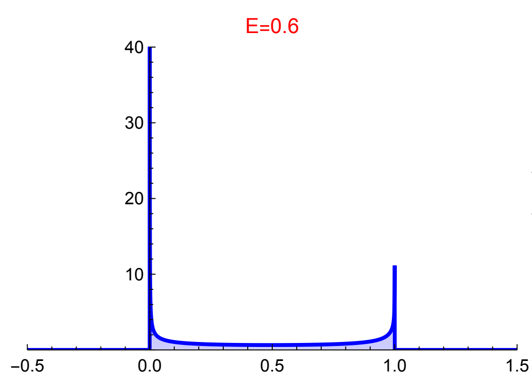


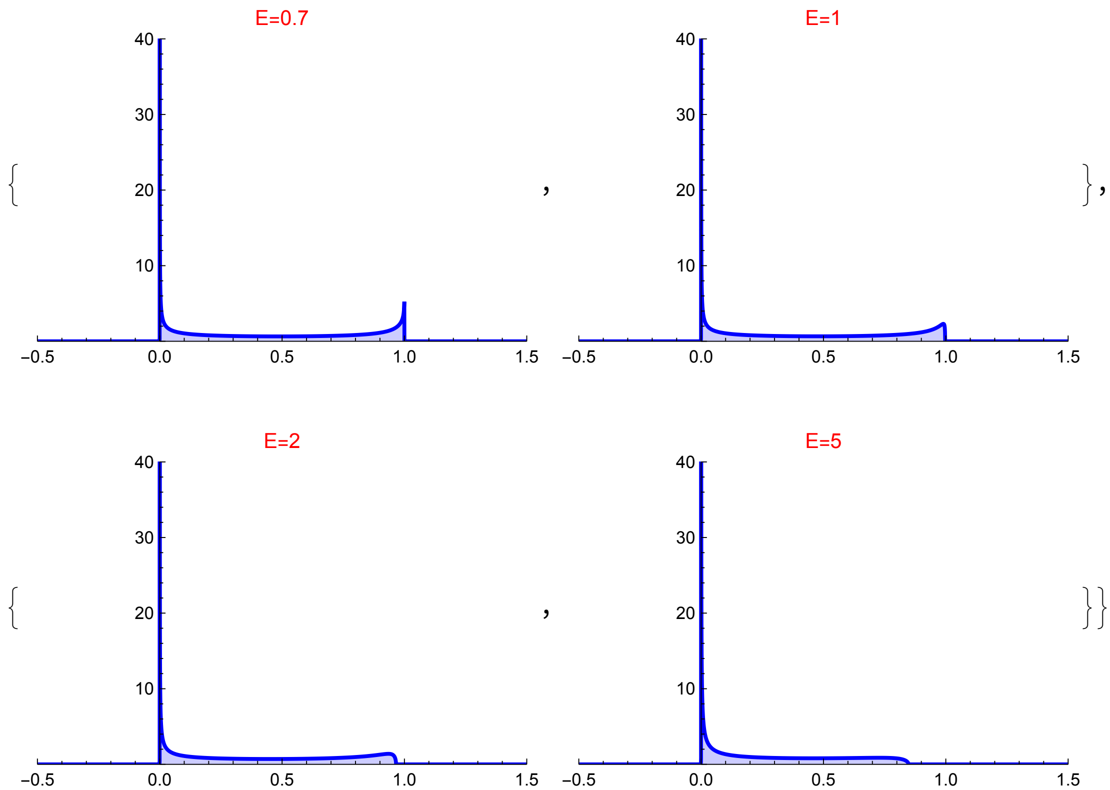
},

{ 
E=-0.1
40
30
20
10
-0.5
0.0
0.5
1.0
1.5
, 
E=0
40
30
20
10
-0.5
0.0
0.5
1.0
1.5
},

{ 
E=0.05
40
30
20
10
-0.5
0.0
0.5
1.0
1.5
, 
E=0.1
40
30
20
10
-0.5
0.0
0.5
1.0
1.5
},

{ 
E=0.15
40
30
20
10
-0.5
0.0
0.5
1.0
1.5
, 
E=0.4
40
30
20
10
-0.5
0.0
0.5
1.0
1.5
},

{ 
E=0.5
40
30
20
10
-0.5
0.0
0.5
1.0
1.5
, 
E=0.6
40
30
20
10
-0.5
0.0
0.5
1.0
1.5
},

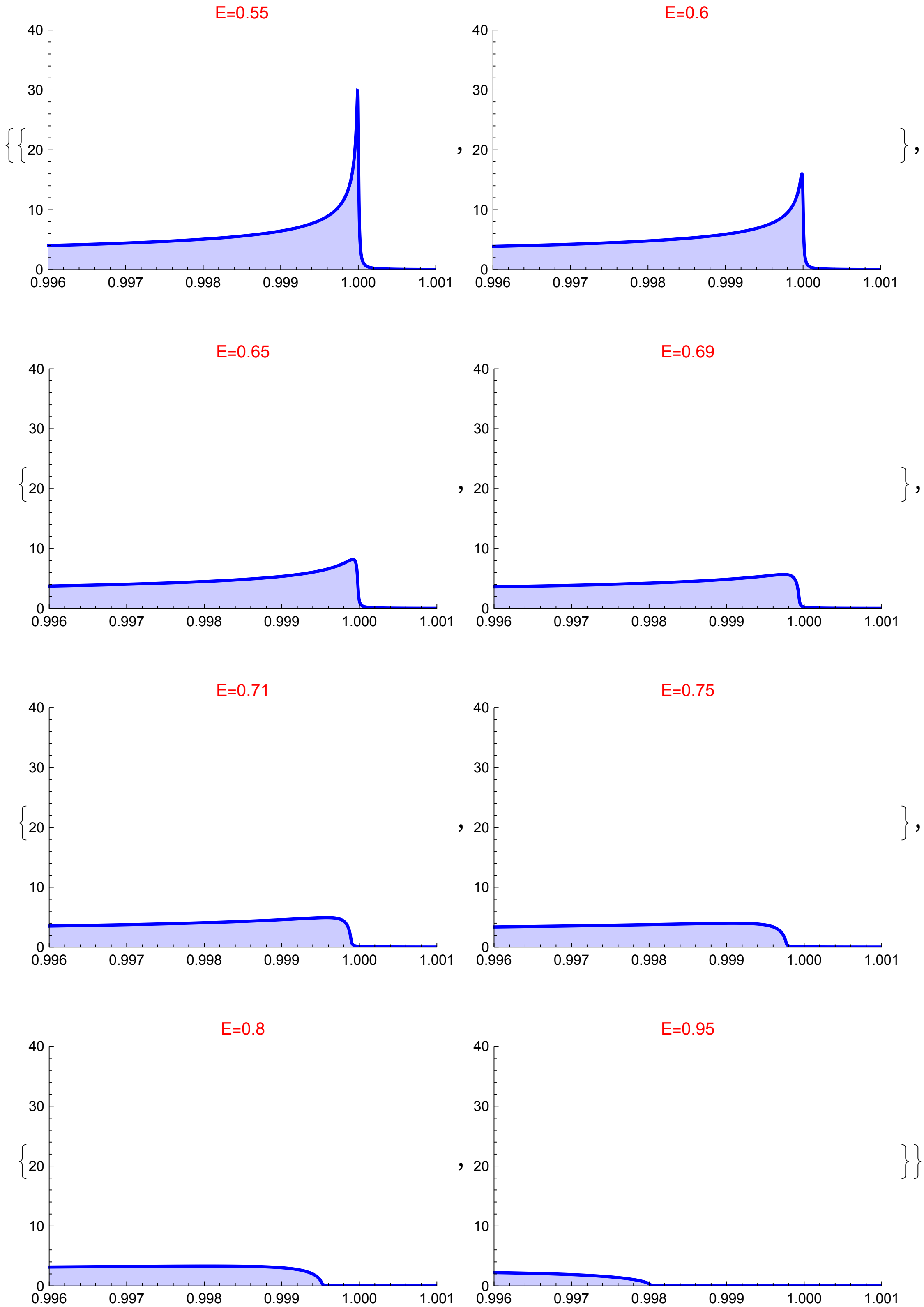


Density of states around z=1 for En between 0.6 and 0.7 (critical values)

In[341]:=

```
p = {}; EnList = {0.55, 0.6, 0.65, 0.69, 0.71, 0.75, 0.8, 0.95};
Do[plotLabel = "E=" <> ToString[EnList〚ind〛];
  p = Append[p, DoSplot[N[LLP /. En → EnList〚ind〛], {.996, 1.001},
     SolutionAccuracy → 0.0001, Iterations → 50 000, ImaginaryPart → 0.00001,
     GridNumber → 5000, PlotRange → {{.996, 1.001}, {0, 40}}, Ticks → Automatic,
     PlotLegends → Automatic, PlotLabel → Style[plotLabel, FontColor → Red]]]
  , {ind, Length[EnList]}];
Table[Table[p〚ind * 2 + j〛, {j, 2}], {ind, 0, Length[EnList] / 2 - 1}]
```

Out[343]=

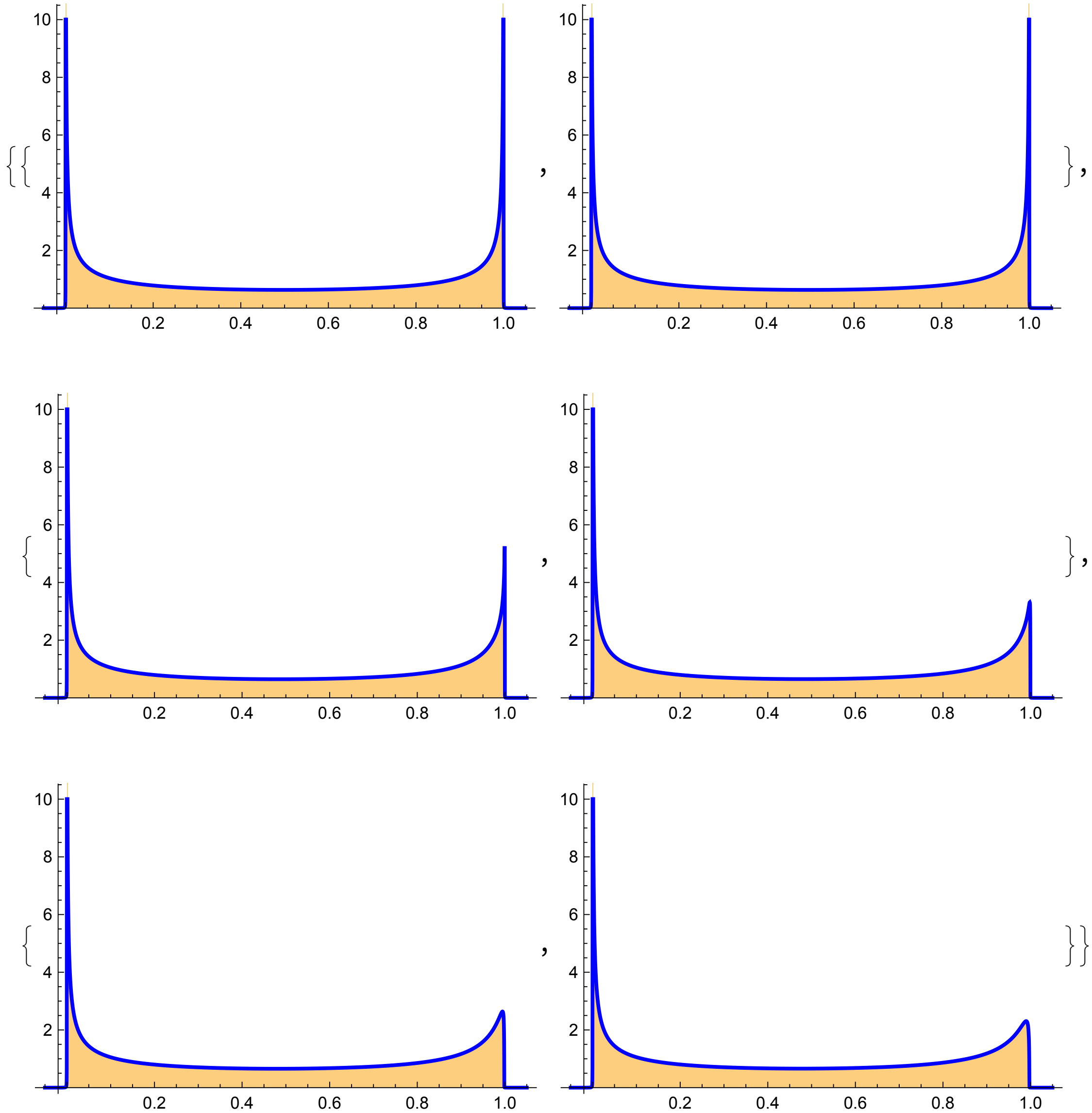

E=0.55
E=0.6
E=0.65
E=0.69
E=0.71
E=0.75
E=0.8
E=0.95
0
10
20
30
40
0.996
0.997
0.998
0.999
1.000
1.001

Compare the histogram for the simulated matrix and the DOS for the corresponding MDE (DEL)

In[354]:=

```
p = {};
EnList = {-0.1, 0.0, 0.7, 0.8, 0.9, 1.0};
EnTextList = {"_01", "00", "07", "08", "09", "10"};
Do[fileName = "./ev_40K_interval/Eigenvalues40Kat" <> EnTextList[[ind]] <> "a.wdx";
  plotLabel = "E=" <> ToString[EnList[[ind]]];
  ev = Import[fileName];
  p =
   Append[p, Show[Histogram[Re[ev], 500, "PDF"], DoSplot[N[LLP /. En → EnList[[ind]]],
       {-.05, 1.05}, SolutionAccuracy → 0.0001, Iterations → 50 000,
       ImaginaryPart → 0.00001, GridNumber → 5000, PlotRange → {{-0.5, 1.5}, {0, 10}},
       Ticks → Automatic, PlotLegends → Automatic, Filling → None],
     PlotRange → {{-0.05, 1.05}, {0, 10}}, ImageSize → 270]]
  , {ind, Length[EnList]}];
Table[Table[p[[ind * 2 + j]], {j, 2}], {ind, 0, Length[EnList] / 2 - 1}]
```

Out[356]=



Fano Factor for different E=-0.1, 0, 0.1, 0.1, ..., 0.9, 1, 1.1 (matrix size N=40 000, computations performed on the IST server, loaded from the files)

In[357]:=

```
EnList = Range[-0.1, 1.0, 0.1];
EnTextList = {"_01", "00", "01", "02", "03", "04", "05", "06", "07", "08", "09", "10"};
Do[fileName = "./ev_40K_interval/Eigenvalues40Kat" <> EnTextList〚ind〛 <> "a.wdx";
  ev = Import[fileName];
  Print["Fano factor at E=" <>
    ToString[EnList〚ind〛] <> ": " <> ToString[FanoFactor[Re[ev]]]];
  , {ind, Length[EnList]}];
```

```
Fano factor at E=-0.1: 0.249543
Fano factor at E=0.: 0.249237
Fano factor at E=0.1: 0.24955
Fano factor at E=0.2: 0.250428
Fano factor at E=0.3: 0.2519
Fano factor at E=0.4: 0.253912
Fano factor at E=0.5: 0.256423
Fano factor at E=0.6: 0.259419
Fano factor at E=0.7: 0.262821
Fano factor at E=0.8: 0.266585
Fano factor at E=0.9: 0.27066
Fano factor at E=1.: 0.274972
```

Compare (roughly) the DOS and the arcsine distribution, globally and on some short intervals, near edges and inside the bulk

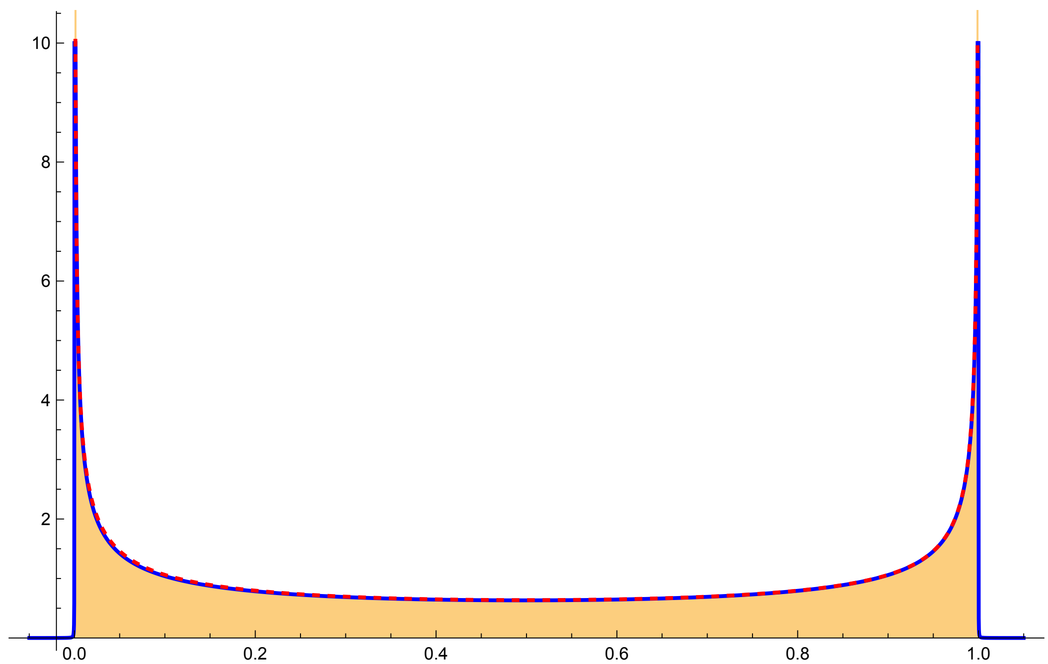
In[360]:=

```
arcsineplot[aa_, bb_] := ListLinePlot[Table[{aa + i * (bb - aa) / 1000,
     1 / (N[Pi] * Sqrt[(aa + i * (bb - aa) / 1000) * (1 - (aa + i * (bb - aa) / 1000))])},
    {i, 1, 999}], PlotStyle → {Red, Thick, Dashed}, PlotRange → Full];
```

In[363]:=

```
ev = Import["./ev_40K_interval/Eigenvalues40Kat_01a.wdx"];
Show[Histogram[Re[ev], 500, "PDF"], DoSplot[N[LLP /. En → -0.1],
  {-.05, 1.05}, SolutionAccuracy → 0.0001, Iterations → 50 000,
  ImaginaryPart → 0.00001, GridNumber → 5000, PlotRange → {{-0.5, 1.5}, {0, 10}},
  Ticks → Automatic, PlotLegends → Automatic, Filling → None],
 arcsineplot[0, 1], PlotRange → {{-0.05, 1.05}, {0, 10}}, ImageSize → 540]
```

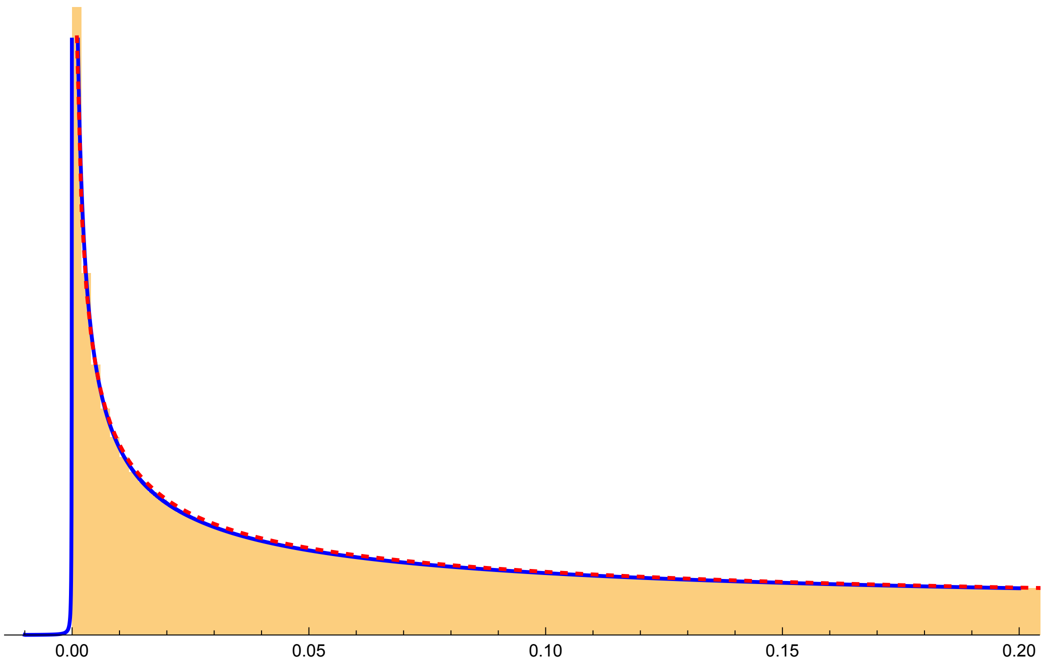
Out[363]=



In[364]:=

```
ev = Import["./ev_40K_interval/Eigenvalues40Kat_01a.wdx"];
Show[Histogram[Re[ev], 500, "PDF"], DoSplot[N[LLP /. En → -0.1],
   {-.01, 0.2}, SolutionAccuracy → 0.0001, Iterations → 50 000,
   ImaginaryPart → 0.00001, GridNumber → 5000, PlotRange → {{-0.01, 0.2}, {0, 10}},
   Ticks → Automatic, PlotLegends → Automatic, Filling → None],
 arcsineplot[0, 1], PlotRange → {{-0.01, 0.2}, {0, 10}}, ImageSize → 540]
```

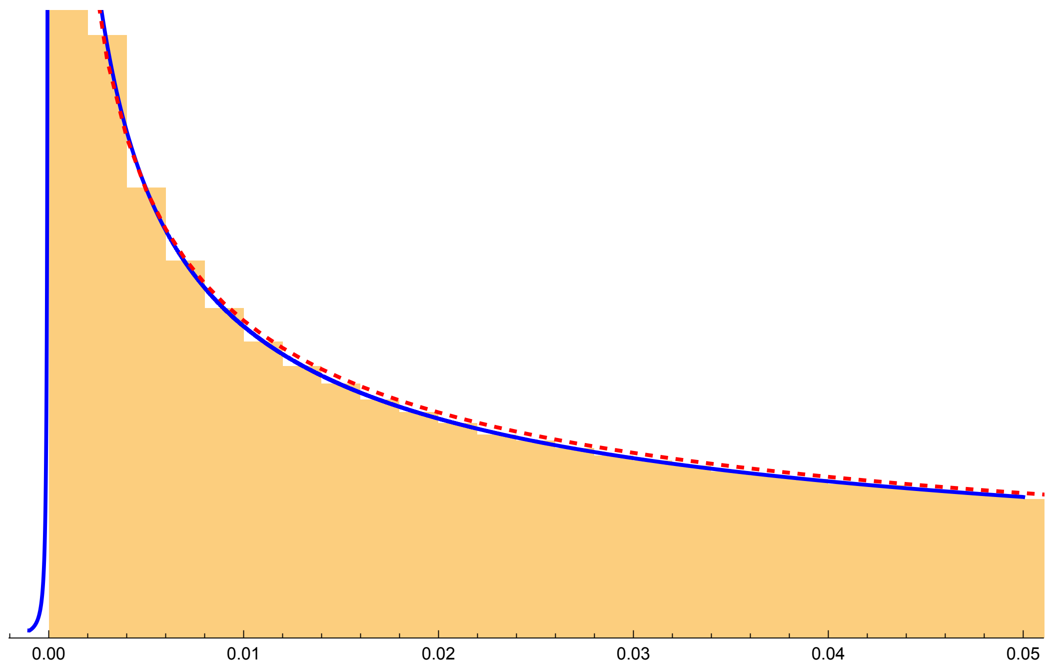
Out[364]=



In[366]:=

```
ev = Import["./ev_40K_interval/Eigenvalues40Kat_01a.wdx"];
Show[Histogram[Re[ev], 500, "PDF"], DoSplot[N[LLP /. En → -0.1], {-.001, 0.05},
   SolutionAccuracy → 0.0001, Iterations → 50 000, ImaginaryPart → 0.00001,
   GridNumber → 5000, PlotRange → {{-0.001, 0.05}, {0, 10}},
   Ticks → Automatic, PlotLegends → Automatic, Filling → None],
 arcsineplot[0, 1], PlotRange → {{-0.001, 0.05}, {0, 6}}, ImageSize → 540]
```

Out[366]=



Plot the Fano factor as a function of parameter ϕ (load the values of the Fano factor computed on the IST server)

In[ ]:=

```
dataF = {};
```

In[392]:=

```
Do[fanophi =
   Import[StringTemplate["./fano_phi_50_9k/phi_`a`50_9k.wdx"][<|"a" → k|>]];
 dataF = Join[dataF, {{fanophi[[1]], fanophi[[3]]}}];, {k, 1, 49}]
```

In[393]:=

```
ListPlot[dataF, TicksStyle →
  {{FontSize → 12, FontFamily → "Serif"}, {FontSize → 12, FontFamily → "Serif"}},
 AxesLabel → {"ϕ", None}, LabelStyle → {FontSize → 12, FontFamily → "Serif"},
 PlotLabel → "F(ϕ, E=0, γ=1)" , PlotStyle → Black, ImageSize → 540]
```

Out[393]=