AI and Deep Learning

# Linear Regression & Back-propagation

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# Agenda

- Neuron and Regression
- Loss/Error/Cost Function
- Learning and Updating Weights
- Gradient/Slope
- Computation Graph
- Forward Propagation, Backpropagation

# Re-gression

“

After spending most of their time in the ocean, salmons **go back** home(river) where they were born.

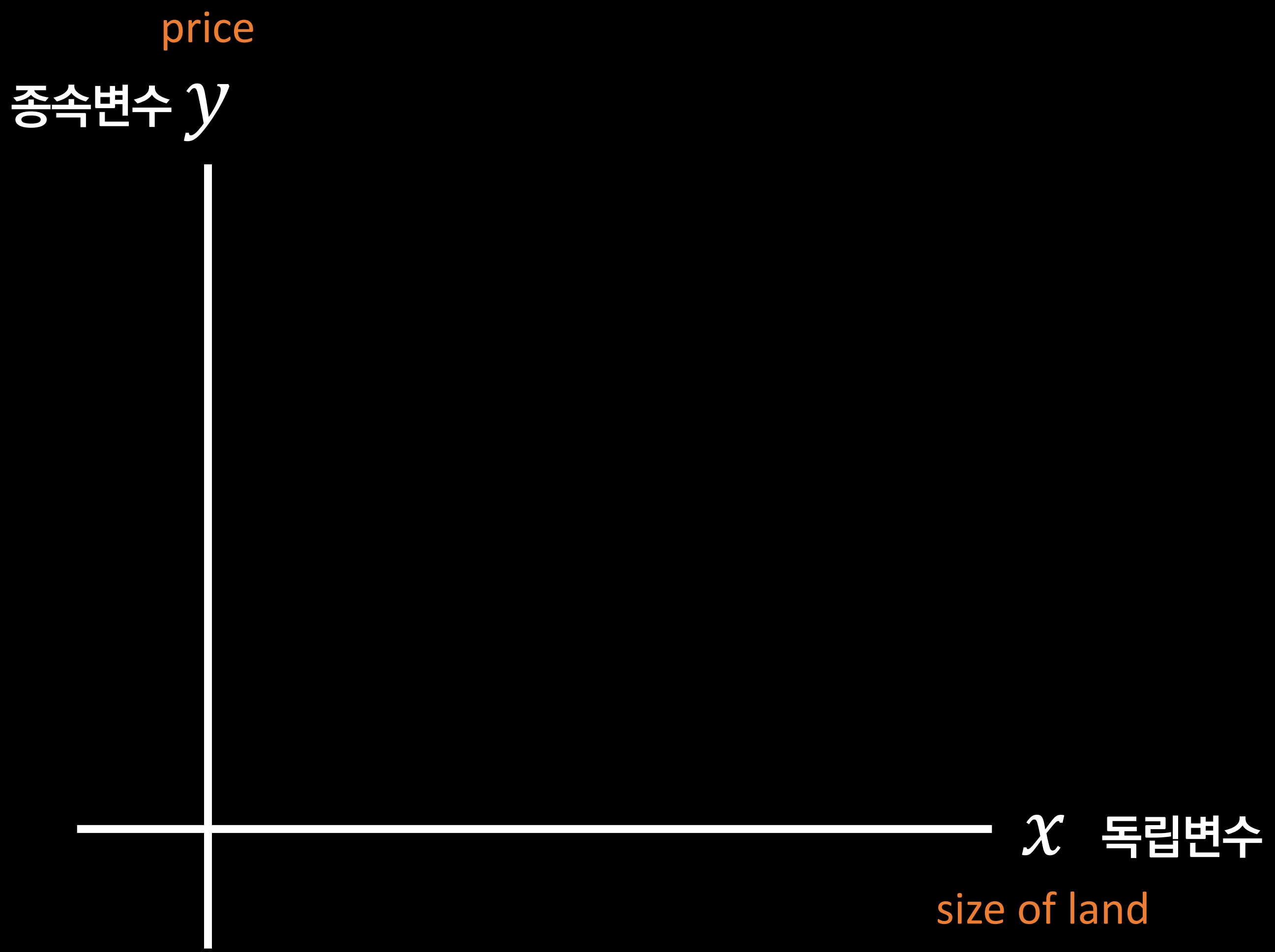
# Regression(회귀)

- Going back
- To describe a natural **phenomena**
- A term frequently used in anthropology**(인류학)** to present a natural tendency

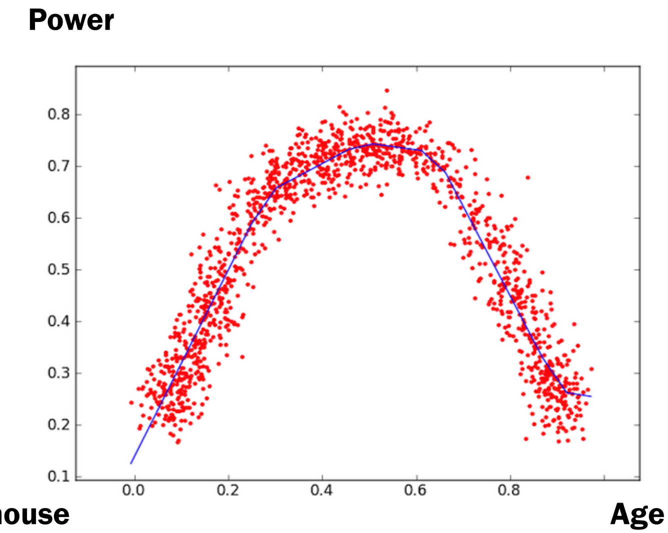
What is 'a proposed explanation for a phenomenon'?

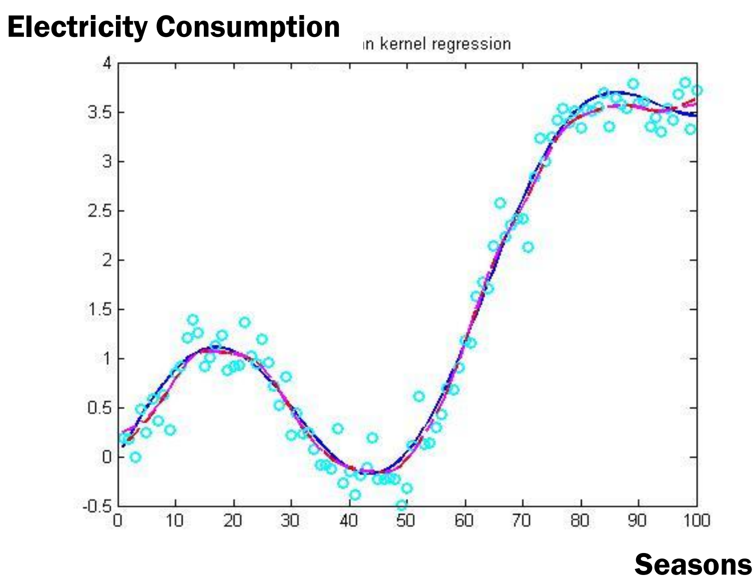
# Regression(회귀)

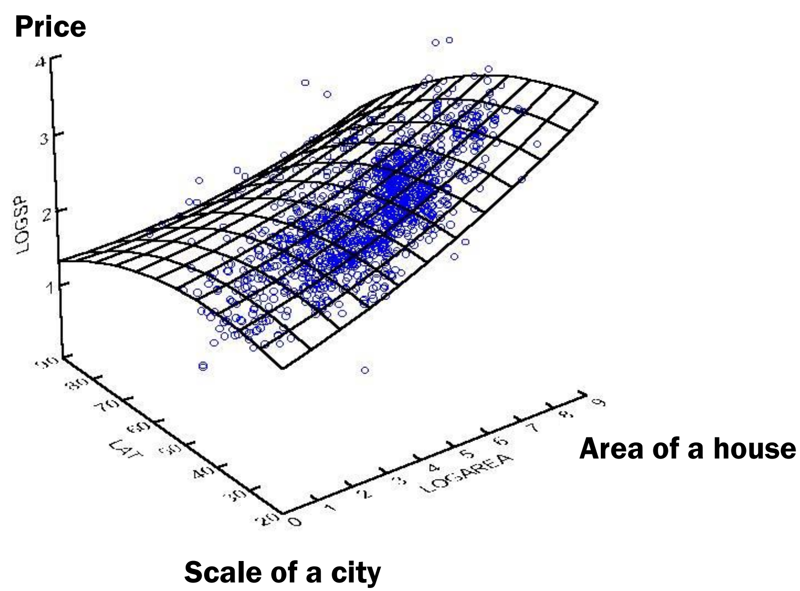
- Statistical measure to determine the relationship between one **dependent** variable (usually denoted by $Y$, **종속변수**) and a series of other **independent** variables ($X$, **독립변수**).


price
종속변수 $y$
$x$ 독립변수
size of land

# Examples of Regression

# **Linear** Regression

- A linear or a non-linear regression model?
- It is not about the relationship between the independent variable and the dependent variable.
- If the hypothesis(dependent variable) is a **linear combination of independent variables and coefficients**, then it is a linear model.

$$h = w_1 \cdot x_1 + w_2 \cdot x_2$$

**Coefficient:** 변수에 붙어있는 상수 $w$

# Lab

# Linear Regression

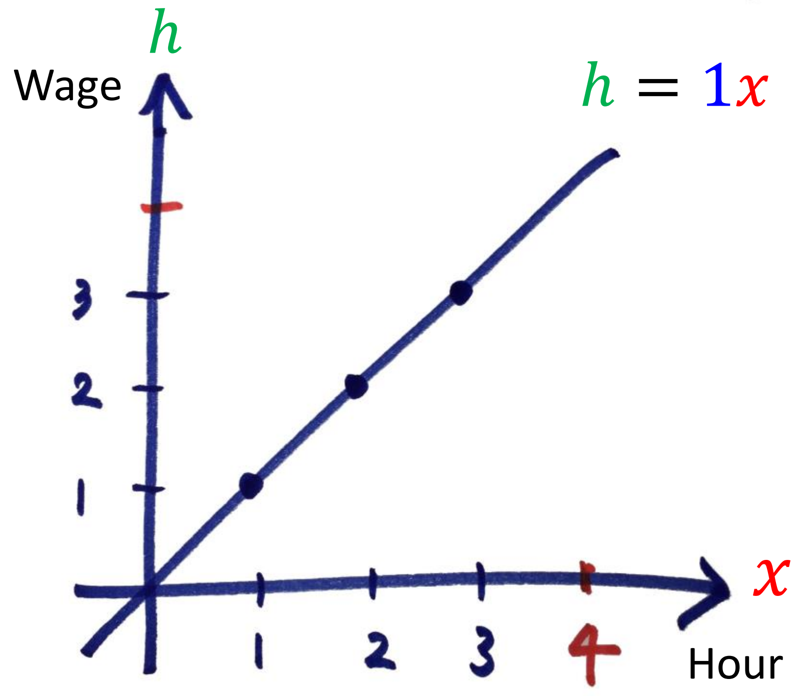
using desmos

그래핑 계산기

# www.**desmos**.com

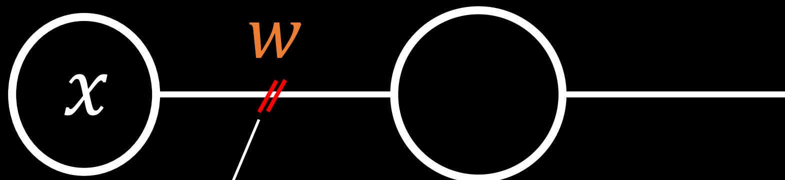
1. Draw a point(data) (1, 1)
2. Add (2, 2), (-1, -1), (-2, -2)
3. $h = x$
4. $h = 2x$
5. $h = wx$ (**rotation**)
6. Move all of the points by adding 1 to y
7. $h = wx + 1$ (**shifting**)
8. $h = wx + b$ (**rotation** and **shifting**)

“ Hypothesis representing the experience (4 points) www.**desmos**.com

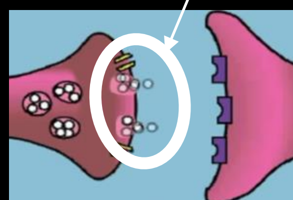


$$h = wx$$

# Neuron and regression




$$h = wx$$



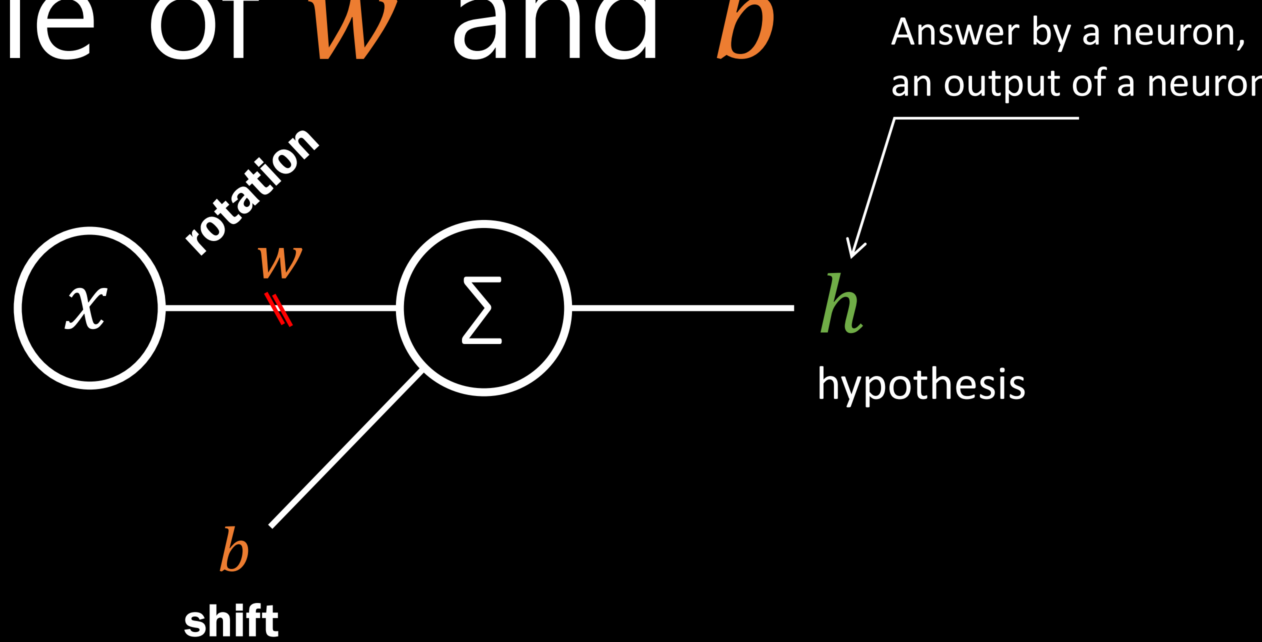
- $w$ : Neuro-transmitter (신경전달물질의 양)
- if large / if small / if not exists (**rotation**)

# Hypothesis

$$h = wx$$
$$h = wx + b$$

- An answer by a neuron
- $h$ypothesis : a proposed explanation for a phenomenon (regression).
- Not proved yet, but it can represent the regression well after updating $w$.
- $b$? **shift** for better linear regression representation

# The role of $w$ and $b$




$$h = \underline{w}x + b$$

linear combination of coefficients

⟶ linear regression

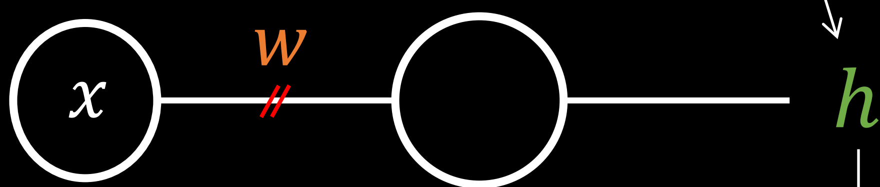
# How to update $w$(learning)

- Scolding or blaming the neuron if it is wrong
- The neuron gets stress and automatically updates $w$ to answer well next time so that the error(difference) decreases.
- How can we calculate the error/loss?

# Error function

loss function
difference function

answer by a neuron




ground truth
Target value

$$E = |w \cdot x - y|$$

Why absolute?

# Error function

The error is the difference between a neuron's answer and its ground truth.

$$E = |h\text{ypothesis} - y|$$

$$E = |w \cdot x - y|$$

| $x_i$ | $y_i$ |
|---|---|
| 1 | 1 |

'1 hour working, then 1 USD'

$$E = |w \cdot 1 - 1|$$

Supervised Learning

지도학습

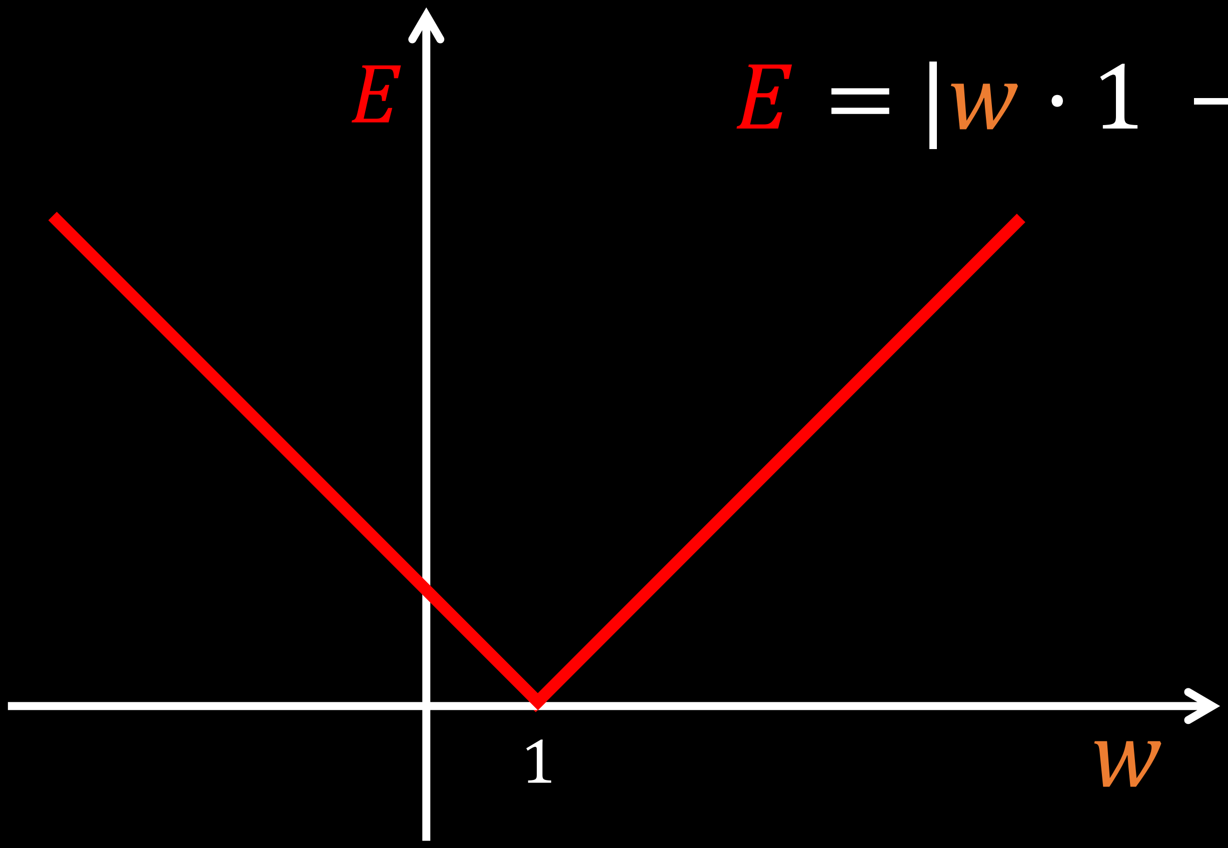
# www.desmos.com

1. Mark (1, 1)
2. $h = w \cdot x$
3. $E = w \cdot 1 - 1$
4. $E = |w \cdot 1 - 1|$
5. $(w, E)$

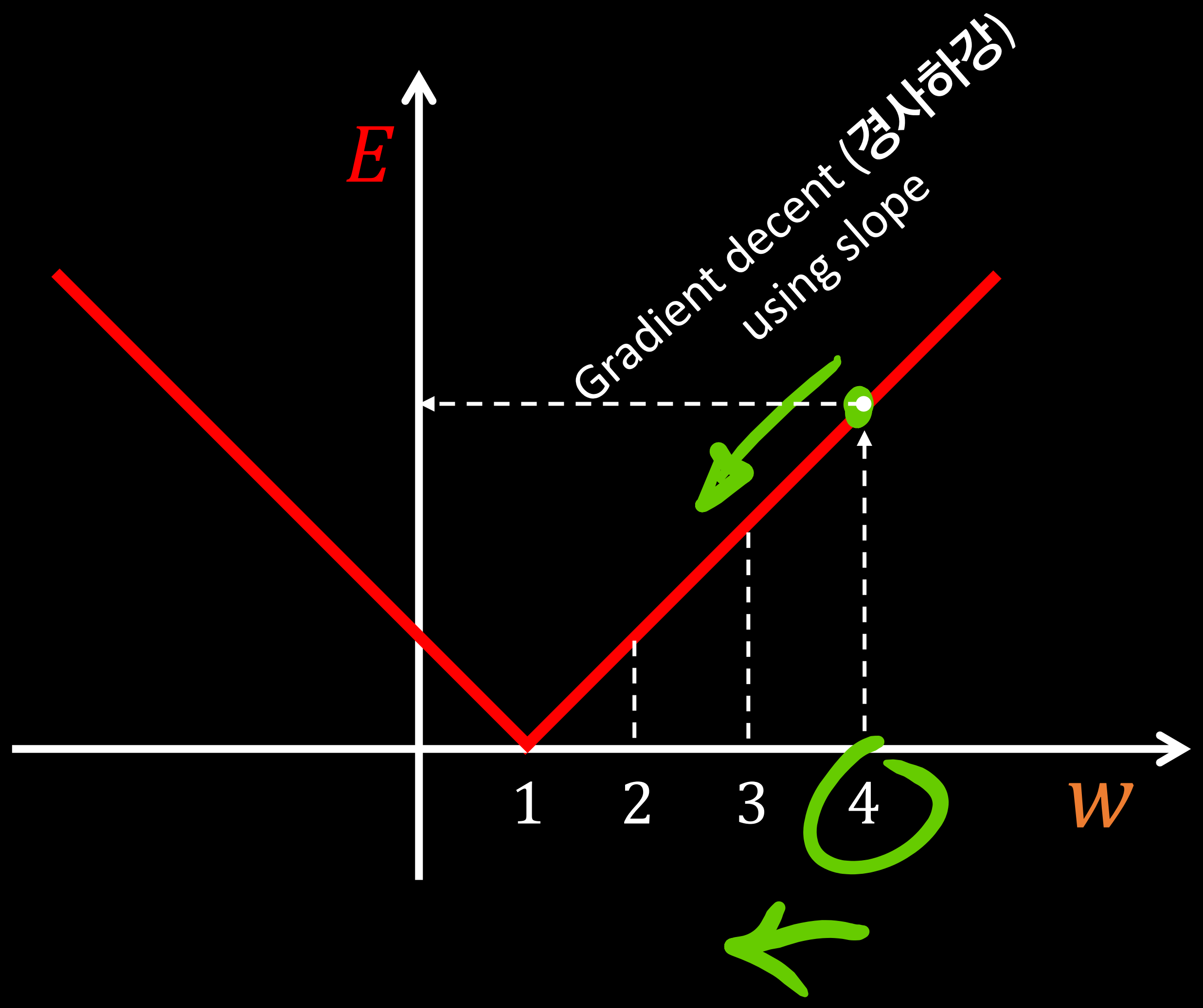
desmos

# Error Function of $w$

$$E = |w \cdot 1 - 1|$$



# How to update $w$

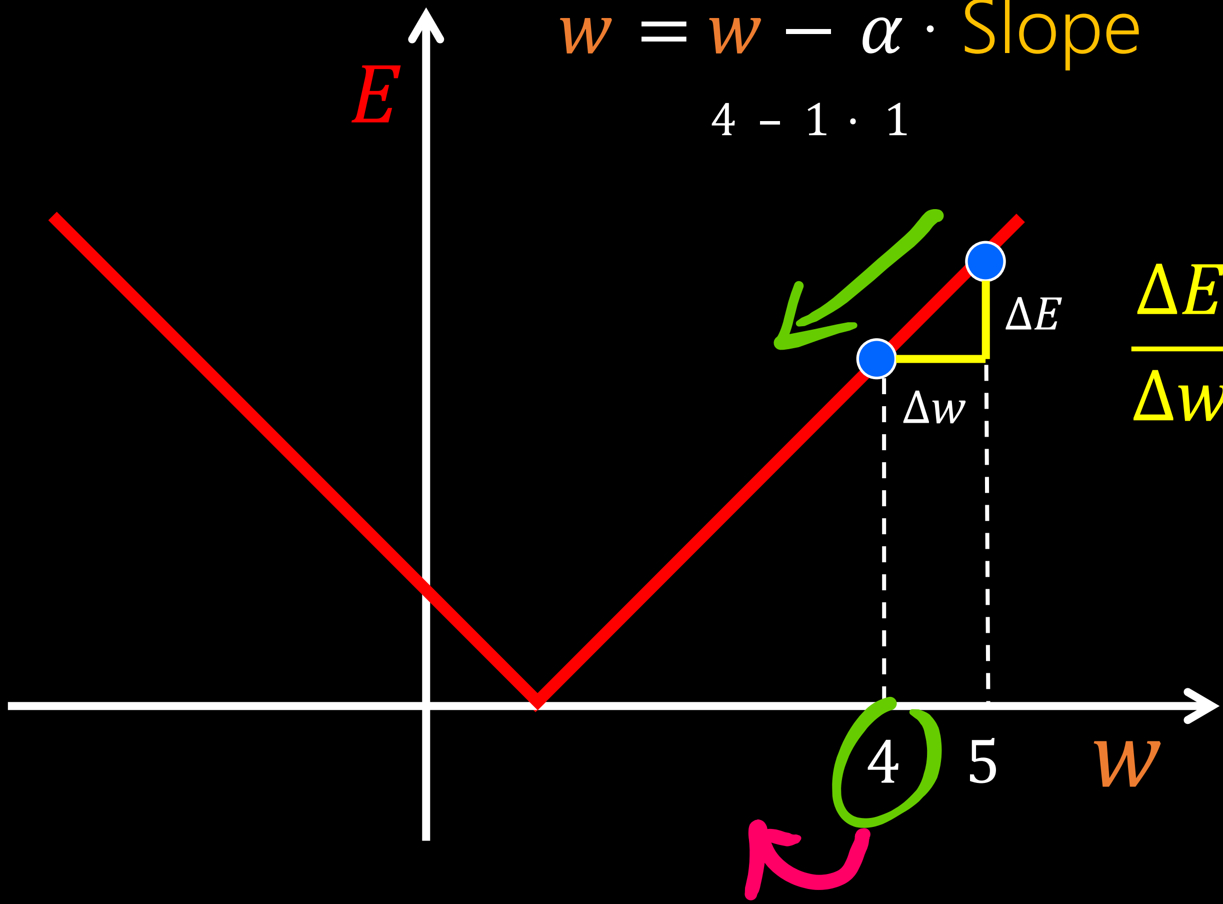


# How to update $w$

$$w = w - \alpha \cdot \text{Slope}$$

learning rate (ex, 1)

$4 - 1 \cdot 1$




$$\frac{\Delta E}{\Delta w} = \frac{+1}{+1}$$

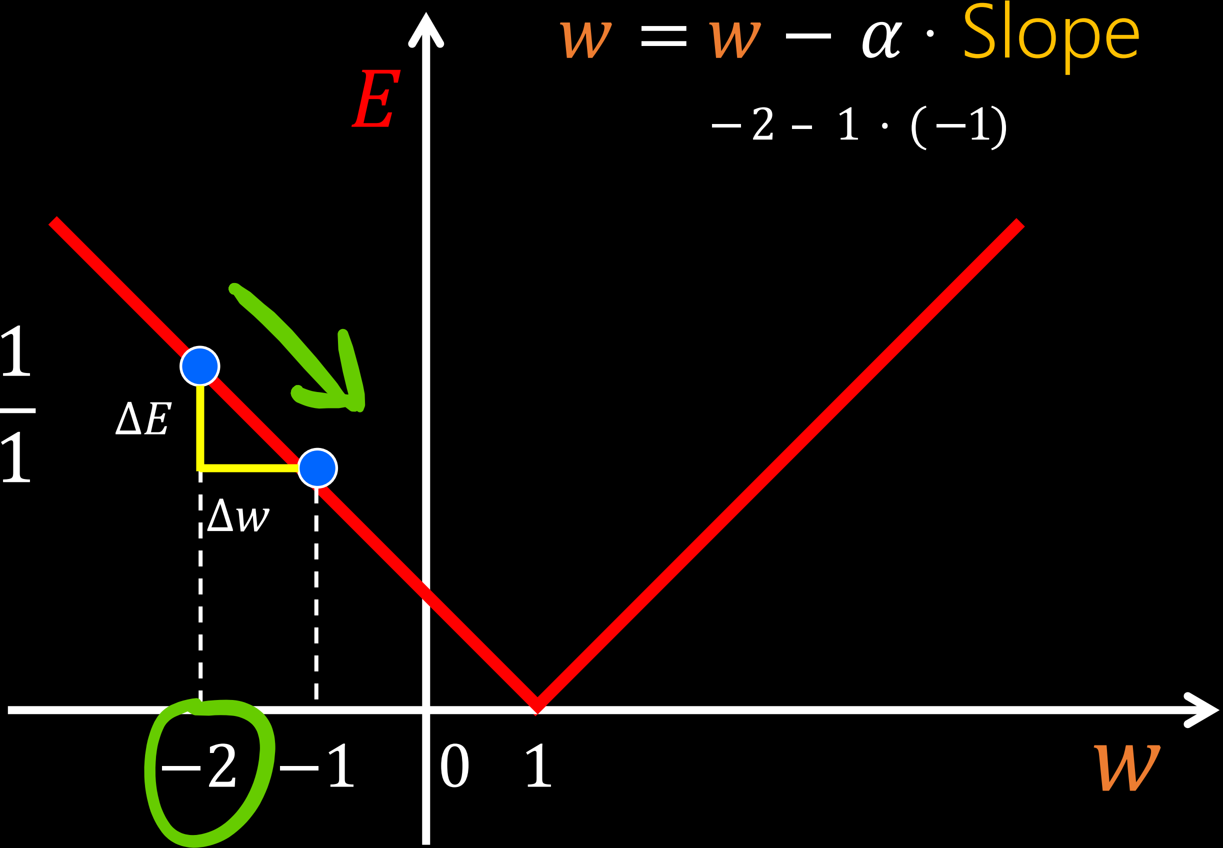
# How to update $w$

learning rate (ex, 1)

$$w = w - \alpha \cdot \text{Slope}$$

$$-2 - 1 \cdot (-1)$$

$$\frac{\Delta E}{\Delta w} = \frac{-1}{+1}$$



randomized

$$w = 4, \alpha = 1, Slope = 1$$

$w = w - \alpha \cdot$ Slope

$4 - 1 \cdot 1 \longrightarrow 3$ Error $E$ = 2

$3 - 1 \cdot 1 \longrightarrow 2$ Error $E$ = 1

$2 - 1 \cdot 1 \longrightarrow 1$ Error $E$ = 0

When $w$ is 1, then the $E$ is 0.
AI model training ended!

randomized

$$w = -2, \alpha = 1, Slope = -1$$

$w = w - \alpha \cdot$ Slope

$-2 - 1 \cdot (-1) \longrightarrow -1$ Error $E$ = 2

$-1 - 1 \cdot (-1) \longrightarrow 0$ Error $E$ = 1

$0 - 1 \cdot (-1) \longrightarrow 1$ Error $E$ = 0

When $w$ is 1, then the $E$ is 0.
AI model training ended!

$$w = -2, \alpha = 2, Slope = -1$$

$w = w - \alpha \cdot$ Slope

$-2 - 2 \cdot (-1) \longrightarrow 0$ Error $E$ = 1

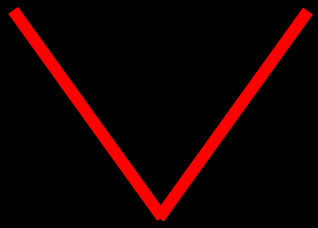
$0 - 2 \cdot (-1) \longrightarrow 2$ Error $E$ = 1

$2 - 2 \cdot (1) \longrightarrow 0$ Error $E$ = 1

$0 - 2 \cdot (-1) \longrightarrow 2$ Error $E$ = 1

$2 - 2 \cdot (1) \longrightarrow 0$ Error $E$ = 1

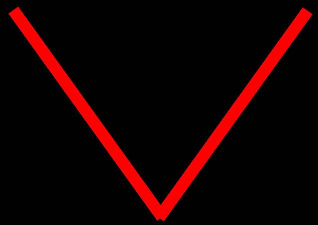
AI model training is never-ending!



Absolute Error

**L1** loss function

# Issues in the **L1** loss

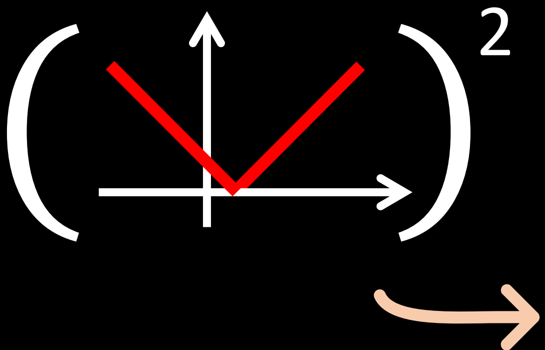


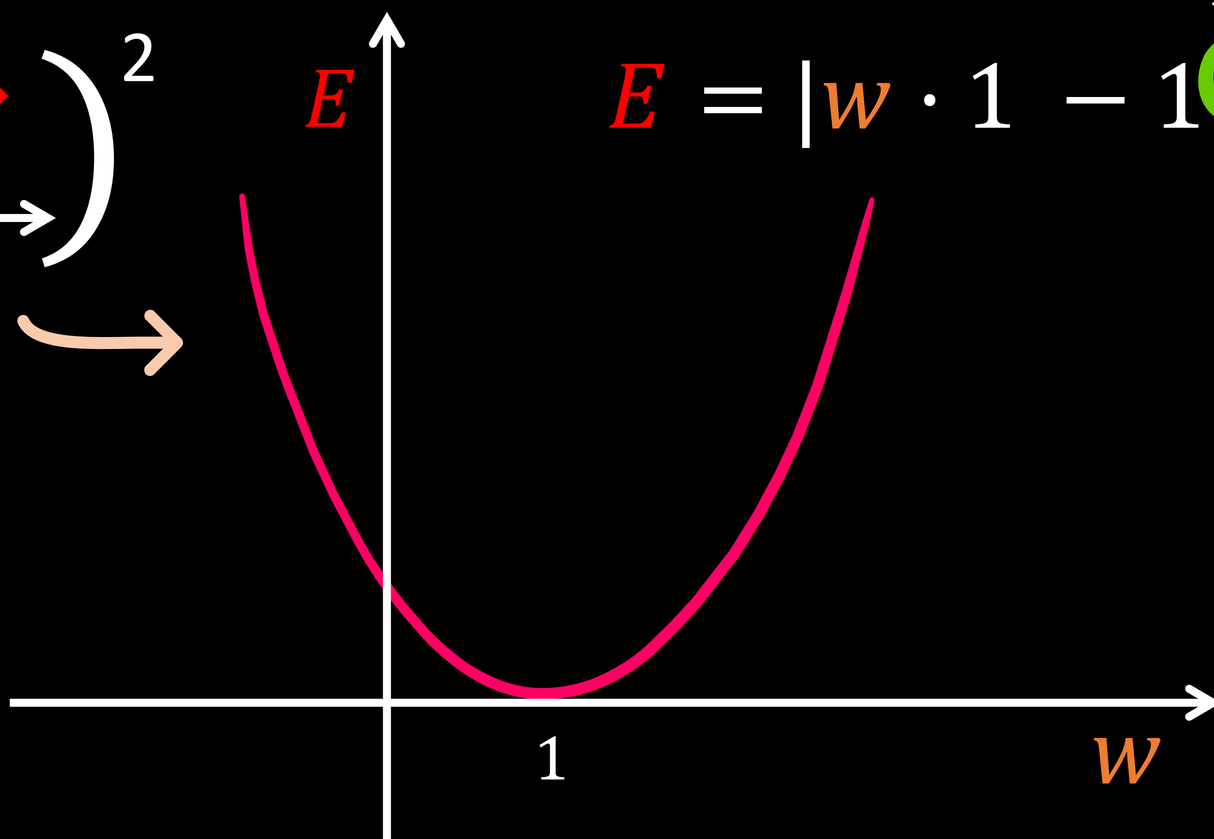
- Always the same slope on both sides regardless of the value of $w$
- Therefore, the same speed in the error decrease
- No guarantee to get the proper value of $w$ that gives 0 or minimum error.

# Square Error

square

$$E = |w \cdot 1 - 1|^2$$

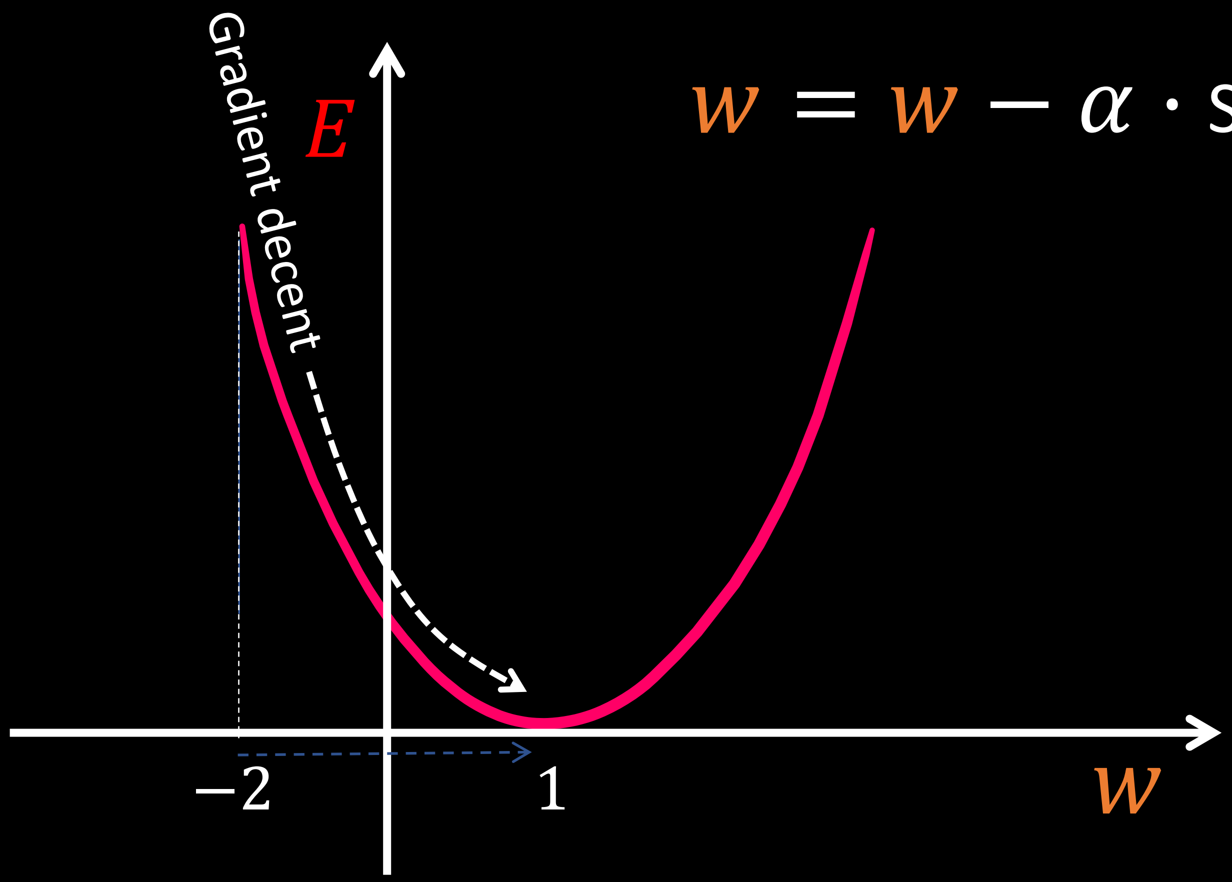




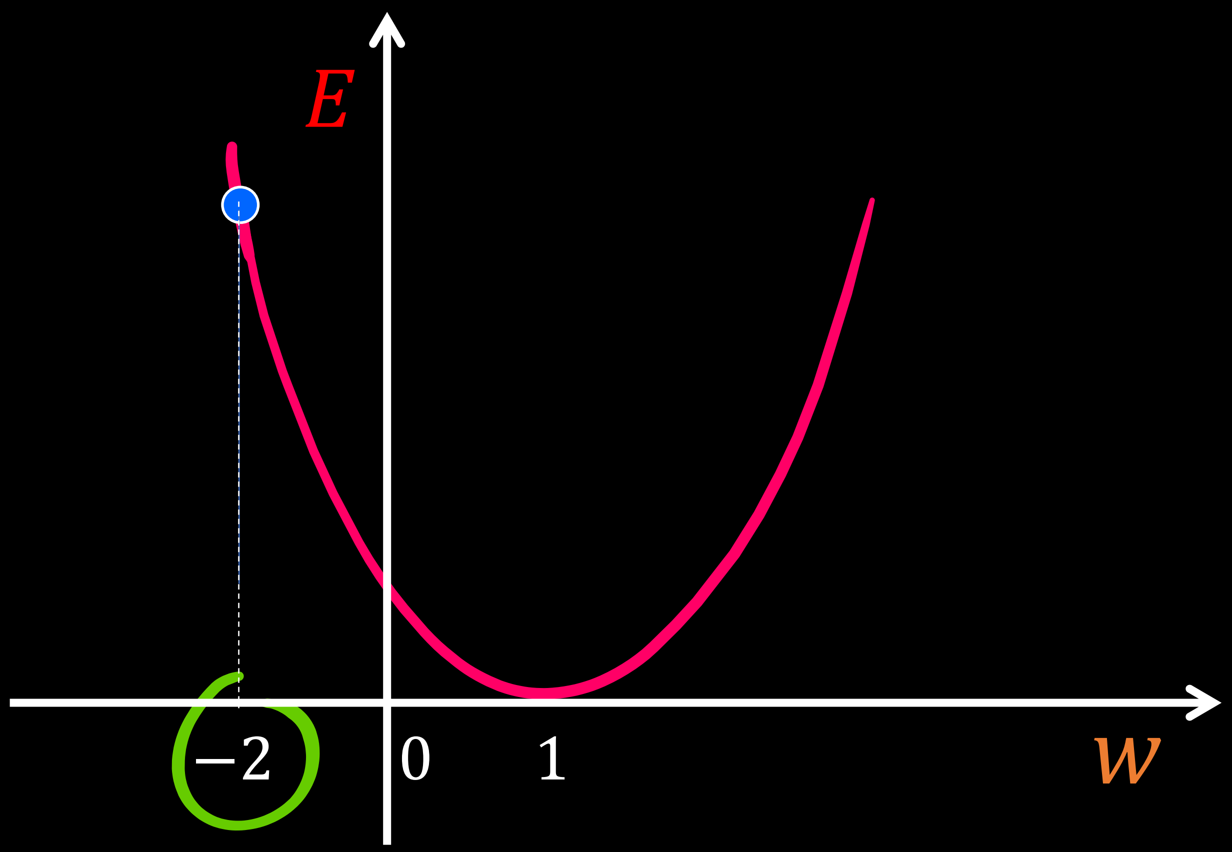
# How to update $w$

$$\frac{\Delta E}{\Delta w}$$

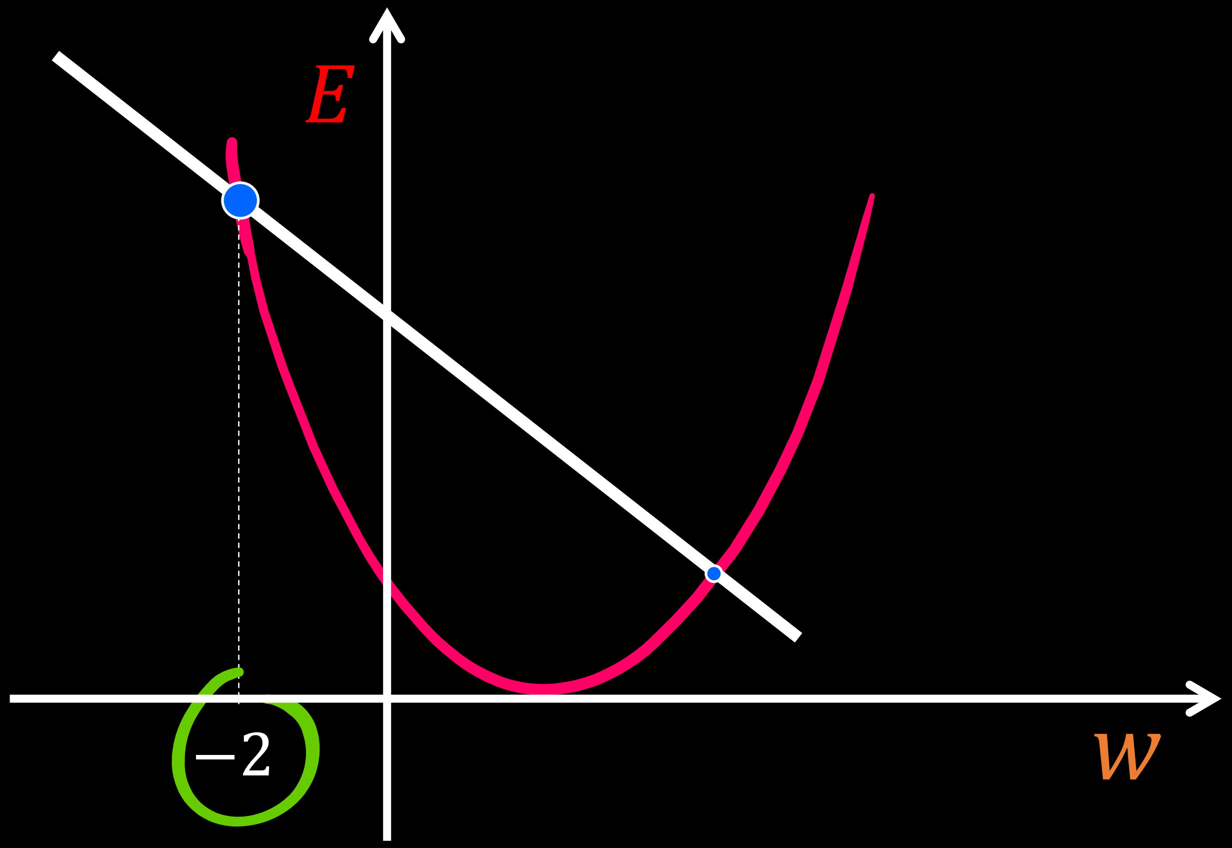
$$w = w - \alpha \cdot \text{Slope}$$



# How to update $w$

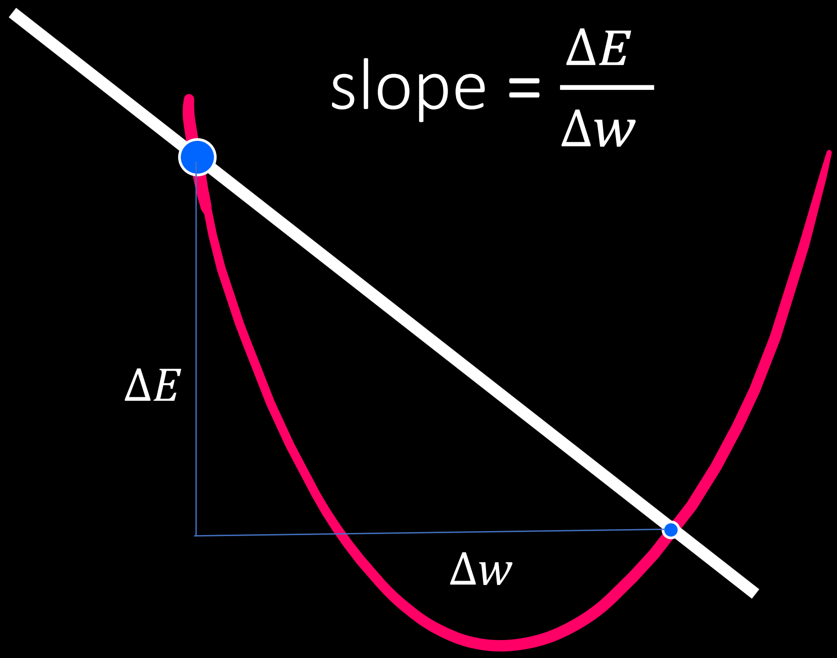


# How to update $w$

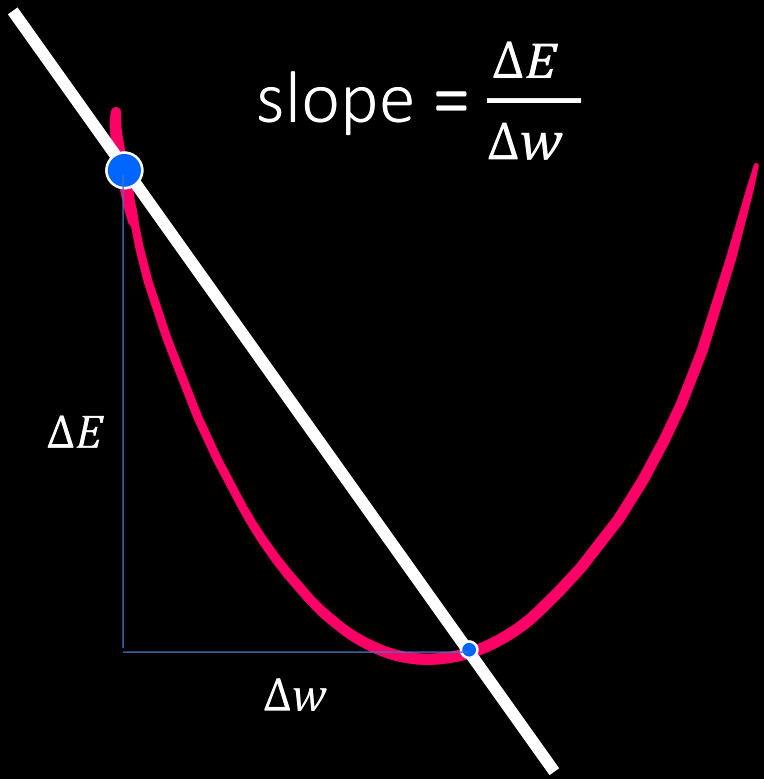


# How to update $w$

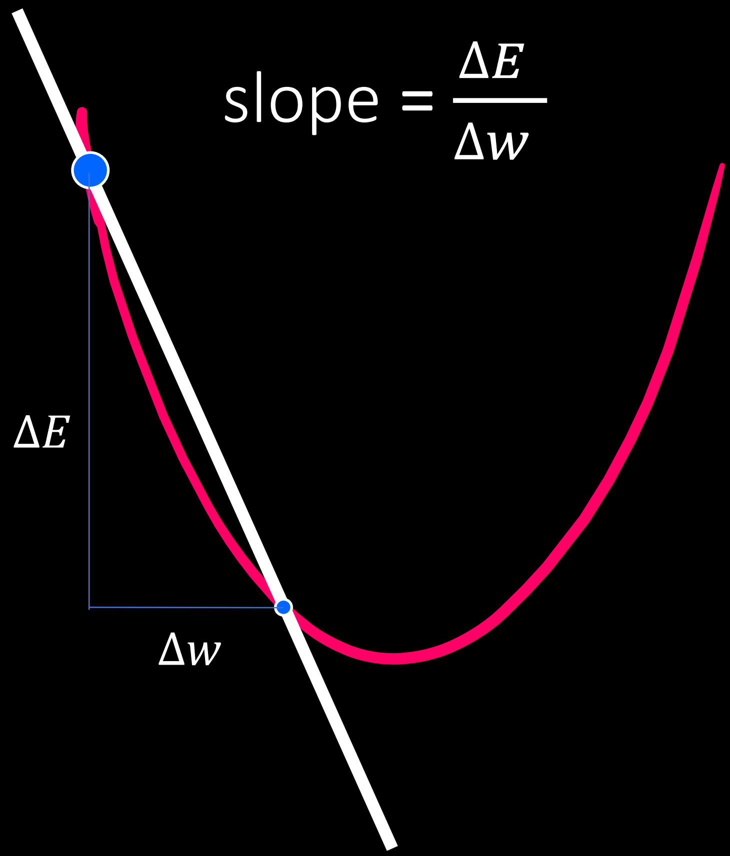
$$\text{slope} = \frac{\Delta E}{\Delta w}$$



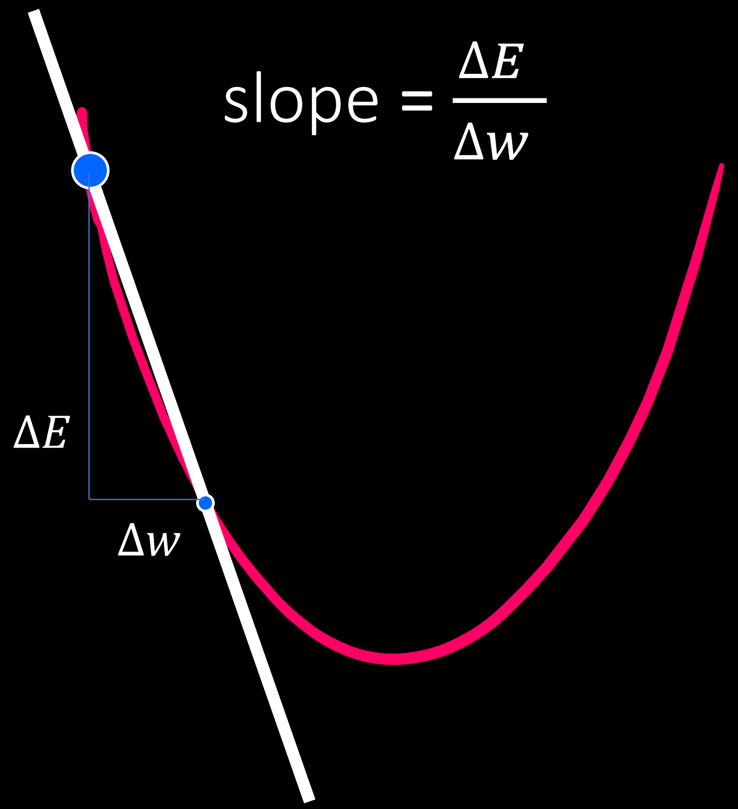
# How to update $w$



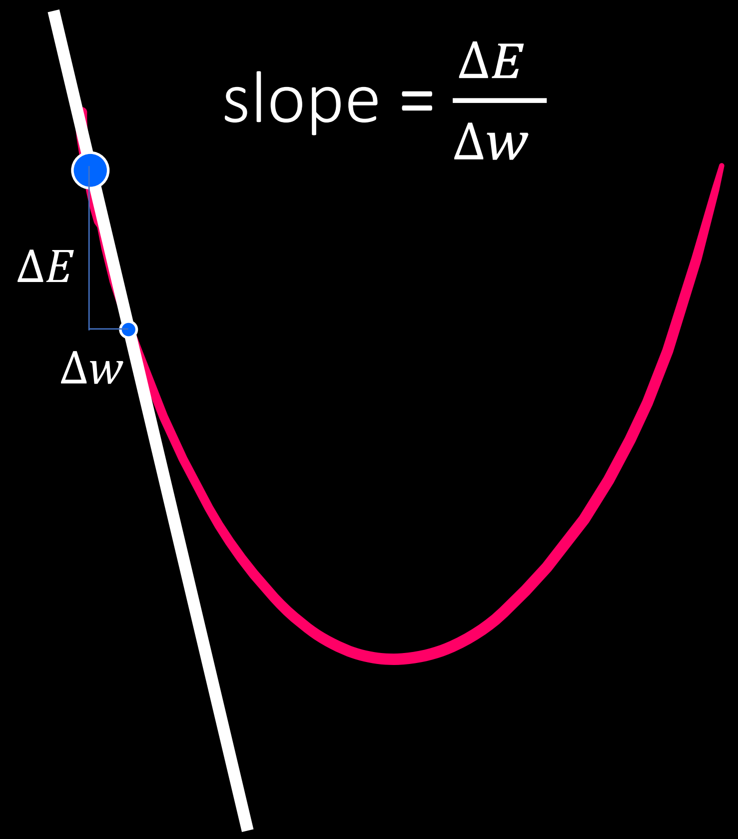
# How to update $w$



# How to update $w$



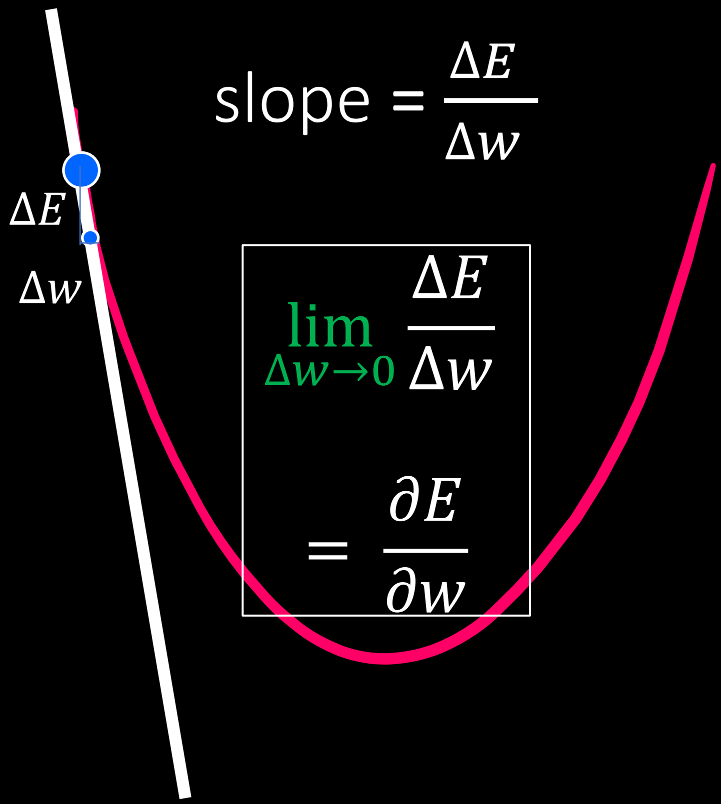
# How to update $w$



# How to update $w$

접선·Tangent line

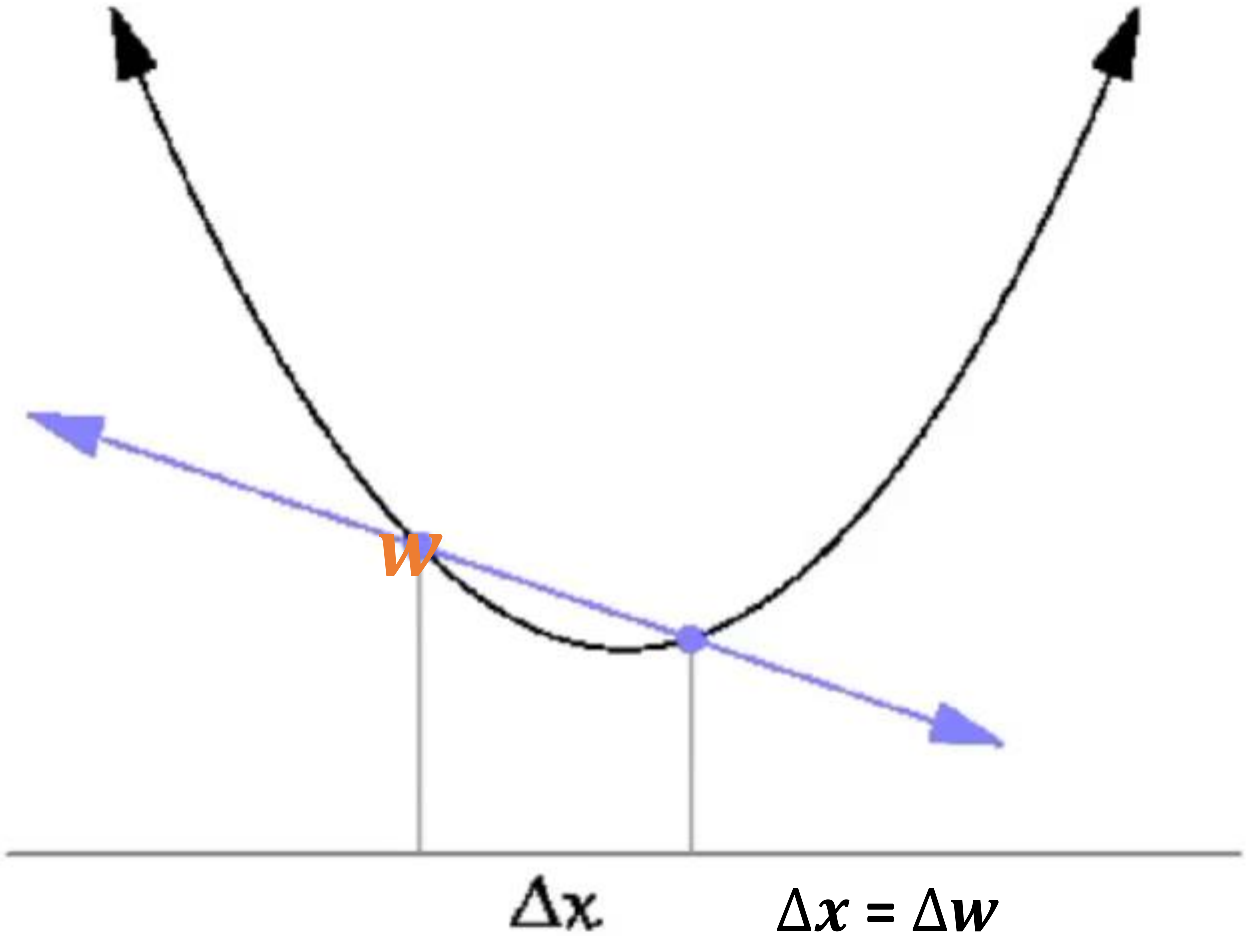
slope = $\frac{\Delta E}{\Delta w}$




$$\lim_{\Delta w \to 0} \frac{\Delta E}{\Delta w}$$

$$= \frac{\partial E}{\partial w}$$

# How to update $w$

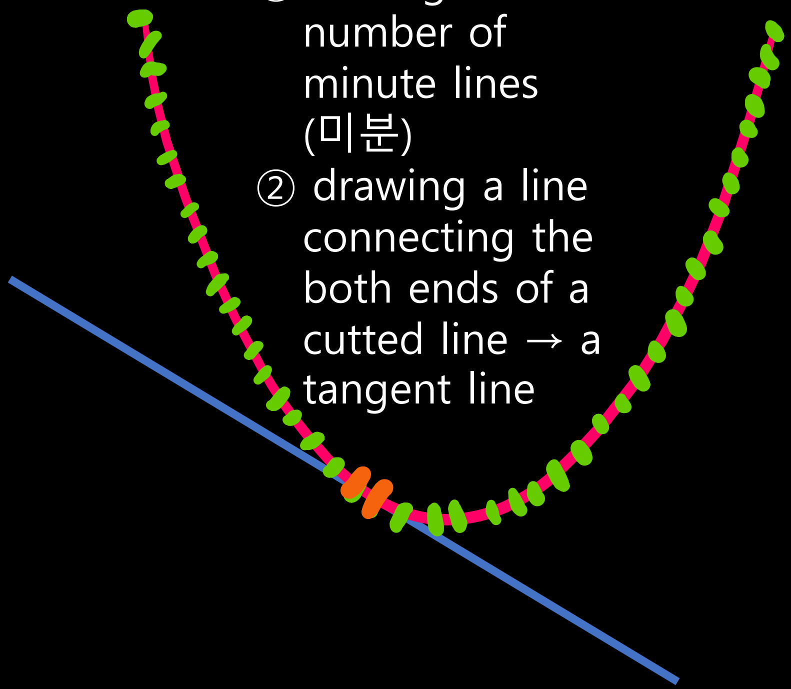


# How to update $w$

Numerical differentiation

① cutting into a number of minute lines (미분)

② drawing a line connecting the both ends of a cutted line → a tangent line



$$\lim_{\Delta w \to 0} \frac{\Delta E}{\Delta w}$$

$$= \frac{\partial E}{\partial w}$$

= 미분

# How to update $w$

$$w = w - \alpha * \text{Slope}$$

$$w = w - \alpha \frac{\partial E}{\partial w}$$

$\alpha =$ learning rate(ex, 0.1)

Squared Error

# L2 Loss function

# Advantages of **L2** loss

- Fast movement from both sides and **slow** fine tuning at the valley(center) area
- Different slope/gradient according to the value of $w$
- Steep slope means that the error is big and $w$ is far from the optimal area.
- We can get the slope(gradient) at any place(differentiable).

# In case of Absolute Error

- Always the same slope in the error graph regardless of the value of $w$
- Therefore, the same speed in the movement
- Not sure to get the w value which gives 0 error or almost 0
- No way to guess where the current $w$ is.
- No differentiable when $w$ is 1

# Multiple Data and Error

For 3 instances of data

| $x_i$ | $y_i$ |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

$$E = \frac{1}{3}\sum_{i=1}^{3}(wx_i - y_i)^2$$

Mean Square Error

desmos

Add (2, 2), (3, 3)

$$E = \frac{1}{3}\sum_{i=1}^{3}(wx_i - y_i)^2$$

Draw $(w, E)$

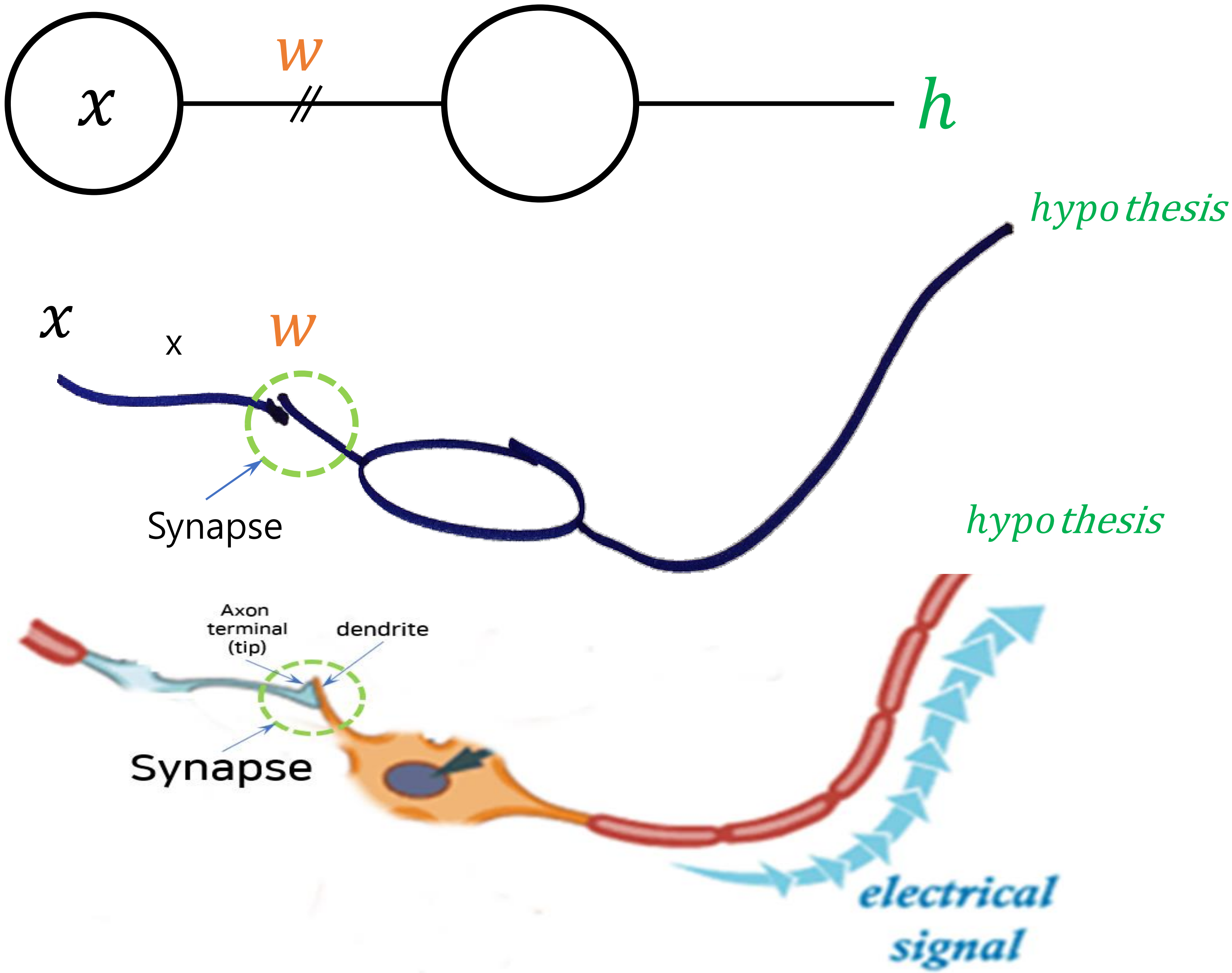
# Multiple Data

In case of $m$ instances,

An answer by a neuron

$$E = \frac{1}{m}\sum_{i=1}^{m}(wx_i - y_i)^2$$

Ground truth



x
w
h
hypothesis
x
x
w
Synapse
hypothesis
Axon terminal (tip)
dendrite
Synapse
electrical signal

# The meaning of slope

**Steep** slope $\frac{\Delta E}{\Delta w}$ $\frac{8}{1}$

The error $E$ will change drastically when we change $w$.

**Gentle** slope $\frac{\Delta E}{\Delta w}$ $\frac{1}{10}$

The error $E$ changes just a little bit when we change $w$.

Therefore, $\lim_{\Delta w \to 0} \frac{\Delta E}{\Delta w} \rightarrow \frac{\partial E}{\partial w}$

# Gradient(slope)

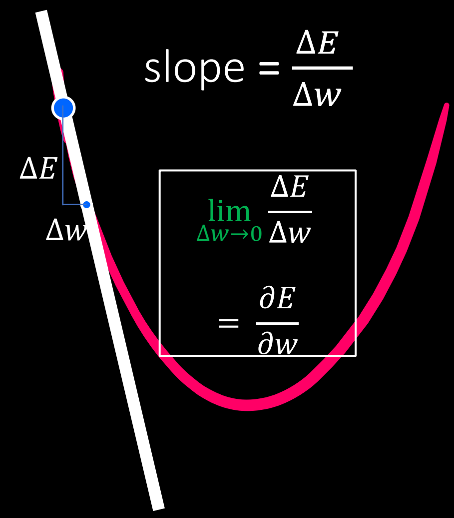
means the influence of $w$ change on error $E$.

# (Q) Compute the influence.

$$E = (wx - y)^2$$

when data $(x, y)$ is $(1, 1)$ and the value of $w$ is 3.

# Method1 numerical gradient




$$E = (w \cdot 1 - 1)^2$$

$w$: 3 -> $E$: 4

$w$: 3.00001 -> $E$: 4.00004

$\Delta w$=0.00001

$\Delta E$=0.00004

$$\frac{\Delta E}{\Delta w} = \frac{0.00004}{0.00001} = 4$$

Slope =

Influence of $w$ change = 4

# Method2 derivative, differential equation

$$E = (w \cdot 1 - 1)^2$$

$$\lim_{\Delta w \to 0} \frac{\Delta E}{\Delta w} = \frac{\partial E}{\partial w} = \frac{\partial}{\partial w}(w \cdot 1 - 1)^2$$

$$= 2(w \cdot 1 - 1)$$

Therefore, when $w = 3$,
the gradient is $2(3 - 1) = 4$

# How to update $w$ (Learning)

1. Initialize $w$ with a random value (*ex, 3*)

Loop

2. Get the gradient(slope) of $w$ change on $E$
3. Update $w$ using the below eq:

$$W = W - \alpha * \mathrm{slope}$$

Parameter Tuning

4. Go to step 2

# TensorFlow

- Machine learning framework by Google
- Tuning parameters including $w$ automatically for us
- Define $w$ to be tuned by TensorFlow.
- Define hypothesis($h$) and cost_function($E$)

# Linear Regression using TF

hypo=$w * x$

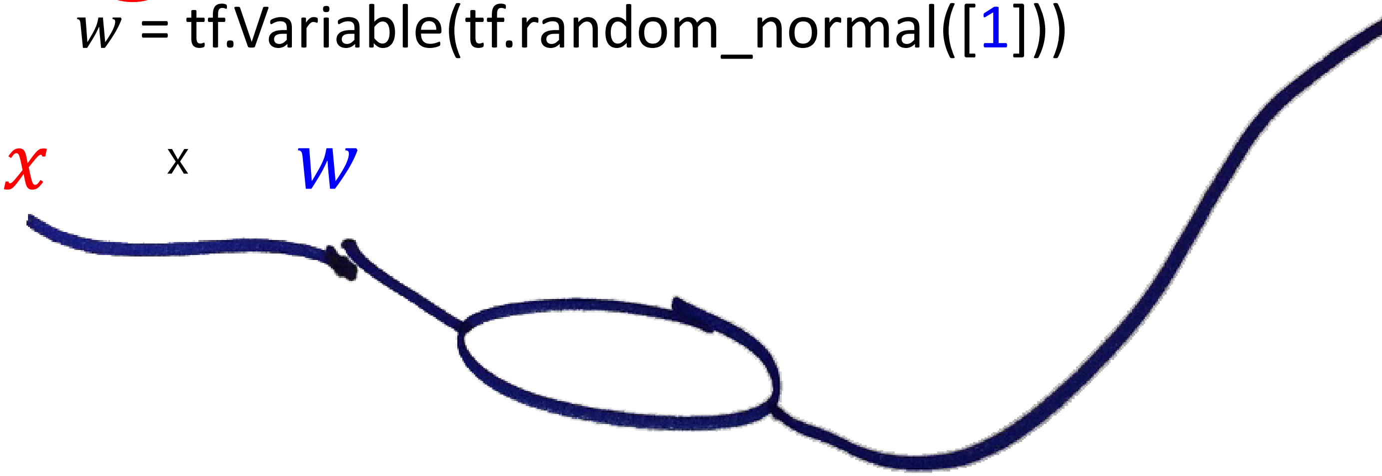
③

$w$ = tf.Variable(tf.random_normal([1]))

$h$ ④

$x$ x $w$

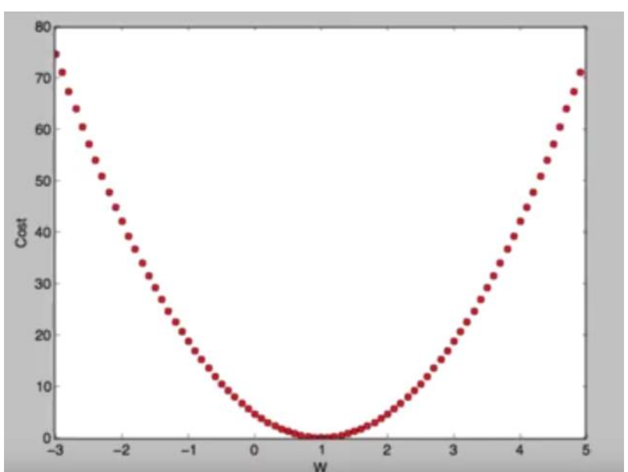
y = [1]



①

x = [1]

②




cost_function = (hypo – y) ** 2

⑤

$$E = (\text{hypo} - y)^2$$

# Download myml.git

https://github.com/yungbyun/myml.git

1) Run DOS prompt
2) git clone https://github.com/yungbyun/myml.git
3) Open using PyCharm (File | Open...)

# Lab 01.py

# Finding $w$ in linear regression

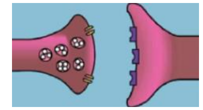
You also can find the source code here at Kaggle.com.
https://github.com/yungbyun/myml/blob/master/01.simple_with_keras

# [FYI] Change the first line as below:

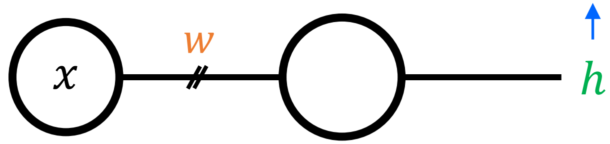
```
import tensorflow as tf
```

```
import tensorflow.compat.v1 as tf
tf.disable_v2_behavior()
```

```python
import tensorflow as tf
```



$$E = |w \cdot x - y| ** 2$$




```python
#----- training data
x_data = [1]
y_data = [1]
```

```python
#----- a neuron / neural network
w = tf.Variable(tf.random_normal([1]))
hypo = w * x_data
```

train operation to update $w$ to minimize error($E$)

```python
#----- learning
cost = (hypo - y_data) ** 2

train = tf.train.GradientDescentOptimizer(learning_rate=0.01).minimize(cost)

sess = tf.Session()
sess.run(tf.global_variables_initializer())

for i in range(1001):
    sess.run(train) #1-run, 1-update of w -> 1001 updates

    if i % 100 == 0:
        print('w:', sess.run(w), 'cost:', sess.run(cost))
```

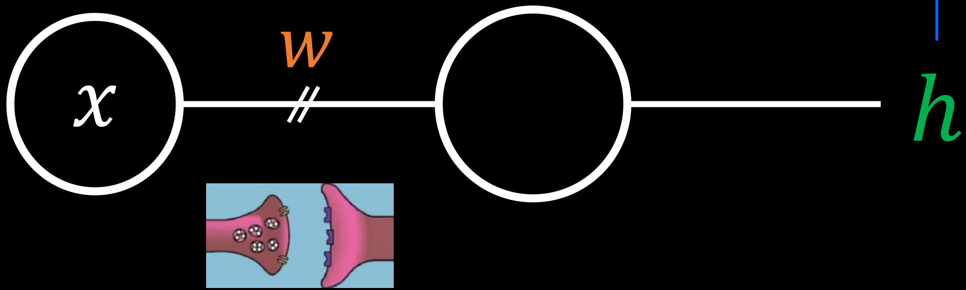
```python
#----- testing(prediction)
x_data = [2]
print(sess.run(x_data * w))
```

`sess.run(train)`

# How to update $w$ in TensorFlow

## Computation Graph

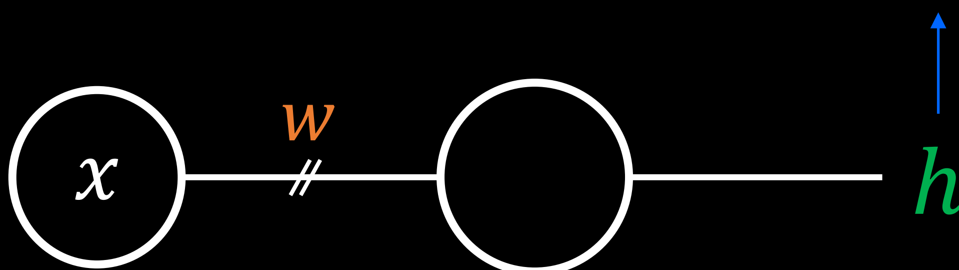
$$E = (h - y)^2$$




| $x$ | $y$ |
|---|---|
| 1 | 1 |
| 2 | 2 |

a.csv

$$E = (wx - y)^2$$



# Loss/Error function

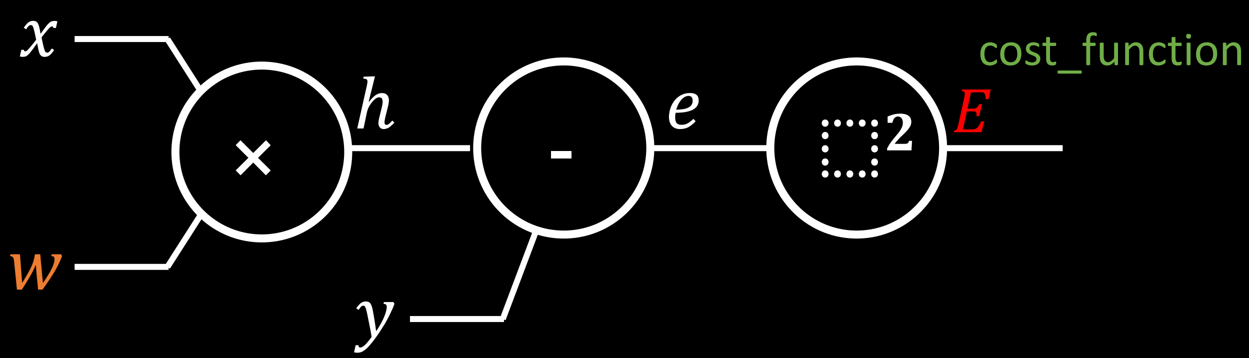
$$E = (wx - y)^2$$

- The part representing a neuron
- Where is a synapse?
- Which one is an input data?
- The output of a neuron
- Find hypothesis
- Find a correct answer or ground truth.
- Imagine $E$ having multiple inputs.

# Computation Graph of $E$

$$E = (w \cdot x - y)^2$$

```
hypo = w * x
cost_function(E) = (hypo - y) ** 2)
```

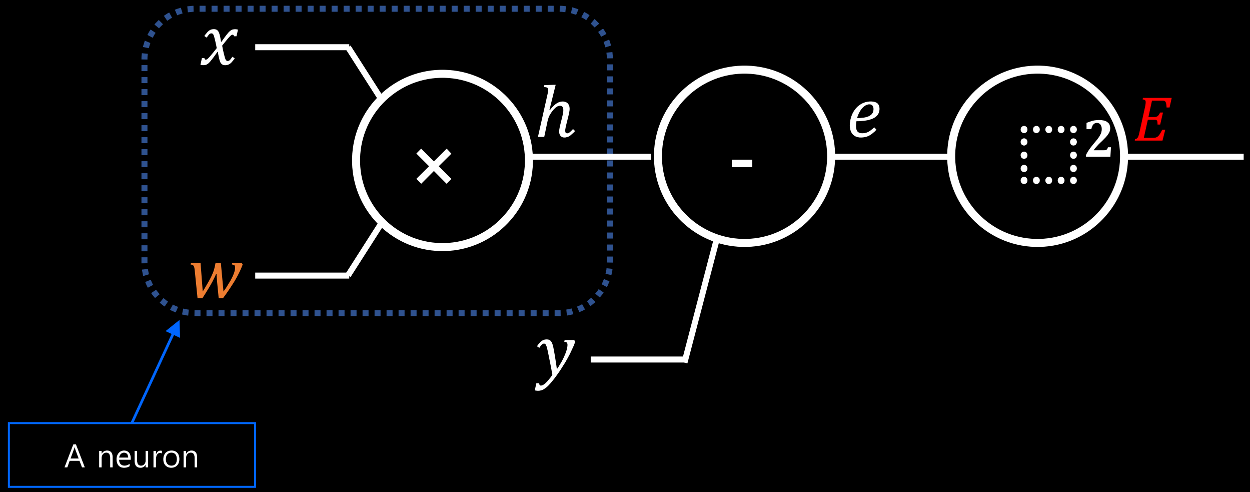



| $x$ | $y$ |
|---|---|
| 1 | 1 |
| 2 | 2 |

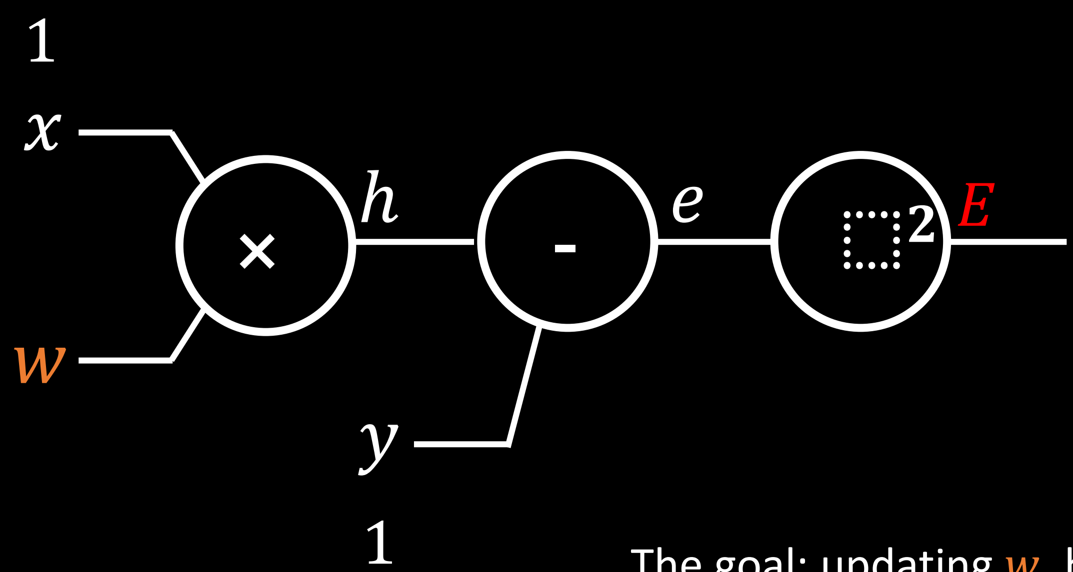
a.csv

# Computation Graph of $E$

$$E = (w \cdot x - y)^2$$



# Computation Graph of $E$

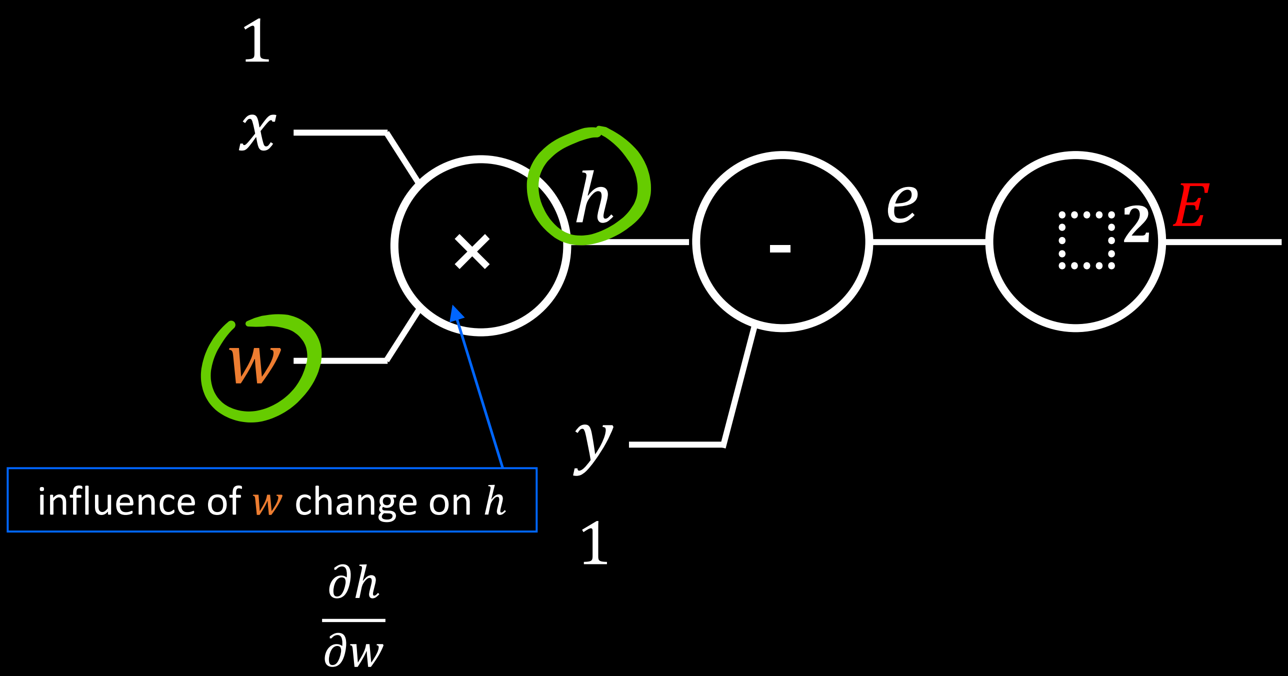



The goal: updating $w$, how?

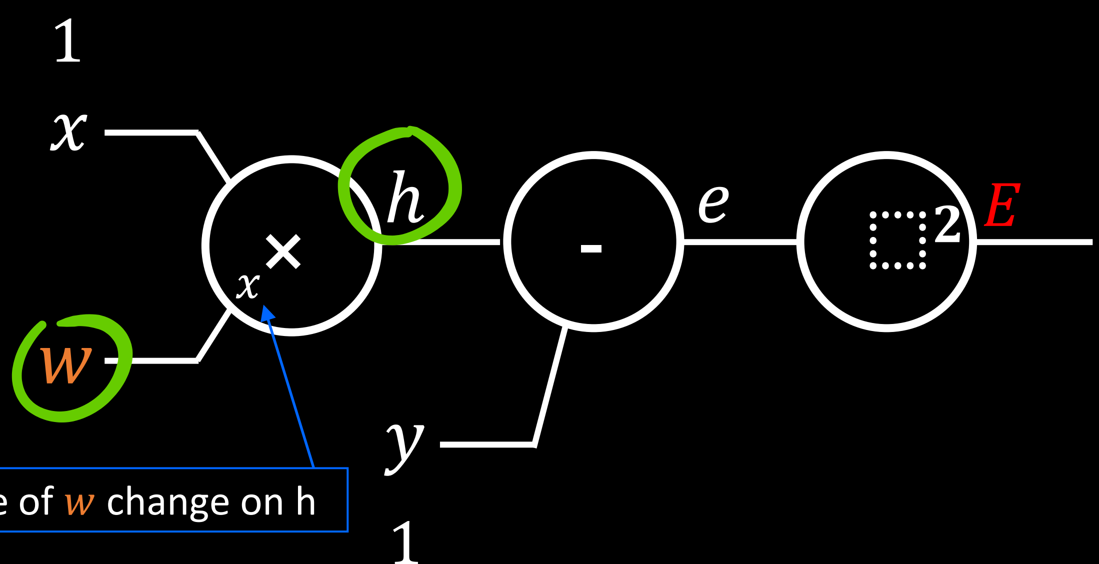
| $x$ | $y$ |
|---|---|
| 1 | 1 |

a.csv

# Computation Graph of $E$



# Computation Graph of $E$

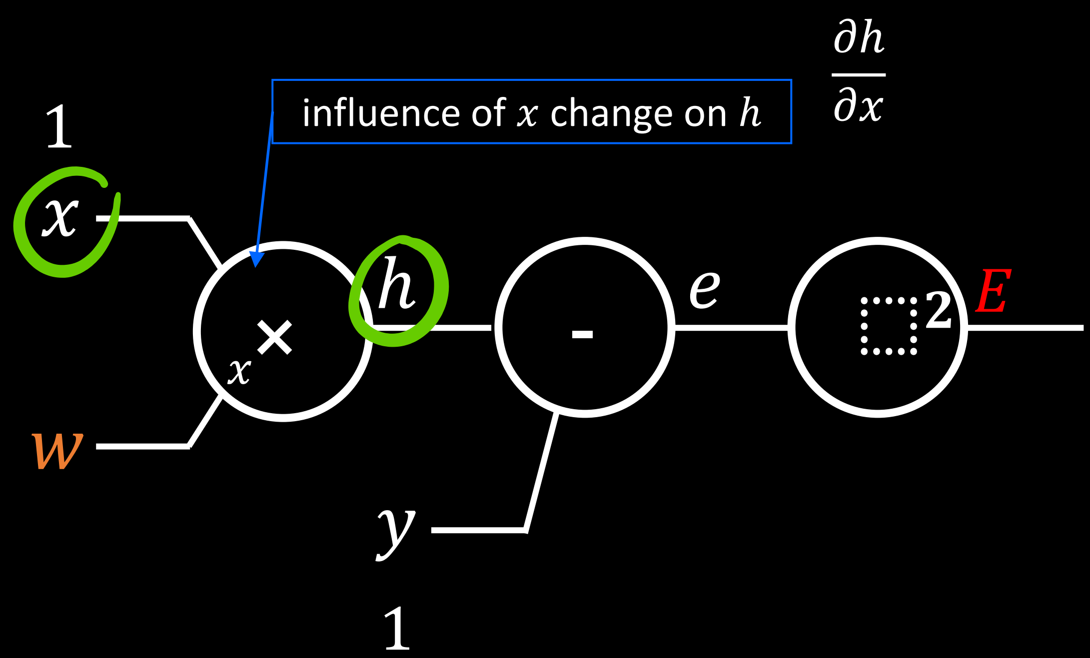



$$\frac{\partial h}{\partial w}$$

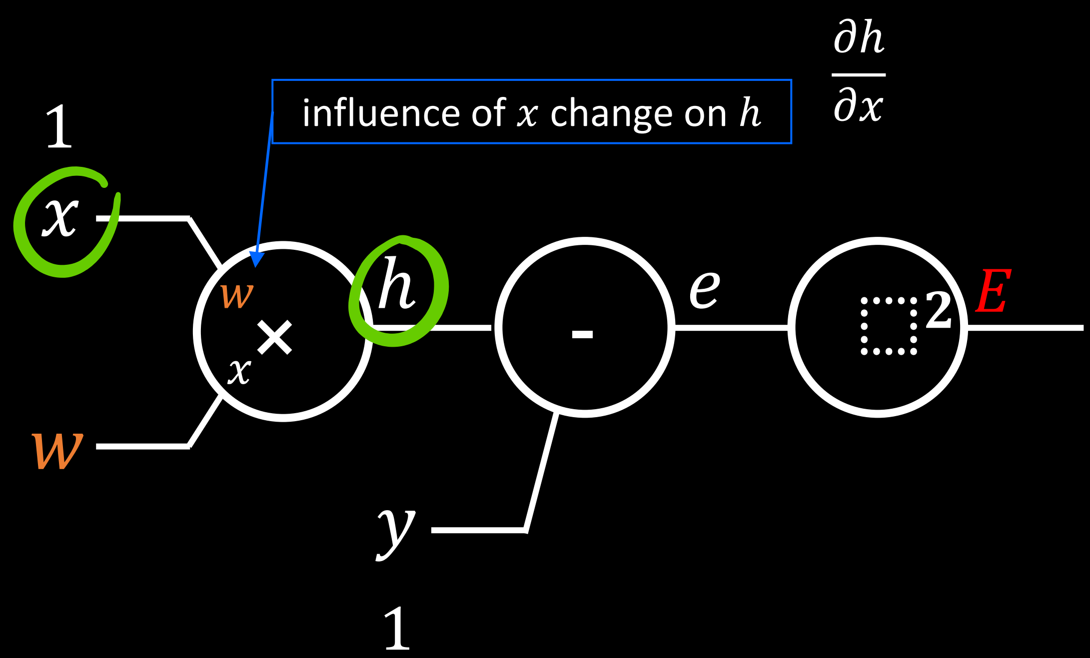
# Local gradient

지역 기울기

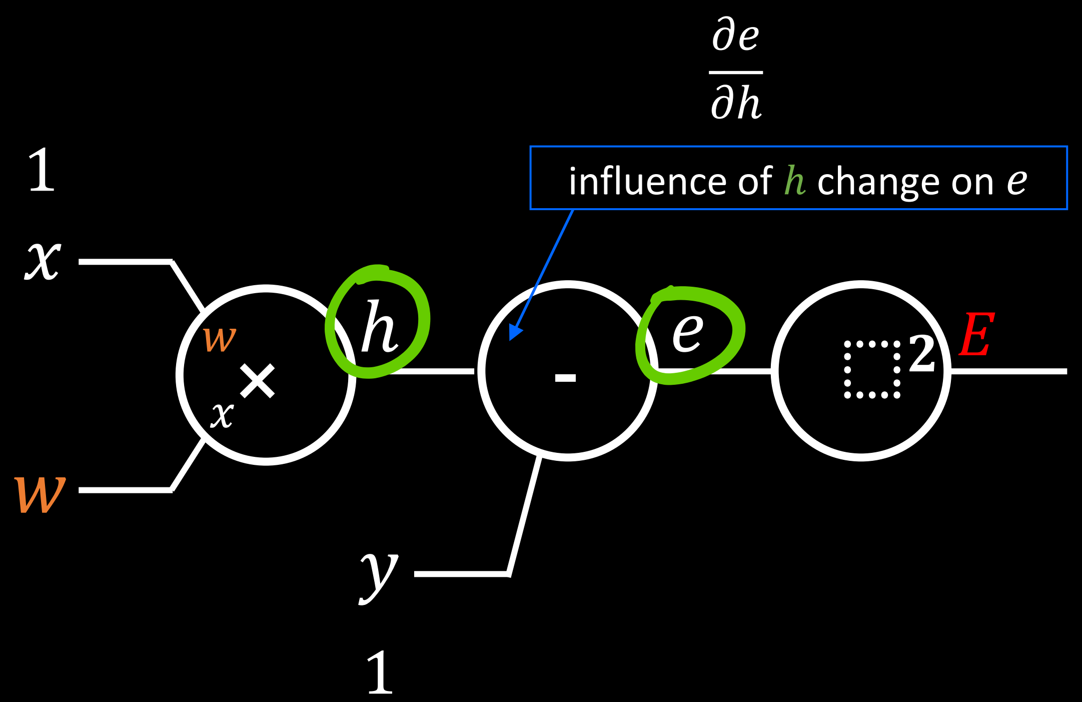
# Computation Graph of $E$



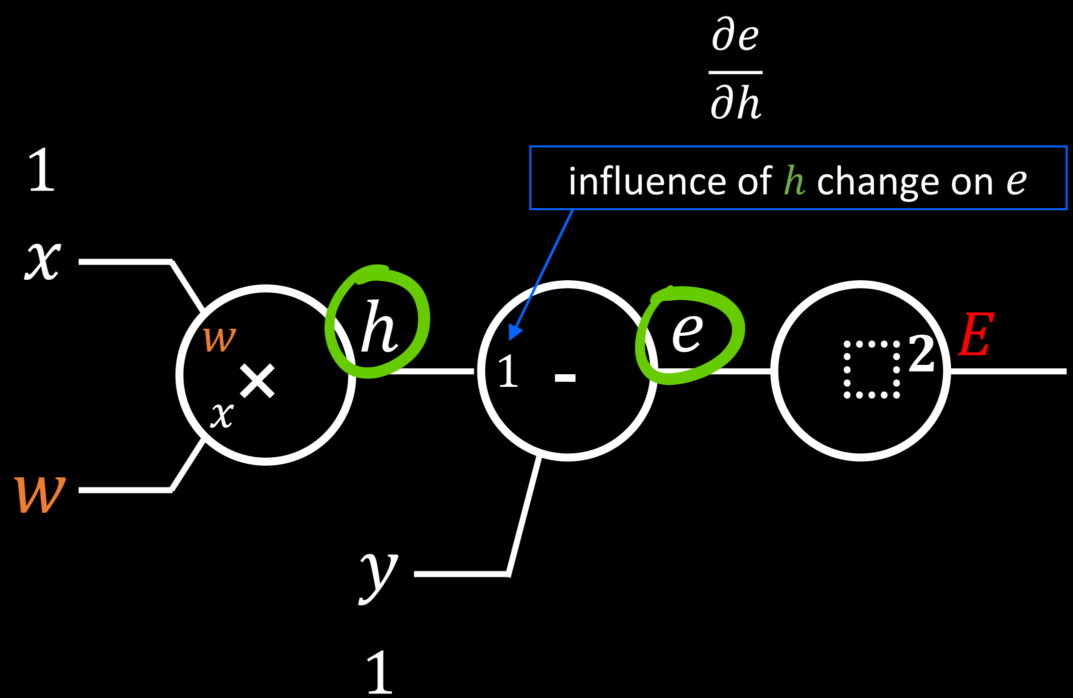
# Computation Graph of $E$



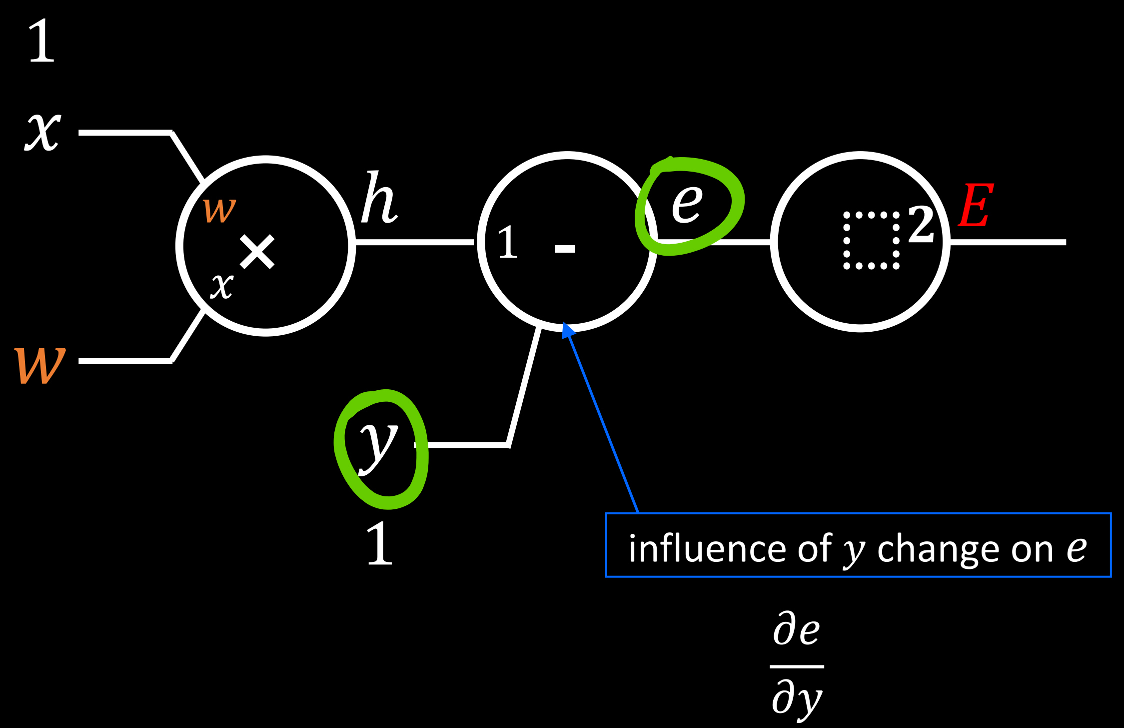
# Computation Graph of $E$



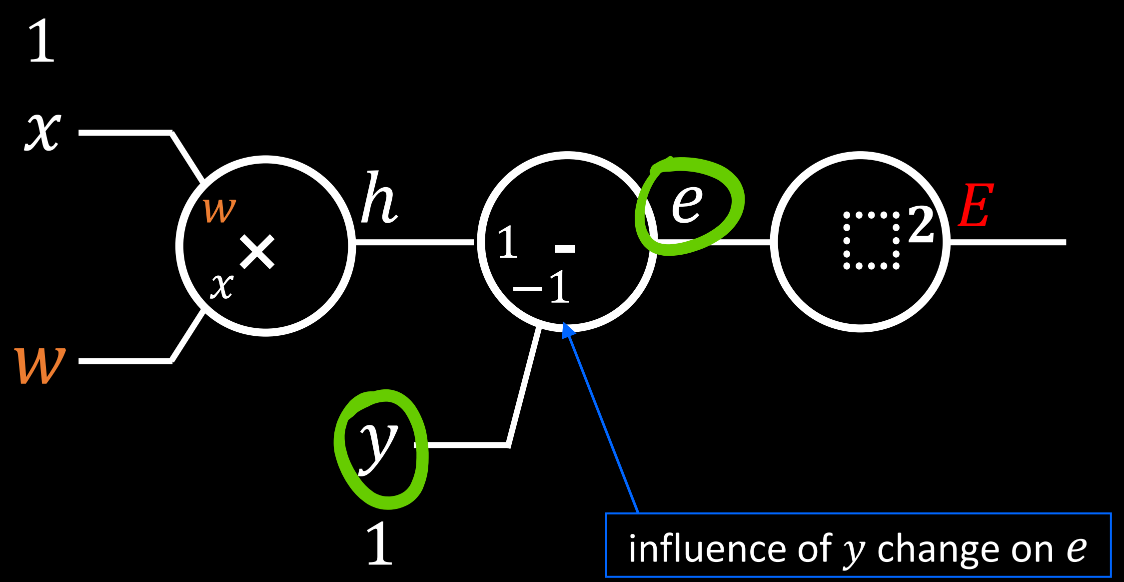
# Computation Graph of $E$



# Computation Graph of $E$

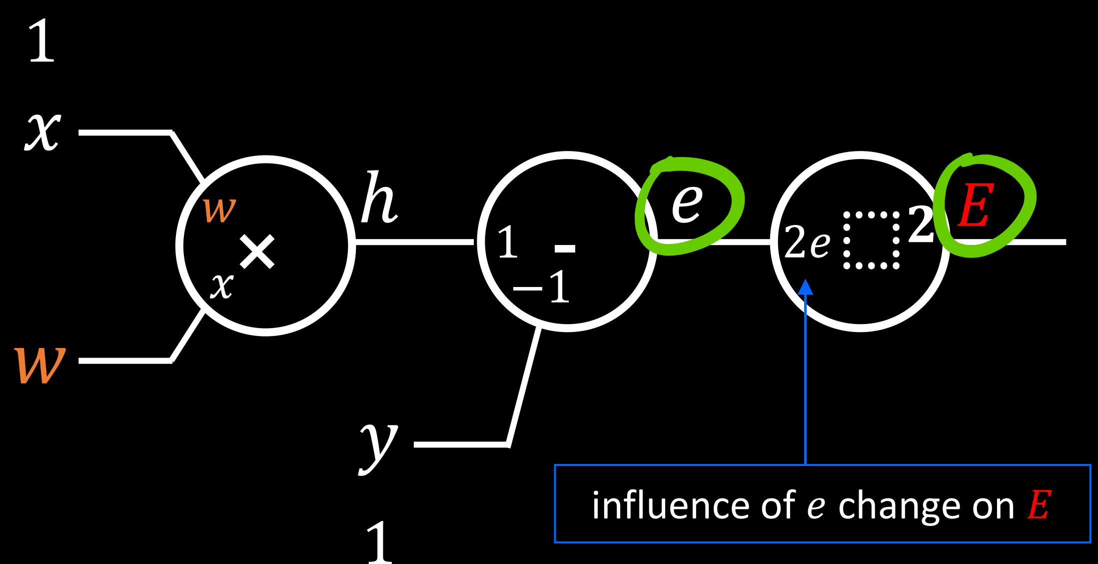


# Computation Graph of $E$



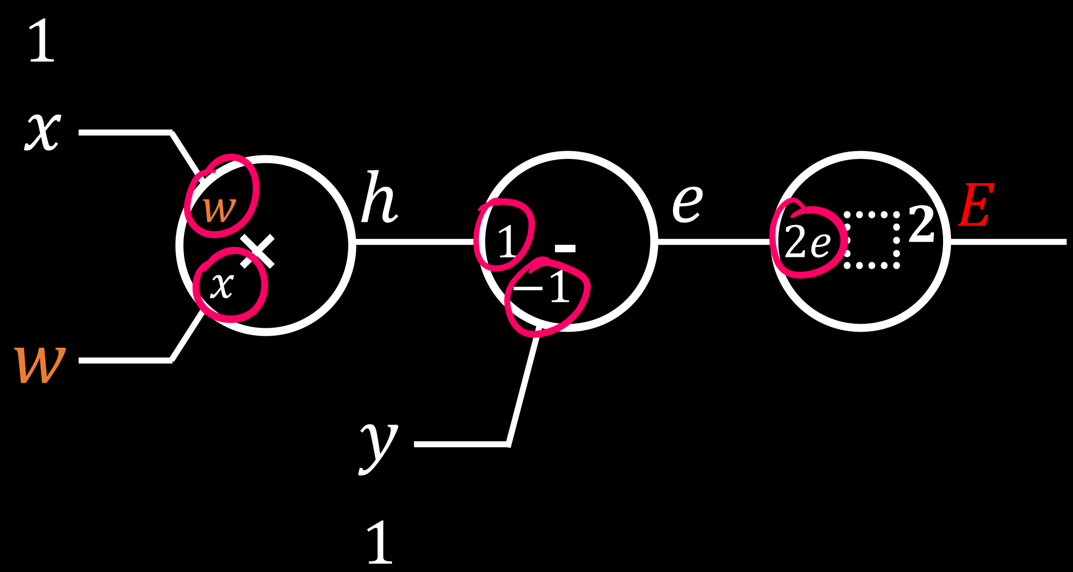

$$\frac{\partial e}{\partial y}$$

# Computation Graph of $E$




$$\frac{\partial E}{\partial e}$$

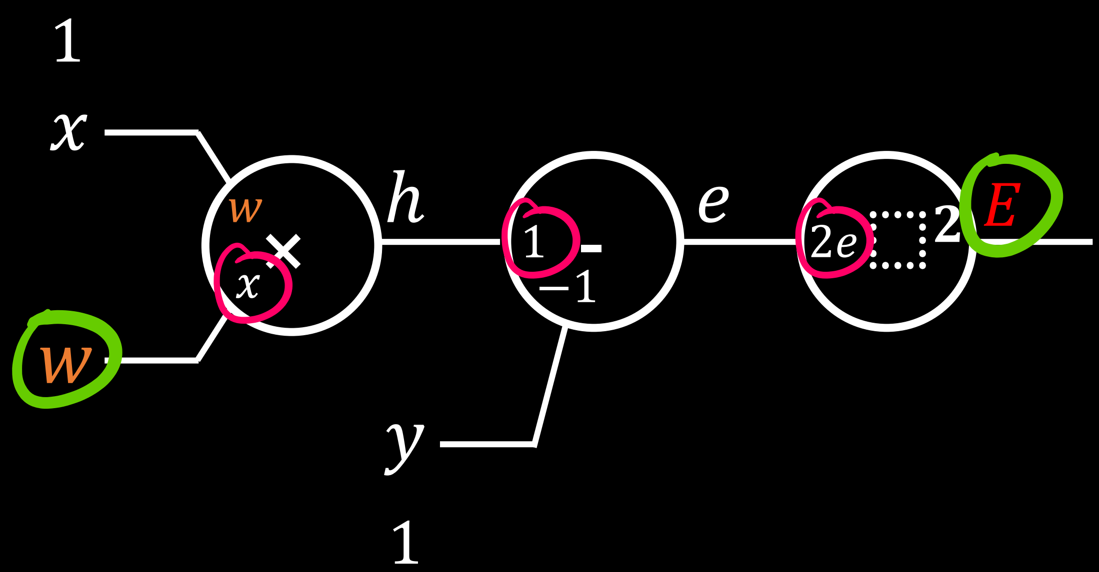
# 5 Local Gradients in gates




“

How can we get the influence of $w$ change on $E$?

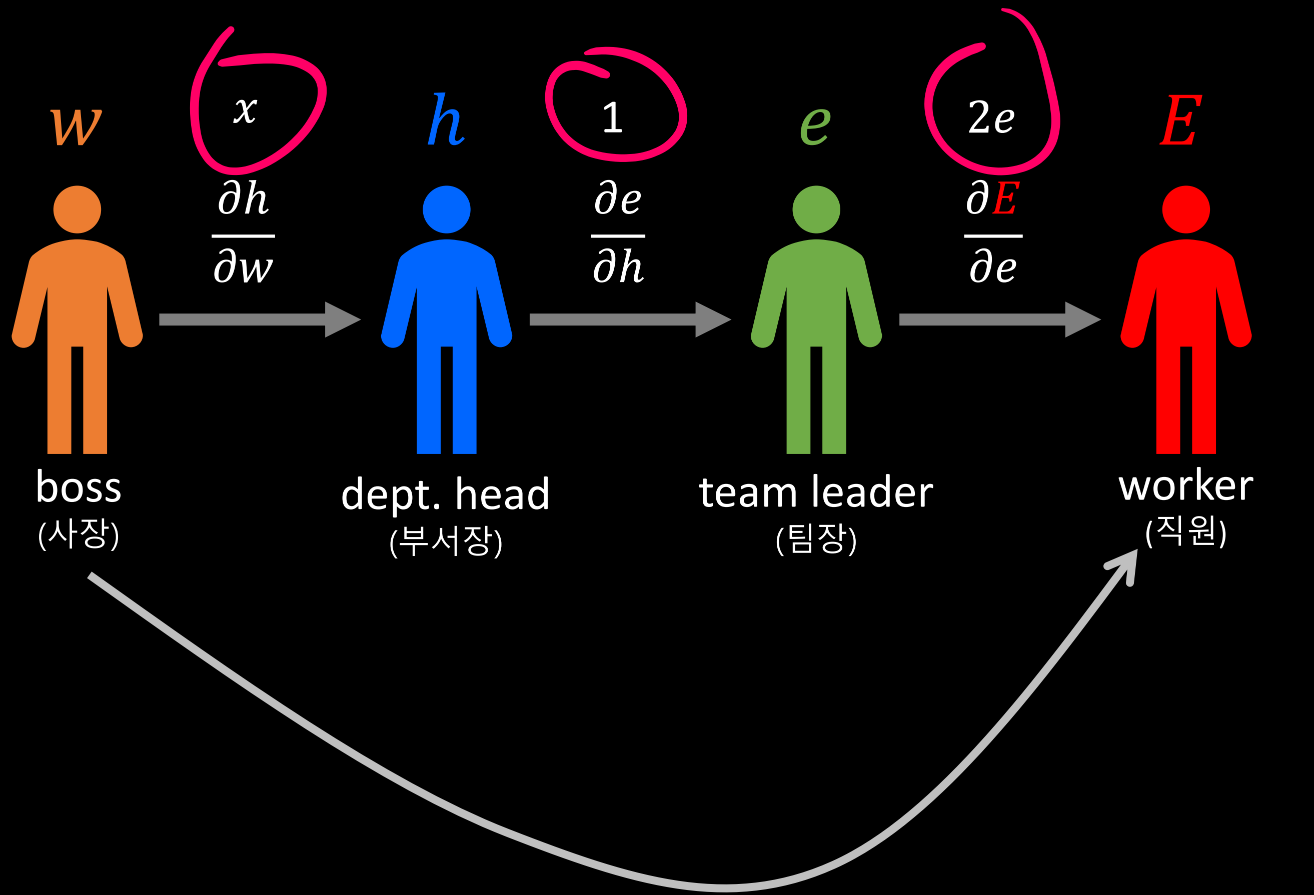
# 3 Local Gradients in gates




“

How can we get the influence of $w$ change on $E$?

# Influence between persons



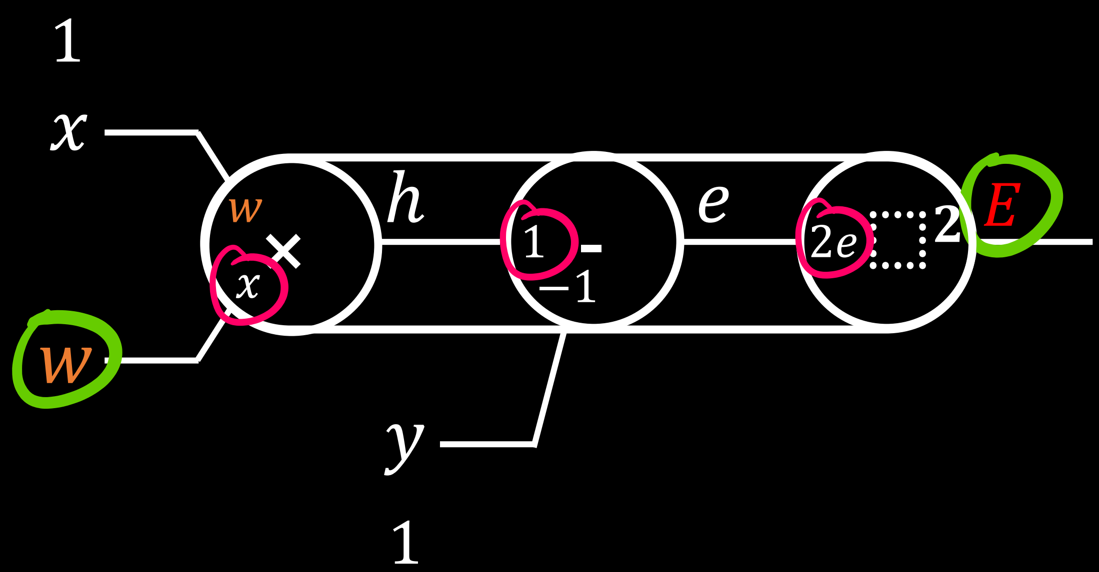

$$\frac{\partial E}{\partial w} = \frac{\partial h}{\partial w} \cdot \frac{\partial e}{\partial h} \cdot \frac{\partial E}{\partial e} = x \cdot 1 \cdot 2e$$

# The influence of $w$ change on $E$

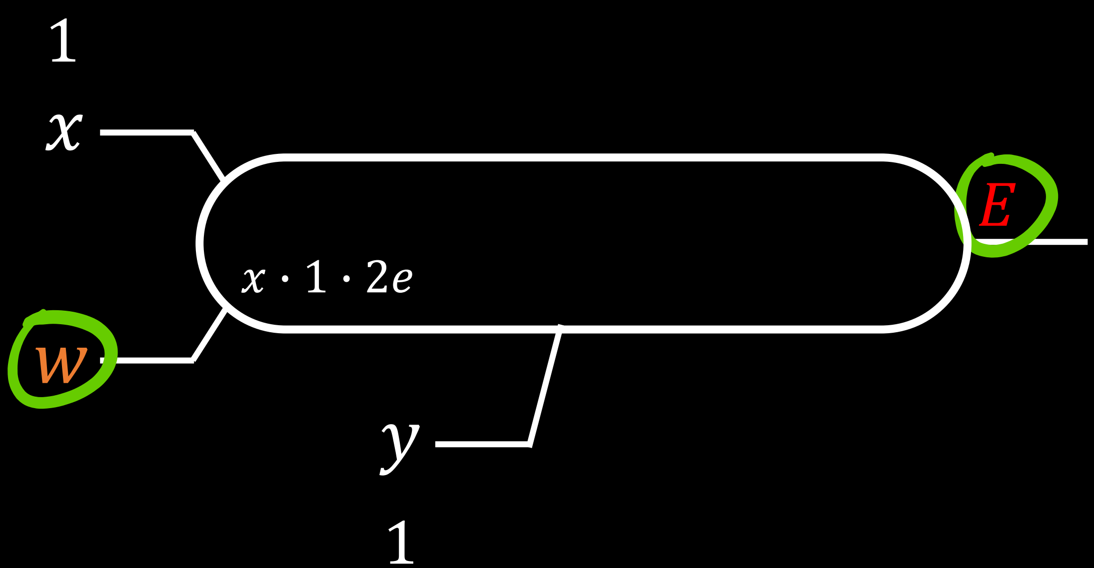
$$\frac{\partial E}{\partial w} = \frac{\partial h}{\partial w} \times \frac{\partial e}{\partial h} \times \frac{\partial E}{\partial e}$$

**Chain rule**

# Merging gates

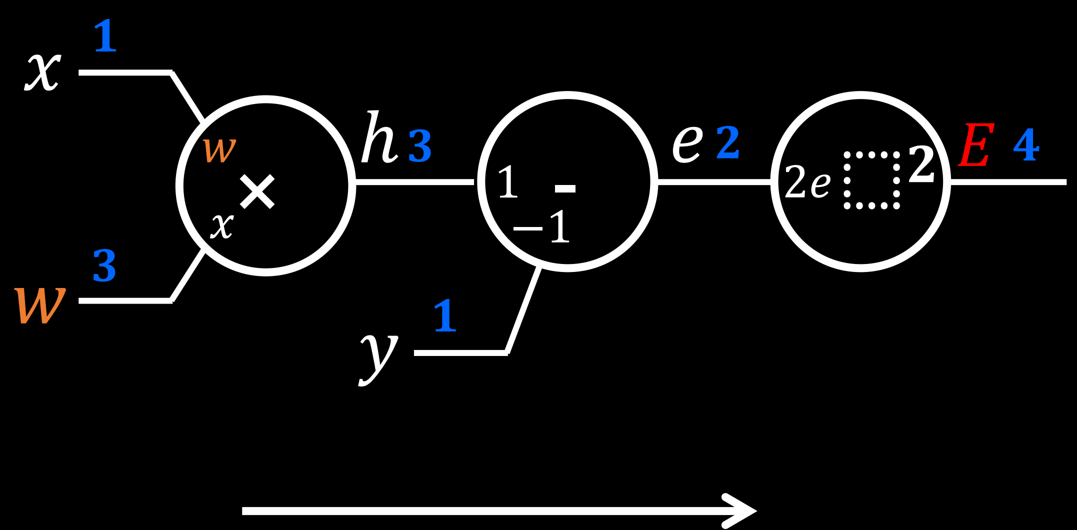


# *Composite* gates



# $Forward$ propagation

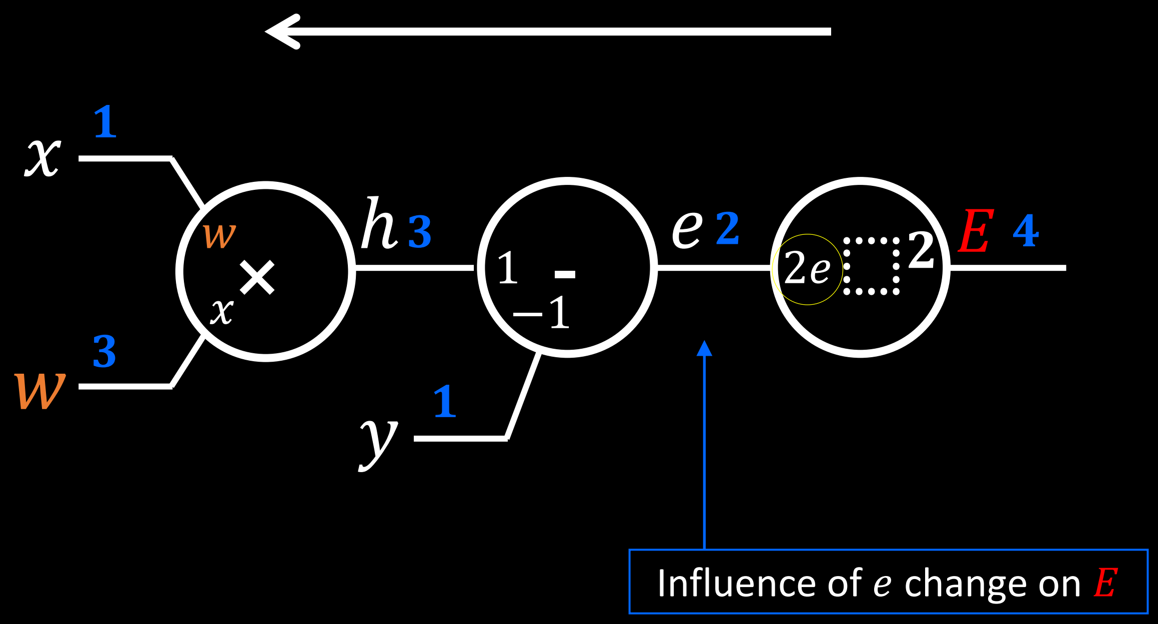
Let $(x, y) = (1, 1)$ and $w = 3$, then compute $E$.



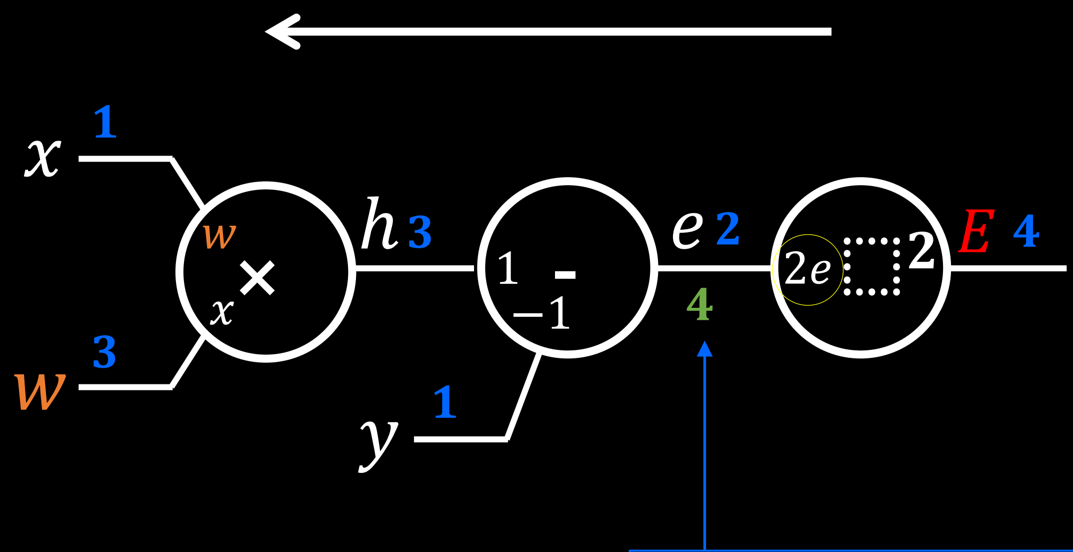
Error is big(4),
so, let's update $w$

using *back*-propagation.

# *Back*-propagation



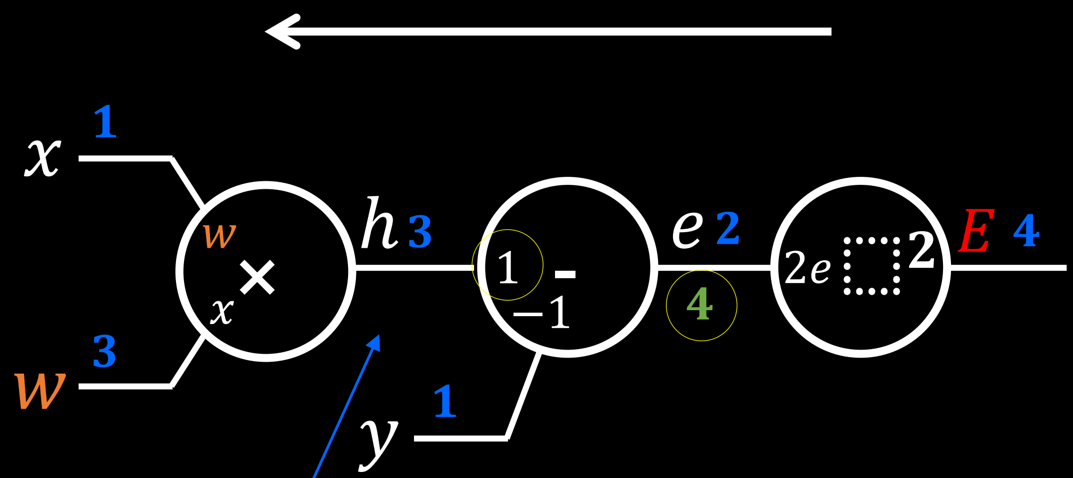
# *Back*-propagation




Influence of $e$ change on $E$

$$\frac{\partial E}{\partial e} = 2e$$

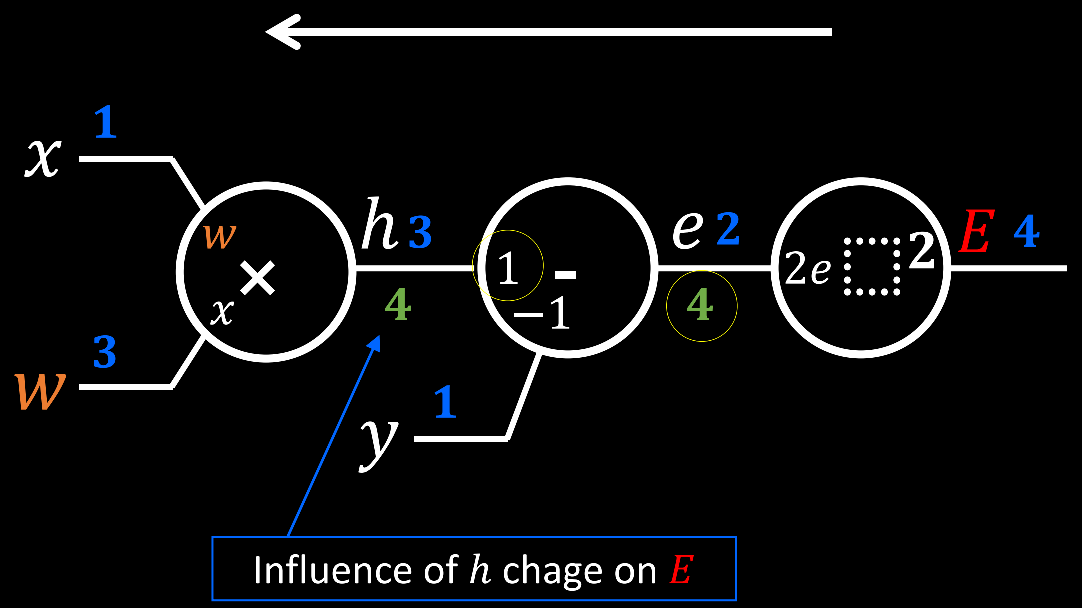
# $Back$-propagation



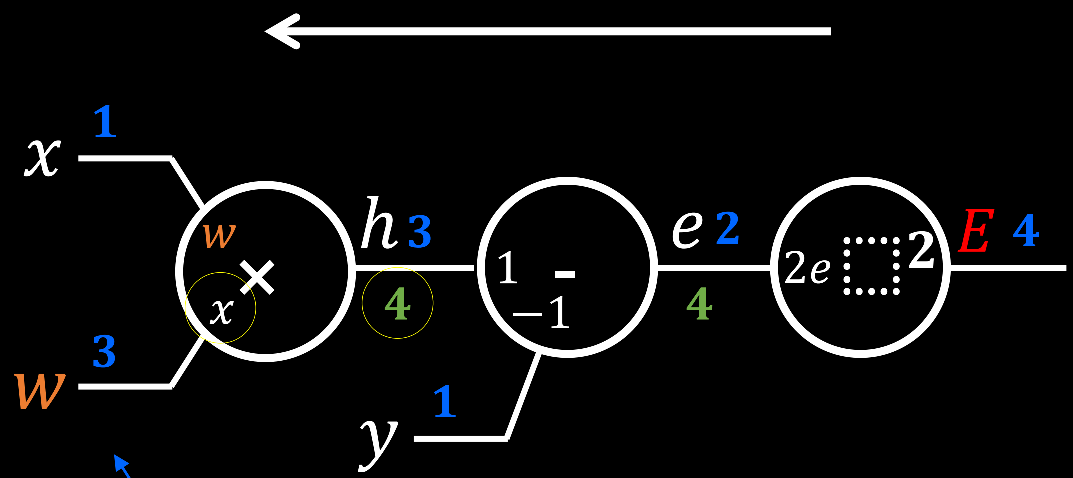

Influence of $h$ chage on $E$

$$\frac{\partial E}{\partial h} = \frac{\partial e}{\partial h} \cdot \frac{\partial E}{\partial e} = 1 \cdot 4$$

# $Back$-propagation

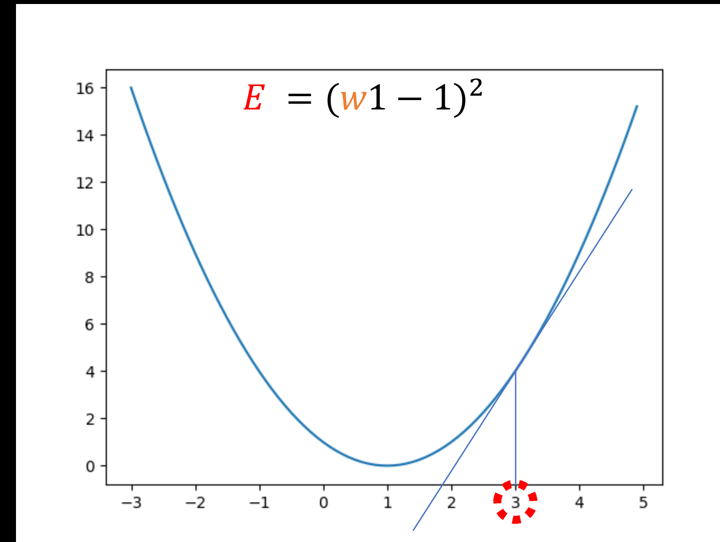


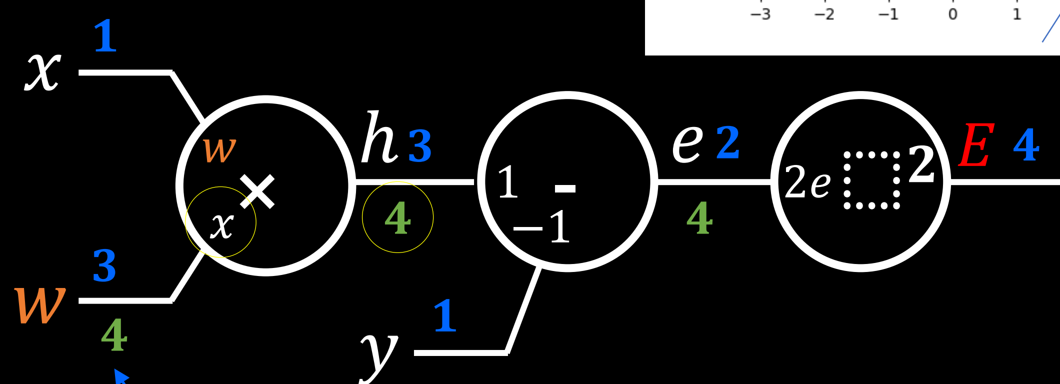
# $Back$-propagation




Influence of $w$ change on $E$

$$\frac{\partial E}{\partial w} = \frac{\partial h}{\partial w} \cdot \frac{\partial E}{\partial h} = 1 \cdot 4$$




Influence of $w$ change on $E$

$$\frac{\partial E}{\partial w}$$

$Back$-propagation,
the process to apply
chain rules

$$\frac{\partial E}{\partial w}$$

Gradient decent (경사하강)
using slope

$$w = 3 - 0.1 * 4 \quad \frac{\partial E}{\partial w}$$

$$w = 2.6$$

Tuned parameter after 1 step learning

After enough number of steps(epochs), the parameter $w$ will be optimized properly.

# *Back*–propagation

by Paul Webros (1974, 1982) and

Geoffrey Hinton (1986)

Prof. Univ. of Toronto,
Google Brain
Yann LeCun (his post doc)

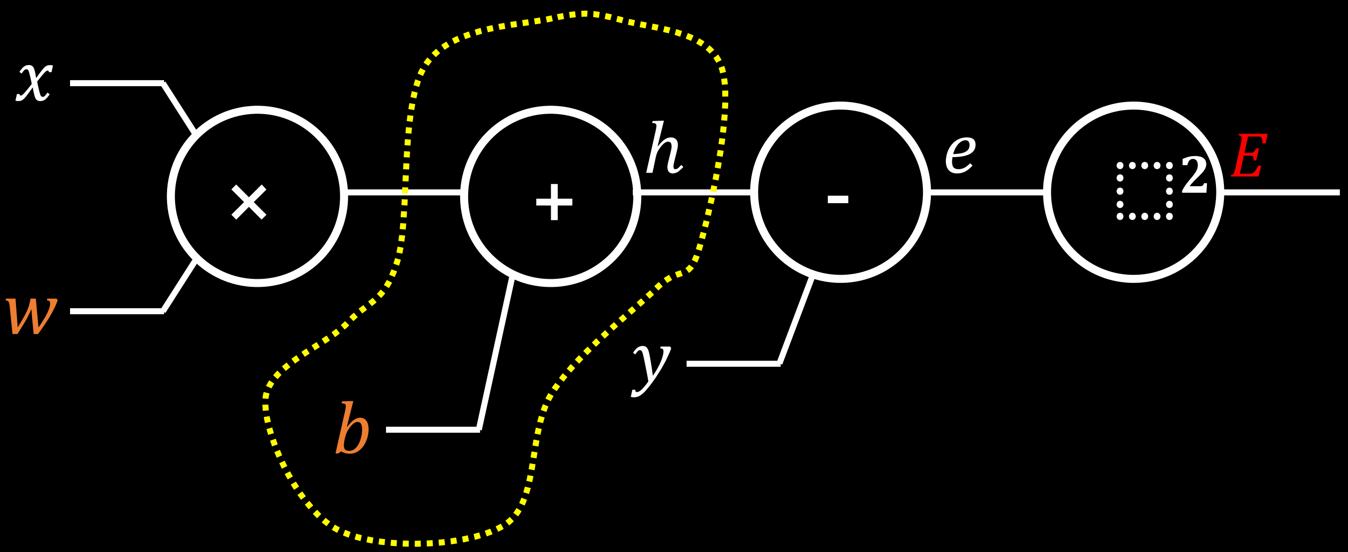
```python
import tensorflow as tf

#----- training data
x_data = [1]
y_data = [1]

#----- a neuron / neural network
w = tf.Variable(tf.random_normal([1]))
hypo = w * x_data

#----- learning
cost = (hypo - y_data) ** 2

train = tf.train.GradientDescentOptimizer(learning_rate=0.01).minimize(cost)

sess = tf.Session()
sess.run(tf.global_variables_initializer())

for i in range(1001):
    sess.run(train) # 1-run, 1-update of w -> 1001 updates

    if i % 100 == 0:
        print('w:', sess.run(w), 'cost:', sess.run(cost))

#----- testing(prediction)
x_data = [2]
print(sess.run(x_data * w))
```

train operation to update $w$ to minimize cost(error)

# **Extension** of the Graph

- adding bias/shift $b$ (one more plus gate)

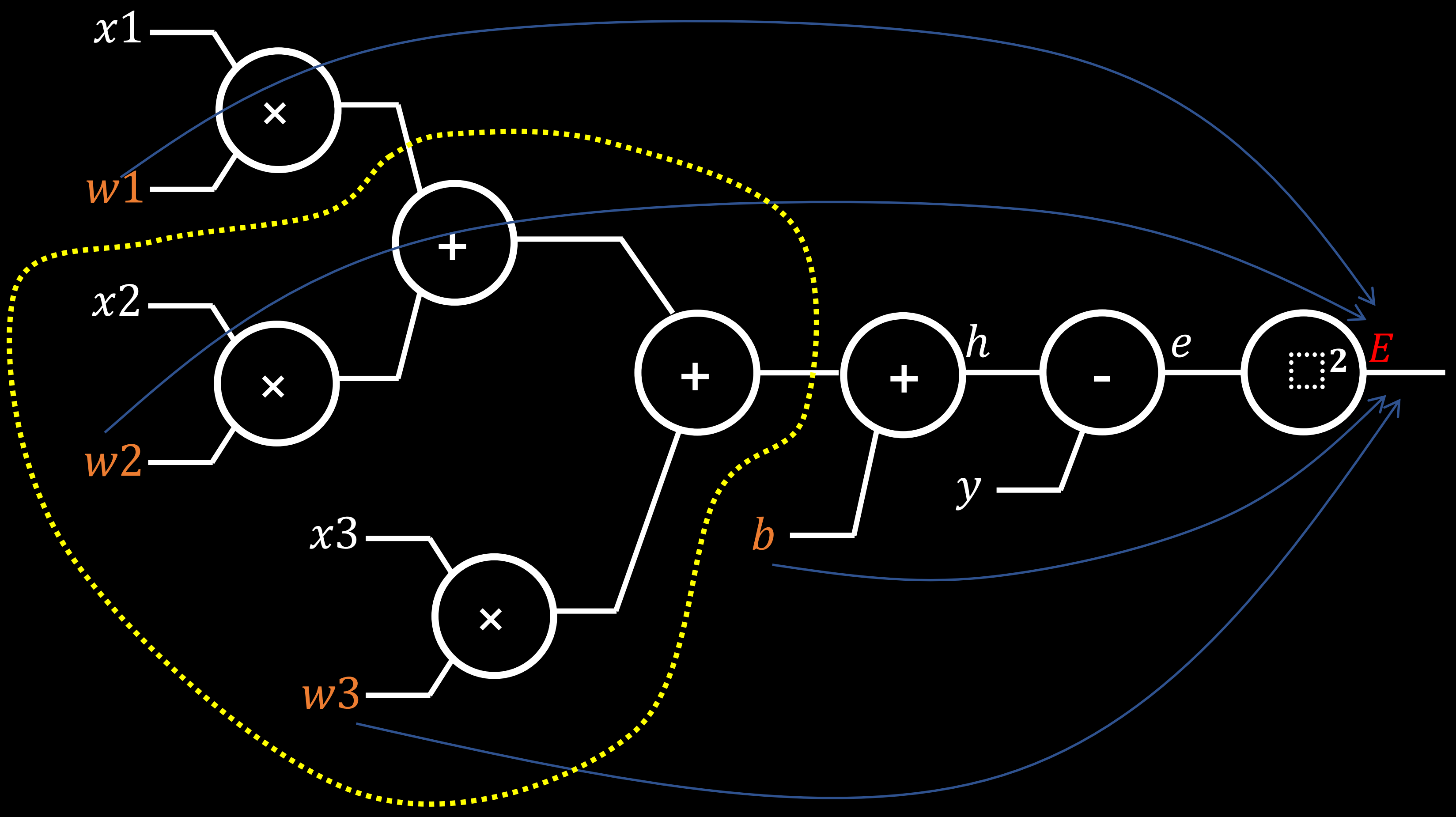
$$E = ((wx \boxed{+ b}) - y)^2$$



# **Extension** of the Graph

- a neuron with 3 inputs (2 more + gate)

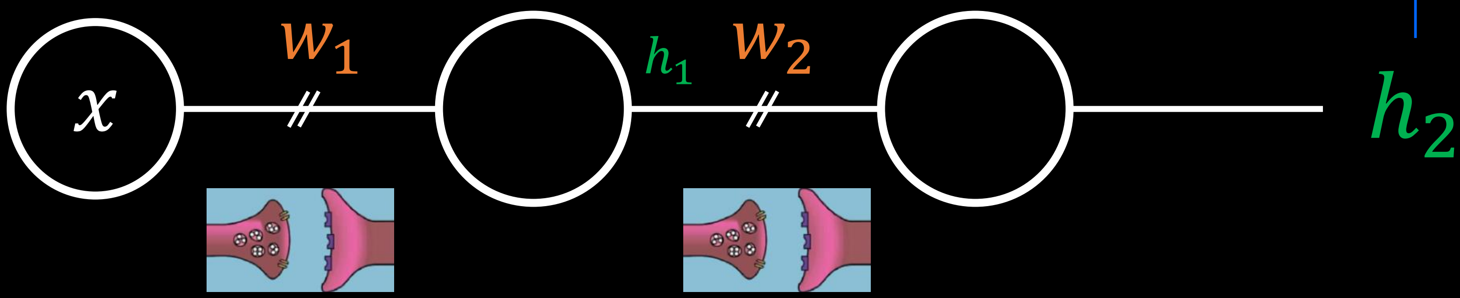
$$E = ((w1x1 \boxed{+ w2x2 + w3x3} + b) - y)^2$$



# **Extension** of the Graph

- Two neurons, 3-layer

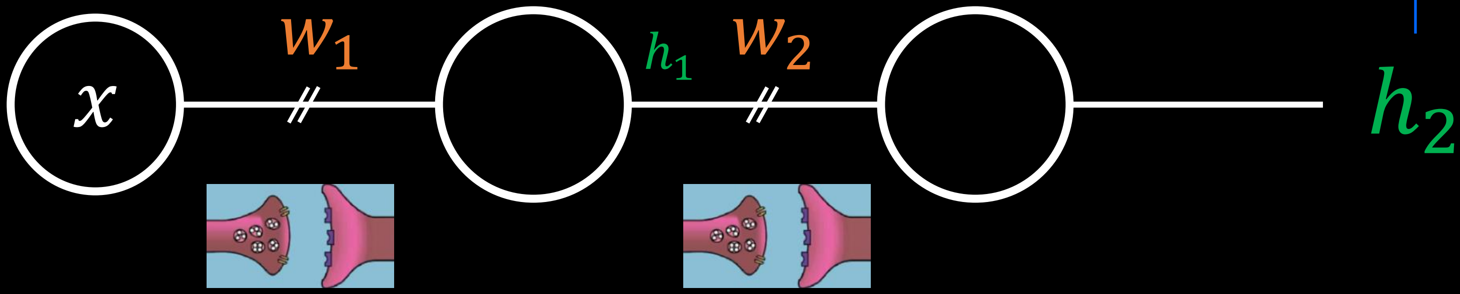
$$E = (w_2 h_1 - y)^2$$



# **Extension** of the Graph

- Two neurons, 3-layer

$$E = (w_2 \cdot (w_1 x) - y)^2$$

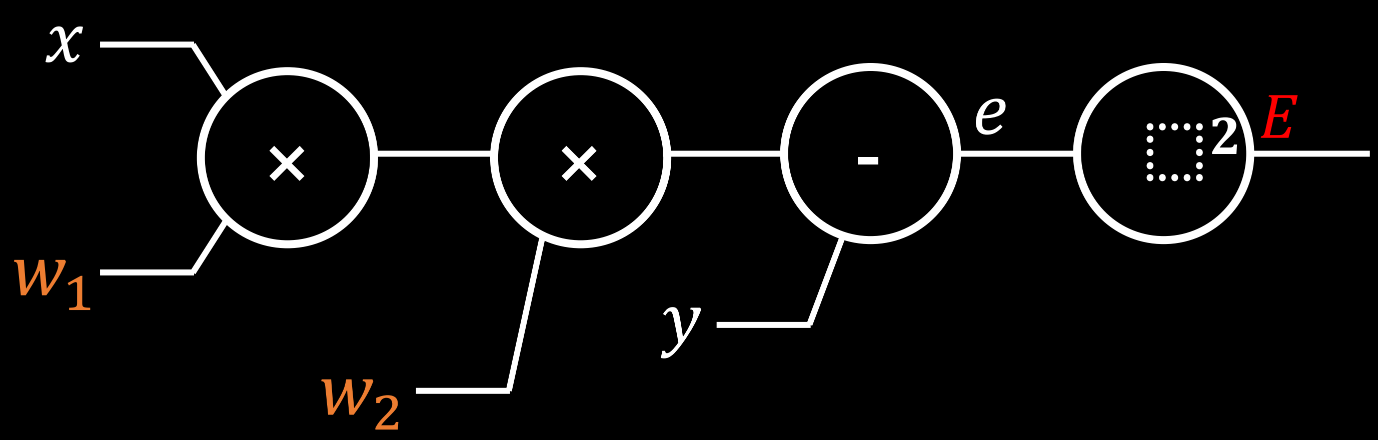



The hypothesis($h_2$) is the linear combination of coefficients($w_1$, $w_2$), so it is a linear model.

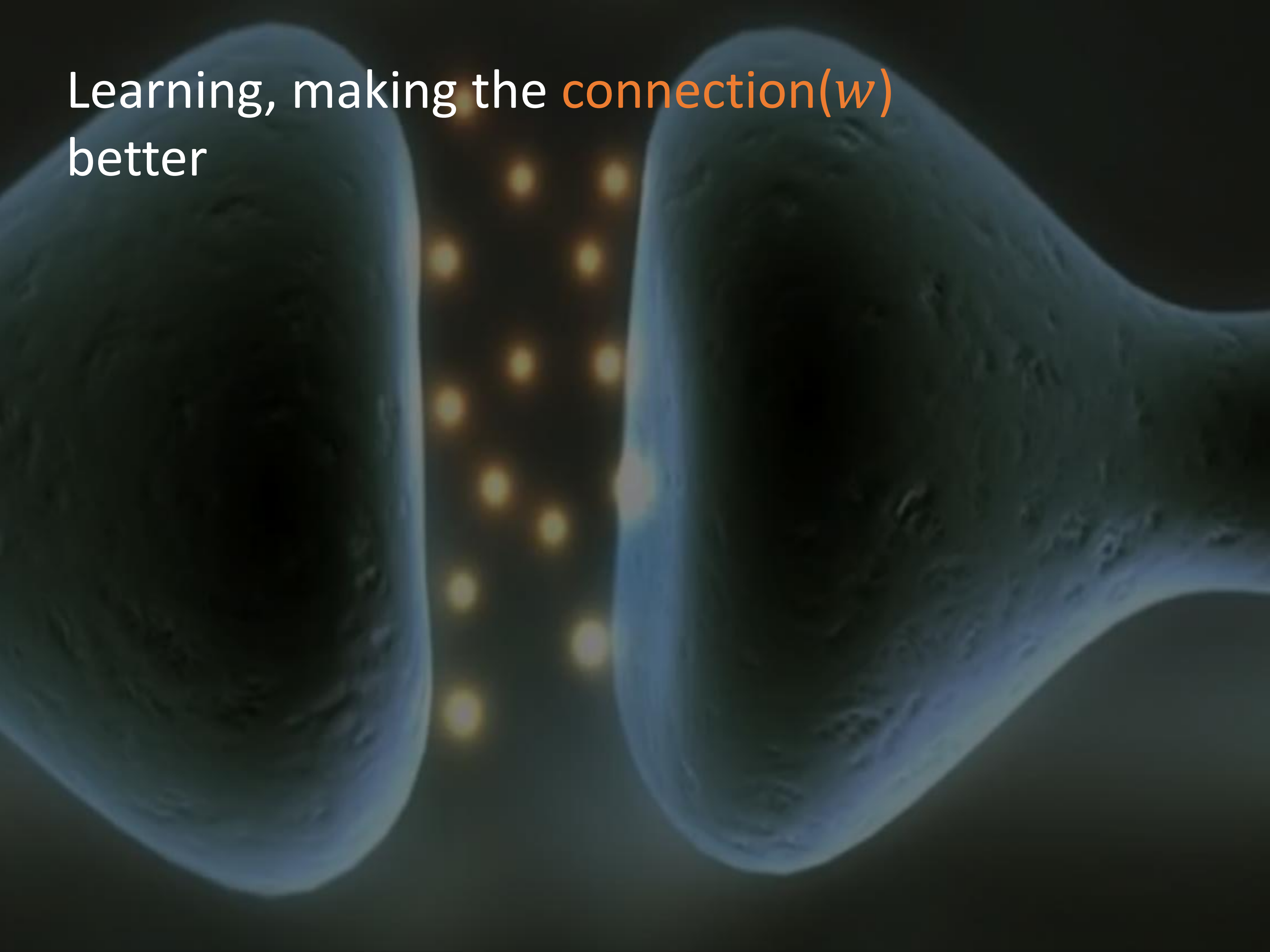
# **Extension** of the Graph

- Two neurons, 3-layer

$$E = (w_2 \cdot (w_1 x) - y)^2$$

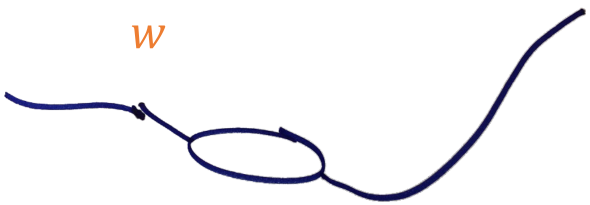


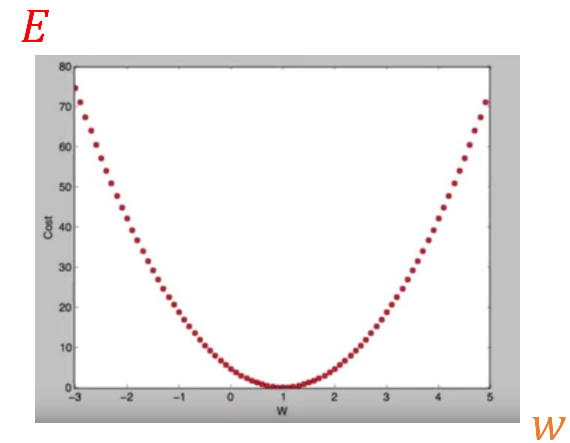
## Learning, making the connection($w$) better



# Cost(Error) graph

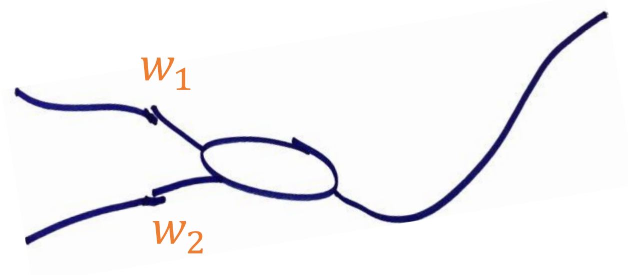
$$E = (w \cdot 1 - 1)^2$$

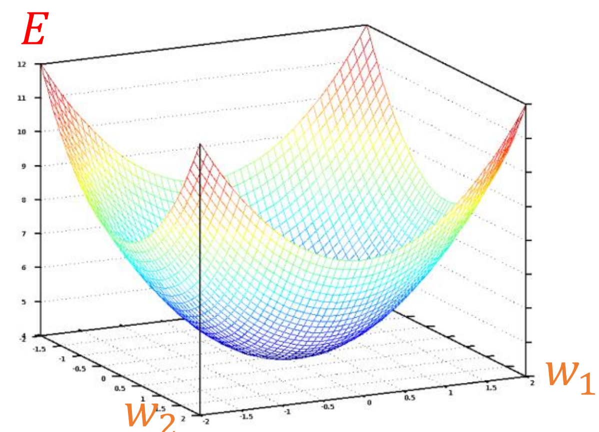



convex function

$$E = (\boldsymbol{w_1} \cdot 1 + \boldsymbol{w_2} \cdot 1 - 1)^2$$




convex function

# Lab 02.with_bias.py

# Parameter tuning including bias

# Lab 03.py

# Using multiple data

# Lab 04.py

# Training a neuron having multiple inputs