MATH 180A - INTRODUCTION TO PROBABILITY
PRACTICE MIDTERM #1

WINTER 2021

Name (Last, First): ____________________

Student ID: ____________________

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT.

THIS EXAM WILL BE SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS ON THE PAPER PROVIDED. DO NOT REMOVE ANY OF THE PAGES.

1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space.

(a) Suppose that $A, B \in \mathcal{F}$ satisfy

$$\mathbb{P}(A) + \mathbb{P}(B) > 1.$$

Making no further assumptions on A and B, prove that $A \cap B \neq \emptyset$.

(b) Prove that A is independent from itself if and only if $\mathbb{P}(A) \in \{0, 1\}$.

2. Suppose we have an urn with 10 red balls, 15 blue balls, and 20 green balls. We draw three balls without replacement uniformly at random.

(a) What is the probability that we have more red balls than non-red balls?
(b) What is the probability that we have more red balls than green balls?
(c) What is the probability that at least two of the three balls have the same color?

3. A box contains 3 coins, two of which are fair and the third has probability 3/4 of coming up heads. A coin is chosen randomly from the box and tossed 3 times.

(a) What is the probability that all 3 tosses are heads?
(b) Given that all three tosses are heads, what is the probability that the biased coin was chosen?

4. Let X be a discrete random variable taking the values $\{1, 2, \dots, n\}$ all with equal probability. Let Y be another discrete random variable taking values in $\{1, 2, \dots, n\}$. Assume that X and Y are independent. Show that $\mathbb{P}(X = Y) = \frac{1}{n}$. (Hint: you do not need to know the distribution of Y to calculate this.)

5. Consider a point $P = (X, Y)$ chosen uniformly at random inside of the triangle in $\mathbb{R}^2$ that has vertices $(1, 0)$, $(0, 1)$, and $(0, 0)$. Let $Z = \max(X, Y)$ be the random variable defined as the maximum of the two coordinates of the point. For example, if $P = (\frac{1}{2}, \frac{1}{3})$, then $Z = \max(X, Y) = \frac{1}{2}$. Determine the cumulative distribution function of Z. Determine if Z is a continuous random variable, a discrete random variable, or neither. If continuous, determine the probability density function of Z. If discrete, determine the probability mass function of Z. If neither, explain why.

(Hint: Draw a picture.)