

# Lecture 0: Introduction

Yi, Yung (이용)

EE210: Probability and Introductory Random Processes  
KAIST EE

August 22, 2022

- Course logistics
- Why this course?

- Yi, Yung (이융)
- Office: N1, 810
- Homepage: <https://yung-web.github.io/home/>
- E-mail: [yiyung@kaist.edu](mailto:yiyung@kaist.edu)
- Computer Division
- In KAIST EE since 2008

- non-real-time online ( $\leq 50\%$ ) + real-time offline/online ( $\geq 50\%$ )

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- realtime offline/online: Watch lecture videos in the classroom or in the zoom, with asking and answering questions.
- No attendance check!

- Method 1:

<https://yung-web.github.io/home/courses/probability.html>

# Accessing Lecture Videos and Slides



- Method 1:

<https://yung-web.github.io/home/courses/probability.html>

- Method 2: (a) Type **Yung Yi** in the google, (b) visit his [GitHub homepage](#), (c) find the links on [Course](#).

Google search results for "yung yi":

- Search term: yung yi
- Number of results: 약 20,600,000개 (0.62초)
- Language: 한국어 검색결과만 검색합니다. 환경설정에서 검색 언어를 지정할 수 있습니다.
- Links:
  - <https://scholar.google.com/citations> - Yung Yi - Google Scholar
  - <https://ee.kaist.ac.kr/node/> - Yung Yi - KAIST ELECTRICAL ENGINEERING
  - <https://yung-web.github.io/> - Yung Yi, KAIST - GitHub Pages
  - [https://openreview.net/profile?id=Yung\\_Yi](https://openreview.net/profile?id=Yung_Yi) - Yung Yi | OpenReview
  - <https://dl.acm.org/profile> - Yung Yi - Home - ACM Digital Library



**Short Bio:** Yung Yi received his B.S. and the M.S. in the School of Computer Science and Engineering from Seoul National University in 1997 and 1999, respectively, and his Ph.D. in the Department of Electrical and Computer Engineering at the University of Texas at Austin in 2006. From 2006 to 2008, he was a post-doctoral research associate in the Department of Electrical Engineering at Princeton University. He is currently a KAIST Chair Professor at the Department of Electrical Engineering at KAIST, South Korea. His current research interests include applied machine learning, design and analysis of wired/wireless networking systems. He was the recipient of two best paper awards at SECON 2013 and ACM MobiHoc 2013. He was the co-recipient of IEEE William R. Bennett Award, 2016.

## LANADA (Laboratory of Network Architecture, Design, and Analysis)

LANADA is a research group which I currently lead. Currently, we do not hire new graduate students.

## Students advised (PhD)

- Jinsung Lee, 2012, Postdoc at U. of Colorado
- Jae Sung Jeong, 2014, Sony Ericsson, Sweden
- Joonhyun Lee, 2014, Hanyang Univ.
- Hyejin Park, 2015, National Security Research Inst.
- Donggyu Yun, 2016, Naver
- Sodhwan Lee, 2016, ETRI
- Jhyeong Lee, 2016, SK Telecom
- Hyeyoung Jang, 2016, Dongguk Univ.
- Jungsul Ok, 2017, POSTECH
- Hyeojeong Lee, 2017, Samsung

## Education

- Ph.D.: Dept. of Electrical and Computer Engineering, University of Texas at Austin, 2006
- M.S.: Dept. of Computer Science and Engineering, Seoul National University, 1999
- B.S.: Dept. of Computer Science and Engineering, Seoul National University, 1997

## Position

- KAIST Chair Professor [KAIST 지정학과 교수]: Dept. of Electrical Engineering, KAIST, 2021 - Current
- Full Professor: Dept. of Electrical Engineering, KAIST, 2012.2 - Current
- Associate Professor: Dept. of Electrical Engineering, KAIST, 2011.8 - 2012.2
- Assistant Professor: Dept. of Electrical Engineering, KAIST, 2008.8 - 2011.8
- Postdoctoral Research Associate: Dept. of Electrical Engineering Princeton University, 2006.8 - 2008.8

## Courses

- Probability and Introductory Random Process Video included, Undergraduate
- Data Structures for Electrical Engineers, Undergraduate
- Mathematics for Machine Learning, Undergraduate
- Computer Network, Undergraduate
- Complex Network Analysis: Ecosystems and Rumours Video included, Graduate

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N1 Building,  
Daegu, South Korea  
Phone: +82 42  
Fax: +82 42  
Email: ieste@kaist.edu  
Office Hours

## Memorabilia



2020 Spring

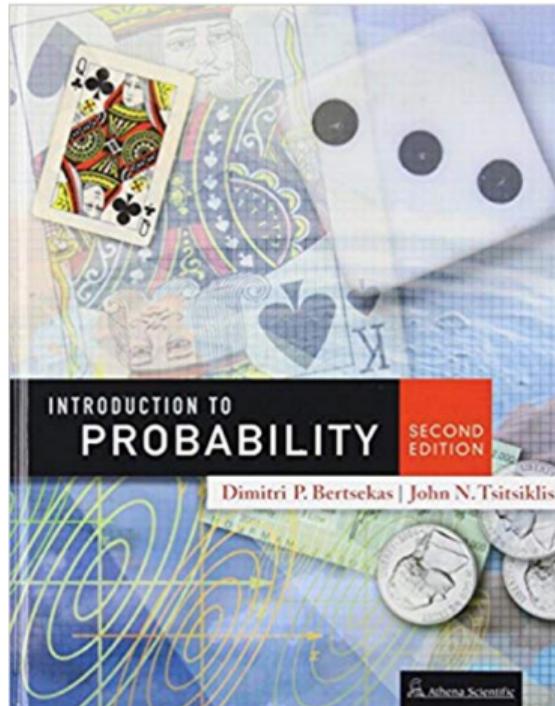
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- Homework upload
- Score upload and all the grade-related things

- Introduction to Probability  
(2nd edition)
  - MIT course textbook
  - Dimitri P. Bertsekas and John N. Tsitsiklis
- You can order it from Yes24, Aladin, Kyobo
  - Yes24: <http://www.yes24.com/Product/Goods/3995311>
  - Aladin: <https://www.aladin.co.kr/shop/wproduct.aspx?ItemId=12945615>
  - Kyobo: <http://www.kyobobook.co.kr/product/detailViewEng.laf?ejkGb=ENG&mallGb=ENG&barcode=9781886529380&orderClick=LAG&Kc=>



- <http://athenasc.com/probbook.html>

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- Links to the MIT courses

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- You can find the urls (2006, 2010, 2013) for the MIT lectures based on the same textbook, where there are many useful resources (recitation problems, homework problems, old exam problems, etc)
- Some of my lecture slides are based on theirs, but my slides are largely modified/reorganized/edited in many places for our purpose.

1. Probabilistic model (1/2 week)
2. Conditioning and Independence (1/2 week)
3. Random Variable, Part I (Discrete Random Variable) (1.5 week)
4. Random Variable, Part II (Continuous Random Variable) (1.5 week)
5. Random Variable, Part III (Advanced Topic on Random Variable) (1.5 week)
6. Limit of Scaled Sum of Random Variables: Central Limit Theorem and Weak Law of Large Numbers (1.5 week)
7. Random Process: Bernoulli and Poisson Processes (2 week)
8. Random Process: Markov Chain (2 week)
9. Introduction to Statistical Inference (2 week)

- 2 Exams (mid-term and final)

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- Homeworks
  - All problems are from exercise problems in the textbook.
  - We do NOT check whether you copy your solution from the problem solutions or not.

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- 9 Homeworks for each of 9 chapters.

- Read **ALL** the emails and sms from KLMS.

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- OK not to be present in the classroom? Yes

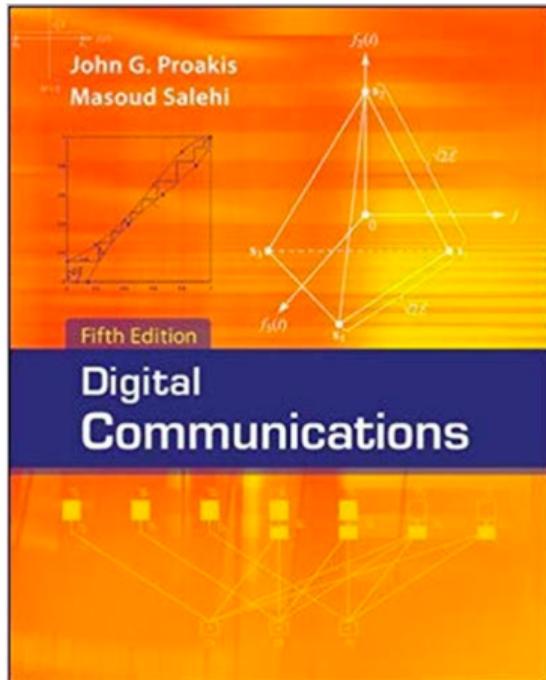
- Read **ALL** the emails and sms from KLMS.
- OK not to be present in the classroom? Yes
- OK that my homework solutions is sams as those in the solutions book? Yes
- Can I ask for a personal meeting to ask quetions or get other general advices? Yes.  
Send me an email.

Questions?

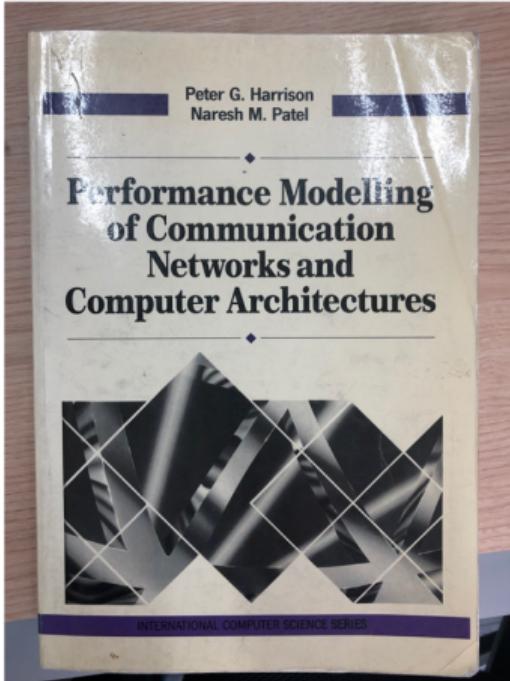
- Many things are "probabilistic"

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- Assume that you are a designer of the following engineering systems. Good design?
  - a web server
  - a communication device like mobile phones
  - an AI-based image classifier

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- Assume that you are a designer of the following engineering systems. Good design?
  - a web server
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- From an engineering point of view,
  - System input
  - Algorithms in systems
  - Analysis of systems



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Preface

**Chapter 1 Essentials of Probability Theory**

- 1.1 Sample space, events and probability
- 1.2 Conditional probability
- 1.3 Independence

Exercises

**Chapter 2 Random Variables and Distributions**

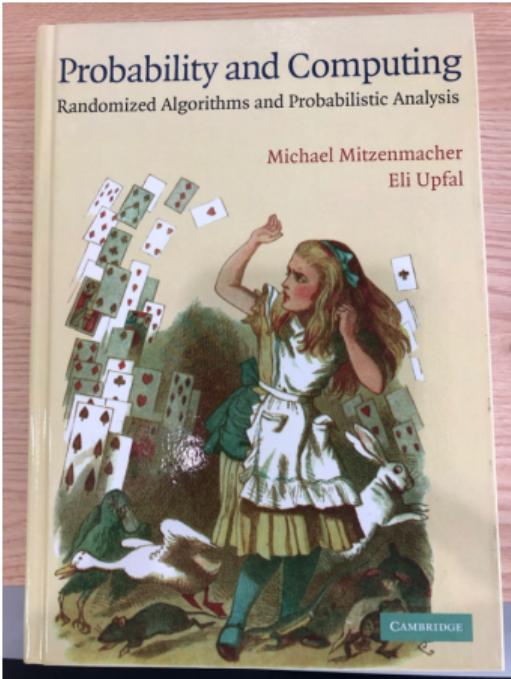
- 2.1 Probability distribution functions
- 2.2 Discrete random variables
- 2.3 Continuous random variables
- 2.4 Joint random variables
- 2.5 Conditional distributions
- 2.6 Independence and sums

Exercises

**Chapter 3 Expected Values and Moments**

- 3.1 Expectation
- 3.2 Generating functions and transforms
- 3.3 Asymptotic properties

Exercises



Preface

**1 Events and Probability**

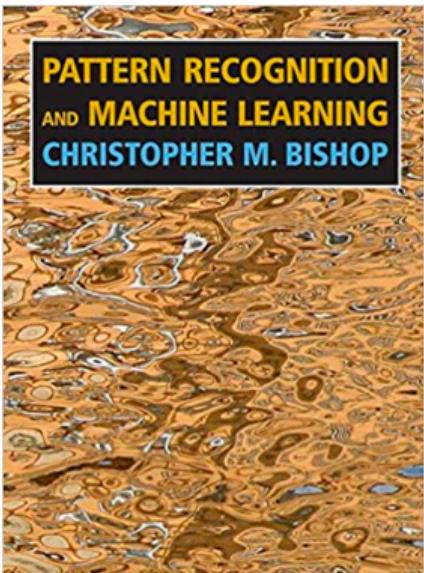
- 1.1 Application: Verifying Polynomial Identities
- 1.2 Axioms of Probability
- 1.3 Application: Verifying Matrix Multiplication
- 1.4 Application: A Randomized Min-Cut Algorithm
- 1.5 Exercises

**2 Discrete Random Variables and Expectation**

- 2.1 Random Variables and Expectation
  - 2.1.1 Linearity of Expectations
  - 2.1.2 Jensen's Inequality
- 2.2 The Bernoulli and Binomial Random Variables
- 2.3 Conditional Expectation
- 2.4 The Geometric Distribution
  - 2.4.1 Example: Coupon Collector's Problem
- 2.5 Application: The Expected Run-Time of Quicksort
- 2.6 Exercises

**3 Moments and Deviations**

- 3.1 Markov's Inequality
- 3.2 Variance and Moments of a Random Variable
  - 3.2.1 Example: Variance of a Binomial Random Variable
- 3.3 Chebyshev's Inequality
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These days, every area in CS and EE is directly or indirectly related to machine learning!

- Designer's perspective?

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- We will take this exciting journey from the next lecture!

Questions?

