CHE 524 Process Control Laboratory
Group 1 – Prelab for Multivariate Process Control
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This lab we performed close loop system identification using Matlab and transfer function. Assuming that the system fit a first order a first order response.

$$G_p = \frac{K_p}{\tau_p s + 1}$$

And we use a PI controller,

$$G_c = K_c + \frac{K_c}{\tau_I s}$$

We develop a transfer function and used lsqcurvefit to find the process parameter τ_p and K_p.

Close loop System Identification

Since this experiment is conducted in close loop condition, We need a new model to describe system behavior.

$$\frac{T'(s)}{T_{sp}(s)} = \frac{G_c G_p}{1 + G_c G_p} = \frac{\left(K_c + \frac{K_c}{\tau_I s}\right)\left(\frac{K_p}{\tau_p s + 1}\right)}{1 + \left(K_c + \frac{K_c}{\tau_I s}\right)\left(\frac{K_p}{\tau_p s + 1}\right)} \times \frac{\tau_p s + 1}{\tau_p s + 1}$$

$$= \frac{\left(K_c + \frac{K_c}{\tau_I s}\right)K_p}{\tau_p s + 1 + \left(K_c + \frac{K_c}{\tau_I s}\right)K_p} = \frac{K_c K_p + \frac{K_c K_p}{\tau_p s}}{\tau_p s + 1 + K_c K_p + \frac{K_c K_p}{\tau_I s}} \times \frac{s}{s}$$

$$\frac{T'(s)}{T_{sp}(s)} = \frac{K_c K_p s + \frac{K_c K_p}{\tau_I}}{\tau_p s^2 + \left(K_c K_p + 1\right)s + \frac{K_c K_p}{\tau_I}}$$

$$T'(s) = \frac{K_c K_p s + \frac{K_c K_p}{\tau_I}}{\tau_p s^2 + \left(K_c K_p + 1\right)s + \frac{K_c K_p}{\tau_I}} \times T_{sp}(s) = \frac{K_c K_p s + \frac{K_c K_p}{\tau_I}}{\tau_p s^2 + \left(K_c K_p + 1\right)s + \frac{K_c K_p}{\tau_I}} \times \frac{M}{s}$$

$T'(s)$ has the form of

$$F(s) = \frac{M(as+b)}{s(cs^2+ds+e)}$$

Using Matlab symbolic math tool box,

```
EDU>> syms a b c d e M s t
EDU>> F=(M*(a*s+b))/(c*s^3+d*s^2+e*s)

F =

(M*(b + a*s))/(c*s^3 + d*s^2 + e*s)

EDU>> ilaplace(F,t)

ans =

(M*b)/e - (M*b*exp(-(d*t)/(2*c))*(cosh((t*(d^2/4 - c*e)^(1/2))/c) - (c*sinh((t*(d^2/4 - c*e)^(1/2))/c)*(d/(2*c) +
(M*a*e - M*b*d)/(M*b*c)))/(d^2/4 - c*e)^(1/2)))/e
```

Adding appropriate symbol for element and element multiplication, we obtained formula X, a time domain model that describes PI and first order (PIAFO) behaviour.

```
X=(M.*b)/e - (M.*b.*exp(-(d.*t)./(2.*c)).*(cosh((t.*(d.^2/4 -
c.*e).^(1/2))/c) - (c.*sinh((t.*(d.^2./4 - c.*e).^(1/2))./c).*(d./
(2.*c) + (M.*a.*e - M.*b.*d)/(M.*b.*c)))/(d^2/4 - c.*e)^(1/2))).*/e
```

Comparing F(s) and PIAFO model we obtained,

$$\begin{aligned} a &= K_c K_p \\ b &= \frac{K_c K_p}{\tau_I} \\ c &= \tau_p \\ d &= K_c K_p + 1 \\ e &= \frac{K_c K_p}{\tau_I} \end{aligned}$$

Developing a Matlab function for PIAFO model

A PIFAO Model requires the knowledge of time (s), step change (M), control gain (Kc), control integrative action (τ_I), process gain (Kp), process time constant (τ_p)

```
function X=PIAFO(x,M,kc,tauI,t)

kp=x(1)
taup=x(2)

a =kc.*kp
b =kc.*kp./tauI
c=taup
d=kc*kp+1
e=b
X=(M.*b)/e - (M.*b.*exp(-(d.*t)./(2.*c)).*(cosh((t.*(d.^2/4 - c.*e).^(1/2))/c) - (c.*sinh((t.*(

end
```

We have the knowledge of all except process gain and process time delay. In order to use optimization algorithm to solve for them, we put Kp and taup in a column vector x where x is

$$x = \begin{matrix} K_p \\ \tau_p \end{matrix}$$

We want to use lsqcurvefit to solve for Kp and taup. See appendix for the full Matlab script. The script is explained below.

The function takes the following form:

Syntax

```
x = lsqcurvefit(fun,x0,xdata,ydata)
```

where fun is the model we want to fit using function handle. X0 is the initial guess for vector x, xdata is the independent variables and ydata is the dependent variable. Since our PIFAO model have many parameters but we are only solving for 2 of them, we use function handle to tell Matlab which parameters we want to solve.

parameter to solve

pass the other parameters to the function without optimization

```
22 -    fun=@(x,t)PIAFO(x,M,kc,tauI,t)
```

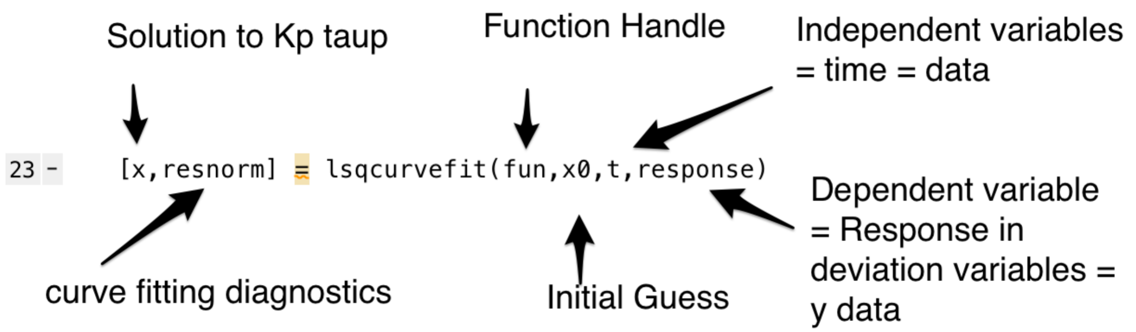
Independent variable = time = xdata

Then we called the lsqcurefit function using our function handle (fun), initial guess where

$$x_0 = \begin{matrix} 0.3 \\ 5 \end{matrix}$$

Which is determined by trial and error.

```
[x,resnorm] = lsqcurvefit(fun,x0,t,response)
```



We determined that the software take sample every 6 seconds. Instead of importing time vector every time, we create a column time vector that starts from 0 with increment of 6 until it match the length of the response data.

```
t=(0:6:6*(length(response)-1))'
```

The response vector, step change, and Kc and τ_I, title of the plot are defined in the beginning of the code

```
%input file
response=PS40T50
%title of the plot
name='fitted plot for pressure setpoint change 40 to 50'
%what is the step change and control parameters
M=10
kc=1
tauI=0.5
%initial guess
x0= [0.3;5]
```

Result of Close Loop System Identification

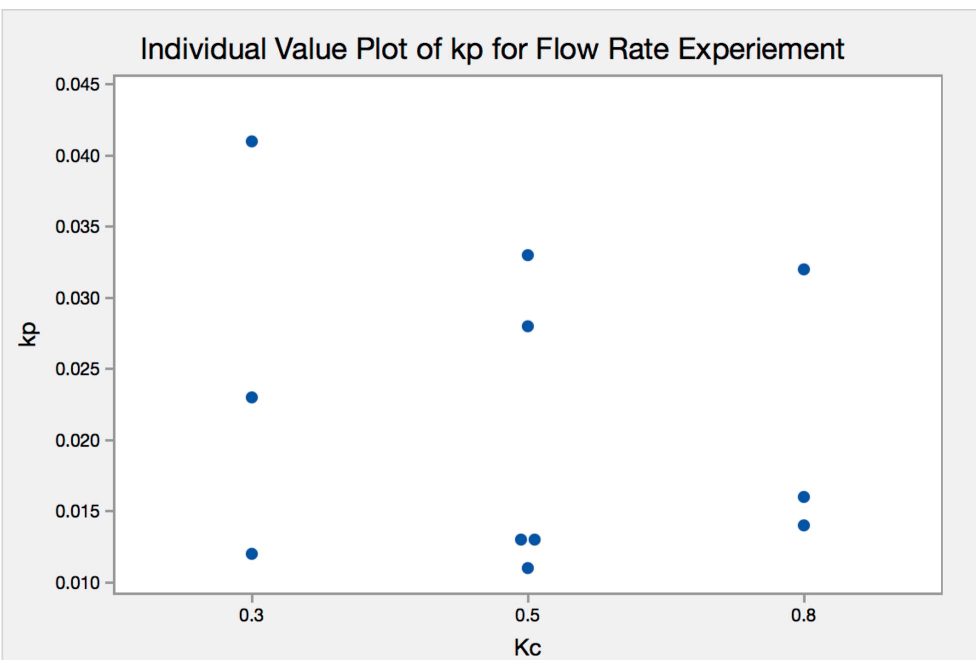
Flow rate Experiment

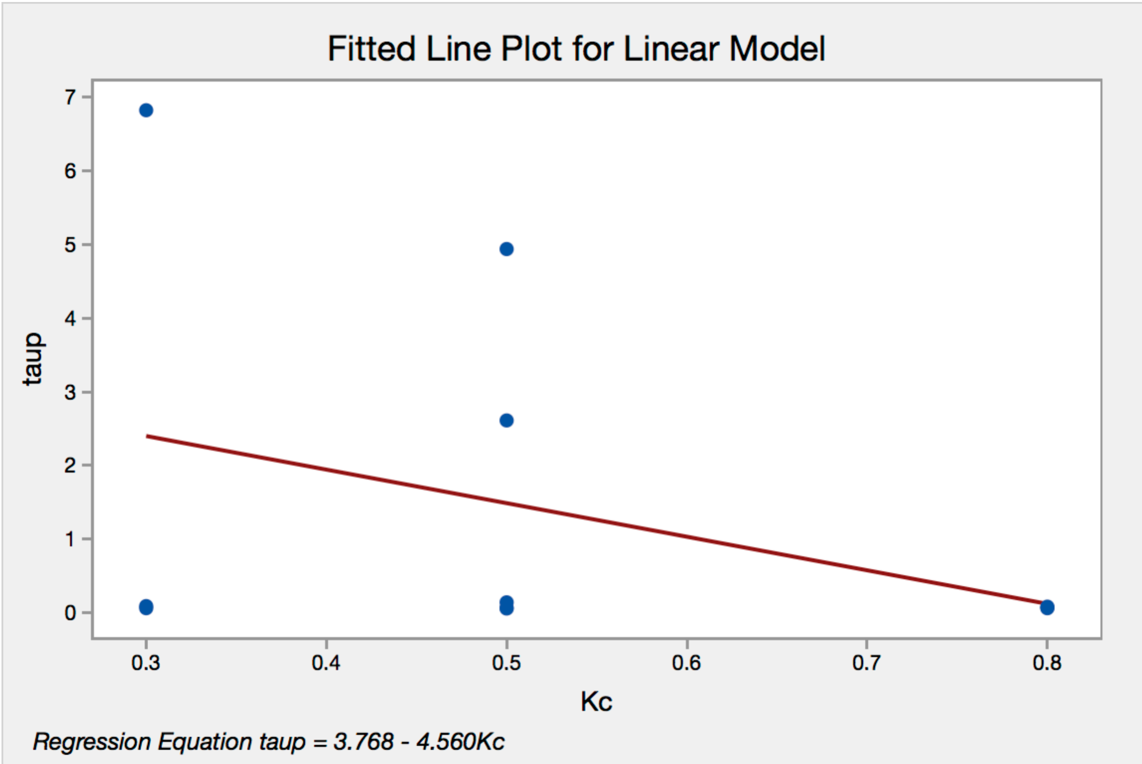
	Vector Name	Kp	τ_p
Kc = 0.5	FL20T30	0.011	2.611
	FL30T40	0.028	0.064
	FL40T50	0.013	4.94
	FL50T60		
	FL60T40	0.033	0.056
	FL40T20	0.013	0.14

Kc=0.3	FL20T30	0.012	6.826
	FL30T40	0.023	0.089
	FL40T50	0.041	0.06
kc=0.8	FL20T30	0.014	0.081
	FL30T40	0.032	0.06
	FL40T50	0.016	0.068
Average		0.021	1.363
Standard Deviation		0.010	2.396
Upper bound		0.0276	2.7793
Lower bound		0.015	0.000

Statistics

Variable	Kc	Total Count	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
kp	0.3	3	3	0	0.025333	0.008452	0.014640	0.012000	0.012000	0.023000	0.041000	0.041000
	0.5	5	5	0	0.019600	0.004534	0.010139	0.011000	0.012000	0.013000	0.030500	0.033000
	0.8	3	3	0	0.020667	0.005696	0.009866	0.014000	0.014000	0.016000	0.032000	0.032000





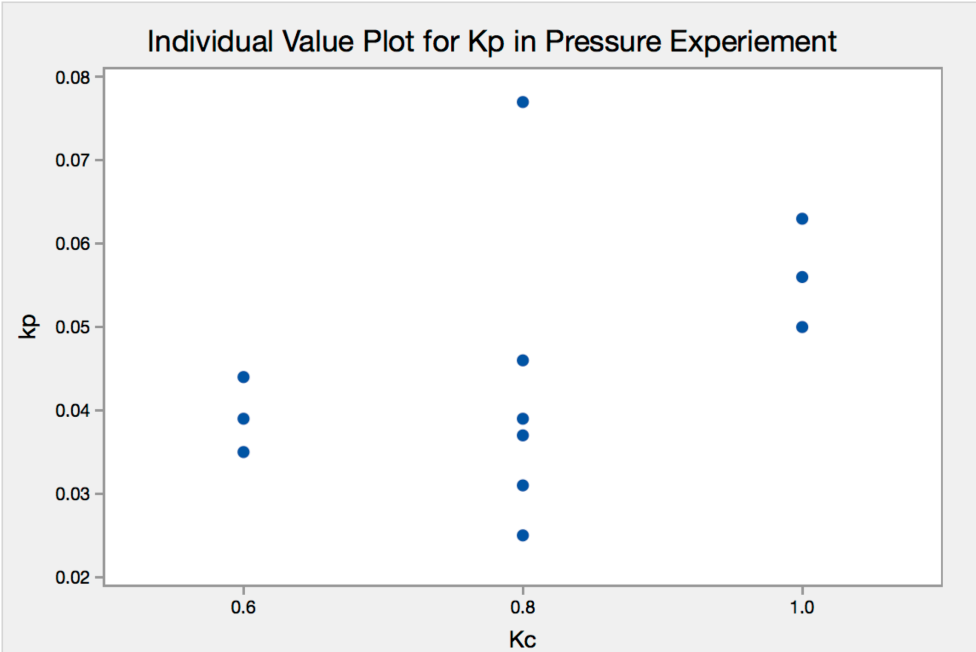
We determined that process gain (Kp) is between 0.015 - 0.0276 and process time constants is between 0~2.77. The average is 0.021 and1.363, respectively. Experiment conducted in low Kc has higher variance than when Kc is high. However, the variation at the 3 conditions is negligible.

Pressure Experiment

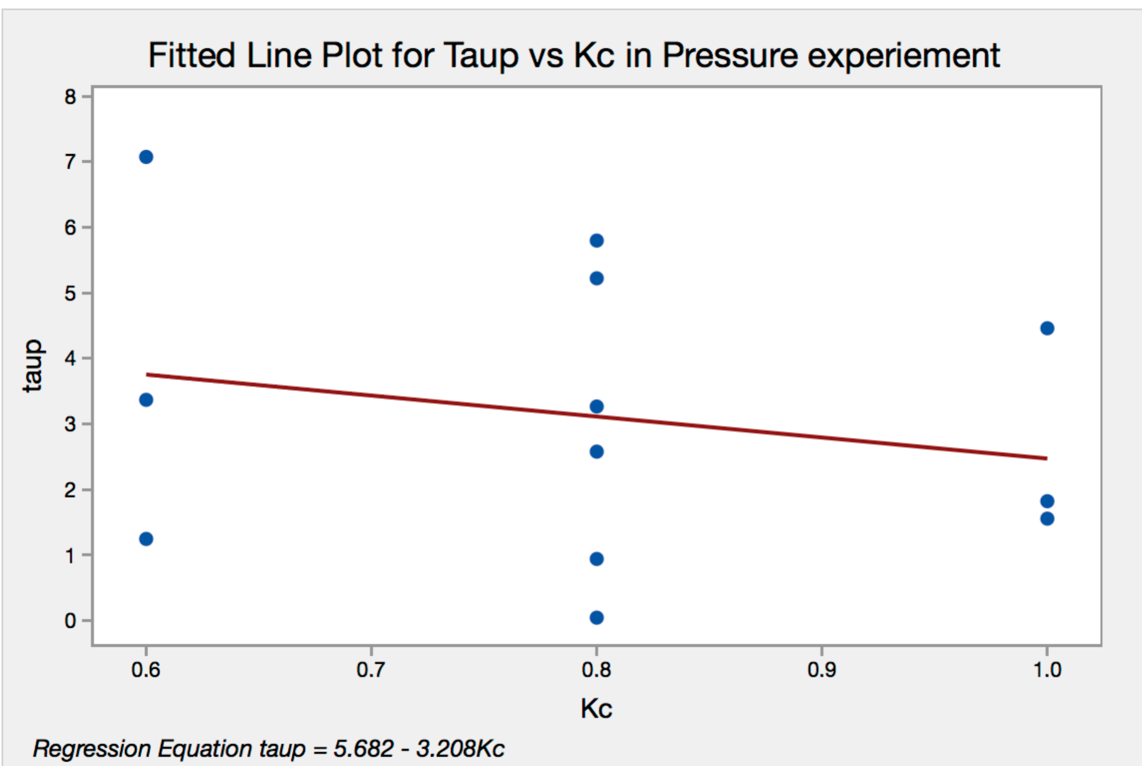
tauc=0.5	Vector Name	kp	taup
kc=0.8	PS20T30	0.025	0.047
	PS30T40	0.037	5.797
	PS40T50	0.039	0.943
	PS50T60	0.046	3.267
	PS60T40	0.077	2.579
	PS40T20	0.031	5.222
kc=0.6	PS20T30	0.044	1.248
	PS30T40	0.039	7.073
	PS40T50	0.035	3.369
kc=1.0	PS20T30	0.063	4.46
	PS30T40	0.05	1.823
	PS40T50	0.056	1.557
Avg		0.045	3.115
STDEV		0.015	2.157
Upper bound		0.053	4.336
Lower bound		0.037	1.895

Statistics

Variable	Kc	Total Count	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
kp	0.6	3	3	0	0.039333	0.002603	0.004509	0.035000	0.035000	0.039000	0.044000	0.044000
	0.8	6	6	0	0.042500	0.007491	0.018349	0.025000	0.029500	0.038000	0.053750	0.077000
	1.0	3	3	0	0.056333	0.003756	0.006506	0.050000	0.050000	0.056000	0.063000	0.063000



We determined that the process gain for the pressure experiment is between 0.037~0.053, the process time constant is 1.895~4.336. We found that experiment runs at Kc=0.8 have the highest variance and the experiment runs at Kc=1 have the highest average process gain. Estimation of process time constant decreased as Kc increase.



Conclusion for Close loop System Identification

For flow rate experiment, we determined that process gain (Kp) is between 0.015 - 0.0276 and process time constants is between 0~2.77. The average is 0.021 and 1.363, respectively.

For pressure MIMO experiment, we determined that the process gain for the pressure experiment is between 0.037~0.053, the process time constant is 1.895~4.336. The average is 0.045 and 3.115, respectively.

We found that for both experiments, Kp are not affected by Kc, but τ_p decreases with increasing Kc for both experiment.

Effect of Chaining Kc
Flowrate Experiement

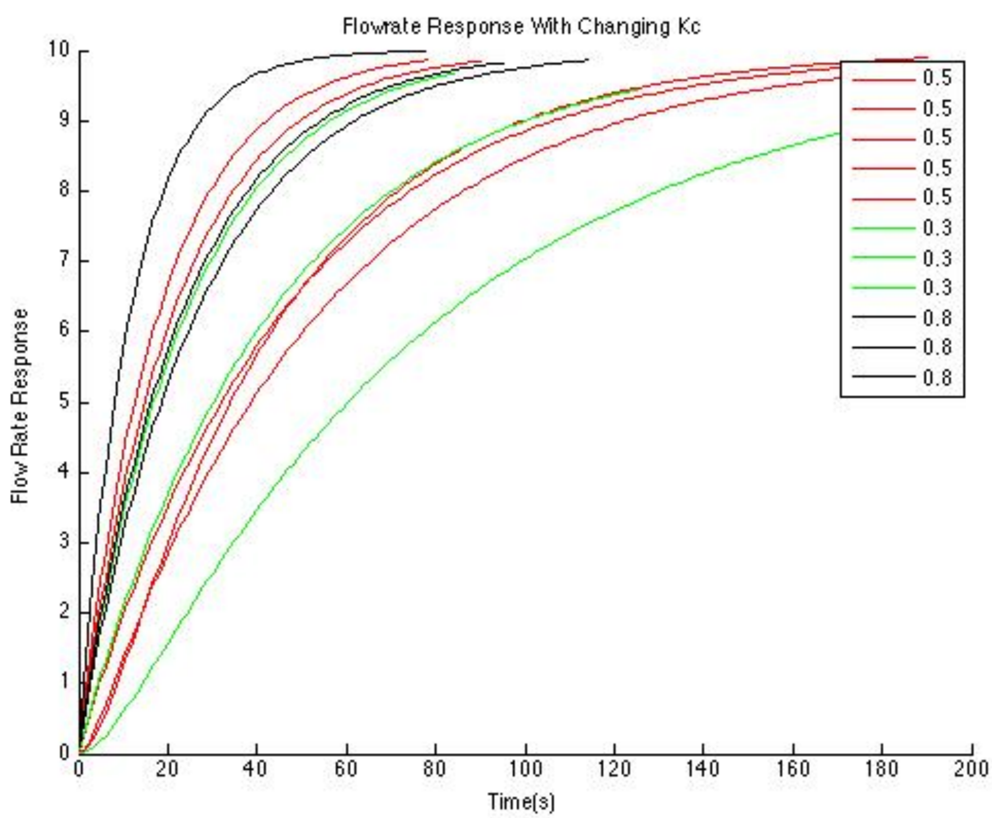
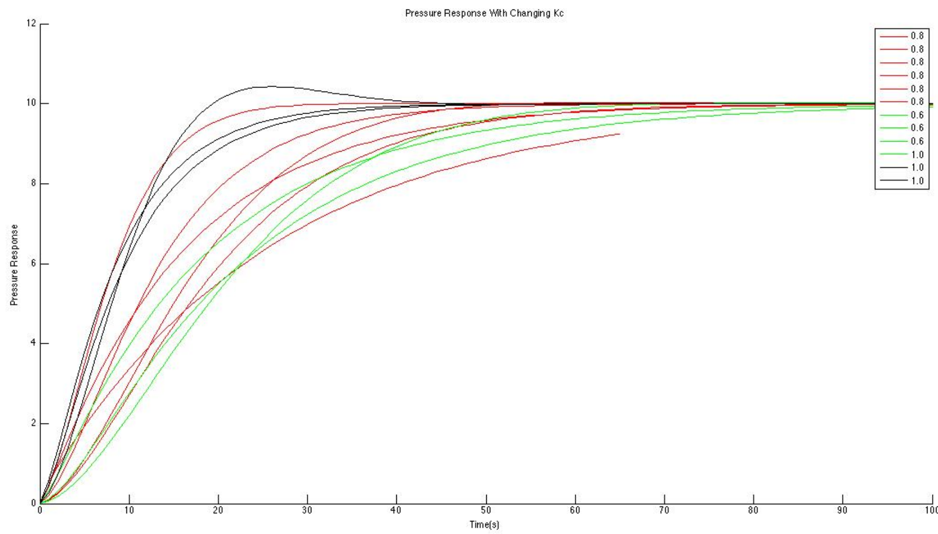


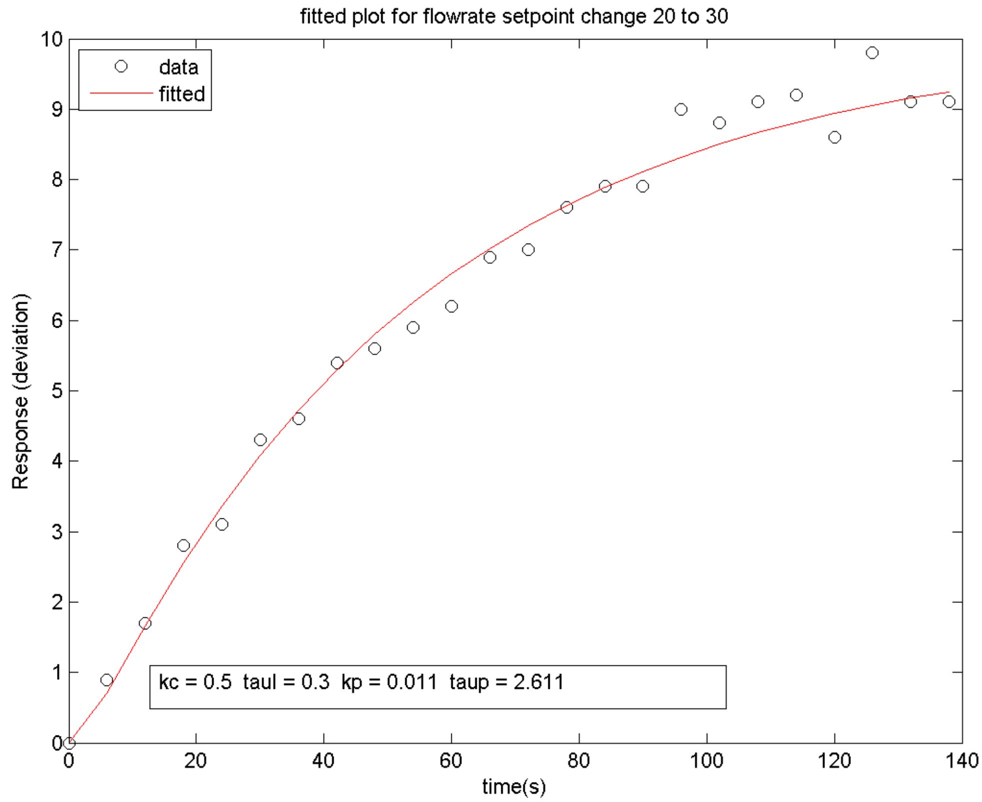
Figure compares all flow rate response with varying Kc. Though the data overlap with each other, it is clear that with increasing Kc, flow rate response are faster, and with lower variance between each runs.

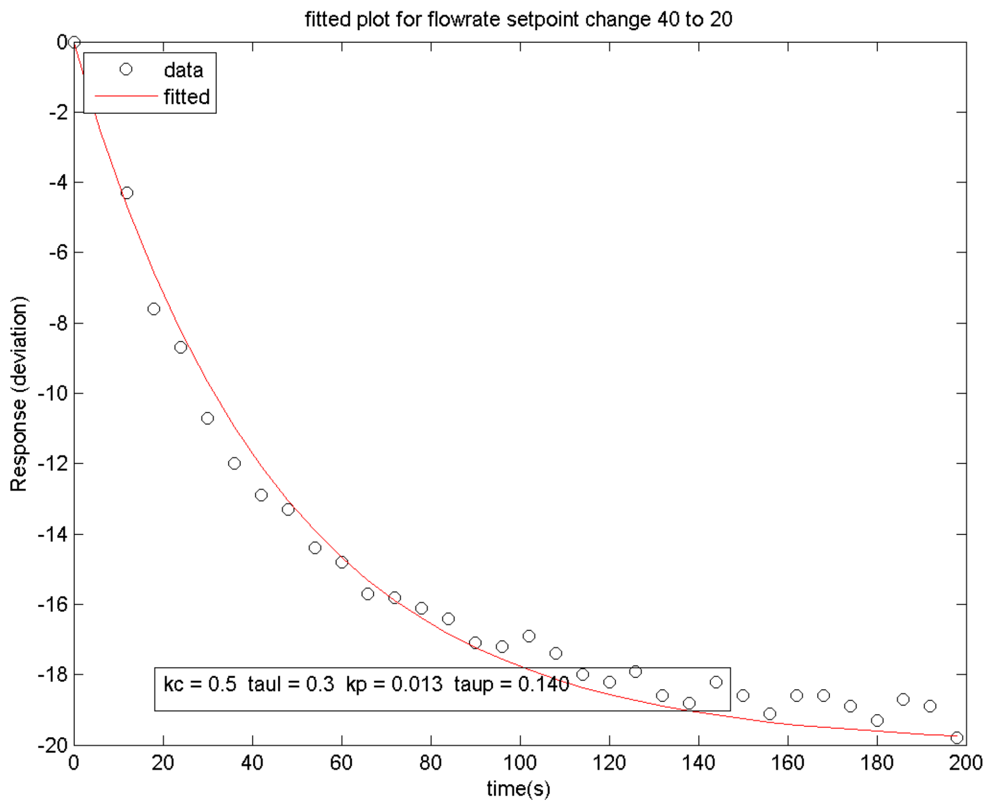
Pressure Experiment

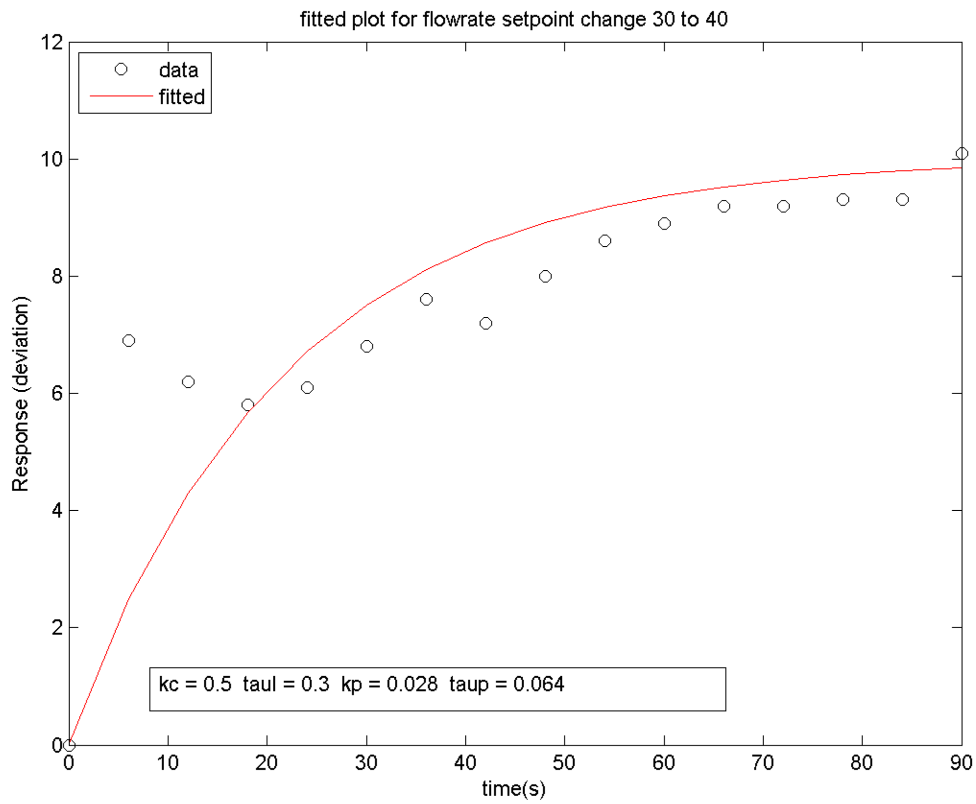


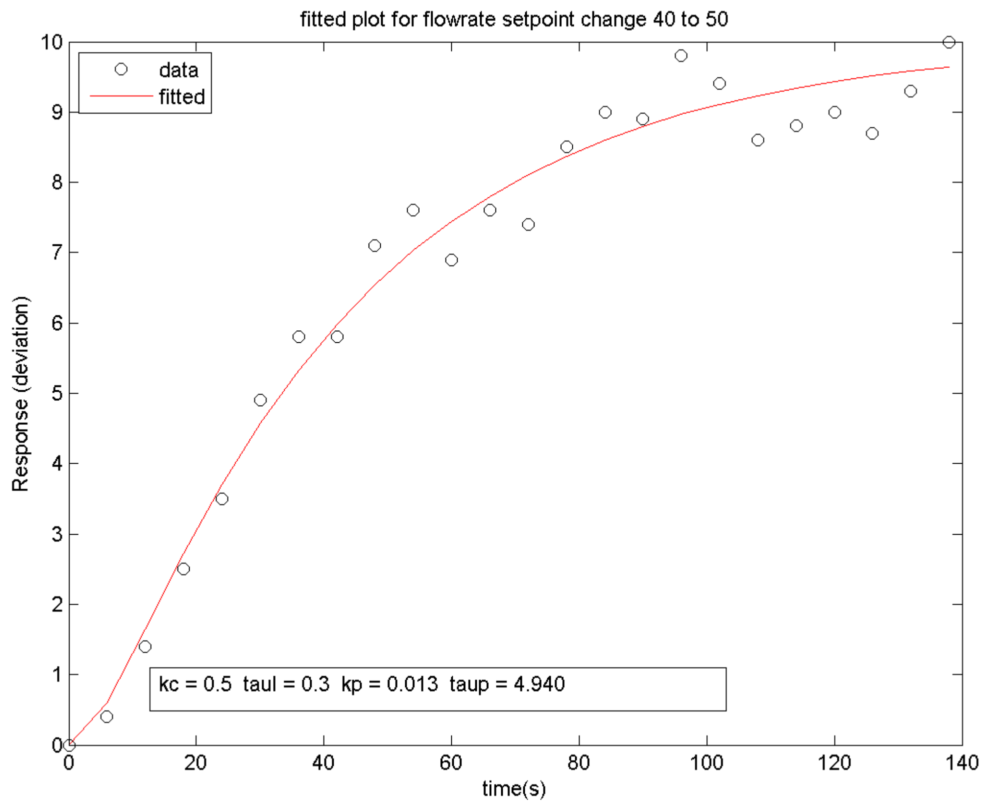
Appendix A

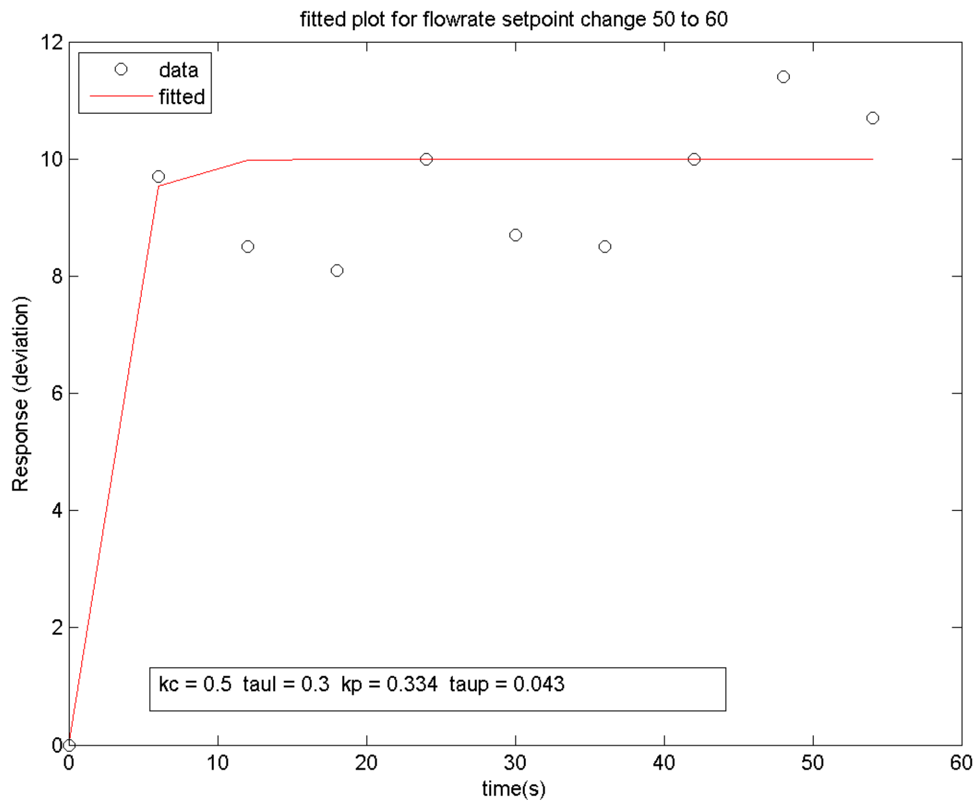
Flow rate experiment

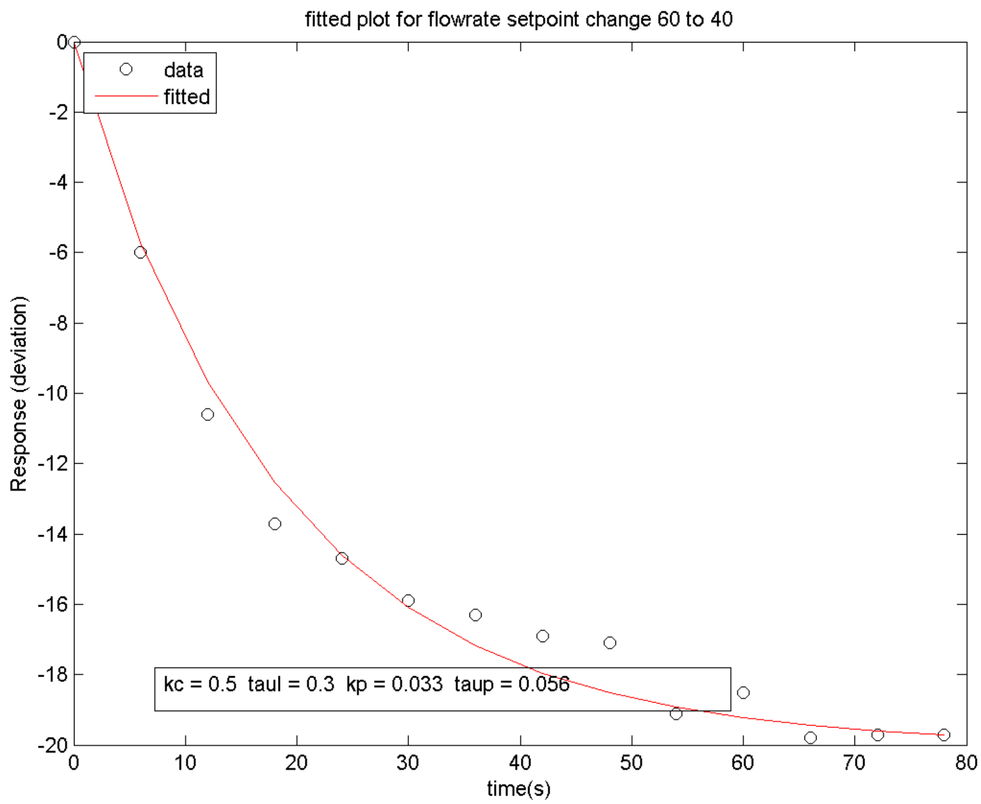

fitted plot for flowrate setpoint change 20 to 30
data
fitted
kc = 0.5 taul = 0.3 kp = 0.011 taup = 2.611
Response (deviation)
time(s)
0
1
2
3
4
5
6
7
8
9
10
20
40
60
80
100
120
140

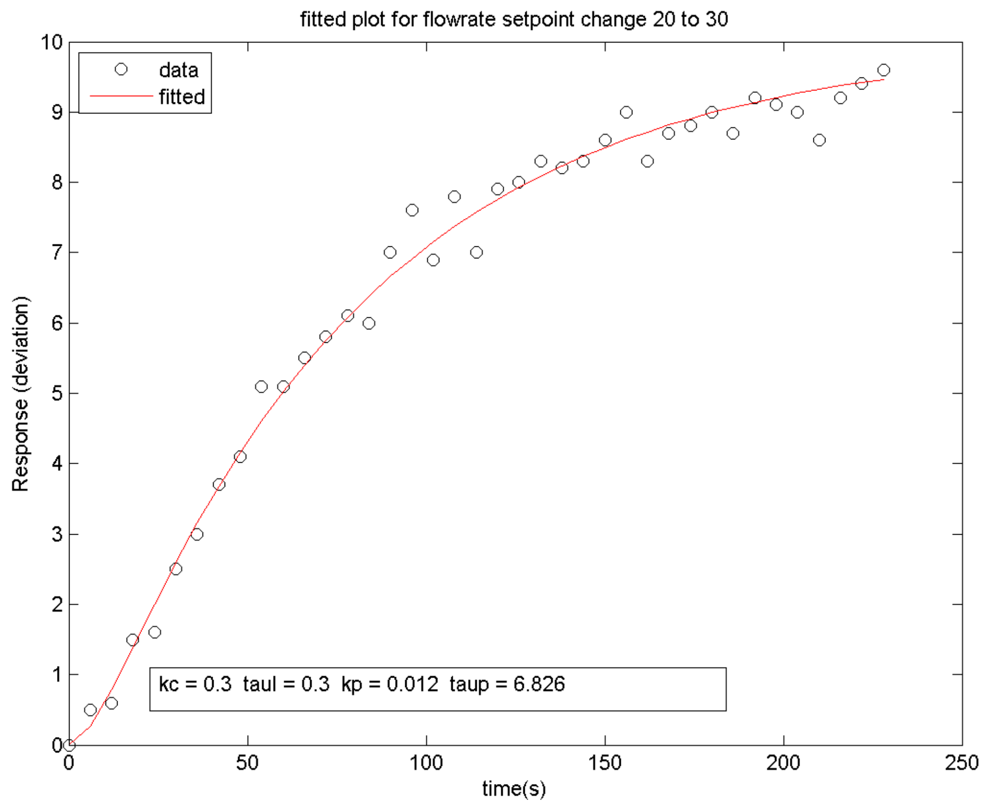

fitted plot for flowrate setpoint change 40 to 20
data
fitted
kc = 0.5 taul = 0.3 kp = 0.013 taup = 0.140
Response (deviation)
time(s)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
0
20
40
60
80
100
120
140
160
180
200

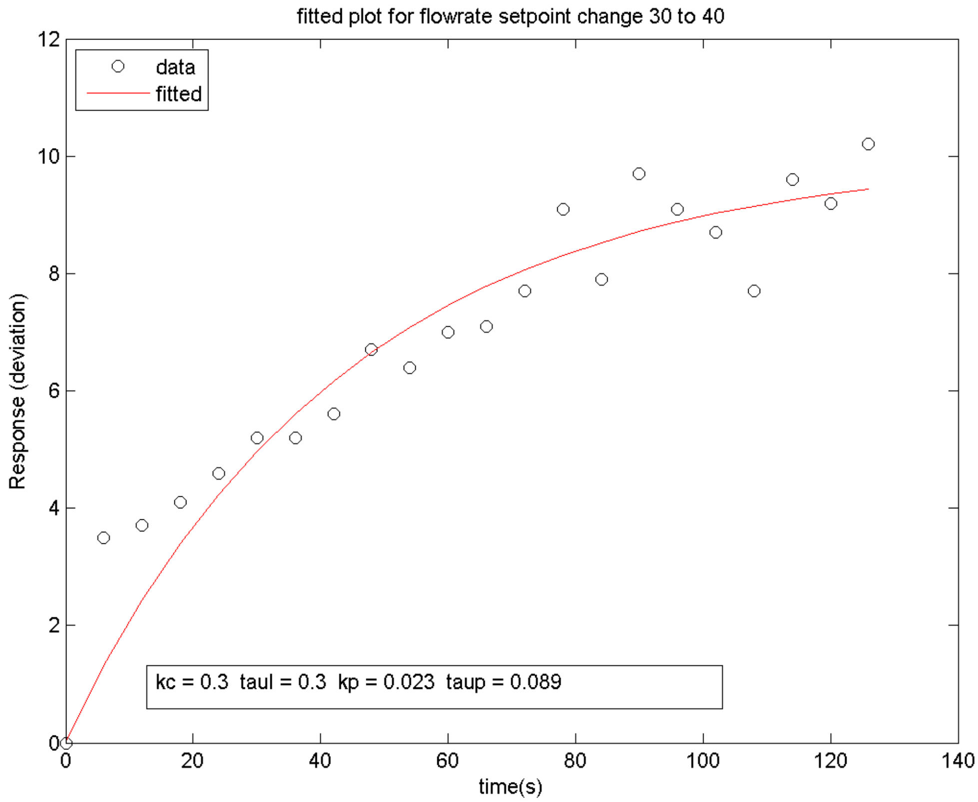

fitted plot for flowrate setpoint change 30 to 40
data
fitted
kc = 0.5 taul = 0.3 kp = 0.028 taup = 0.064
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
10
20
30
40
50
60
70
80
90


fitted plot for flowrate setpoint change 40 to 50
data
fitted
kc = 0.5 taul = 0.3 kp = 0.013 taup = 4.940
Response (deviation)
time(s)
0
20
40
60
80
100
120
140
0
1
2
3
4
5
6
7
8
9
10

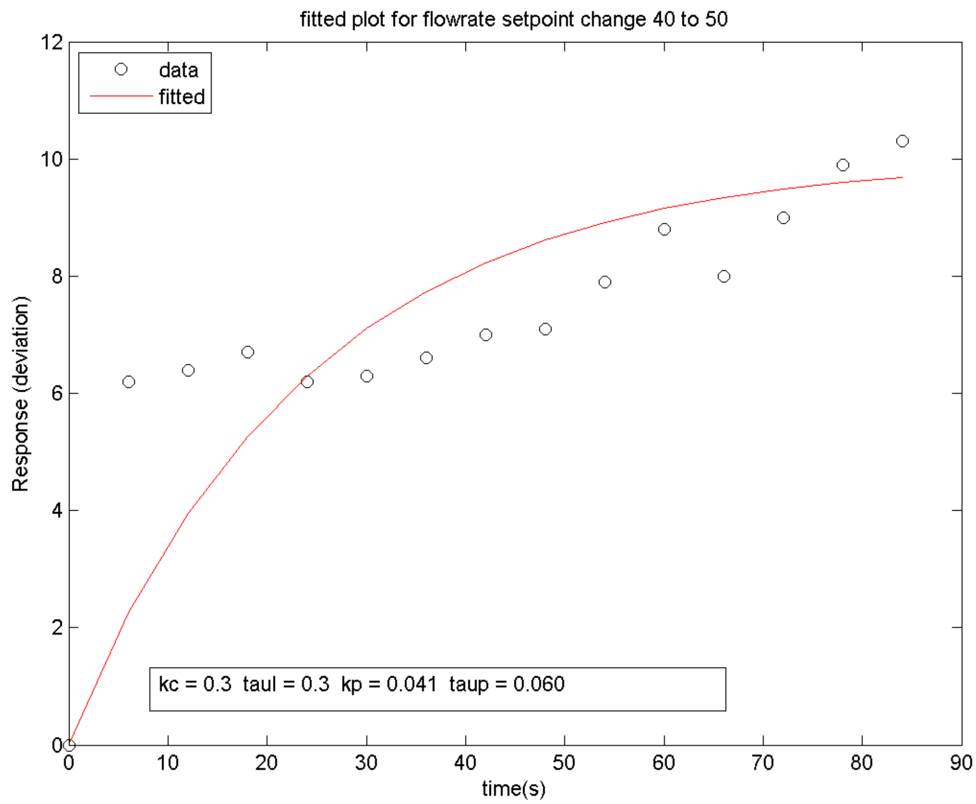

fitted plot for flowrate setpoint change 50 to 60
data
fitted
kc = 0.5 taul = 0.3 kp = 0.334 taup = 0.043
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
10
20
30
40
50
60


fitted plot for flowrate setpoint change 60 to 40
data
fitted
kc = 0.5 taul = 0.3 kp = 0.033 taup = 0.056
Response (deviation)
time(s)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
0
10
20
30
40
50
60
70
80

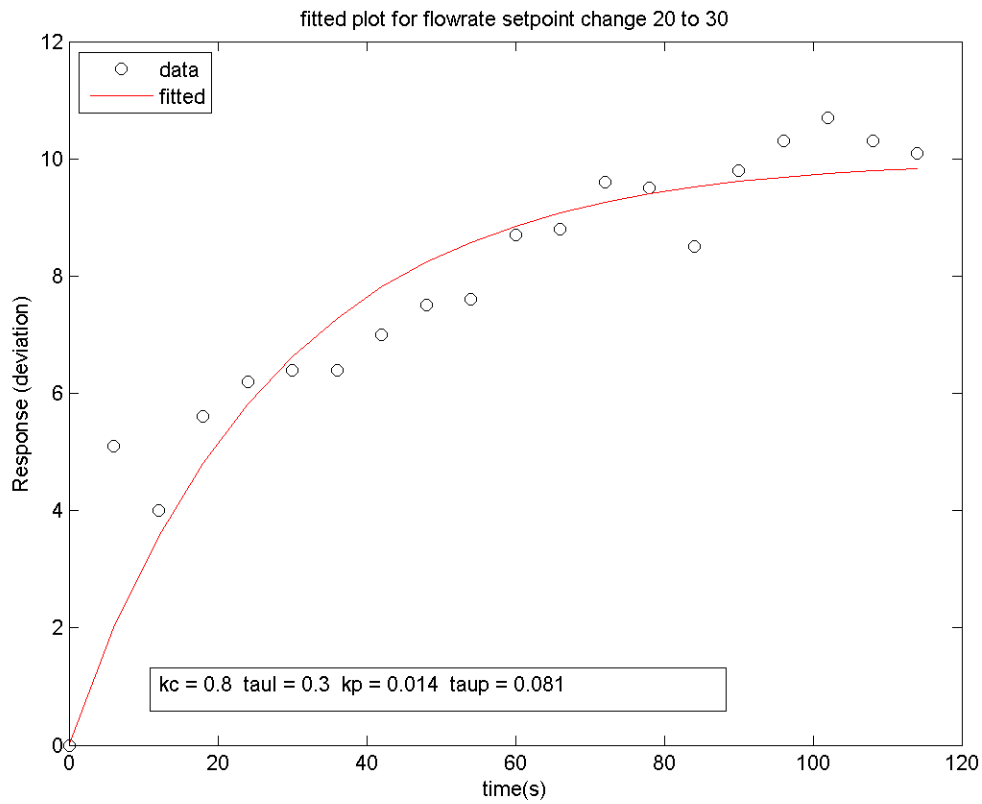

fitted plot for flowrate setpoint change 20 to 30
data
fitted
kc = 0.3 taul = 0.3 kp = 0.012 taup = 6.826
Response (deviation)
time(s)
0
1
2
3
4
5
6
7
8
9
10
0
50
100
150
200
250



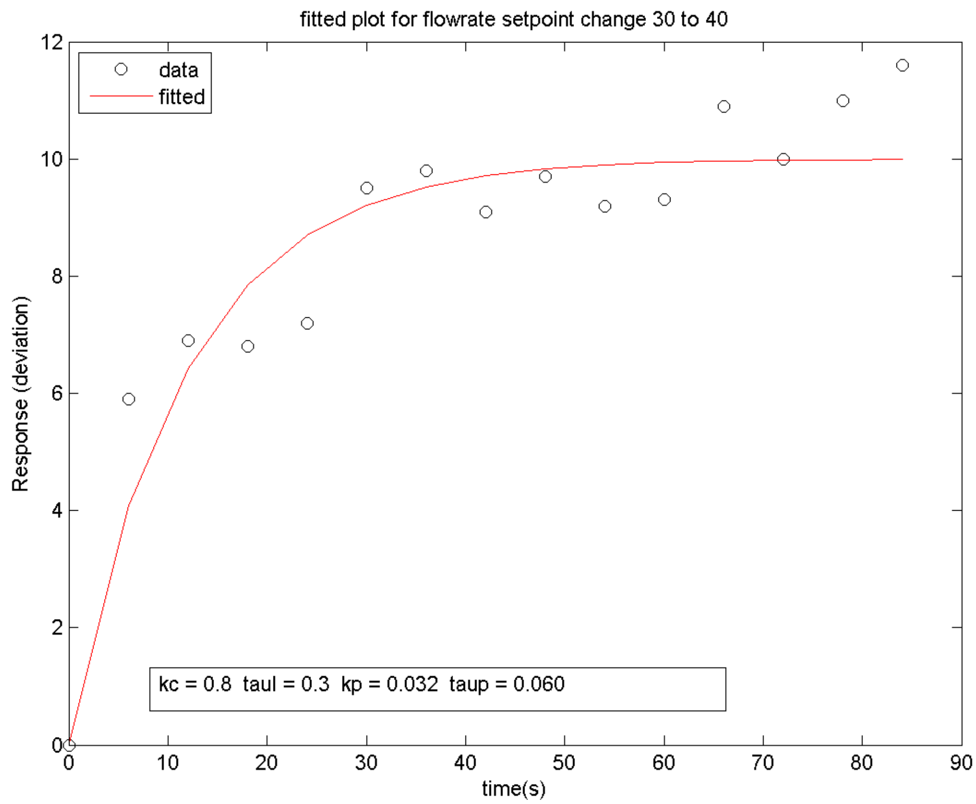
fitted plot for flowrate setpoint change 30 to 40
data
fitted
kc = 0.3 taul = 0.3 kp = 0.023 taup = 0.089
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
20
40
60
80
100
120
140

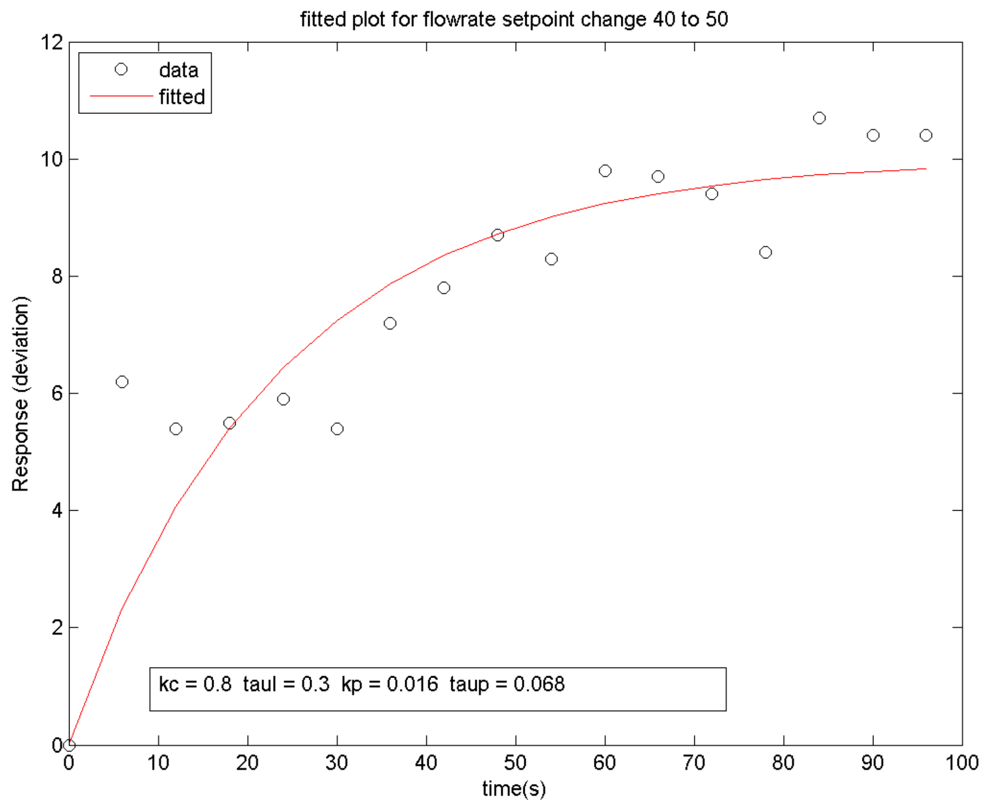


fitted plot for flowrate setpoint change 40 to 50
data
fitted
kc = 0.3 taul = 0.3 kp = 0.041 taup = 0.060
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
10
20
30
40
50
60
70
80
90

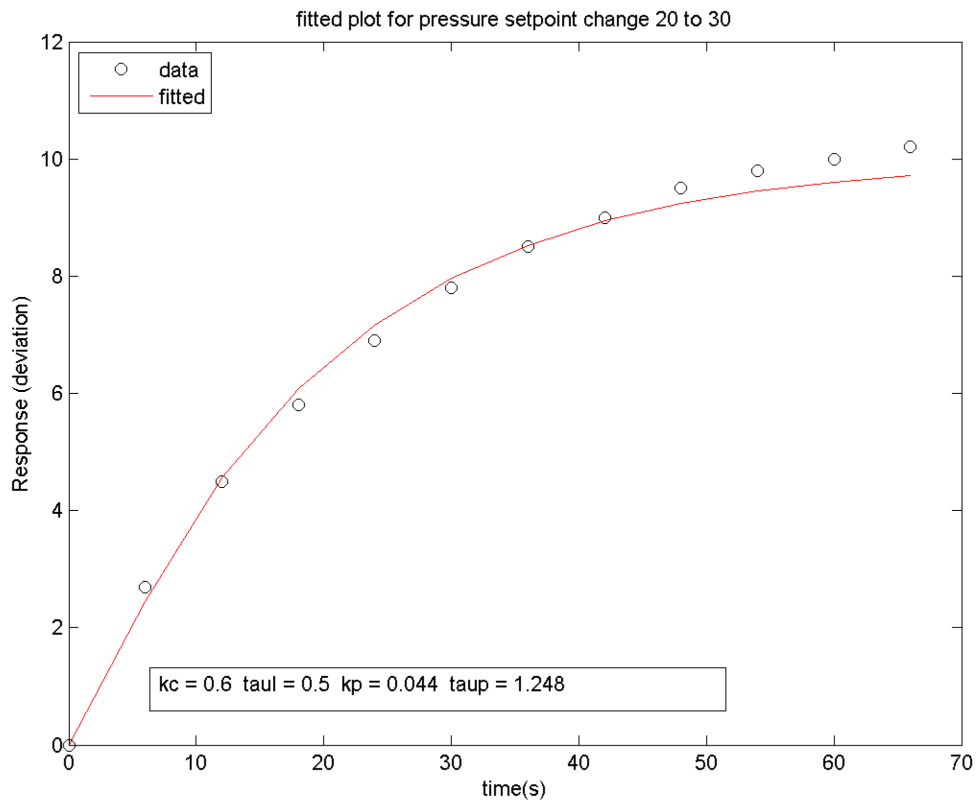


fitted plot for flowrate setpoint change 20 to 30
data
fitted
kc = 0.8 taul = 0.3 kp = 0.014 taup = 0.081
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
20
40
60
80
100
120

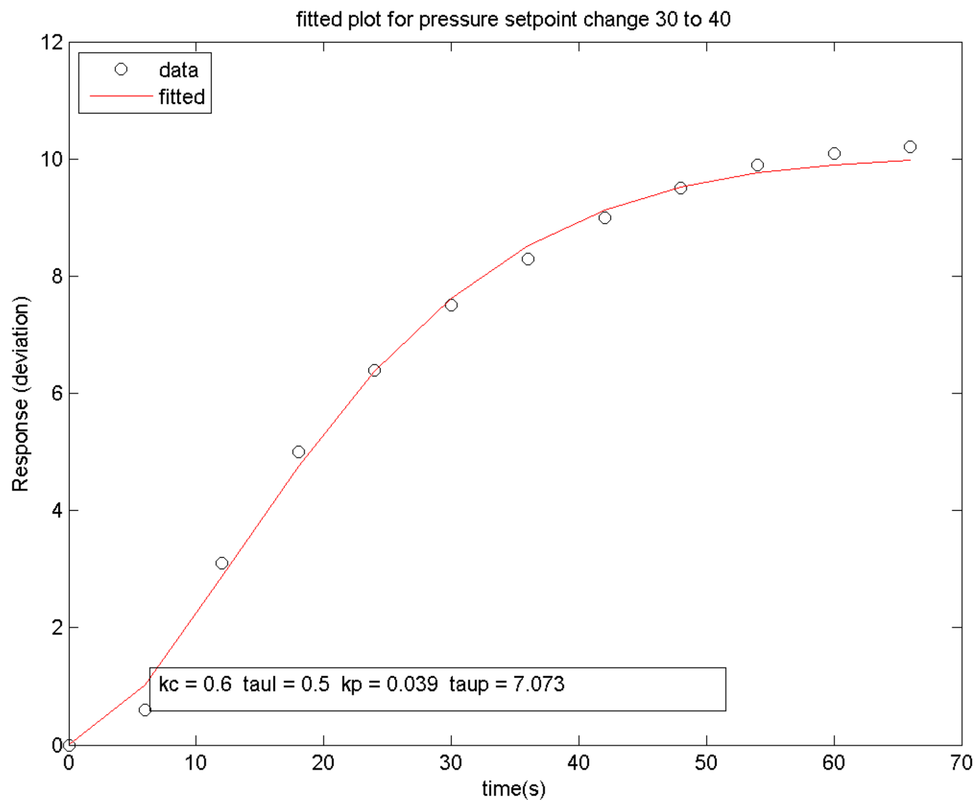

fitted plot for flowrate setpoint change 30 to 40
data
fitted
kc = 0.8 taul = 0.3 kp = 0.032 taup = 0.060
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
10
20
30
40
50
60
70
80
90

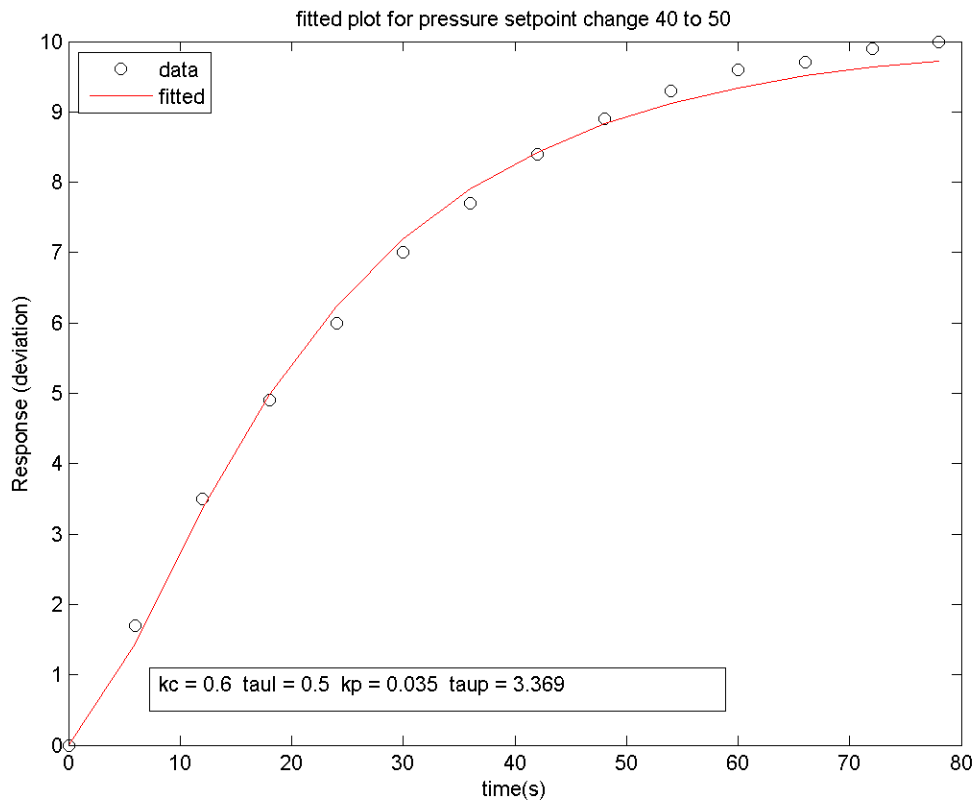


fitted plot for flowrate setpoint change 40 to 50
data
fitted
kc = 0.8 taul = 0.3 kp = 0.016 taup = 0.068
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
10
20
30
40
50
60
70
80
90
100

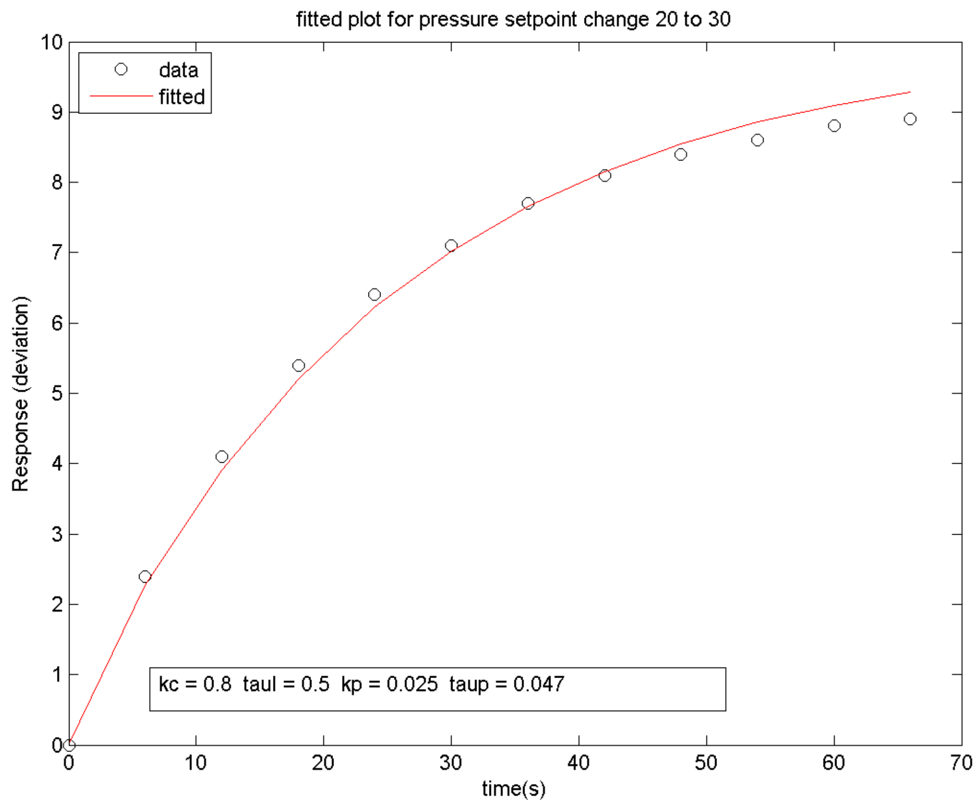


fitted plot for pressure setpoint change 20 to 30
data
fitted
kc = 0.6 taul = 0.5 kp = 0.044 taup = 1.248
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
10
20
30
40
50
60
70

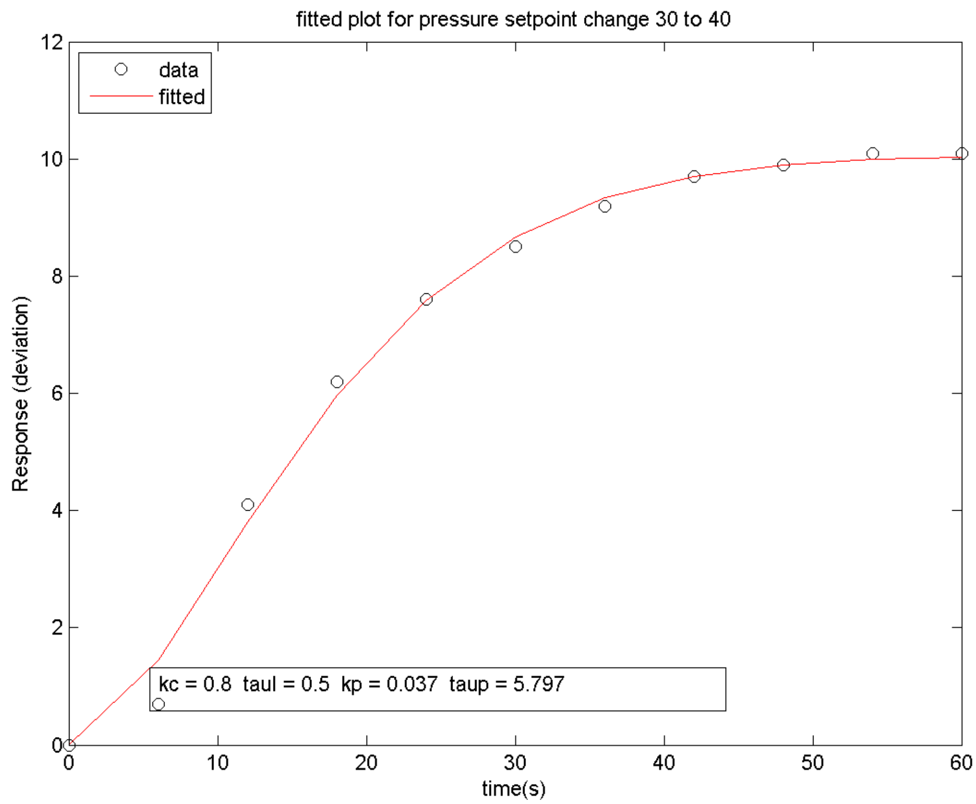

fitted plot for pressure setpoint change 30 to 40
data
fitted
kc = 0.6 taul = 0.5 kp = 0.039 taup = 7.073
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
10
20
30
40
50
60
70



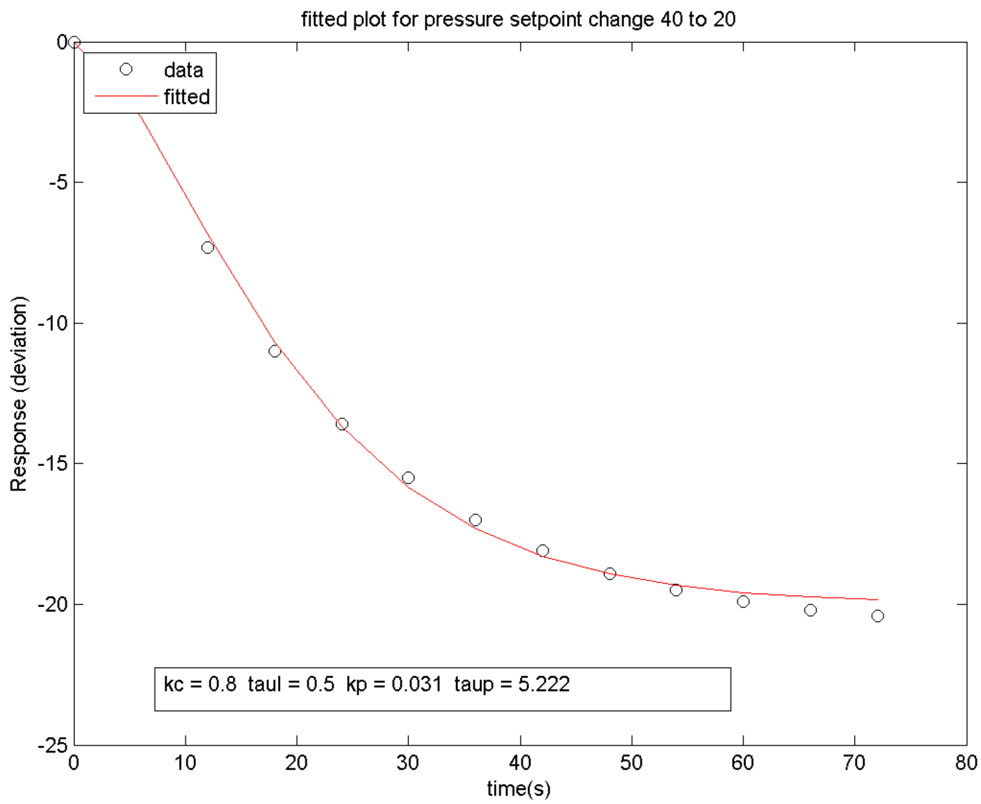
fitted plot for pressure setpoint change 40 to 50
data
fitted
kc = 0.6 taul = 0.5 kp = 0.035 taup = 3.369
Response (deviation)
time(s)
0
10
20
30
40
50
60
70
80
1
2
3
4
5
6
7
8
9
10

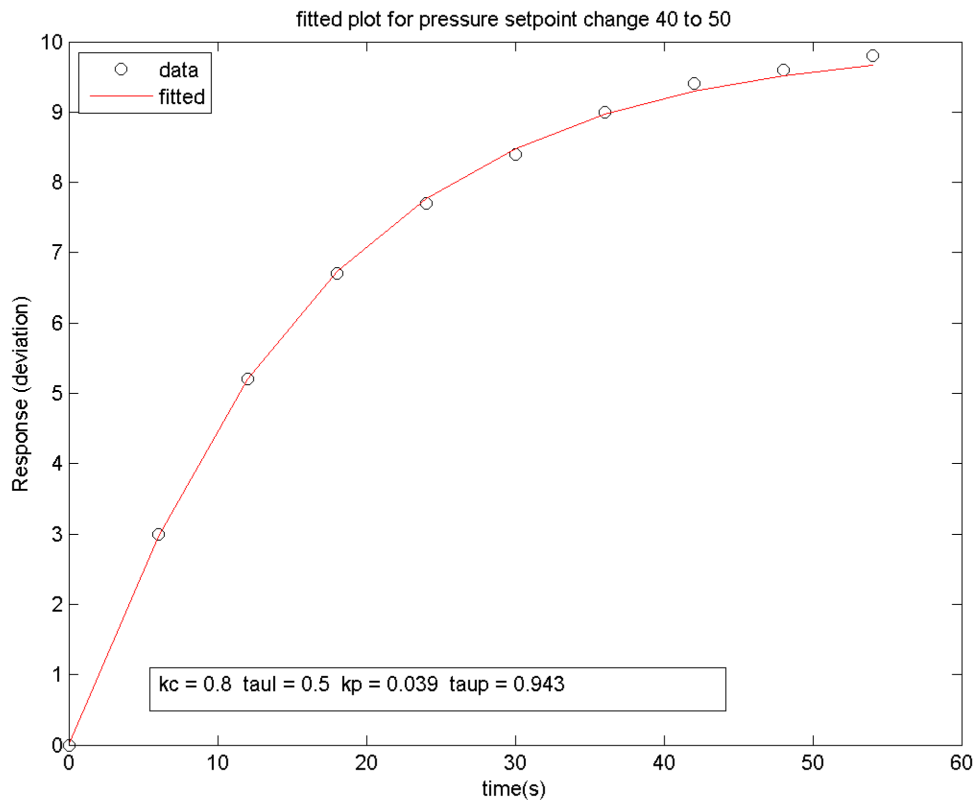


fitted plot for pressure setpoint change 20 to 30
data
fitted
kc = 0.8 taul = 0.5 kp = 0.025 taup = 0.047
Response (deviation)
time(s)
0
1
2
3
4
5
6
7
8
9
10
0
10
20
30
40
50
60
70

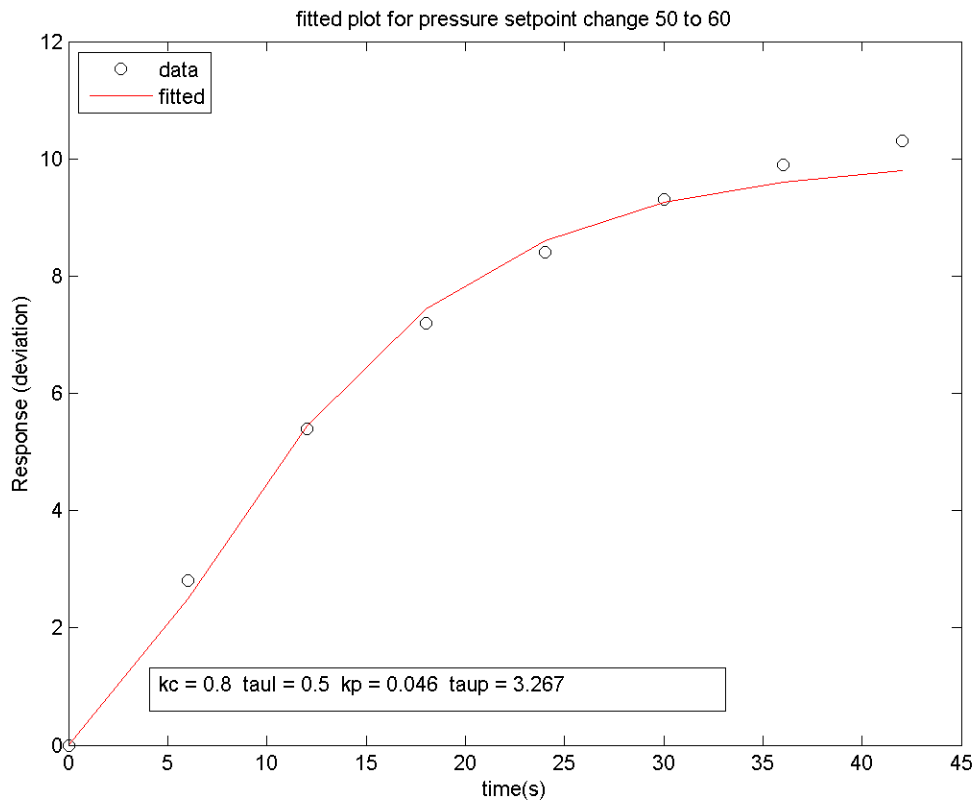


fitted plot for pressure setpoint change 30 to 40
data
fitted
kc = 0.8 taul = 0.5 kp = 0.037 taup = 5.797
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
10
20
30
40
50
60

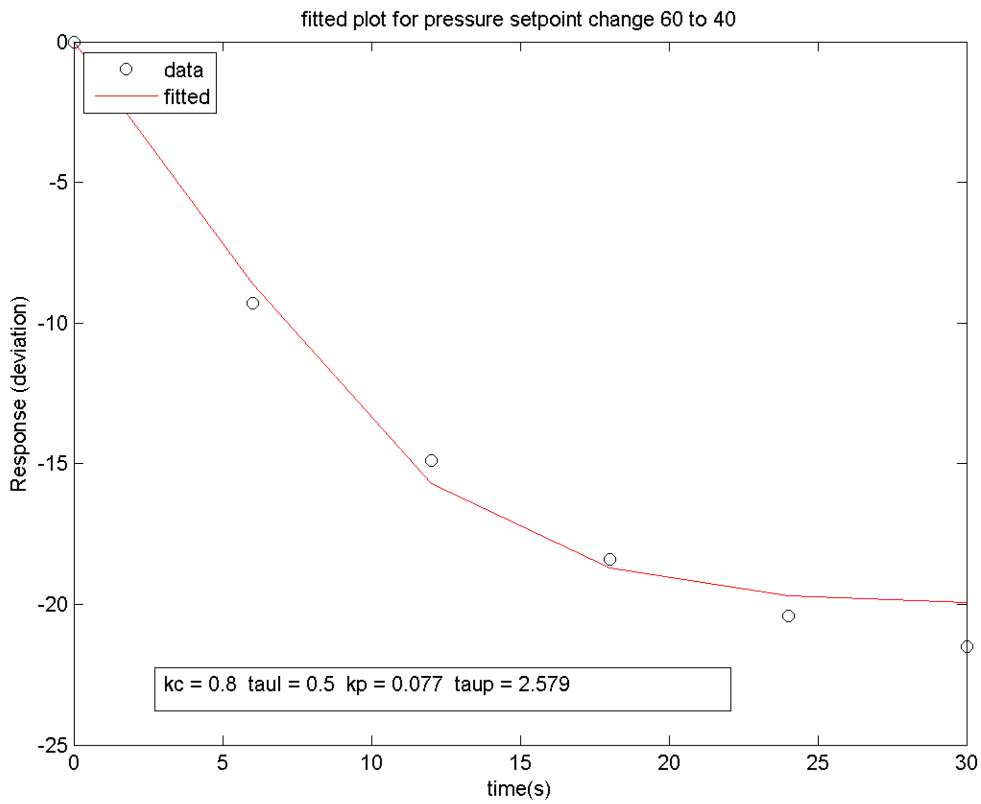

fitted plot for pressure setpoint change 40 to 20
data
fitted
kc = 0.8 taul = 0.5 kp = 0.031 taup = 5.222
Response (deviation)
time(s)
0
-5
-10
-15
-20
-25
0
10
20
30
40
50
60
70
80

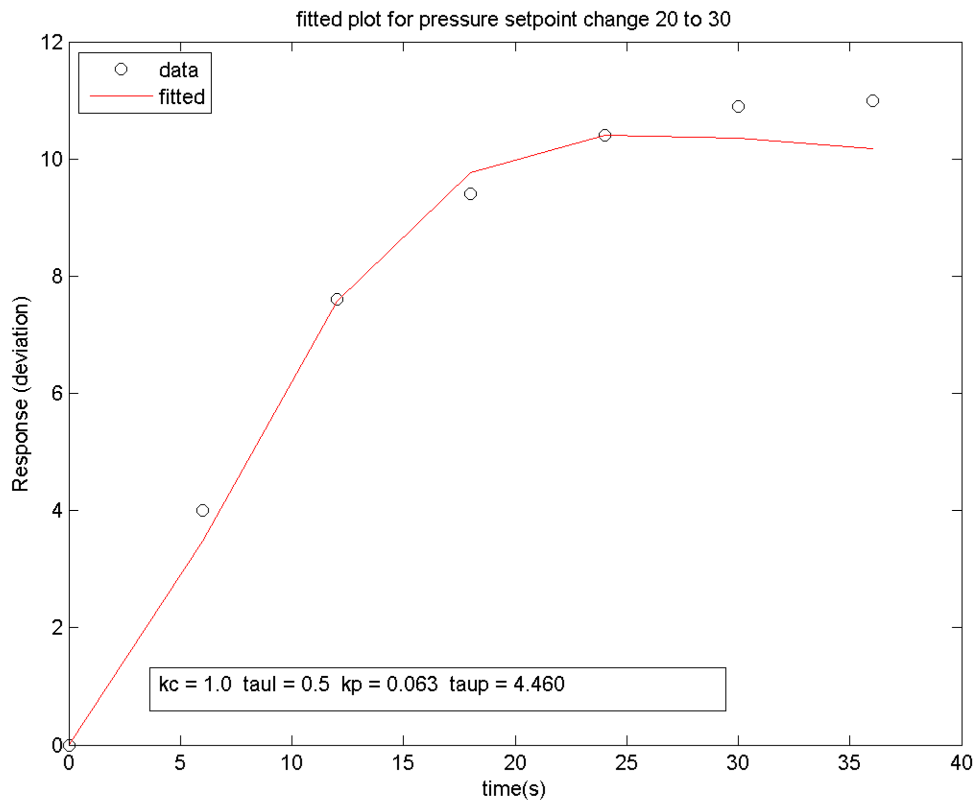


fitted plot for pressure setpoint change 40 to 50
data
fitted
kc = 0.8 taul = 0.5 kp = 0.039 taup = 0.943
Response (deviation)
time(s)
0
10
20
30
40
50
60
1
2
3
4
5
6
7
8
9
10

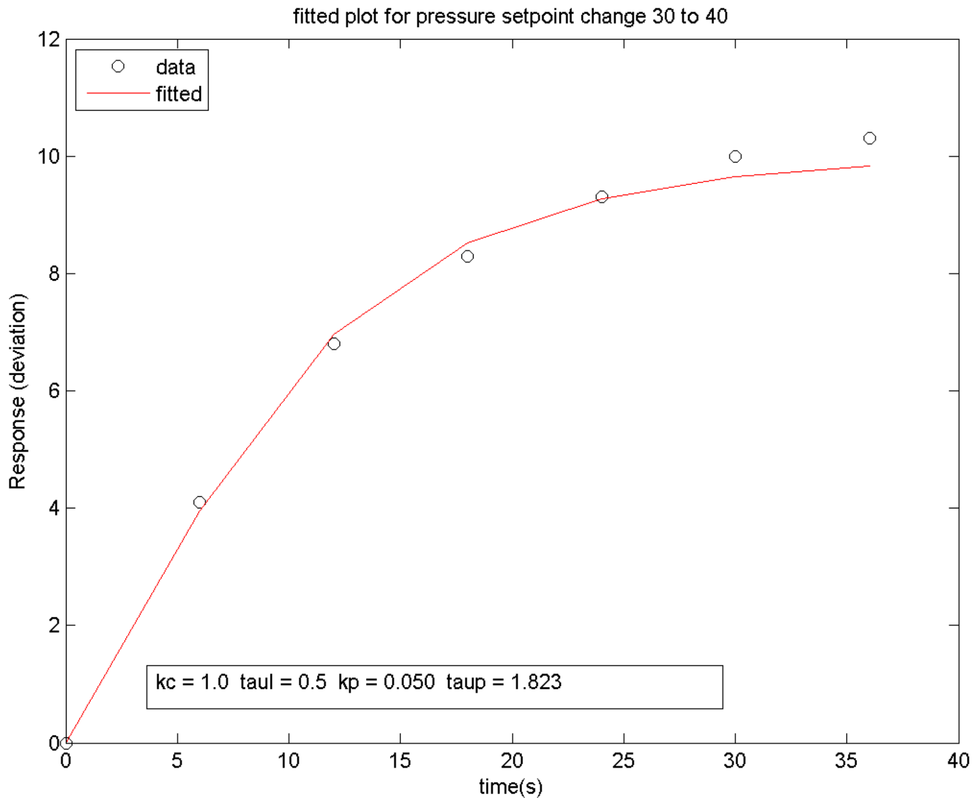


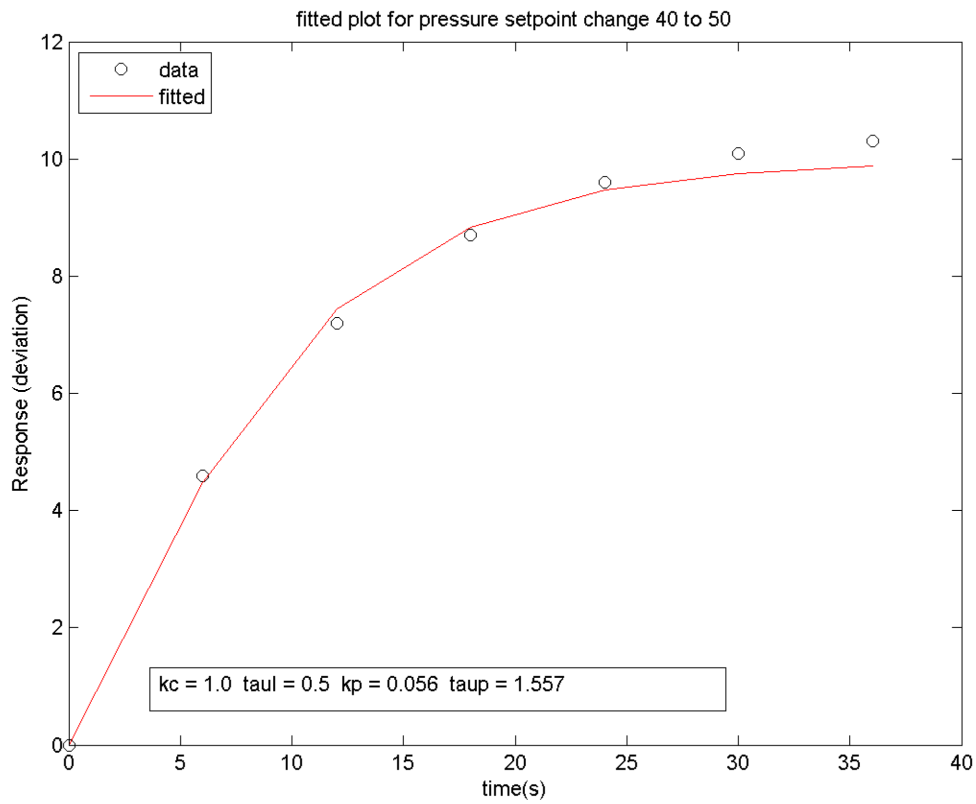
fitted plot for pressure setpoint change 50 to 60
data
fitted
kc = 0.8 taul = 0.5 kp = 0.046 taup = 3.267
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
5
10
15
20
25
30
35
40
45


fitted plot for pressure setpoint change 60 to 40
data
fitted
kc = 0.8 taul = 0.5 kp = 0.077 taup = 2.579
Response (deviation)
time(s)
0
-5
-10
-15
-20
-25
0
5
10
15
20
25
30



fitted plot for pressure setpoint change 20 to 30
data
fitted
kc = 1.0 taul = 0.5 kp = 0.063 taup = 4.460
Response (deviation)
time(s)
0
2
4
6
8
10
12
5
10
15
20
25
30
35
40


fitted plot for pressure setpoint change 30 to 40
data
fitted
kc = 1.0 taul = 0.5 kp = 0.050 taup = 1.823
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
5
10
15
20
25
30
35
40


fitted plot for pressure setpoint change 40 to 50
data
fitted
kc = 1.0 taul = 0.5 kp = 0.056 taup = 1.557
Response (deviation)
time(s)
0
2
4
6
8
10
12
0
5
10
15
20
25
30
35
40