

MATH 180A HOMEWORK #3

WINTER 2021

Due date: **Sunday 1/31/2021 11:59 PM** (via Gradescope)

1. (ASV*, Exercise 2.18) - 2 points.

We choose a number from the set $\{10, 11, 12, \dots, 99\}$ uniformly at random.

- (a) Let X be the first digit and Y the second digit of the chosen number. Show that X and Y are independent random variables.
- (b) Let X be the first digit of the chosen number and Z the sum of the two digits. Show that X and Z are not independent.

2. (ASV, Exercise 1.36) - 2 points.

- (a) Let (X, Y) denote a uniformly chosen random point inside the unit square

$$[0, 1]^2 = [0, 1] \times [0, 1] = \{(x, y) : 0 \leq x, y \leq 1\}.$$

Let $0 \leq a < b \leq 1$. Find the probability $\mathbb{P}(a < X < b)$, that is, the probability that the x -coordinate X of the chosen point lies in the interval (a, b) .

- (b) What is the probability $\mathbb{P}(|X - Y| \leq 1/4)$?

3. (ASV, Exercise 3.20) - 2 points.

Let $c > 0$ and $X \sim \text{Unif}[0, c]$. Show that the random variable $Y = c - X$ has the same cumulative distribution function as X and hence also the same density function.

4. (ASV, Exercise 3.39) - 2 points.

Parts (a) and (b) ask for an example of a random variable X whose cumulative distribution function $F(x)$ satisfies $F(1) = 1/3$, $F(2) = 3/4$, and $F(3) = 1$.

- (a) Make X discrete and give its probability mass function.
- (b) Make X continuous and give its probability density function.

5. (ASV, Exercise 3.41) - 3 points.

We produce a random real number X through the following two-stage experiment. First roll a fair die to get an outcome Y in the set $\{1, 2, \dots, 6\}$. Then, if $Y = k$, choose X uniformly in the interval $(0, k]$. Find the cumulative distribution function $F(s)$ and the probability density function $f(s)$ of X for $3 < s < 4$.

6. (ASV, Exercise 3.46) - 3 points.

A stick of length ℓ is broken at a uniformly chosen random location. We denote the length of the smaller piece by X .

- (a) Find the cumulative distribution function of X .
- (b) Find the probability density function of X .

*Introduction to Probability, by David F. Anderson, Timo Seppäläinen, and Benedek Valkó

7. (*ASV, Exercise 2.20*) - 3 points.

A fair die is rolled repeatedly. Use precise notation of probabilities of events and random variables for the solutions to the questions below.

- (a) Write down a precise sum expression for the probability that the first five rolls give a three at most two times.
- (b) Calculate the probability that the first three does not appear before the fifth roll.
- (c) Calculate the probability that the first three appears before the twentieth roll but not before the fifth roll.

8. (*ASV, Exercise 2.21*) - 3 points.

Jane must get at least three of the four problems on the exam correct to get an A. She has been able to do 80% of the problems on old exams, so she assumes that the probability she gets any problem correct is 0.8. She also assumes that the results on different problems are independent.

- (a) What is the probability she gets an A?
- (b) If she gets the first problem correct, what is the probability she gets an A?