

MATH 180A HOMEWORK #1

WINTER 2021

Due date: **Friday 1/15/2021 11:59 PM** (via Gradescope)

1. (ASV*, *Exercise 1.20*) - 3 points.

We roll a fair die four times.

- (a) Describe the sample space Ω and the probability measure P that model this experiment. To describe P , give the value of $P\{\omega\}$ for each outcome $\omega \in \Omega$.
- (b) Let A be the event that there are at least two fives among the four rolls. Let B be the event that there is at most one five among the four rolls. Find the probabilities $P(A)$ and $P(B)$ by finding the ratio of the number of favorable outcomes to the total (i.e., $P(A) = \frac{\#A}{\#\Omega}$ and $P(B) = \frac{\#B}{\#\Omega}$).
- (c) What is the set $A \cup B$? What equality should $P(A)$ and $P(B)$ satisfy? Check that your answers to part (b) satisfy this equality.

2. (ASV, *Exercise 1.4*) - 3 points.

Every day a kindergarten class chooses randomly one of the 50 state flags to hang on the wall, without regard to previous choices. We are interested in the flags that are chosen on Monday, Tuesday and Wednesday of next week.

- (a) Describe a sample space Ω and a probability measure P to model this experiment.
- (b) What is the probability that the class hangs Wisconsin's flag on Monday, Michigan's flag on Tuesday, and California's flag on Wednesday?
- (c) What is the probability that Wisconsin's flag will be hung at least two of the three days?

3. (ASV, *Exercise 1.26*) - 3 points.

10 men and 5 women are meeting in a conference room. Four people are chosen at random from the 15 to form a committee.

- (a) What is the probability that the committee consists of 2 men and 2 women?
- (b) Among the 15 is a couple, Bob and Jane. What is the probability that Bob and Jane both end up on the committee?
- (c) What is the probability that Bob ends up on the committee but Jane doesn't?

4. (ASV, *Exercise 1.34*) - 2 points.

Pick a uniformly chosen random point inside a unit square (a square of sidelength 1) and draw a circle of radius $1/3$ around the point. Find the probability that the circle lies entirely inside the square.

5. (ASV, *Exercise 1.40*) - 3 points.

An urn contains 1 green ball, 1 red ball, 1 yellow ball and 1 white ball. I draw 4 balls with replacement. What is the probability that there is at least one

**Introduction to Probability*, by David F. Anderson, Timo Seppäläinen, and Benedek Valkó

color that is repeated exactly twice? *Hint:* Use inclusion-exclusion with events $G = \{\text{exactly two balls are green}\}$, $R = \{\text{exactly two balls are red}\}$, etc.

6. (ASV, Exercise 1.14) - 2 points.

Assume that $P(A) = 0.4$ and $P(B) = 0.7$. Making no further assumptions on A and B , show that $P(AB)$ satisfies $0.1 \leq P(AB) \leq 0.4$.

7. (ASV, Exercise 1.43) - 4 points.

Show that for any events A_1, A_2, \dots, A_n ,

$$P(A_1 \cup \dots \cup A_n) \leq \sum_{k=1}^n P(A_k).$$

Hint: Obtain the case $n = 2$ from inclusion-exclusion, and more generally apply the $n = 2$ case repeatedly. Or formulate a proof by induction.