

# Lecture 0: Introduction

Yi, Yung (이웅)

EE210: Probability and Introductory Random Processes  
KAIST EE

August 25, 2021

- Course logistics
- Why this course?

- Yi, Yung (이웅)
- Office: N1, 810
- Homepage: <https://yung-web.github.io/home/>
- E-mail: [yiyung@kaist.edu](mailto:yiyung@kaist.edu)
- Computer Division
- In KAIST EE since 2008

- Basically, **Non-real-time** online class, but with a little bit of real-time zoom classes (if necessary)
- All lecture videos have already been pre-recorded. Now available in [YouTube](#).
  - No class attendance check
  - You can watch the videos **anytime** and **anywhere**.
  - You can adjust the speed of studying the materials according to your schedule, as long as you submit the homeworks until the announced deadlines and you show up at the right time for the exams.
- Sometimes, we will open **real-time** zoom classes (maybe biweekly)
  - To answer students' questions
  - To give important announcements
  - etc

# Watching Lecture Videos



- Method 1:

<https://yung-web.github.io/home/courses/probability.html>

- Method 2: (a) Type Yung Yi in the google, (b) visit his GitHub homepage, (c) find the links on Course.

Google search results for 'yung yi':

- Yung Yi - Google Scholar
- Yung Yi - KAIST ELECTRICAL ENGINEERING
- Yung Yi - Home - ACM Digital Library

Yung Yi's GitHub profile page:

**Short Bio:** Yung Yi received his B.S. and the M.S. in the School of Computer Science and Engineering from Seoul National U. in 1997 and 1999, respectively, and his Ph.D. in the Department of Electrical and Computer Engineering at the University o in 2006. From 2006 to 2008, he was a post-doctoral research associate in the Department of Electrical Engineering at Prince he is a KAIST Chair Full professor at the Department of Electrical Engineering at KAIST, South Korea. His current research i applied machine learning, design and analysis of wired/wireless networking systems. He was the recipient of two best paper SECON 2013 and ACM MobiHoc 2013. He was the co-recipient of IEEE William R. Bennett Award, 2016.

**LANADA (LAboratory of Network Architecture, Design, and Analysis)**  
LANADA is a research group which I currently lead. Currently, we do not hire new graduate students.

**Education**

- Ph.D: Dept. of Electrical and Computer Engineering, University of Texas at Austin, 2006
- M.S: Dept. of Computer Science and Engineering, Seoul National University, 1999
- B.S: Dept. of Computer Science and Engineering, Seoul National University, 1997

**Position**

- KAIST Chair Professor (KAIST 지정석교수): Dept. of Electrical Engineering, KAIST, 2021 - Current
- Full Professor: Dept. of Electrical Engineering, KAIST, 2018.2 - Current
- Associate Professor: Dept. of Electrical Engineering, KAIST, 2011.8 - 2018.2
- Assistant Professor: Dept. of Electrical Engineering, KAIST, 2008.8 - 2011.8
- Postdoctoral Research Associate: Dept. of Electrical Engineering Princeton University, 2006.8 - 2008.0

**Courses**

- Probability and Introductory Random Process (video included), Undergraduate
- Data Structures for Electrical Engineers, Undergraduate
- Mathematics for Machine Learning, Undergraduate
- Computer Network, Undergraduate
- Complex Network Analysis: Epidemics and Rumours (video included), Graduate

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Phone: +82 42  
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edu  
Office Hours

**Memor.**

2020 Spring

- Questions on the course contents
  - Upload them in the **Q&A board in KLMS**.
  - Your question will also be very helpful to other classmates
  - Answers to your questions will be given **at the board** (if simple), or via separate **lecture videos** or **a real-time zoom session**.
- Real-time Zoom Session
  - Open when necessary and it will be recorded so as to be accessible throughout this semester.
  - The zoom link information will be given in advance at the corresponding week.
- Personal Office Hours
  - Will arrange by an **individual request**.
  - Send an email to **yiyung@kaist.edu** for appointment.
  - Questions on the course and other any types of advices that may help you

- Lecture videos and slides:

<https://yung-web.github.io/home/courses/probability.html>

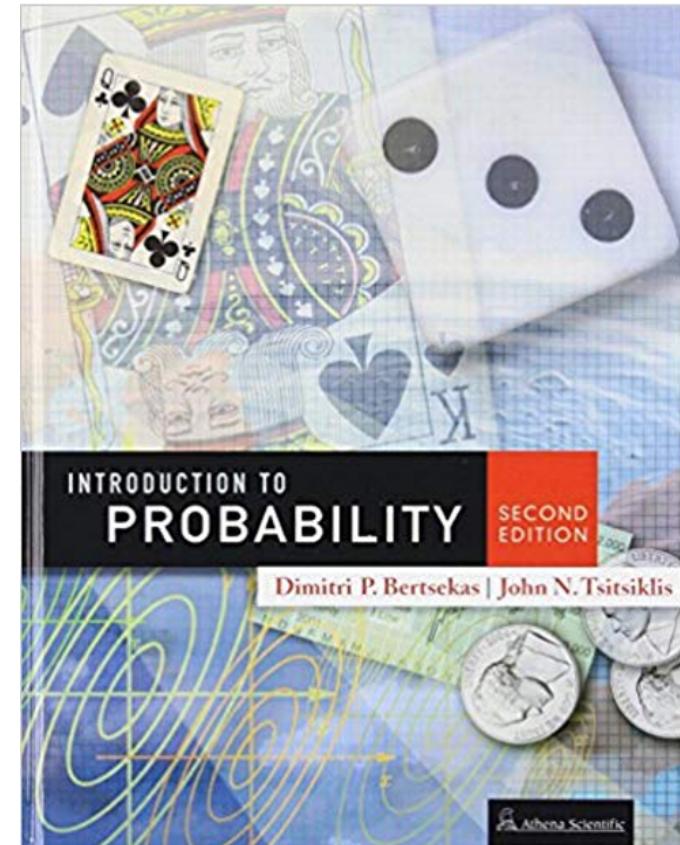
- All other contents: KLMS, <http://klms.kaist.ac.kr/>

- To ask questions about everything
- To check your score on each homework/exam
- To see all the announcements about the class
- ...

- **Important: Please regularly check!**

- Emails
- SMS
- KLMS announcements

- Introduction to Probability  
(2nd edition)
  - MIT course textbook
  - Dimitri P. Bertsekas and John N. Tsitsiklis
- You can order it from Yes24, Aladin, Kyobo
  - Yes24: <http://www.yes24.com/Product/Goods/3995311>
  - Aladin: <https://www.aladin.co.kr/shop/wproduct.aspx?ItemId=12945615>
  - Kyobo: <http://www.kyobobook.co.kr/product/detailViewEng.laf?ejkGb=ENG&mallGb=ENG&barcode=9781886529380&orderClick=LAG&Kc=>



1. Probabilistic model (1/2 week) (08/31)
2. Conditioning and Independence (1/2 week) (09/02)
3. Random Variable, Part I (Discrete Random Variable) (1.5 week) (09/07, 09/09, 09/14)  
**Exam 1 (09/16)** \_\_\_\_\_
4. Random Variable, Part II (Continuous Random Variable) (1.5 week) (09/23, 09/28, 09/30)
5. Random Variable, Part III (Advanced Topic on Random Variable) (1.5 week) (10/05, 10/07, 10/12)  
**Exam 2 (10/14)** \_\_\_\_\_
6. Limit of Scaled Sum of Random Variables: Central Limit Theorem and Weak Law of Large Numbers (1.5 week) (10/19, 10/21, 10/26)
7. Random Process: Bernoulli and Poisson Processes (2 week) (10/28, 11/2, 11/4, 11/9)  
**Exam 3 (11/11)** \_\_\_\_\_
8. Random Process: Markov Chain (2 week) (11/16, 11/18, 11/23, 11/25)
9. Introduction to Statistic Inference (2 week) (11/30, 12/2, 12/7, 12/9)  
**Exam 4 (12/14)** \_\_\_\_\_

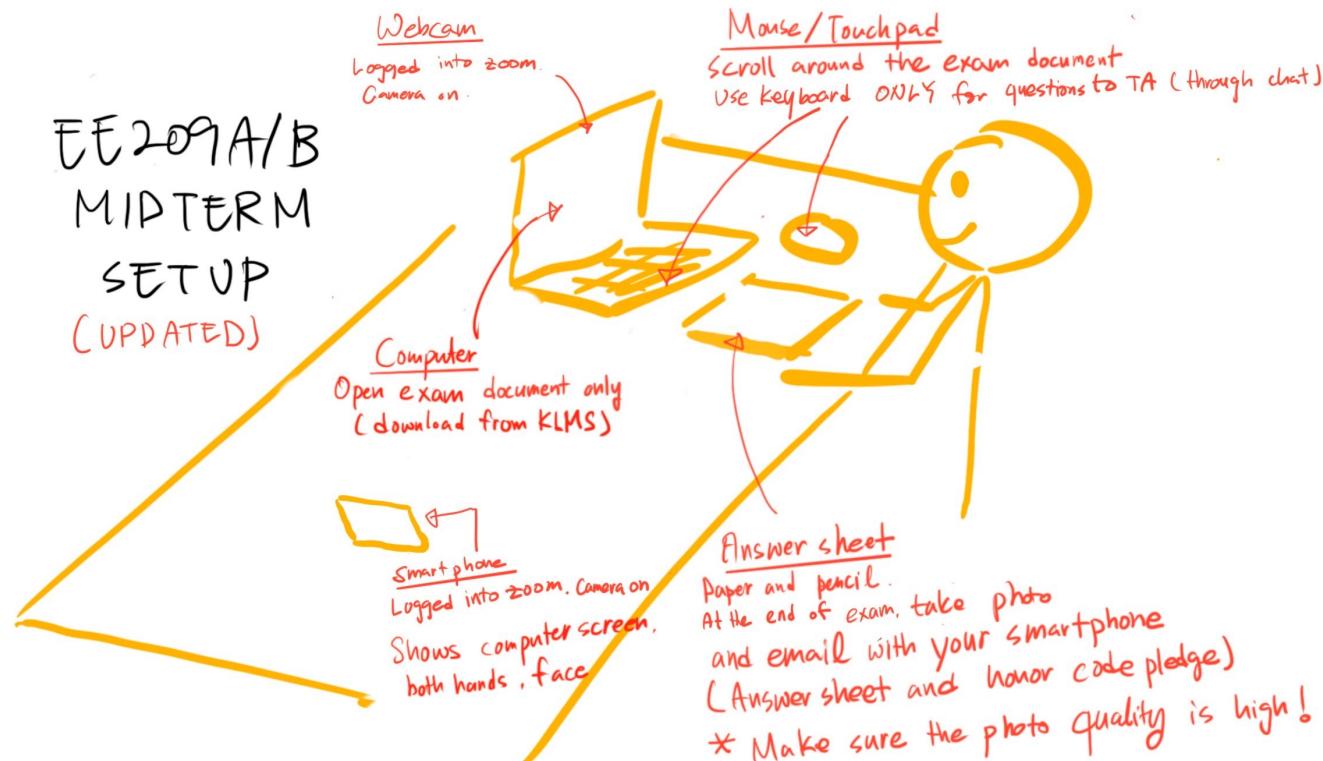
- On-line lectures at MIT and EdX
  - MIT: <http://bit.ly/2PkvYdr>
  - EdX: <http://bit.ly/3pHmZRd>
  - You can find the urls (2006, 2010, 2013) for the MIT lectures based on the same textbook, where there are many useful resources (recitation problems, homework problems, old exam problems, etc)
  - Some of my lecture slides are based on theirs, but my slides are largely modified/reorganized/edited in many places for our purpose.

- 4 Exams
- Schedule Again
  - Exam 1: 09/16, 7:00PM - 10:00PM
  - Exam 2: 10/14, 7:00PM - 10:00PM
  - Exam 3: 11/11, 7:00PM - 10:00PM
  - Exam 4: 12/14, 7:00PM - 10:00PM
  - Check your calendar and empty your schedule at these times.
- 9 Homeworks for each of 9 chapters.
  - Will take just about 2-3 hours if you understand the course contents very well.
  - Try to finish as soon as possible soon after you watch the lecture videos.

- Homeworks: total 36%,  $36/9 = 4$  points for each homework
- Exams: total 64%,  $64/4 = 16$  points for each exam
- Letter or Pass/Fail
  - You can decide until the date of Exam 2
  - No extension
- Cheating
  - **Zero tolerance** (very serious)
  - Honor code from EE department

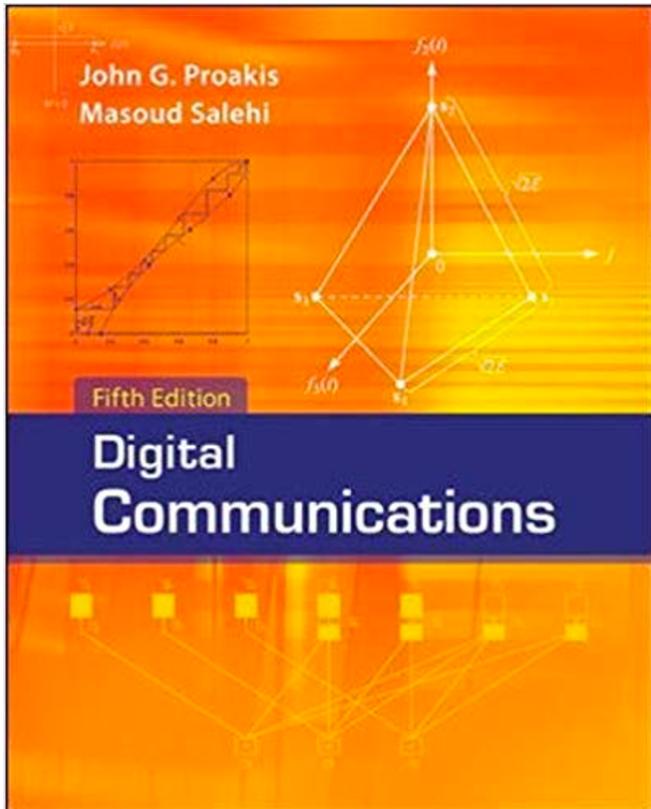
# How to Take Exam

- Will announce more details when you take exams
- Earlier experience from the EE209 class

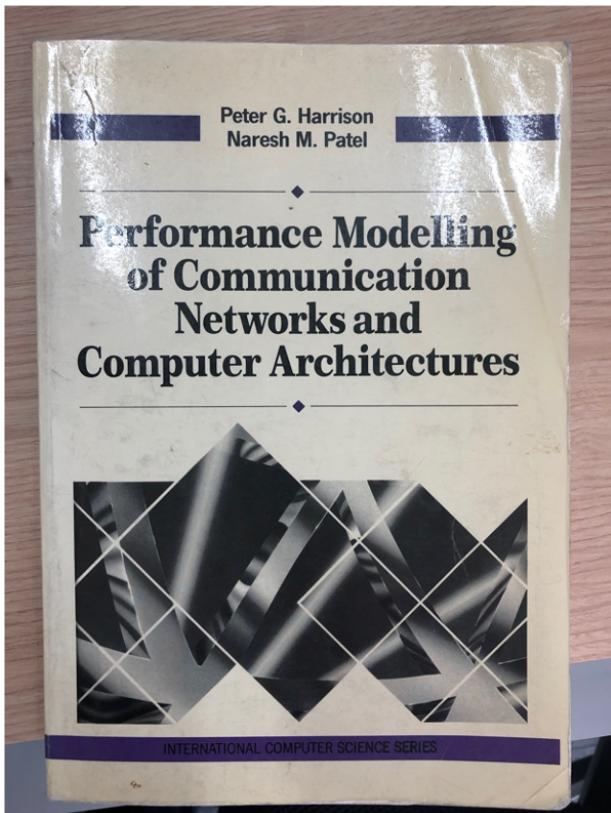


Questions?

- Many things are "probabilistic"
- Assume that you are a designer of the following engineering systems. Good design?
  - a web server
  - a communication device like mobile phones
  - an AI-based image classifier
- From an engineering point of view,
  - System input
  - Algorithms in systems
  - Analysis of systems



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Preface

**Chapter 1 Essentials of Probability Theory**

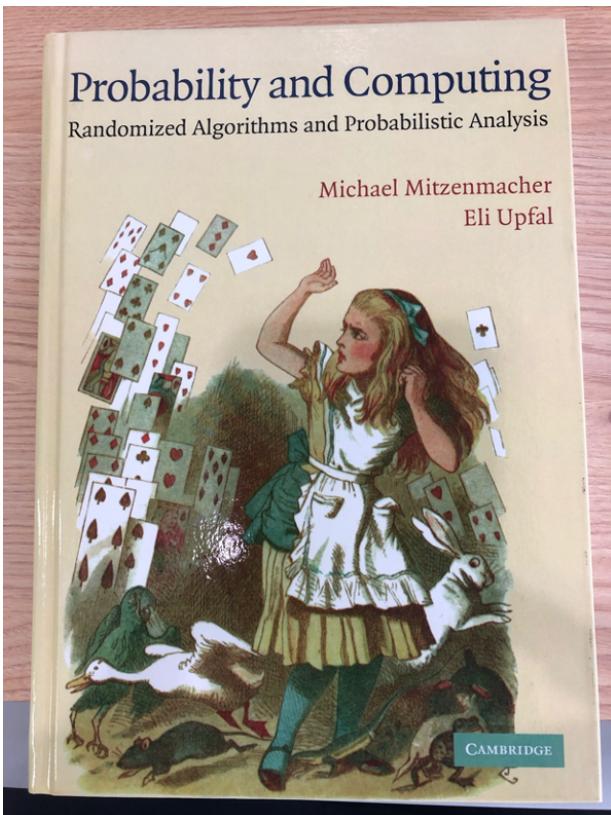
- 1.1 Sample space, events and probability
- 1.2 Conditional probability
- 1.3 Independence
- Exercises

**Chapter 2 Random Variables and Distributions**

- 2.1 Probability distribution functions
- 2.2 Discrete random variables
- 2.3 Continuous random variables
- 2.4 Joint random variables
- 2.5 Conditional distributions
- 2.6 Independence and sums
- Exercises

**Chapter 3 Expected Values and Moments**

- 3.1 Expectation
- 3.2 Generating functions and transforms
- 3.3 Asymptotic properties
- Exercises



*Preface*

**1 Events and Probability**

- 1.1 Application: Verifying Polynomial Identities
- 1.2 Axioms of Probability
- 1.3 Application: Verifying Matrix Multiplication
- 1.4 Application: A Randomized Min-Cut Algorithm
- 1.5 Exercises

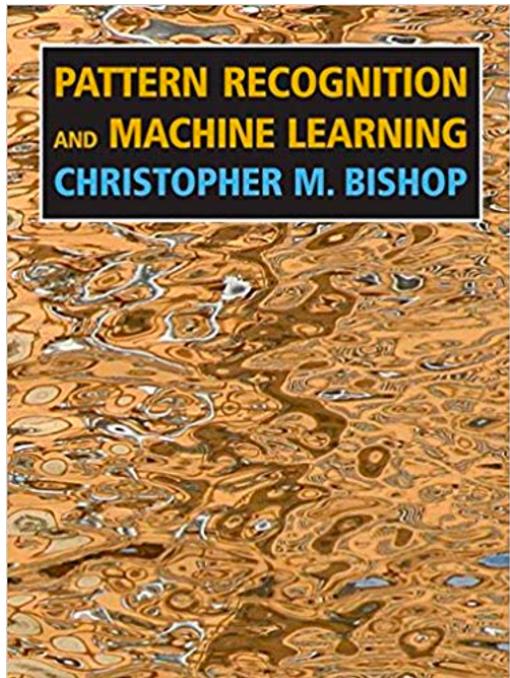
**2 Discrete Random Variables and Expectation**

- 2.1 Random Variables and Expectation
  - 2.1.1 Linearity of Expectations
  - 2.1.2 Jensen's Inequality
- 2.2 The Bernoulli and Binomial Random Variables
- 2.3 Conditional Expectation
- 2.4 The Geometric Distribution
  - 2.4.1 Example: Coupon Collector's Problem
- 2.5 Application: The Expected Run-Time of Quicksort
- 2.6 Exercises

**3 Moments and Deviations**

- 3.1 Markov's Inequality
- 3.2 Variance and Moments of a Random Variable
  - 3.2.1 Example: Variance of a Binomial Random Variable
- 3.3 Chebyshev's Inequality
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- 3.4 Application: A Randomized Algorithm for Computing the
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- 3.5 Exercises

# Textbook: Machine Learning



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These days, every area in CS and EE is directly or indirectly related to machine learning!

- Designer's perspective?
- In the year of 2021, suppose that unfortunately there is no theory of mathematically studying the *uncertainty* of some phenomena, events, etc.
- You have to design such a theory called "probability". How are you going to do it?  
Where are you going to start?
- You just have other basic mathematical theories such as set theory.
- You need to get used to the *English terms* on probability (e.g., sample space = 표본공간, probability density function = 확률밀도함수).
- We will take this exciting journey from the next lecture!

Questions?

