Lecture 0: Introduction

Yi, Yung (이융)

EE210: Probability and Introductory Random Processes
KAIST EE

MONTH DAY, 2021

Outline

- Course logistics
- Why necessary to take the course of probability and random process?

Instructor

- Yi, Yung (이융)
- Office: N1, 810
- `http://lanada.kaist.ac.kr`, `yiyung@kaist.edu`
- Computer Division
- A professor in KAIST EE since 2008
- Office hours: TBA

TAs

- A
- B
- C

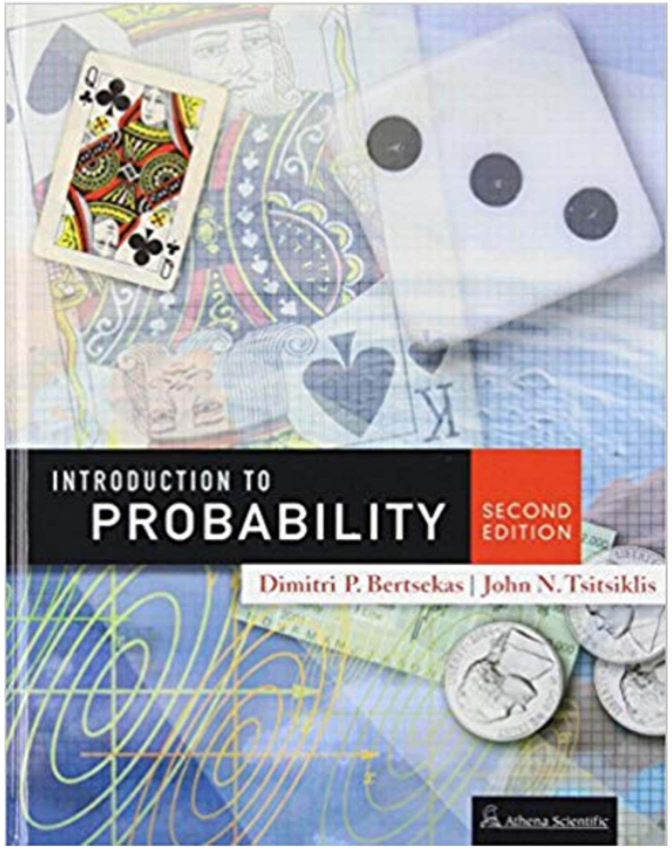
- Mailing list: `ee210@lanada.kaist.ac.kr`
  - Please use KLMS for the questions about the lecture contents
  - This mailing list can be used for individual issues

Course Homepage

- `http://klms.kaist.ac.kr/`
- To download course materials
- To ask questions about everything
- To check your score on each homework/exam
- To see all the announcements about the class

Textbook

- Introduction to Probability (2nd edition)
  - MIT course textbook
  - Dimitri P. Bertsekas and John N. Tsitsiklis



Contents and Web Resources

- Three Parts
  - Part I: Fundamentals of Probability
  - Part II: Inference and Limit Theorems
  - Part III: Random Processes
- On-line lectures at MIT and EdX
  - MIT: `http://bit.ly/2PkvYdr`
  - EdX: `http://bit.ly/3pHmZRd`
  - You can find older urls (2006, 2010, 2013) for this lecture, where there are many useful resources (recitation problems, homework problems, old exam problems, etc)
  - My lecture slides: based on theirs, but largely modified/reorganized/edited in many places for KAIST students

Grading

KAIST EE

- In-class quiz (sometimes)
- Basically, weekly homework, but often bi-weekly
- 3 Exams (2 mid-terms and 1 final)

- Class participation
- Grading portions: A (X%), B (Y%), C(Z%), D(W%), F ...
- Online lectures due to COVID-19 may change how to grade.

How to Communicate

- Most should be via KLMS
  - Technical questions about lectures, homework, and etc
- Please DO NOT individually send emails to Prof. Yung Yi and TAs (or making calls or sending KakaoTalk msgs) about the technical questions (course contents, homework, etc)
  - All the questions need to be shared among the students.
  - TAs and Prof. Yung Yi will handle your questions as soon as possible.
  - But, you can send an email to Prof. Yung Yi for the things that need to be individually discussed.

Questions?

Why Probability?

- Many things are ”probabilistic”

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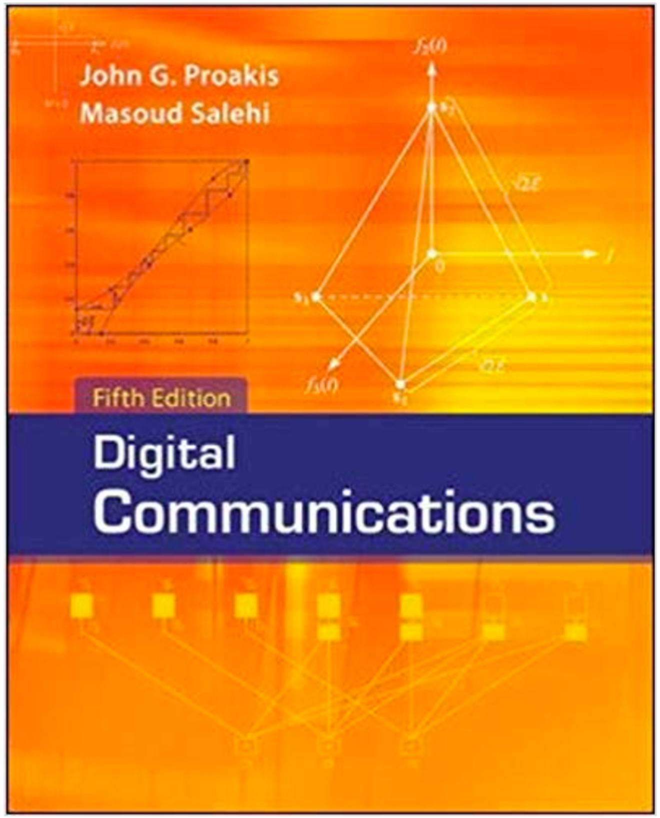
- Assume that you are a designer of the following engineering systems. Good design?
  - a web server
  - a communication device like mobile phones
  - an AI-based image classifier

Why Probability?

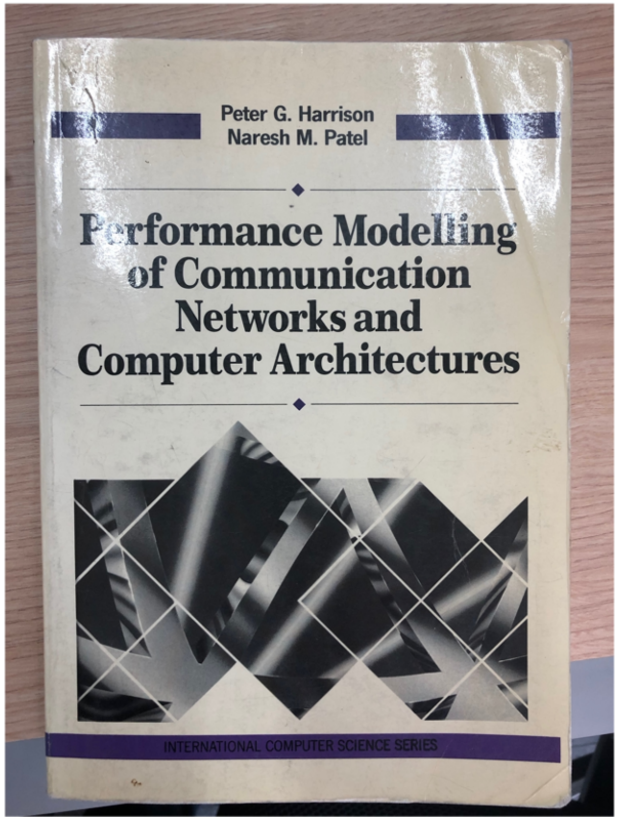
- Many things are "probabilistic"
- Assume that you are a designer of the following engineering systems. Good design?
  - a web server
  - a communication device like mobile phones
  - an AI-based image classifier
- From an engineering point of view,
  - System input
  - Algorithms in systems
  - Analysis of systems

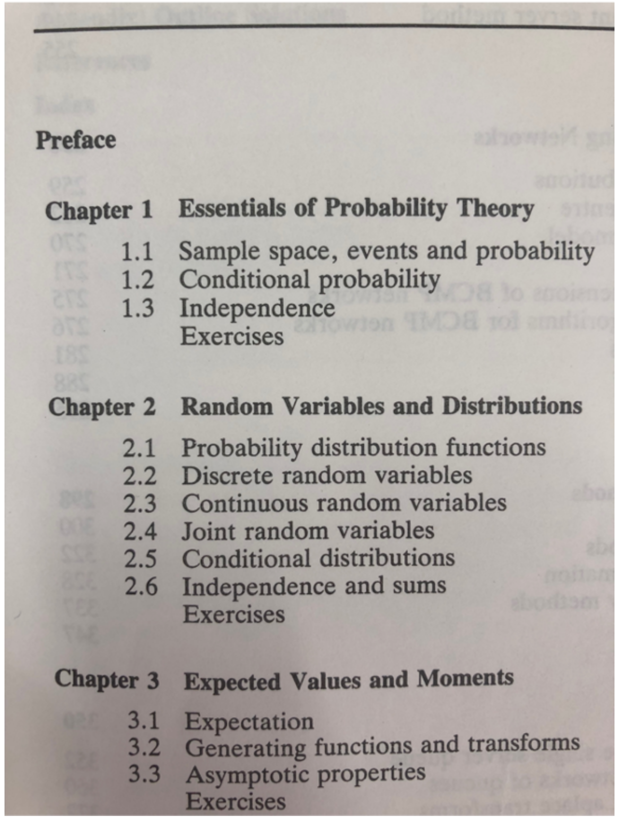
Textbook: Digital Communication



	Communications	13
1-5	Overview of the Book	16
1-6	Bibliographical Notes and References	16
2	**Probability and Stochastic Processes**	17
2-1	Probability	17
2-1-1	Random Variables, Probability Distributions, and Probability Densities	22
2-1-2	Functions of Random Variables	28
2-1-3	Statistical Averages of Random Variables	33
2-1-4	Some Useful Probability Distributions	37
2-1-5	Upper bounds on the Tail Probability	53
2-1-6	Sums of Random Variables and the Central Limit Theorem	58
2-2	Stochastic Processes	62
2-2-1	Statistical Averages	64
2-2-2	Power Density Spectrum	67
2-2-3	Response of a Linear Time-Invariant System to a Random Input Signal	68
2-2-4	Sampling Theorem for Band-Limited Stochastic Processes	72
2-2-5	Discrete-Time Stochastic Signals and Systems	74
2-2-6	Cyclostationary Processes	75
2-3	Bibliographical Notes and References	77
	Problems	77

Textbook: Computer Networking





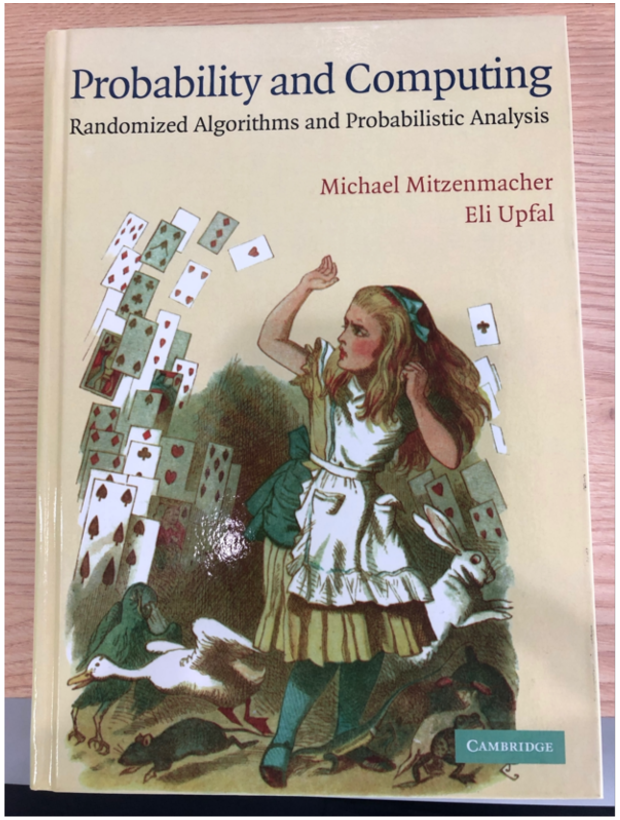
Preface

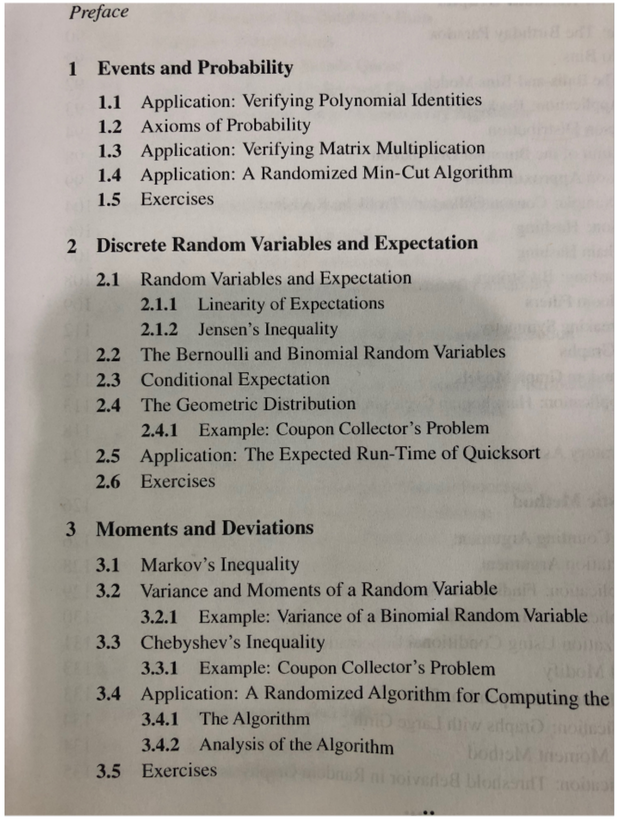
Chapter 1 Essentials of Probability Theory
1.1 Sample space, events and probability
1.2 Conditional probability
1.3 Independence
Exercises

Chapter 2 Random Variables and Distributions
2.1 Probability distribution functions
2.2 Discrete random variables
2.3 Continuous random variables
2.4 Joint random variables
2.5 Conditional distributions
2.6 Independence and sums
Exercises

Chapter 3 Expected Values and Moments
3.1 Expectation
3.2 Generating functions and transforms
3.3 Asymptotic properties
Exercises

Textbook: Algorithm and Computing




Preface

1 Events and Probability

1.1 Application: Verifying Polynomial Identities
1.2 Axioms of Probability
1.3 Application: Verifying Matrix Multiplication
1.4 Application: A Randomized Min-Cut Algorithm
1.5 Exercises

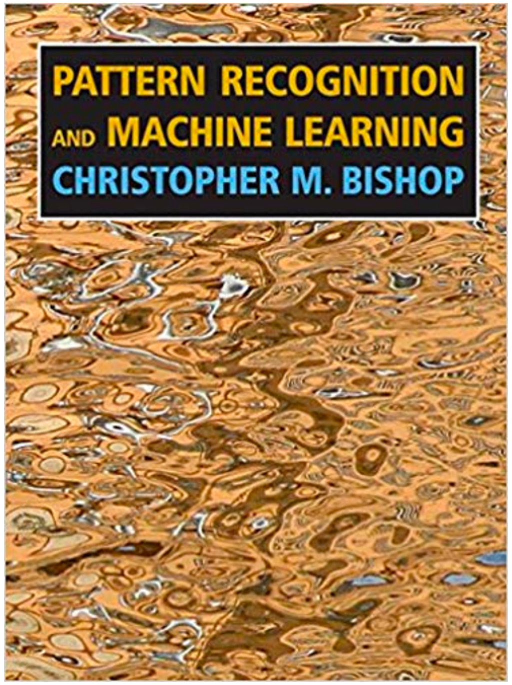
2 Discrete Random Variables and Expectation

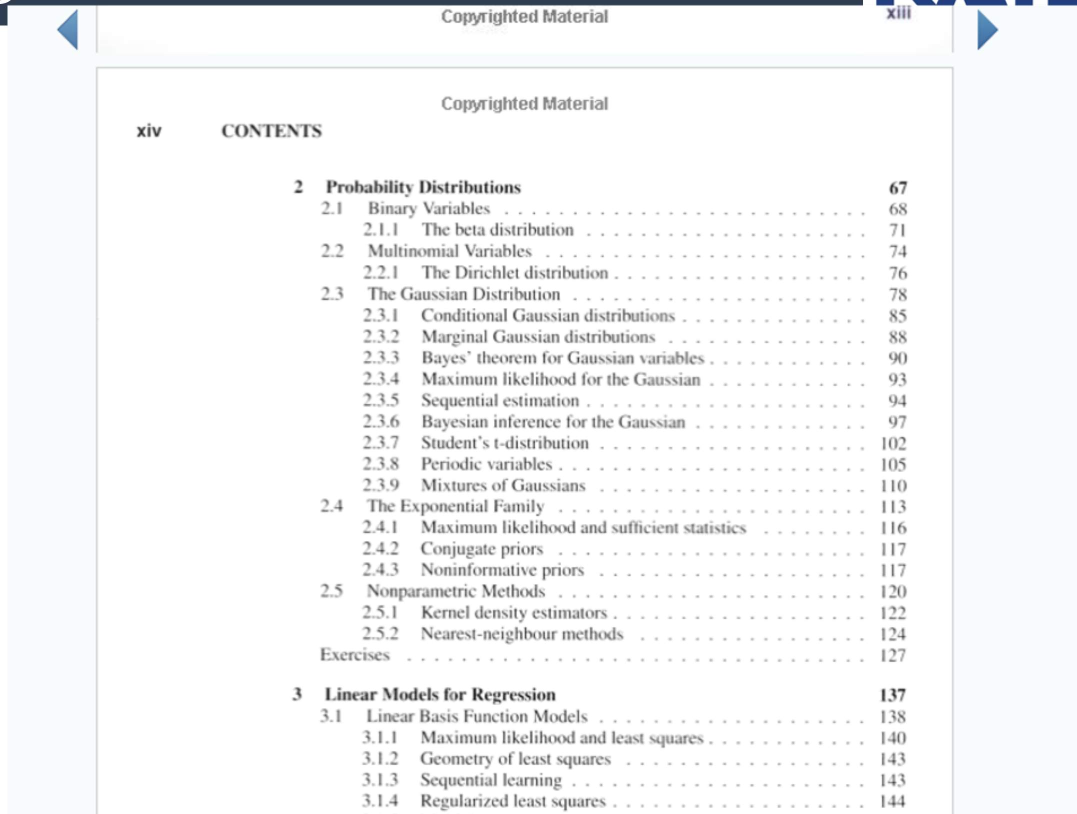
2.1 Random Variables and Expectation
2.1.1 Linearity of Expectations
2.1.2 Jensen's Inequality
2.2 The Bernoulli and Binomial Random Variables
2.3 Conditional Expectation
2.4 The Geometric Distribution
2.4.1 Example: Coupon Collector's Problem
2.5 Application: The Expected Run-Time of Quicksort
2.6 Exercises

3 Moments and Deviations

3.1 Markov's Inequality
3.2 Variance and Moments of a Random Variable
3.2.1 Example: Variance of a Binomial Random Variable
3.3 Chebyshev's Inequality
3.3.1 Example: Coupon Collector's Problem
3.4 Application: A Randomized Algorithm for Computing the
3.4.1 The Algorithm
3.4.2 Analysis of the Algorithm
3.5 Exercises

Textbook: Machine Learning





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2 Probability Distributions 67
2.1 Binary Variables . . . 68
2.1.1 The beta distribution . . . 71
2.2 Multinomial Variables . . . 74
2.2.1 The Dirichlet distribution . . . 76
2.3 The Gaussian Distribution . . . 78
2.3.1 Conditional Gaussian distributions . . . 85
2.3.2 Marginal Gaussian distributions . . . 88
2.3.3 Bayes' theorem for Gaussian variables . . . 90
2.3.4 Maximum likelihood for the Gaussian . . . 93
2.3.5 Sequential estimation . . . 94
2.3.6 Bayesian inference for the Gaussian . . . 97
2.3.7 Student's t-distribution . . . 102
2.3.8 Periodic variables . . . 105
2.3.9 Mixtures of Gaussians . . . 110
2.4 The Exponential Family . . . 113
2.4.1 Maximum likelihood and sufficient statistics . . . 116
2.4.2 Conjugate priors . . . 117
2.4.3 Noninformative priors . . . 117
2.5 Nonparametric Methods . . . 120
2.5.1 Kernel density estimators . . . 122
2.5.2 Nearest-neighbour methods . . . 124
Exercises . . . 127

3 Linear Models for Regression 137
3.1 Linear Basis Function Models . . . 138
3.1.1 Maximum likelihood and least squares . . . 140
3.1.2 Geometry of least squares . . . 143
3.1.3 Sequential learning . . . 143
3.1.4 Regularized least squares . . . 144

These days, every area in CS and EE is directly or indirectly related to machine learning!

How to take this course? A designer's perspective

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- You need to get used to the English terms on probability (e.g., sample space = 표본공간, probability density function = 확률밀도함수).
- We will take this exciting journey from the next lecture!

Questions?