

## Lecture 0: Introduction

Yi, Yung (이용)

EE210: Probability and Introductory Random Processes  
KAIST EE

August 31, 2021

- Course logistics
- Why this course?

August 31, 2021 1 / 21

August 31, 2021 2 / 21

Instructor

- Yi, Yung (이용)
- Office: N1, 810
- Homepage: <https://yung-web.github.io/home/>
- E-mail: [yiyung@kaist.edu](mailto:yiyung@kaist.edu)
- Computer Division
- In KAIST EE since 2008

- Basically, **Non-real-time** online class, but with a little bit of real-time zoom classes (if necessary)
- All lecture videos have already been pre-recorded. Now available in [YouTube](#).
  - No class attendance check
  - You can watch the videos **anytime** and **anywhere**.
  - You can adjust the speed of studying the materials according to your schedule, as long as you submit the homeworks until the announced deadlines and you show up at the right time for the exams.
- Sometimes, we will open **real-time zoom** classes (maybe biweekly)
  - To answer students' questions
  - To give important announcements
  - etc

August 31, 2021 3 / 21

August 31, 2021 4 / 21

- Method 1:  
<https://yung-web.github.io/home/courses/probability.html>
- Method 2: (a) Type **Yung Yi** in the google, (b) visit his [GitHub homepage](#), (c) find the links on [Course](#).

The screenshot shows a Google search results page for 'yung yi'. The top result is a GitHub profile for 'yung yi' which includes a photo, a brief bio, and sections for 'Education', 'Students advised (PhD)', 'Position', and 'Courses'. Below the GitHub link are other links related to Yung Yi's work at KAIST.

This is a detailed screenshot of the GitHub homepage for Yung Yi. It features a large photo of him, a brief bio, and sections for 'Education', 'Students advised (PhD)', 'Position', and 'Courses'. The 'Courses' section lists various probability-related courses he has taught or is teaching.

August 31, 2021 5 / 21

- Questions on the course contents
  - Upload them in the [Q&A board in KLMS](#).
  - Your question will also be very helpful to other classmates
  - Answers to your questions will be given [at the board](#) (if simple), or via separate [lecture videos](#) or [a real-time zoom session](#).

### Real-time Zoom Session

- Open when necessary and it will be recorded so as to be accessible throughout this semester.
- The zoom link information will be given in advance at the corresponding week.

### Personal Office Hours

- Will arrange by an [individual request](#).
- Send an email to [yiyung@kaist.edu](mailto:yiyung@kaist.edu) for appointment.
- Questions on the course and other any types of advices that may help you

August 31, 2021 6 / 21

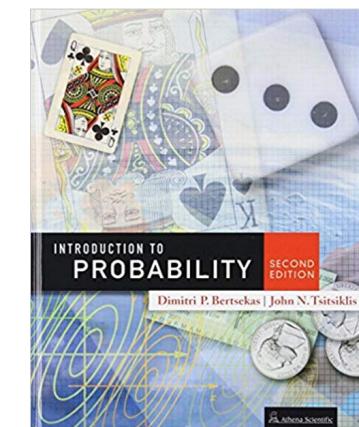
## Most Course-related Contents and Announcements

- Lecture videos and slides:  
<https://yung-web.github.io/home/courses/probability.html>
- All other contents: KLMS, <http://klms.kaist.ac.kr/>
  - To ask questions about everything
  - To check your score on each homework/exam
  - To see all the announcements about the class
  - ...
- Important: Please regularly check!**
  - Emails
  - SMS
  - KLMS announcements

August 31, 2021 7 / 21

## Textbook

- Introduction to Probability (2nd edition)
  - MIT course textbook
  - Dimitri P. Bertsekas and John N. Tsitsiklis
- You can order it from Yes24, Aladin, Kyobo
  - Yes24: <http://www.yes24.com/Product/Goods/3995311>
  - Aladin: <https://www.aladin.co.kr/shop/wproduct.aspx?ItemId=12945615>
  - Kyobo: <http://www.kyobobook.co.kr/product/detailViewEng.laf?ejkGb=ENG&mallGb=ENG&barcode=9781886529380&orderClick=LAG&Kc=>



August 31, 2021 8 / 21

1. Probabilistic model (1/2 week) (08/31)
2. Conditioning and Independence (1/2 week) (09/02)
3. Random Variable, Part I (Discrete Random Variable) (1.5 week) (09/07, 09/09, 09/14)  
**Exam 1 (09/23)**
4. Random Variable, Part II (Continuous Random Variable) (1.5 week) (09/16, 09/28, 09/30)
5. Random Variable, Part III (Advanced Topic on Random Variable) (1.5 week) (10/05, 10/07, 10/12)  
**Exam 2 (10/14)**
6. Limit of Scaled Sum of Random Variables: Central Limit Theorem and Weak Law of Large Numbers (1.5 week) (10/19, 10/21, 10/26)
7. Random Process: Bernoulli and Poisson Processes (2 week) (10/28, 11/2, 11/4, 11/9)  
**Exam 3 (11/11)**
8. Random Process: Markov Chain (2 week) (11/16, 11/18, 11/23, 11/25)
9. Introduction to Statistic Inference (2 week) (11/30, 12/2, 12/7, 12/9)  
**Exam 4 (12/16)**

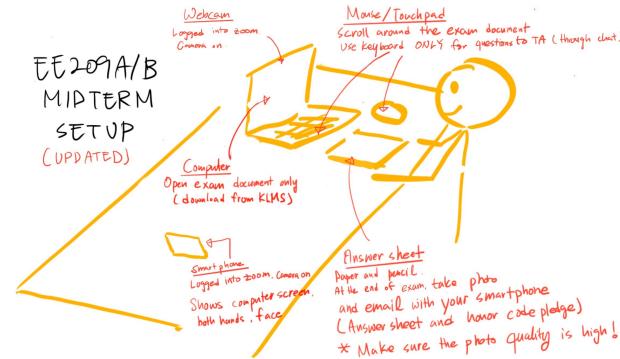
- On-line lectures at MIT and EdX
  - MIT: <http://bit.ly/2PkVYdr>
  - EdX: <http://bit.ly/3pHmZRd>
  - You can find the urls (2006, 2010, 2013) for the MIT lectures based on the same textbook, where there are many useful resources (recitation problems, homework problems, old exam problems, etc)
  - Some of my lecture slides are based on theirs, but my slides are largely modified/reorganized/edited in many places for our purpose.

## More about Exams and Homeworks

- 4 Exams
- Schedule Again
  - Exam 1: **09/23**, 7:00PM - 10:00PM
  - Exam 2: **10/14**, 7:00PM - 10:00PM
  - Exam 3: **11/11**, 7:00PM - 10:00PM
  - Exam 4: **12/16**, 9:00AM - 12:00PM
  - Check your calendar and empty your schedule at these times.
- 9 Homeworks for each of 9 chapters.
  - Will take just about **2-3 hours** if you understand the course contents very well.
  - Try to finish as soon as possible soon after you watch the lecture videos.

- Homeworks: total 37%, 4 points for each of the first 8 homeworks and 5 points for the last homework
- Exams: total 63%,  $63/3 = 21$  points for each exam
  - You can exclude one exam that you don't want to be graded
- Letter or Pass/Fail
  - You can decide until the date of Exam 2, no extension
  - **Minimum requirement for Pass:** C+ or higher
  - **Minimum requirement for C+ or higher**
    - ▶ Take all the exams, and the homework score  $\geq 18$
- Cheating
  - **Zero tolerance** (very serious)
  - Honor code from EE department

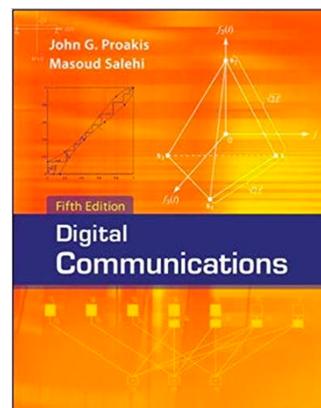
- Will announce more details when you take exams
- Earlier experience from the EE209 class



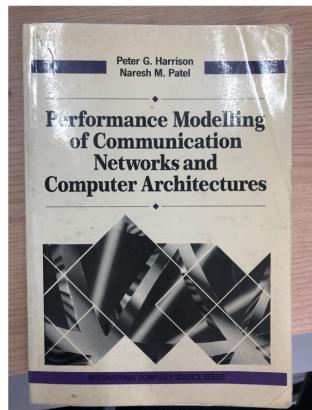
## Questions?

## Why Probability?

- Many things are "probabilistic"
- Assume that you are a designer of the following engineering systems. Good design?
  - a web server
  - a communication device like mobile phones
  - an AI-based image classifier
- From an engineering point of view,
  - System input
  - Algorithms in systems
  - Analysis of systems



Communications	13
1-1 Overview of the Book	16
1-6 Bibliographical Notes and References	16
<b>2 Probability and Stochastic Processes</b>	17
2-1 Probability	17
2-1-1 Random Variables, Probability Distributions, and Probability Densities	22
2-1-2 Functions of Random Variables	28
2-1-3 Statistical Averages of Random Variables	33
2-1-4 Some Useful Probability Distributions	37
2-1-5 Upper Bounds on the Tail Probability	53
2-1-6 Sums of Random Variables and the Central Limit Theorem	58
2-2 Stochastic Processes	62
2-2-1 Statistical Averages	64
2-2-2 Power Density Spectrum	67
2-2-3 Response of a Linear Time-Invariant System to a Random Input Signal	68
2-2-4 Sampling Theorem for Band-Limited Stochastic Processes	72
2-2-5 Discrete-Time Stochastic Signals and Systems	74
2-2-6 Cyclostationary Processes	75
2-3 Bibliographical Notes and References	77
Problems	77



Preface

**Chapter 1 Essentials of Probability Theory**

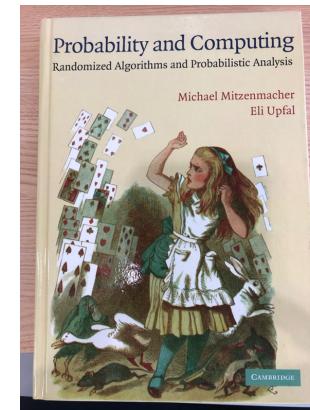
- 1.1 Sample space, events and probability
- 1.2 Conditional probability
- 1.3 Independence
- Exercises

**Chapter 2 Random Variables and Distributions**

- 2.1 Probability distribution functions
- 2.2 Discrete random variables
- 2.3 Continuous random variables
- 2.4 Joint random variables
- 2.5 Conditional distributions
- 2.6 Independence and sums
- Exercises

**Chapter 3 Expected Values and Moments**

- 3.1 Expectation
- 3.2 Generating functions and transforms
- 3.3 Asymptotic properties
- Exercises



Preface

**1 Events and Probability**

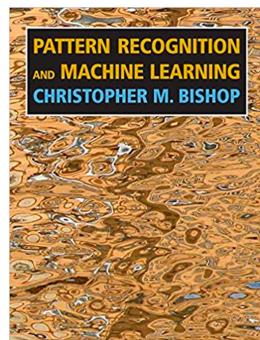
- 1.1 Application: Verifying Polynomial Identities
- 1.2 Axioms of Probability
- 1.3 Application: Verifying Matrix Multiplication
- 1.4 Application: A Randomized Min-Cut Algorithm
- 1.5 Exercises

**2 Discrete Random Variables and Expectation**

- 2.1 Random Variables and Expectation
- 2.1.1 Linearity of Expectations
- 2.1.2 Jensen's Inequality
- 2.2 The Bernoulli and Binomial Random Variables
- 2.3 Conditional Expectation
- 2.4 The Geometric Distribution
- 2.4.1 Example: Coupon Collector's Problem
- 2.5 Application: The Expected Run-Time of Quicksort
- 2.6 Exercises

**3 Moments and Deviations**

- 3.1 Markov's Inequality
- 3.2 Variance and Moments of a Random Variable
- 3.2.1 Example: Variance of a Binomial Random Variable
- 3.3 Chebychev's Inequality
- 3.3.1 Example: Coupon Collector's Problem
- 3.4 Application: A Randomized Algorithm for Computing the
- 3.4.1 The Algorithm
- 3.4.2 Analysis of the Algorithm
- 3.5 Exercises



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xiv CONTENTS

Copyrighted Material	xiii
2 Probability Distributions	67
2.1 Binary Variables	68
2.1.1 The beta distribution	71
2.2 Multinomial Variables	74
2.2.1 The Dirichlet distribution	76
2.3 The Normal Distribution	78
2.3.1 Conditional Gaussian distributions	85
2.3.2 Marginal Gaussian distributions	88
2.3.3 Bayes' theorem for Gaussian variables	90
2.3.4 Maximum likelihood for the Gaussian	93
2.3.5 Sequential estimator	94
2.3.6 Bayesian inference for the Gaussian	97
2.3.7 Student's t-distribution	102
2.3.8 Periodic distributions	105
2.3.9 Mixture of Gaussians	110
2.4 The Exponential Family	113
2.4.1 Maximum likelihood and sufficient statistics	116
2.4.2 Conjugate priors	117
2.4.3 Noninformative priors	117
2.5 Nonparametric Methods	120
2.5.1 Kernel density estimators	122
2.5.2 Nearest-neighbour methods	124
Exercises	127
3 Linear Models for Regression	137
3.1 Linear Function Models	138
3.1.1 Maximum likelihood and least squares	140
3.1.2 Geometry of least squares	143
3.1.3 Sequential learning	143
3.1.4 Regularized least squares	144

- Designer's perspective?
- In the year of 2021, suppose that unfortunately there is no theory of mathematically studying the *uncertainty* of some phenomena, events, etc.
- You have to design such a theory called "probability". How are you going to do it? Where are you going to start?
- You just have other basic mathematical theories such as set theory.
- You need to get used to the *English terms* on probability (e.g., sample space = 표본공간, probability density function = 확률밀도함수).
- We will take this exciting journey from the next lecture!

These days, every area in CS and EE is directly or indirectly related to machine learning!

Questions?

