

**MATH 180A - INTRODUCTION TO PROBABILITY
PRACTICE MIDTERM #1 ADDITIONAL PROBLEMS**

WINTER 2021

Name (Last, First): _____

Student ID: _____

**REMEMBER THIS EXAM IS GRADED BY A
HUMAN BEING. WRITE YOUR SOLUTIONS
NEATLY AND COHERENTLY, OR THEY RISK
NOT RECEIVING FULL CREDIT.**

**THIS EXAM WILL BE SCANNED. MAKE SURE
YOU WRITE ALL SOLUTIONS ON THE PAPER
PROVIDED. DO NOT REMOVE ANY OF THE
PAGES.**

1. At a political meeting there are 7 progressives and 7 conservatives. We choose five people uniformly at random to form a committee (president, vice-president and 3 regular members whose positions cannot be distinguished).

- (a) Let A be the event that we end up with more conservatives than progressives. What is the probability of A ?
- (b) Let B be the event that Ronald, representing conservatives, becomes the president, and Felix, representing liberals, becomes the vice-president. What is the probability of B ?

2. Let A, B be events in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

- (a) Suppose that A, B satisfy $A \cap B = \emptyset$. If A and B are independent, what can you say about $\mathbb{P}(A)$ and $\mathbb{P}(B)$?
- (b) Suppose that $\mathbb{P}(A) = 0.5$ and $\mathbb{P}(B) = 0.8$. What possible range of values can $\mathbb{P}(A \cap B)$ have?

3. Suppose that we have two possibly unfair coins: the first coin takes heads with probability $p \in (0, 1)$ and tails with probability $1 - p$; the second coin takes heads with probability $q \in (0, 1)$ and tails with probability $1 - q$. The coins are independent of each other and consecutive flips are also independent.

Consider the following game. Flip both coins simultaneously. If the coins land on the same side (for example, both land on heads), then you win. Otherwise, then you lose. Let X be the number of times you play the game until you win. For example, X is equal to 1 if you win on your first play of the game. Determine the probability mass function of X .

4. Consider a point $P = (X, Y)$ chosen uniformly at random inside of the unit square $[0, 1]^2 = [0, 1] \times [0, 1] = \{(x, y) : 0 \leq x, y \leq 1\}$. Let $Z = \min(X, Y)$ be the random variable defined as the minimum of the two coordinates of the point. For example, if $P = (\frac{1}{2}, \frac{1}{3})$, then $Z = \min(\frac{1}{2}, \frac{1}{3}) = \frac{1}{3}$. Determine the cumulative distribution function of Z . Determine if Z is a continuous random variable, a discrete random variable, or neither. If continuous, determine the probability density function of Z . If discrete, determine the probability mass function of Z . If neither, explain why.

(Hint: Draw a picture.)

5. You shoot an arrow (uniformly at random) at a round target of radius 50 cm. If you hit a point at a distance ≤ 10 cm from the center of the target, you are awarded 10 points; if the point you hit is between 10 and 20 cm from the center, you get 5 point; if the point you hit is between 20 and 30 cm from the center, you get 3 points; if the point you hit is between 30 and 40 points from the center you get 1 point; if you hit a point which is 40 cm or more from the center you get 0 points. Let X be a random variable that gives the total number of points you get after two independent shots.

- (a) Is the random variable X continuous, discrete, neither or both?
- (b) If X is continuous or discrete, compute the p.m.f./p.d.f. of X .
- (c) Compute and plot the c.d.f. of the random variable X .