Tips for a good system engineer and/or a good programmer

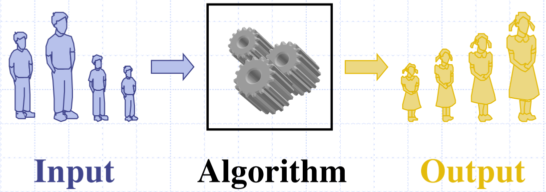
- Computer systems
  - Whatever you want to do in your computer, there are ways
    - Fast searching of how to do them in google, and courage to try them in your systems
    - People often tend to try only what they know
  - No fear about using new tools and commands
- Programming
  - Not a technique, but a science (감으로 하는 것이 아님)
  - Clearly know what a language provides and understand the underlying principles in relation to its interaction with computer internals

EE 205

Data Structure and Algorithms for Electrical Engineering

Lecture 3. Analysis of Algorithms

Yung Yi



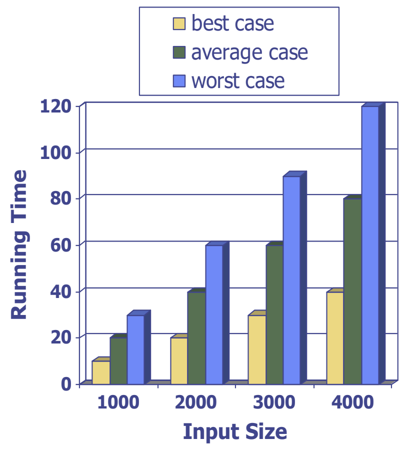
An **algorithm** is a step-by-step procedure for solving a problem in a finite amount of time.

What are we going to learn?

- Need to say that some algorithms are "better" than others
- Criteria for evaluation
  - Structure of programs (simplicity, elegance, OO, etc.)
  - Running time
  - Memory space
  - What else???

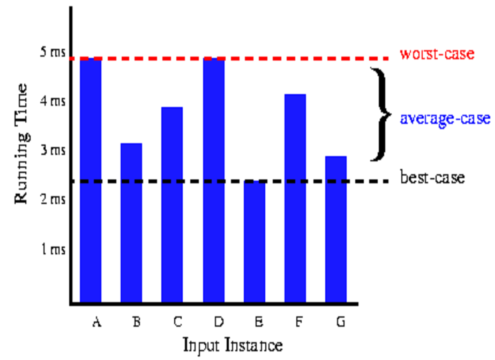
Running Time (§3.1)

- Most algorithms transform input objects into output objects.
- The running time of an algorithm typically grows with the input size.
- Average-case running time is often difficult to determine.
  - Why?
- We focus on the worst case running time.
  - Easier to analyze
  - Crucial to applications such as games, finance and robotics



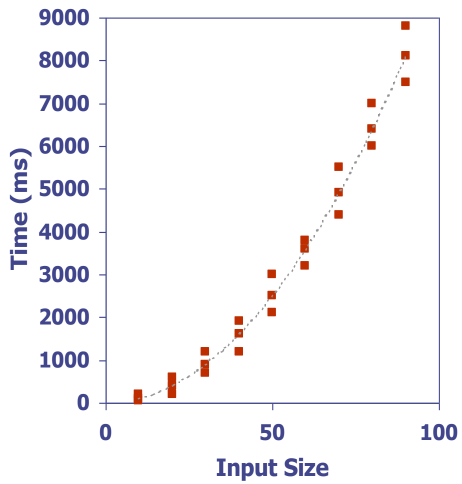
Average Case vs. Worst Case

- The average case running time is harder to analyze because you need to know the probability distribution of the input.
- In certain apps (air traffic control, weapon systems,etc.), knowing the worst case time is important.



Experimental Approach

- Write a program implementing the algorithm
- Run the program with inputs of varying size and composition
- Use a wall clock to get an accurate measure of the actual running time
- Plot the results



Limitations of Experiments

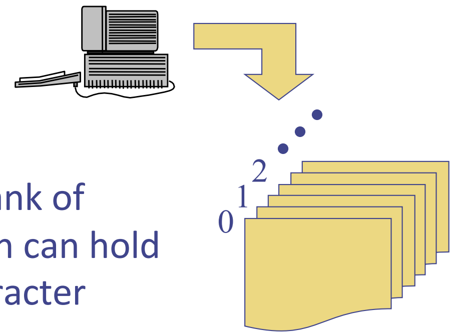
- It is necessary to implement the algorithm, which may be difficult and often time-consuming
- Results may not be indicative of the running time on other inputs not included in the experiment.
- In order to compare two algorithms, the same hardware and software environments must be used
  - Restrictions

Theoretical Analysis

- Uses a high-level description of the algorithm instead of an implementation
- Characterizes running time as a function of the input size, *n*.
- Takes into account all possible inputs
- Allows us to evaluate the speed of an algorithm *independent of* the hardware/software environment

The Random Access Machine (RAM) Model

- A **CPU**



- A potentially unbounded bank of **memory** cells, each of which can hold an arbitrary number or character
- Memory cells are numbered and accessing any cell in memory takes unit time.

Pseudocode (§4.2.3)

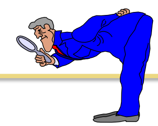
- High-level description of an algorithm
- More structured than english prose
- Less detailed than a program
- Preferred notation for describing algorithms
- Hides program design issues

Example: find the max element of an array

```
Algorithm arrayMax(A, n)
  Input array A of n integers
  Output maximum element of A

  currentMax ← A[0]
  for i ← 1 to n − 1 do
    if A[i] > currentMax then
      currentMax ← A[i]
  return currentMax
```


Pseudocode Details



- Control flow
  - **if** … **then** … [**else** …]
  - **while** … **do** …
  - **repeat** … **until** …
  - **for** … **do** …
  - Indentation replaces braces
  -
- Method declaration

  Algorithm *method* (*arg* [, *arg*…])

  Input …

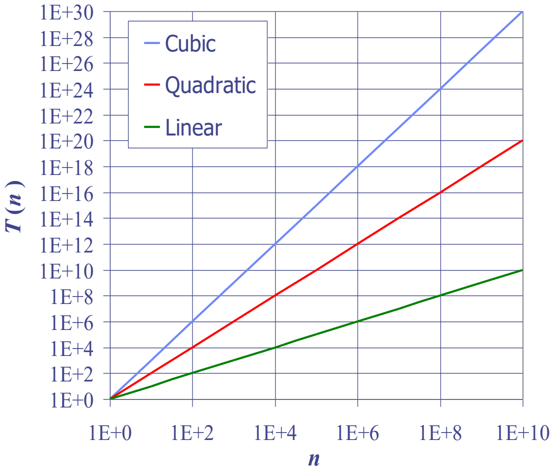
  Output …
- Method call

  var.method (*arg* [, *arg*…])
- Return value

  return *expression*
- Expressions
  - ← Assignment (like = in C, C++)
  - = Equality testing (like == in C, C++)
  - n^2 Superscripts and other mathematical formatting allowed

Seven Important Functions (§3.3)

- Seven functions that often appear in algorithm analysis:
  - Constant ≈ 1
  - Logarithmic ≈ $\log n$
  - Linear ≈ n
  - N-Log-N ≈ $n \log n$
  - Quadratic ≈ n^2
  - Cubic ≈ n^3
  - Exponential ≈ 2^n
- In a log-log chart, the slope of the line corresponds to the growth rate of the function



Primitive Operations

- Basic computations performed by an algorithm
- Identifiable in pseudocode
- Largely independent from the programming language
- Exact definition not important (we will see why later)
- Assumed to take a constant amount of time in the RAM model
- Examples:
  - Evaluating an expression
  - Assigning a value to a variable
  - Indexing into an array
  - Calling a method
  - Returning from a method

Counting Primitive Operations (§3.4)

- By inspecting the pseudocode, we can determine the maximum number of primitive operations executed by an algorithm, as a function of the input size

Algorithm *arrayMax*(A, n)	# operations
currentMax ← $A[0]$	2
for $i \leftarrow 1$ **to** $n - 1$ **do**	$2n$
if $A[i] > currentMax$ **then**	$2(n-1)$
currentMax ← $A[i]$	$2(n-1)$
{ increment counter i }	$2(n-1)$
return *currentMax*	1
Total	$8n - 2$

Estimating Running Time

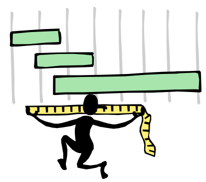
- Algorithm ***arrayMax*** executes $8n - 2$ primitive operations in the worst case. Define:

  a = Time taken by the fastest primitive operation

  b = Time taken by the slowest primitive operation
- Let $T(n)$ be worst-case time of ***arrayMax.*** Then

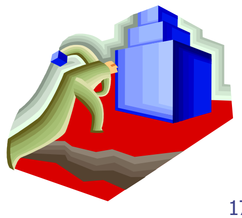
$$a\,(8n - 2) \le T(n) \le b(8n - 2)$$

- Hence, the running time $T(n)$ is bounded by two linear functions



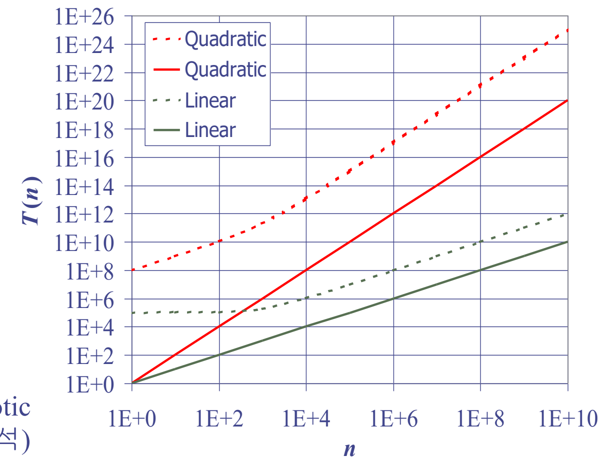
Growth Rate of Running Time

- Changing the hardware/ software environment
  - Affects $T(n)$ by a constant factor, but
  - Does not alter the growth rate of $T(n)$
- The linear growth rate of the running time $T(n)$ is an intrinsic property of algorithm ***arrayMax***



Constant Factors

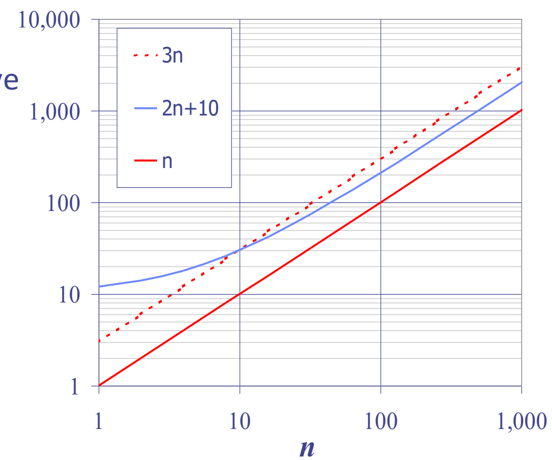
- The growth rate is not affected by
  - constant factors or
  - lower-order terms
- Examples
  - $10^2 n + 10^5$ is a linear function
  - $10^5 n^2 + 10^8 n$ is a quadratic function
- We consider when n is sufficiently large
  - We call this "Asymptotic Analysis" (점근적 분석)



Big-Oh Notation (§4.2.3)

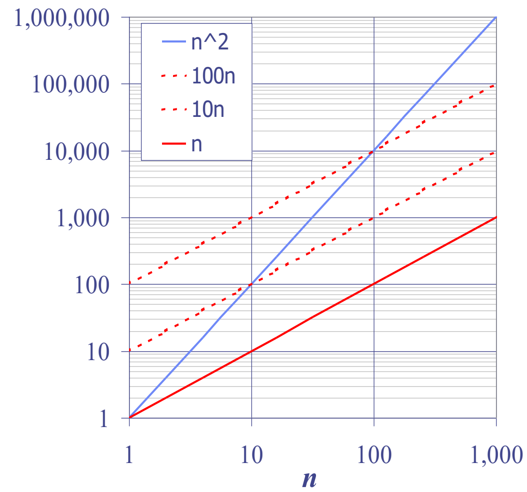
- Given functions $f(n)$ and $g(n)$, we say that $f(n)$ is $O(g(n))$ if there are positive constants
  c and n_0 such that

  $f(n) \leq cg(n)$ for $n \geq n_0$
- Example: $2n + 10$ is $O(n)$
  - $2n + 10 \leq cn$
  - $(c - 2)\, n \geq 10$
  - $n \geq 10/(c - 2)$
  - Pick $c = 3$ and $n_0 = 10$

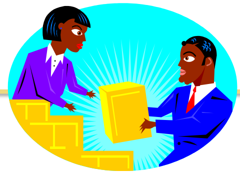


Big-Oh Example

- Example: the function n^2 is not $O(n)$
  - $n^2 \leq cn$
  - $n \leq c$
  - The above inequality cannot be satisfied since c must be a constant



More Big Oh Examples



- $7n-2$
  $7n-2$ is $O(n)$
  need $c > 0$ and $n_0 \geq 1$ such that $7n-2 \leq c \bullet n$ for $n \geq n_0$
  this is true for $c = 7$ and $n_0 = 1$
- $3n^3 + 20n^2 + 5$
  $3n^3 + 20n^2 + 5$ is $O(n^3)$
  need $c > 0$ and $n_0 \geq 1$ such that $3n^3 + 20n^2 + 5 \leq c \bullet n^3$ for $n \geq n_0$
  this is true for $c = 4$ and $n_0 = 21$
- $3 \log n + 5$
  $3 \log n + 5$ is $O(\log n)$
  need $c > 0$ and $n_0 \geq 1$ such that $3 \log n + 5 \leq c \bullet \log n$ for $n \geq n_0$
  this is true for $c = 8$ and $n_0 = 2$
- (Question) $3 \log n + 5$ is $O(n)$? Yes or No?

Big-Oh and Growth Rate

- The big-Oh notation gives an upper bound on the growth rate of a function
- The statement "$f(n)$ is $O(g(n))$" means that the growth rate of $f(n)$ is no more than the growth rate of $g(n)$
- We can use the big-Oh notation to rank functions according to their growth rate

Which is possible?

	$f(n)$ is $O(g(n))$	$g(n)$ is $O(f(n))$
$g(n)$ grows faster	Yes	No
$f(n)$ grows faster	No	Yes
Same growth	Yes	Yes

Big-Oh Rules

- If is $f(n)$ a polynomial of degree d, then $f(n)$ is $O(n^d)$, i.e.,
  1. Drop lower-order terms
  2. Drop constant factors
- Use the smallest possible class of functions
  - Say "$2n$ is $O(n)$" instead of "$2n$ is $O(n^2)$"
- Use the simplest expression of the class
  - Say "$3n + 5$ is $O(n)$" instead of "$3n + 5$ is $O(3n)$"

Asymptotic Algorithm Analysis

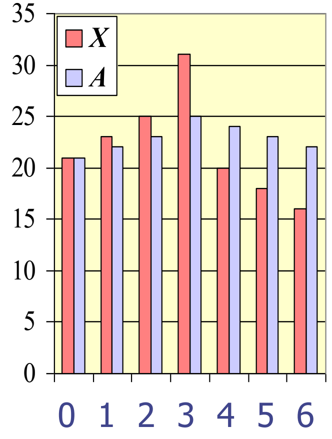
- The asymptotic analysis of an algorithm determines the running time in big-Oh notation
- To perform the asymptotic analysis
  - We find the worst-case number of primitive operations executed as a function of the input size
  - We express this function with big-Oh notation
- Example:
  - We determine that algorithm ***arrayMax*** executes at most $8n - 2$ primitive operations
  - We say that algorithm ***arrayMax*** "runs in $O(n)$ time"
- Since constant factors and lower-order terms are eventually dropped anyhow, we can disregard them when counting primitive operations

Computing Prefix Averages

- We further illustrate asymptotic analysis with two algorithms for prefix averages
- The ***i***-th prefix average of an array X is average of the first $(i + 1)$ elements of X:

  $A[i] = (X[0] + X[1] + \ldots + X[i])/(i+1)$

- Computing the array A of prefix averages of another array X has applications to financial analysis



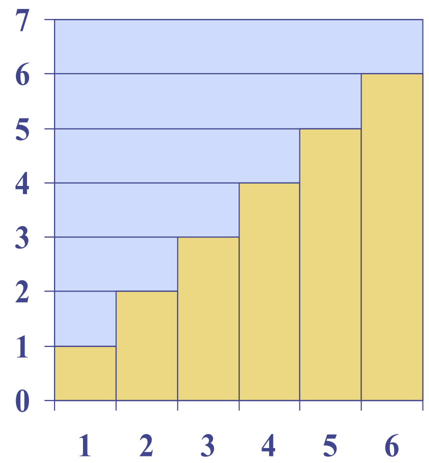
Prefix Averages (Quadratic)

- The following algorithm computes prefix averages in quadratic time by applying the definition

```
Algorithm prefixAverages1(X, n)
   Input array X of n integers
   Output array A of prefix averages of X        #operations
   A ← new array of n integers                      n
   for i ← 0 to n − 1 do                            n
       s ← X[0]                                     n
       for j ← 1 to i do                            1 + 2 + …+ (n − 1)
           s ← s + X[j]                             1 + 2 + …+ (n − 1)
       A[i] ← s / (i + 1)                           n
   return A                                         1
```


Arithmetic Progression

- The running time of ***prefixAverages1*** is $O(1 + 2 + \ldots + n)$
- The sum of the first n integers is $n(n + 1) / 2$
  - There is a simple visual proof of this fact
- Thus, algorithm ***prefixAverages1*** runs in $O(n^2)$ time



Prefix Averages (Linear)

- The following algorithm computes prefix averages in linear time by keeping a running sum

```
Algorithm prefixAverages2(X, n)
   Input array X of n integers
   Output array A of prefix averages of X        #operations
   A ← new array of n integers                      n
   s ← 0                                            1
   for i ← 0 to n − 1 do                            n
       s ← s + X[i]                                 n
       A[i] ← s / (i + 1)                           n
   return A                                         1
```

- Algorithm ***prefixAverages2*** runs in $O(n)$ time

Another Example

Result ← 0; m ← 1;

for I ← 1 to n

    *m ← m*2;*

    for j ← 1 to m do

        result ← result + i*m*j

Math you need to review

- Summations
- Logarithms and Exponents
- Proof techniques
- Basic probability
  - For randomized algorithms (later in this course)

- **properties of logarithms:**

  $\log_b(xy) = \log_b x + \log_b y$

  $\log_b (x/y) = \log_b x - \log_b y$

  $\log_b x^a = a\log_b x$

  $\log_b a = \log_x a / \log_x b$

- **properties of exponentials**:

  $a^{(b+c)} = a^b a^c$

  $a^{bc} = (a^b)^c$

  $a^b / a^c = a^{(b-c)}$

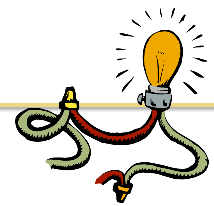
  $b = a^{\log_a b}$

  $b^c = a^{c*\log_a b}$

Relatives of Big-Oh

- **big-Omega**
  - f(n) is Ω(g(n)) if there is a constant c > 0 and an integer constant $n_0 \geq 1$ such that $f(n) \geq c \bullet g(n)$ for $n \geq n_0$
- **big-Theta**
  - f(n) is Θ(g(n)) if there are constants c' > 0 and c'' > 0 and an integer constant $n_0 \geq 1$ such that $c' \bullet g(n) \leq f(n) \leq c'' \bullet g(n)$ for $n \geq n_0$

Intuition for Asymptotic Notation



Big-Oh

- f(n) is O(g(n)) if f(n) is asymptotically **less than or equal** to g(n)

big-Omega

- f(n) is Ω(g(n)) if f(n) is asymptotically **greater than or equal** to g(n)

big-Theta

- f(n) is Θ(g(n)) if f(n) is asymptotically **equal** to g(n)

Examples (1)

- **$5n^2$ is $\Omega(n^2)$**

  $f(n)$ is $\Omega(g(n))$ if there is a constant $c > 0$ and an integer constant $n_0 \geq 1$ such that $f(n) \geq c \bullet g(n)$ for $n \geq n_0$

  let $c = 5$ and $n_0 = 1$

- **$5n^2$ is $\Omega(n)$**

  $f(n)$ is $\Omega(g(n))$ if there is a constant $c > 0$ and an integer constant $n_0 \geq 1$ such that $f(n) \geq c \bullet g(n)$ for $n \geq n_0$

  let $c = 1$ and $n_0 = 1$

Examples (2)

- **$5n^2$ is $\Theta(n^2)$**

  $f(n)$ is $\Theta(g(n))$ if it is $\Omega(n^2)$ and $O(n^2)$. we have already seen the former, for the latter (for $O(n^2)$)recall that $f(n)$ is $O(g(n))$ if there is a constant $c > 0$ and an integer constant $n_0 \geq 1$ such that $f(n) \leq c \bullet g(n)$ for $n \geq n_0$

  Let $c = 5$ and $n_0 = 1$

What do we want for our algorithms?

- Prof. Yung Yi → A graduate student
  - "What is the order of your algorithm?"
  - Answer: nlogn, n^2, n^3, 2^n
- Polynomial order
  - Generally fine.
  - Try to reduce the running time if above or equal to n^3
- There are some problems for which there does NOT exist any polynomial-time algorithm (up to so far)
  - We say that they "NP-hard" or "NP-complete"
  - You will learn formalism for this in the algorithm class