MATH180C: Introduction to Stochastic Processes II

www.math.ucsd.edu/~ynemish/teaching/180c

Today: Renewal processes. Examples

> Q&A: November 6

Next: PK 7.4-7.5, Durrett 3.1

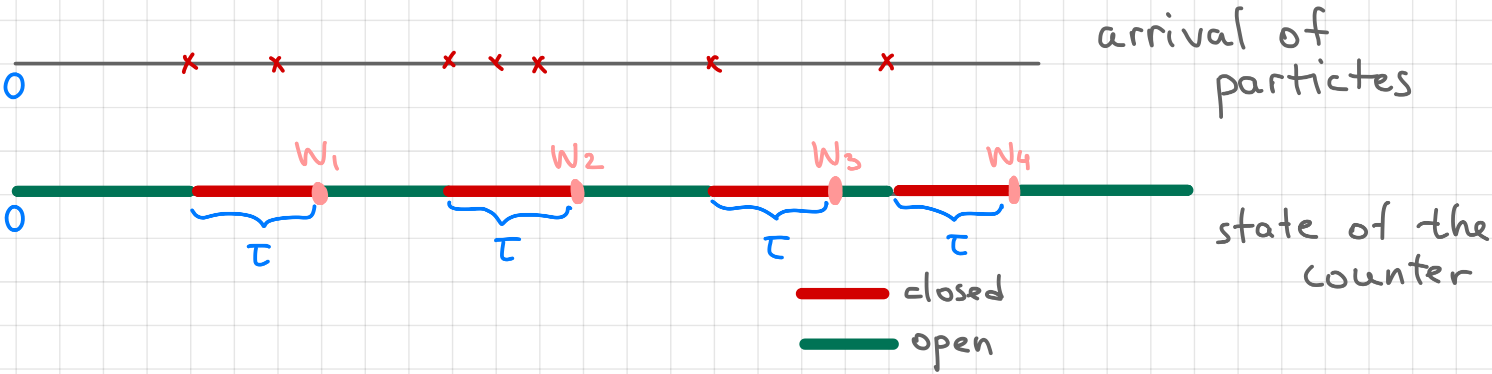
This week:

- Homework 4 (due Friday, November 6, 11:59 PM)

Other renewal processes

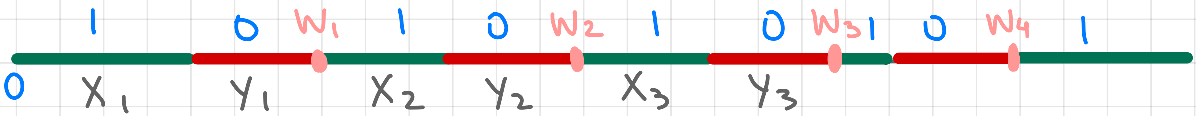
- traffic flow : distances between successive cars are assumed to be i.i.d. random variables

- counter process: particles / signals arrive on a device and lock it for time τ ; particles arrive according to a PP; times at which the counter unlocks form a renewal process



Other renewal processes

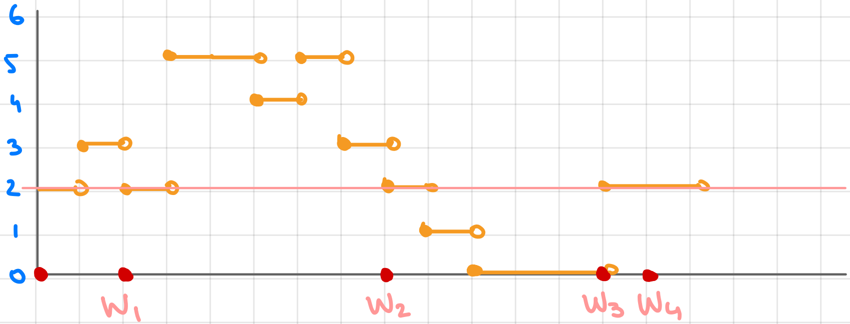
- more generally, if a component has two states (0/1, operating/non-operating etc), switches between them, times spent in 0 are X_i, times spent in 1 are Y_i, $(X_i)_{i=1}^{\infty}$ i.i.d., $(Y_i)_{i=1}^{\infty}$ i.i.d., then the times of switching from 0 to 1 form a renewal process with interrenewal times $X_i + Y_i$



Other renewal processes

- Markov chains : if $(Y_n)_{n\geq 0}$, $Y_n \in \{0,1,\dots\}$ is a recurrent MC starting from $Y_0 = k$, then the times of returns to state k form a renewal process. More precisely define

$$W_1 = \min\{n>0 : Y_n = k\}$$
$$W_p = \min\{n > W_{p-1} : Y_n = k\}$$



Example with $k=2$

Similarly for continuous time MCs.

Strong Markov property!

Other renewal processes

- Queues. Consider a single-server queueing process

(i) if customer arrival times form a renewal process then the times of the starts of successive idle periods generate a second renewal time

(ii) if customes arrive according to a Poisson process, then the times when the server passes from busy to free form a renewal process

Asymptotic behavior

Asymptotic behavior of renewal processes

Let $N(t)$ be a renewal process with interrenewal times X_i, $X_i \in (0, \infty)$.

<u>Thm</u>.

<u>Proof</u>. $N(t)$ is nondecreasing, therefore N_∞ is the total number of events ever happened.

<u>Thm</u> (Pointwise renewal thm).

Elementary Renewal Theorem

Thm. If $M(t) = E(N(t))$ and $E(X_1) = \mu$, then

Proof. (Only for bounded X_i: $\exists\ K$ s.t. $P(X_1 \leq K) = 1$)

First note that

In lecture 11 we showed that ,

so $M(t) =$

$\frac{M(t)}{t} =$

If $X_i \leq K$, then

Asymptotic distribution of $N(t)$

<u>Thm</u>. Let $N(t)$ be a renewal process with $E(X_1) = \mu$ and $Var(X_1) = \sigma^2$, then

1)

2)

No proof.