MATH 180A - INTRODUCTION TO PROBABILITY
PRACTICE FINAL

WINTER 2021

Name (Last, First): ______________________________

Student ID: ______________________________

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT.

THIS EXAM WILL BE SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS ON THE PAPER PROVIDED. DO NOT REMOVE ANY OF THE PAGES.

THE EXAM CONSISTS OF N QUESTIONS. YOU ARE ALLOWED TO USE RESULTS FROM THE TEXTBOOK, HOMEWORK, AND LECTURE.

1. Suppose that $X \sim \text{Geom}(p)$ and $Y \sim \text{Geom}(q)$ are independent random variables. Find the probability $\mathbb{P}(X < Y)$.

2. Suppose that $X \sim \text{Unif}[-2, 1]$. Let $Y = X^2$.

(a) (10 points) Find the CDF of Y.

(b) (5 points) Is Y discrete, continuous, or neither? If discrete, find the p.m.f. If continuous, find the density. If neither, explain why.

3. Suppose that we choose a number N uniformly at random from the set $\{0, \dots, 4999\}$. Let X denote the sum of its digits. For example, if $N = 123$, then $X = 1 + 2 + 3 = 6$. Determine $\mathbb{E}[X]$.

4. Let T be the triangle in $\mathbb{R}^2$ with vertices $(0, 0)$, $(0, 1)$, and $(1, 1)$ (including the interior). Suppose that $P = (X, Y)$ is a point chosen uniformly at random inside of T.

(a) What is the joint density function of (X, Y)? Use this to compute $\text{Cov}(X, Y)$.

(b) Determine if X and Y are independent.

5. Suppose that we roll a fair six-sided die until we roll a 6, at which point we stop. Let N be the number of times that we rolled an odd number before we stopped. For example, we could have the sequence of rolls $(1, 3, 4, 1, 2, 6)$, in which case $N = 3$. Compute the expectation $\mathbb{E}[N]$.

6. Suppose that we have i.i.d. random variables $X_1, X_2, \dots$ with mean zero $\mathbb{E}[X_1] = 0$ and unit variance $\text{Var}(X_1) = 1$. Determine the following limits with precise justifications.

(a)

$$\lim_{n\to\infty} \mathbb{P}\left(-\frac{n}{4} \le X_1 + \cdots + X_n < \frac{n}{2} \right)$$

(b)

$$\lim_{n\to\infty} \mathbb{P}(X_1 + \cdots + X_n = 0)$$