H versus h(x)

The H matrix from the lidar lesson and $h(x)$ equations from the radar lesson are actually accomplishing the same thing; they are both needed to solve $y = z - Hx'$ in the update step.

But for radar, there is no H matrix that will map the state vector x into polar coordinates; instead, you need to calculate the mapping manually to convert from cartesian coordinates to polar coordinates.

Here is the h function that specifies how the predicted position and speed get mapped to the polar coordinates of range, bearing and range rate.

$$h(x') = \begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix} = \begin{pmatrix} \sqrt{{p'}_x^2 + {p'}_y^2} \\ \arctan(p'_y / p'_x) \\ \frac{p'_x v'_x + p'_y v'_y}{\sqrt{{p'}_x^2 + {p'}_y^2}} \end{pmatrix}$$

Hence for radar $y = z - Hx'$ becomes $y = z - h(x')$.

of the position vector ρ which can be defined as $\rho = sqrt(p_x^2 + p_y^2)$.

- $\varphi = atan(p_y/p_x)$. Note that φ is referenced counter-clockwise from the x-axis, so φ from the video clip above in that situation would actually be negative.
- The range rate, $\dot{\rho}$, is the projection of the velocity, v, onto the line, L.

Deriving the Radar Measurement Function

The measurement function is composed of three components that show how the predicted state, $x' = (p'_x, p'_y, v'_x, v'_y)^T$, is mapped into the measurement space, $z = (\rho, \varphi, \dot{\rho})^T$:

The range, ρ, is the distance to the pedestrian which can be defined as:

$$\rho = \sqrt{p_x^2 + p_y^2}$$

φ is the angle between ρ and the x direction and can be defined as:

$$\varphi = \arctan(p_y/p_x)$$

There are two ways to do the range rate $\dot{\rho(t)}$ derivation:

Generally we can explicitly describe the range, ρ, as a function of time:

$$\rho(t) = \sqrt{p_x(t)^2 + p_y(t)^2}$$

The range rate, $\dot{\rho(t)}$, is defined as time rate of change of the range, ρ, and it can be described as the time derivative of ρ:

$$\dot{\rho} = \frac{\partial \rho(t)}{\partial t} = \frac{\partial}{\partial t}\sqrt{p_x(t)^2 + p_y(t)^2} = \frac{1}{2\sqrt{p_x(t)^2+p_y(t)^2}}(\frac{\partial}{\partial t}p_x(t)^2 + \frac{\partial}{\partial t}p_y(t)^2)$$

$$= \frac{1}{2\sqrt{p_x(t)^2+p_y(t)^2}}(2p_x(t)\frac{\partial}{\partial t}p_x(t) + 2p_y(t)\frac{\partial}{\partial t}p_y(t))$$

$\frac{\partial}{\partial t}p_x(t)$ is nothing else than $v_x(t)$, similarly $\frac{\partial}{\partial t}p_y(t)$ is $v_y(t)$. So we have:

$$\dot{\rho} = \frac{\partial \rho(t)}{\partial t} = \frac{1}{2\sqrt{p_x(t)^2+p_y(t)^2}}(2p_x(t)v_x(t) + 2p_y(t)v_y(t)) = \frac{2(p_x(t)v_x(t)+p_y(t)v_y(t))}{2\sqrt{p_x(t)^2+p_y(t)^2}}$$

$$= \frac{p_x(t)v_x(t)+p_y(t)v_y(t)}{\sqrt{p_x(t)^2+p_y(t)^2}}$$

$$\dot{\rho} = \frac{p_x v_x + p_y v_y}{\sqrt{p_x^2 + p_y^2}}$$

The range rate, $\dot{\rho}$, can be seen as a scalar projection of the velocity vector, $\vec{v}$, onto $\vec{\rho}$. Both $\vec{\rho}$ and $\vec{v}$ are 2D vectors defined as:

$$\vec{\rho} = \begin{pmatrix} p_x \\ p_y \end{pmatrix}, \ \vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

The scalar projection of the velocity vector $\vec{v}$ onto $\vec{\rho}$ is defined as:

$$\dot{\rho} = \frac{\vec{v}\vec{\rho}}{|\vec{\rho}|} = \frac{\begin{pmatrix} v_x & v_y \end{pmatrix}\begin{pmatrix} p_x \\ p_y \end{pmatrix}}{\sqrt{p_x^2 + p_y^2}} = \frac{p_x v_x + p_y v_y}{\sqrt{p_x^2 + p_y^2}}$$

where $|\vec{\rho}|$ is the length of $\vec{\rho}$. In our case it is actually the range, so $\rho = |\vec{\rho}|$.

The Next Quiz

$$\begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix} \leftarrow h(x) \begin{pmatrix} p'_x \\ p'_y \\ v'_x \\ v'_y \end{pmatrix}$$

h is a nonlinear function. In the next quiz I would like to check your intuition about what that means.

Search or ask questions in Knowledge.

Ask peers or mentors for help in Student Hub.

NEXT