

Lecture 0: Introduction

Yi, Yung (이웅)

EE210: Probability and Introductory Random Processes
KAIST EE

MONTH DAY, 2021

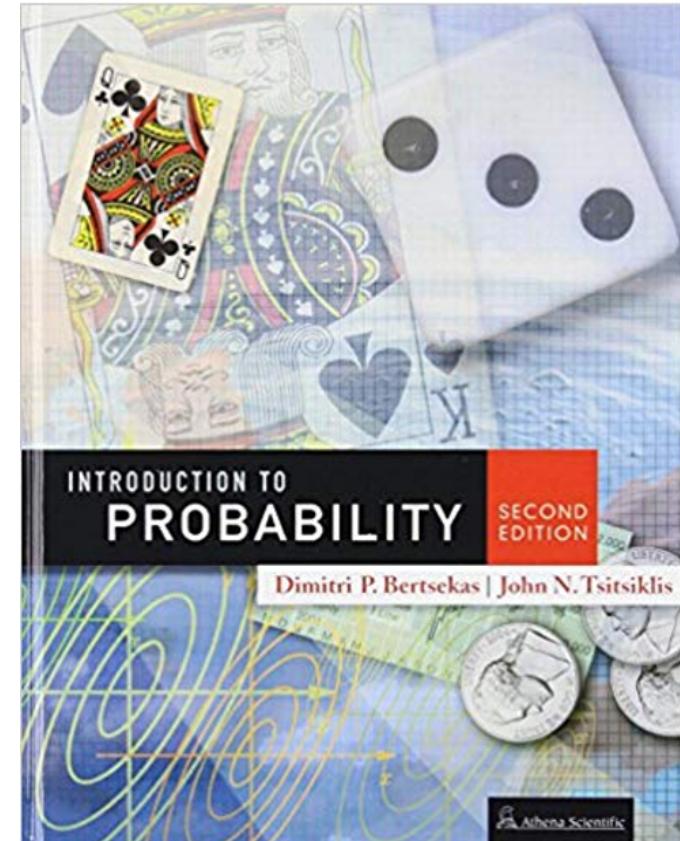
- Course logistics
- Why necessary to take the course of probability and random process?

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- <http://lanada.kaist.ac.kr>, yiyung@kaist.edu
- Computer Division
- A professor in KAIST EE since 2008
- Office hours: TBA

- A
 - B
 - C
-
- Mailing list: ee210@lanada.kaist.ac.kr
 - Please use KLMS for the questions about the lecture contents
 - This mailing list can be used for individual issues

- <http://klms.kaist.ac.kr/>
- To download course materials
- To ask questions about everything
- To check your score on each homework/exam
- To see all the announcements about the class

- Introduction to Probability
(2nd edition)
 - MIT course textbook
 - Dimitri P. Bertsekas and John N. Tsitsiklis



- Three Parts
 - Part I: Fundamentals of Probability
 - Part II: Inference and Limit Theorems
 - Part III: Random Processes
- On-line lectures at MIT and EdX
 - MIT: <http://bit.ly/2PkvYdr>
 - EdX: <http://bit.ly/3pHmZRd>
 - You can find older urls (2006, 2010, 2013) for this lecture, where there are many useful resources (recitation problems, homework problems, old exam problems, etc)
 - My lecture slides: based on theirs, but largely modified/reorganized/edited in many places for KAIST students

- In-class quiz (sometimes)
- Basically, weekly homework, but often bi-weekly
- 3 Exams (2 mid-terms and 1 final)
- Class participation
- Grading portions: A (X%), B (Y%), C(Z%), D(W%), F . . .
- Online lectures due to COVID-19 may change how to grade.

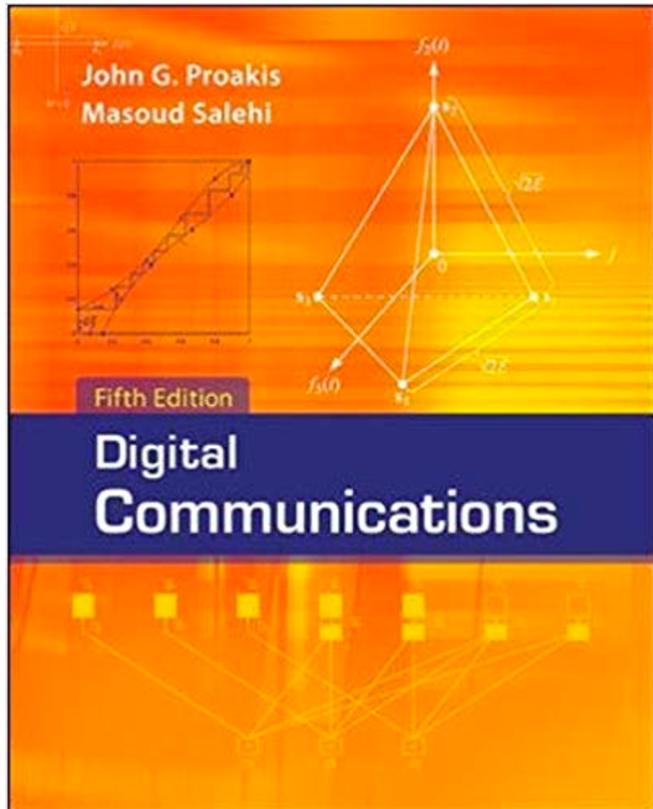
- Most should be via KLMS
 - Technical questions about lectures, homework, and etc
- Please DO NOT individually send emails to Prof. Yung Yi and TAs (or making calls or sending KakaoTalk msgs) about the technical questions (course contents, homework, etc)
 - All the questions need to be shared among the students.
 - TAs and Prof. Yung Yi will handle your questions as soon as possible.
 - But, you can send an email to Prof. Yung Yi for the things that need to be individually discussed.

Questions?

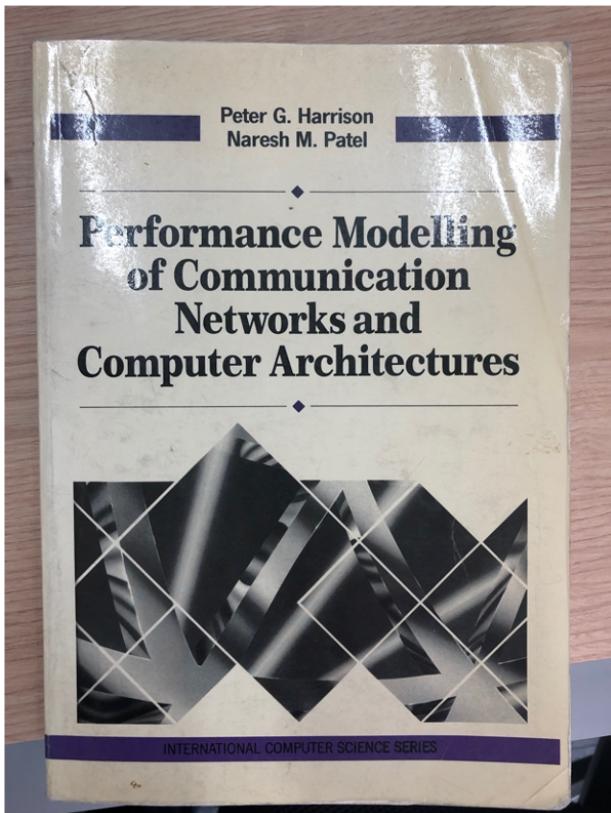
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- Assume that you are a designer of the following engineering systems. Good design?
 - a web server
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 - an AI-based image classifier

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- Assume that you are a designer of the following engineering systems. Good design?
 - a web server
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- From an engineering point of view,
 - System input
 - Algorithms in systems
 - Analysis of systems



Communications	13
1-5 Overview of the Book	16
1-6 Bibliographical Notes and References	16
2 Probability and Stochastic Processes	17
2-1 Probability	17
2-1-1 Random Variables, Probability Distributions, and Probability Densities	22
2-1-2 Functions of Random Variables	28
2-1-3 Statistical Averages of Random Variables	33
2-1-4 Some Useful Probability Distributions	37
2-1-5 Upper bounds on the Tail Probability	53
2-1-6 Sums of Random Variables and the Central Limit Theorem	58
2-2 Stochastic Processes	62
2-2-1 Statistical Averages	64
2-2-2 Power Density Spectrum	67
2-2-3 Response of a Linear Time-Invariant System to a Random Input Signal	68
2-2-4 Sampling Theorem for Band-Limited Stochastic Processes	72
2-2-5 Discrete-Time Stochastic Signals and Systems	74
2-2-6 Cyclostationary Processes	75
2-3 Bibliographical Notes and References	77
Problems	77



Preface

Chapter 1 Essentials of Probability Theory

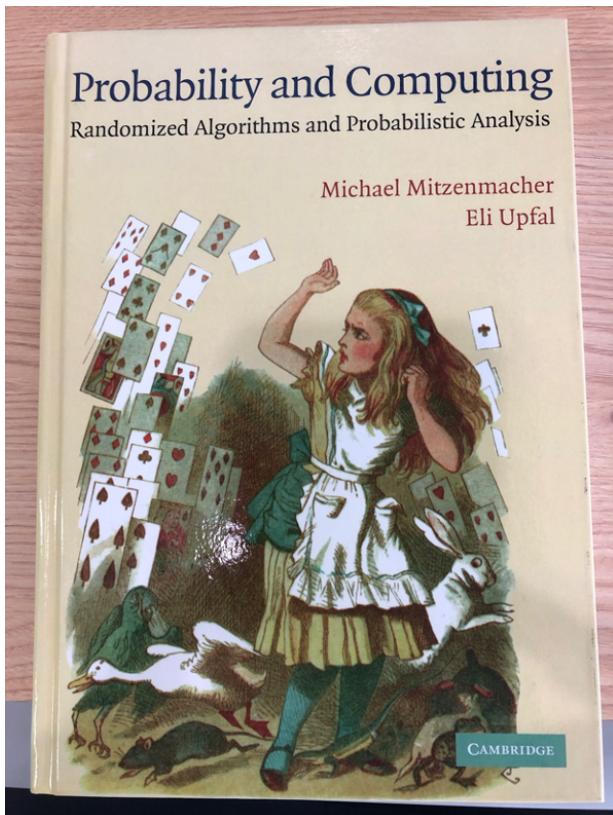
- 1.1 Sample space, events and probability
- 1.2 Conditional probability
- 1.3 Independence
- Exercises

Chapter 2 Random Variables and Distributions

- 2.1 Probability distribution functions
- 2.2 Discrete random variables
- 2.3 Continuous random variables
- 2.4 Joint random variables
- 2.5 Conditional distributions
- 2.6 Independence and sums
- Exercises

Chapter 3 Expected Values and Moments

- 3.1 Expectation
- 3.2 Generating functions and transforms
- 3.3 Asymptotic properties
- Exercises



Preface

1 Events and Probability

- 1.1 Application: Verifying Polynomial Identities
- 1.2 Axioms of Probability
- 1.3 Application: Verifying Matrix Multiplication
- 1.4 Application: A Randomized Min-Cut Algorithm
- 1.5 Exercises

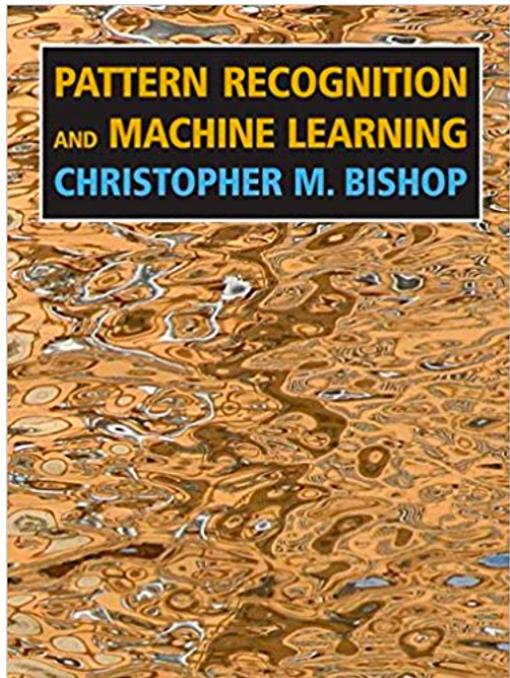
2 Discrete Random Variables and Expectation

- 2.1 Random Variables and Expectation
 - 2.1.1 Linearity of Expectations
 - 2.1.2 Jensen's Inequality
- 2.2 The Bernoulli and Binomial Random Variables
- 2.3 Conditional Expectation
- 2.4 The Geometric Distribution
 - 2.4.1 Example: Coupon Collector's Problem
- 2.5 Application: The Expected Run-Time of Quicksort
- 2.6 Exercises

3 Moments and Deviations

- 3.1 Markov's Inequality
- 3.2 Variance and Moments of a Random Variable
 - 3.2.1 Example: Variance of a Binomial Random Variable
- 3.3 Chebyshev's Inequality
 - 3.3.1 Example: Coupon Collector's Problem
- 3.4 Application: A Randomized Algorithm for Computing the
 - 3.4.1 The Algorithm
 - 3.4.2 Analysis of the Algorithm
- 3.5 Exercises

Textbook: Machine Learning



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xiv	CONTENTS	
2	Probability Distributions	67
2.1	Binary Variables	68
2.1.1	The beta distribution	71
2.2	Multinomial Variables	74
2.2.1	The Dirichlet distribution	76
2.3	The Gaussian Distribution	78
2.3.1	Conditional Gaussian distributions	85
2.3.2	Marginal Gaussian distributions	88
2.3.3	Bayes' theorem for Gaussian variables	90
2.3.4	Maximum likelihood for the Gaussian	93
2.3.5	Sequential estimation	94
2.3.6	Bayesian inference for the Gaussian	97
2.3.7	Student's t-distribution	102
2.3.8	Periodic variables	105
2.3.9	Mixtures of Gaussians	110
2.4	The Exponential Family	113
2.4.1	Maximum likelihood and sufficient statistics	116
2.4.2	Conjugate priors	117
2.4.3	Noninformative priors	117
2.5	Nonparametric Methods	120
2.5.1	Kernel density estimators	122
2.5.2	Nearest-neighbour methods	124
	Exercises	127
3	Linear Models for Regression	137
3.1	Linear Basis Function Models	138
3.1.1	Maximum likelihood and least squares	140
3.1.2	Geometry of least squares	143
3.1.3	Sequential learning	143
3.1.4	Regularized least squares	144
	Exercises	144

These days, every area in CS and EE is directly or indirectly related to machine learning!

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- We will take this exciting journey from the next lecture!

Questions?



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