1. (25 points) Suppose that you are waiting for a bus whose arrival time is distributed as an exponential random variable with mean 1 hour. Once the bus arrives, it takes 1 hour to drive you home. However, if you wait 1 hour for the bus and it still has not arrived, you decide to give up on the bus and walk home, which takes 10 hours. Let Y be the amount of time (in hours) that it takes for you to get home including the time spent waiting for the bus.

(a) (15 points) Calculate the CDF of Y.
(b) (10 points) Calculate the expected value $\mathbb{E}[Y]$.

2. (25 points) Let $X \sim \text{Geom}(p)$, where $p \in (0,1)$. Compute

$$\mathbb{E}\left[\frac{1}{X!}\right],$$

where we recall that $X!$ is the factorial. To receive full credit, your final answer should not contain an infinite series.

3. (25 points) 250000 randomly chosen individuals were interviewed to estimate the unknown fraction $p \in (0,1)$ of the population that like bagels. The resulting estimate is $\hat{p}$. Suppose that we want to construct a 98% confidence interval $(\hat{p}-\varepsilon, \hat{p}+\varepsilon)$. How large must we choose ε? You may leave your answer in terms of the inverse Φ^{-1} of the CDF of the standard normal.

4. (25 points) Let X be the random variable with density

$$f(x) = \begin{cases} 1 & \text{if } x \in (0,1); \\ 0 & \text{if } x \notin (0,1). \end{cases}$$

Let $Y = \ln(\sqrt{X})$.

(a) (15 points) Compute the moment generating function $M_Y(t)$ of Y. Hint: do not try to compute the density of Y.
(b) (10 points) Use the moment generating function to compute the nth moment of Y.