Introduction to Physics– Classical Physics

taught by

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Textbook: Physics for Computer Science Students, by N. Garcia, A. Damask, and S. Schwarz, 2^{nd}, 1998.

Chapter One: Physical Quantities

In physics two fundamental processes are involved:

1. the description of natural phenomena based on experiments, which control variables;
2. mathematical manipulation or theorizing, which is a predictive precess.

Quantities and Units

In classical physics the fundamental parameters in the measurement system are **length, mass,** and **time.**

There are two versions of the metric system in use, the *cgs* (centimeter, gram, second) and the *mks* (meter, kilogram, second).

Conversion of units

$$5280 \text{ ft} = 1 \text{ mi}$$

$$5 \text{ mi} = 5 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}} = 5 \times 5280 \text{ ft} = 26,400 \text{ ft}$$

$$\begin{aligned} 1 \text{ day} &= 1 \text{ day} \left(\frac{24 \text{ h}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) \\ &= 24 \times 60 \text{ min} = 1,440 \text{ min} \end{aligned}$$

In square or cubic units, all measurements must be in the same units.

Powers of 10

$$10^3 = \text{kilo}, 10^{-3} = \text{milli}, 10^{-6} = \text{micro}$$

$$10^{-9} = \text{nano}, 10^{-12} = \text{pico}$$

$$(b \times a^n)(c \times a^m) = bca^{n+m}$$

$$\frac{e \times a^n}{f \times a^m} = \frac{e}{f}a^{n-m}$$

$$(b \times a^n) + (c \times a^n) = (b + c)a^n$$

Accuracy of Numbers

Suppose we wish to find the area of a rectangular surface.

See Figure 1-1.

Our measurement of widths is as 0.4764 ± 0.0001 m and 0.6343 ± 0.0001 m. The largest area is

$$0.4765 \text{ m} \times 0.6344 \text{ m} = 0.3023 \text{ m}^2$$

and the smallest is

$$0.4763 \text{ m} \times 0.6342 \text{ m} = 0.3021 \text{ m}^2$$

We can write the answer as 0.3022 ± 0.0001 m^2.

The accuracy of the product cannot exceed the accuracy of any of the components in the product.

No matter how accurately a given parameter is measured, when is combined arithmetically with another measurement the result is only as accurate as the least-accurate measurement.

Chapter Two: Vectors

- A quantity consisting only of magnitude is called a *scalar* quantity.
- A quantity that has both magnitude and direction and obeys certain algebraic laws is called a *vector* quantity.

Vector Components

See Figure 2-1.

$$R = \sqrt{(4.0 \text{ mi})^2 + (5.0 \text{ mi})^2} = 6.4 \text{ mi}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{4.0 \text{ mi}}{5.0 \text{ mi}} = 0.8$$

or

$$\theta = \arctan 0.8 = 39^\circ$$

See Figure 2-2.

East:

$$5 \text{ mi} + (4.0 \text{ mi}) \times \cos 45^\circ = 5 \text{ mi} + 2.8 \text{ mi} = 7.8 \text{ mi}$$

North:

$$0 \text{ mi} + (4.0 \text{ mi}) \times \sin 45^\circ = 0 \text{ mi} + 2.8 \text{ mi} = 2.8 \text{ mi}$$

See Figure 2-3.

$$R = \sqrt{(7.8 \text{ mi})^2 + (2.8 \text{ mi})^2} = 8.3 \text{ mi}$$

$$\theta = \arctan\left(\frac{2.8 \text{ mi}}{7.8 \text{ mi}}\right) = 19.7^\circ$$

See Figures 2-4, 2-5, 2-6, 2-7.

Example 2-1

A box is pulled by two persons exerting the forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ shown in Fig. 2-8, which $\boldsymbol{F}_1$ is given as 50 lb. Two questions may now be asked. 1. What force $\boldsymbol{F}_2$ must be applied so that the box moves only in the x direction? 2. What single force could replace $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ so that the box moves only in the x direction?

Sol: See Figure 2-9.

Forces on x direction:

$$\boldsymbol{F}_1 : (50 \text{ lb}) \cos 30^\circ = 43.3 \text{ lb}$$

$$\boldsymbol{F}_2 : \boldsymbol{F}_2 \cos 37^\circ = 0.8\boldsymbol{F}_2$$

Forces on y direction:

$$\boldsymbol{F}_1 : -(50 \text{ lb}) \sin 30^\circ = -25.0 \text{ lb}$$

$$\boldsymbol{F}_2 : \boldsymbol{F}_2 \sin 37^\circ = 0.6\boldsymbol{F}_2$$

If the object is going to move in the x direction, then

$$\sum \boldsymbol{F}_y = 0$$

$$-25 \text{ lb} + 0.6\boldsymbol{F}_2 = 0$$

or

$$\boldsymbol{F}_2 = 41.7 \text{ lb}$$

Question 2 can be answered as

$$\sum \boldsymbol{F}_x = 43.3 \text{ lb} + 0.8\boldsymbol{F}_2 = 76.7 \text{ lb}$$

Unit Vectors

See Figure 2-10.

See Figure 2-11.

$$\boldsymbol{F} = (4\boldsymbol{i} + 8\boldsymbol{j} + 5\boldsymbol{k}) \text{ lb}$$

$$F = \sqrt{(4 \text{ lb})^2 + (8 \text{ lb})^2 + (5 \text{ lb})^2} = 10.2 \text{ lb}$$

Dot Product

The *dot product* is defined as

$$\boldsymbol{A} \cdot \boldsymbol{B} \stackrel{\triangle}{=} AB\cos\theta$$

where on the right side A and B are the magnitude of each of the vectors and θ is the angle between them. See Figure 2-12.

$$\boldsymbol{i} \cdot \boldsymbol{j} = \boldsymbol{i} \cdot \boldsymbol{k} = \boldsymbol{j} \cdot \boldsymbol{k} = \cos 90^\circ = 0$$

$$\boldsymbol{i} \cdot \boldsymbol{i} = \boldsymbol{j} \cdot \boldsymbol{j} = \boldsymbol{k} \cdot \boldsymbol{k} = \cos 0^\circ = 1$$

$$\boldsymbol{A} = 3\boldsymbol{i} + 2\boldsymbol{j} \text{ and } \boldsymbol{B} = -\boldsymbol{i} + 3\boldsymbol{j}$$

$$\begin{aligned} \boldsymbol{A} \cdot \boldsymbol{B} &= (3\boldsymbol{i} + 2\boldsymbol{j}) \cdot (-\boldsymbol{i} + 3\boldsymbol{j}) \\ &= 3\boldsymbol{i} \cdot (-\boldsymbol{i}) + 3\boldsymbol{i} \cdot 3\boldsymbol{j} + 2\boldsymbol{j} \cdot (-\boldsymbol{i}) + 2\boldsymbol{j} \cdot 3\boldsymbol{j} \\ &= -3 + 0 + 0 + 6 = 3 \end{aligned}$$

Cross Product

If $\boldsymbol{C}$ is the *cross product* of $\boldsymbol{A}$ and $\boldsymbol{B}$, we have

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$$

and

$$C = AB \sin\theta$$

See Figure 2-13.

The direction of $\boldsymbol{C}$ is perpendicular to both $\boldsymbol{A}$ and $\boldsymbol{B}$ and consequently perpendicular to the plane containing $\boldsymbol{A}$ and $\boldsymbol{B}$(and obey the *right-hand rule*).

See Figures 2-14, 2-15.

Homework: 2-12, 2-16, 2-17.

Chapter Three: Uniformly Accelerated Motion

We introduce certain vector quantities–position, displacement, velocity and acceleration–used to describe the motion of a body.

Speed and Velocity

- The *average speed* is the distance traveled in any direction. Δs, divided by the time Δt, or

$$\overline{\text{speed}} = \frac{\Delta s}{\Delta t}$$

where

$$\Delta(\text{anything}) = \text{final value} - \text{initial value}$$

- The *displacement vector* $\Delta \boldsymbol{r}$ is defined as the vector difference between the final and the initial position vectors, namely,

$$\Delta \boldsymbol{r} = \boldsymbol{r}_f - \boldsymbol{r}_0$$

See Figure 3-1.

- The *average velocity* is defined as the ratio of the displacement vector to the time taken for the displacement to occur, namely,

$$\bar{\boldsymbol{v}} = \frac{\boldsymbol{r}_f - \boldsymbol{r}_0}{t_f - t_0} = \frac{\Delta \boldsymbol{r}}{\Delta t}$$

- The *instantaneous velocity* is defined as

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{r}}{\Delta t} = \frac{d\boldsymbol{r}}{dt}$$

See Figure 3-2.

-

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, \text{ and } v_z = \frac{dz}{dt}$$

where v_x, v_y, and v_z are the Cartesian components of $\boldsymbol{v}$ and x, y, and z are those of $\boldsymbol{r}$.

Acceleration

- If there is a velocity change $\Delta\boldsymbol{v}$ in a certain time Δt, we define the *average acceleration* as

$$\bar{\boldsymbol{a}} = \frac{\Delta\boldsymbol{v}}{\Delta t}$$

- The *instantaneous acceleration* as

$$\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\Delta\boldsymbol{v}}{\Delta t} = \frac{d\boldsymbol{v}}{dt}$$

- $$\boldsymbol{a} = \frac{d}{dt}\left(\frac{d\boldsymbol{r}}{dt}\right) = \frac{d^2\boldsymbol{r}}{dt^2}$$

Example 3-1

The position of a body on the x axis varies as a function of time according to the following equation

$$x(\text{meters}) = (3t + 2t^2)\text{m}$$

Find its velocity and acceleration when $t = 3$ sec.

Sol: Since $\boldsymbol{r} = x$,

$$v = \frac{dx}{dt} = \frac{d}{dt}(3t + 2t^2) = (3 + 4t)\text{m/sec}$$

$$v(t = 3 \text{ sec}) = 3 + 4 \times 3 = 15\text{m/sec}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(3 + 4t) = 4\text{m/sec}^2$$

Linear Motion–Constant Acceleration

- Because displacement, velocity, and acceleration are vectors, we may treat them by the method of Cartesian components.
- Assume that the object moving in the x direction when it starts from or is passing the $x = 0$ point, and we have

$$\bar{v}_x = \frac{x - 0}{t - 0}$$

or

$$x = \bar{v}_x t$$

- The acceleration is the rate of change of the velocity with time. See Figure 3-3.
-

$$v = v_0 + at$$

- Informal derivation of equations associated with displacement, velocity, and acceleration.

  1.
  $$\bar{v} = \frac{v + v_0}{2}$$
  $$x = \frac{v + v_0}{2} t$$

  2.
  $$t = \frac{v - v_0}{a}$$
  $$x = \frac{(v + v_0)}{2} \frac{(v - v_0)}{a}$$

  and
  $$\boxed{v^2 - v_0^2 = 2ax}$$

  3.
  $$x = \frac{v_0 + at + v_0}{2} t$$

$$\boxed{x = v_0 t + \tfrac{1}{2}at^2}$$

- Formal derivation of the above equations:

1.

$$a = \frac{dv}{dt}$$

$$\int_{v_0}^{v} dv = \int_0^t a\ dt$$

$$v - v_0 = at \text{ and } v = v_0 + at$$

2.

$$v = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx = \int_0^t v\ dt$$

$$\int_{x_0}^{x} dx = \int_0^t (v_0 + at)\ dt = v_0 \int_0^t dt + a \int_0^t t\ dt$$

$$\boxed{x - x_0 = v_0 t + \tfrac{1}{2} a t^2}$$

3.

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

$$\int_{v_0}^{v} v\ dv = a\int_{x_0}^{x} dx$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = a(x - x_0)$$

$$\boxed{v^2 - v_0^2 = 2a(x - x_0)}$$

- The acceleration caused by gravity is usually written as the symbol g and has approximate see-level value $g = 9.8$ m/sec^2.

Example 3-2

A boy throws a ball upward with an initial velocity of 12 m/sec. How high does it go?

Sol: We choose the starting point as the origin and the upward direction as positive.

$$v_{0y} = 12 \text{ m/sec},\ v_y = 0 \text{ (at its highest point)},$$

$$a_y = g = -9.8 \text{ m/sec}^2,\ y = ?$$

$$v_y^2 - v_{0y}^2 = 2a_y y$$

$$y = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

Substituting the numerical value for the quantities in the equation,

$$y = \frac{0 - (12 \text{ m/sec})^2}{2(-9.8 \text{ m/sec}^2)} = 7.3 \text{ m}$$

Example 3-3

A boy throws a ball upward with an initial velocity of 12 m/sec and catches it when it returns. How long was it in the air?

Sol: We choose the starting point as the origin and the upward direction as positive.

$$v_{0y} = 12 \text{ m/sec}, \ a_y = -9.8 \text{ m/sec}^2,$$

$$y = 0$$

(vector displacement is zero because it returns to his hand),

$$t = ?$$

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

Using the fact that $y = 0$, we have

$$0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 = v_{0y} + \frac{1}{2}a_y t$$

and

$$t = -\frac{2v_{0y}}{a_y} = -\frac{2 \times 12 \text{ m/sec}}{-9.8 \text{ m/sec}^2} = 2.45 \text{ sec}$$

Projectile Motion (1)

- The *projectile motion* is defined as follows: the object moves in the x direction with its constant initial x velocity but its y velocity is incereaing downward owing to the acceleration of gravity.
- See multiflash photograph.
- $$x = v_{0x}t$$
$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

Example 3-4

A ball moving at 2 m/sec rolls off of a 1-m-high table, Fig. 3-4. How far horizontally from the edge of the table does it land?

Sol: The ball will continue moving in the x direction as long as it is in the air.

$$x_f = \bar{v}_x t_f, \ \bar{v_x} = v_x$$

where t_f is the time that the ball is in the air. We have

$$y_f = -1 \text{ m}, \ v_{0y} = 0,$$

$$a_y = -9.8 \text{ m/sec}^2, \ t_f = ?$$

$$y = v_{0y} t + \frac{1}{2} a_y t^2, \text{ and } y = \frac{1}{2} a_y t^2$$

Thus,

$$t = \pm\sqrt{\frac{2y}{a_y}}$$

$$t_f = \sqrt{\frac{2(-1 \text{ m})}{-9.8 \text{ m/sec}^2}} = 0.45 \text{ sec}$$

$$x_f = 2 \text{ m/sec} \times 0.45 \text{ sec} = 0.9 \text{ m}$$

Projectile Motion (2)

The general formula for the distance that a person can throw a ball or that a gun can fire a projectile. See Figure 3-5.

- $$x_f = \bar{v}_x t_f = v_{0x} t_f \text{ (because } \bar{v}_x = v_{0x})$$
  $$v_{0x} = v_0 \cos\theta \text{ and } x_f = (v_0 \cos\theta) t_f$$

- $$y_f = 0,\ v_{0y} = v_0 \sin\theta,$$
  $$a_y = g = -9.8 \text{ m/sec}^2 m,\ t_f = ?$$
  $$y = v_{0y} t + \frac{1}{2} a_y t^2$$
  $$0 = (v_0 \sin\theta) t_f - \frac{1}{2} g t_f^2$$

$$t_f = \frac{2}{g} v_0 \sin\theta$$

- $$x_f = \frac{v_0^2}{g} 2 \sin\theta \cos\theta$$

$$x_f = \frac{v_0^2}{g} \sin 2\theta$$

since

$$2 \sin\theta \cos\theta = \sin 2\theta$$

Example 3-5

A boy stands on the edge of a roof 10 m above the ground and throws a ball with a velocity of 15 m/sec at an angle of 37° above the horizontal. How far from the building does it land? See Fig. 3-6.

Sol: Let us choose the edge of the roof as the origin of the coordinate system.

1.

$$x_f = \bar{v}_x t_f = v_{0x} t_f$$

$$v_{0x} = v_0 \cos 37^\circ = 15 \text{ m/sec} \times 0.8 = 12 \text{ m/sec}$$

$$x_f = 12 \text{ m/sec} \times t_f$$

2.

$$y_f = -10 \text{ m},$$

$$v_{0y} = 15 \text{ m/sec} \times \sin 37^\circ$$

$$= \quad 15\text{m/sec} \times 0.6 = 9 \text{ m/sec}$$

$$a_y = -9.8 \text{ m/sec}^2, \ t_f = ?$$

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

If $t = t_f$ when $y = y_f = -10$ m, then we have

$$\frac{1}{2}a_y t_f^2 + v_{0y}t_f - y_f = 0$$

$$\begin{aligned} t_f &= \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - (4)(\frac{1}{2}a_y)(-y_f)}}{(2)(\frac{1}{2}a_y)} \\ &= \frac{-v_{0y} \pm \sqrt{v_{0y}^2 + 2a_y y_f}}{a_y} \end{aligned}$$

Subtituting the numerical values for y_f, v_{0y}, and a_y

$$t_f = 2.6 \text{ sec}, \ -0.78 \text{ sec}$$

3.

$$x_f = 12 \text{ m/sec} \times 2.6 \text{ sec} = 31.2 \text{ m}$$

What is v_f? DIY.

Homework: 3.6, 3.8, 3.10, 3.12, 3.14, 3.16, 3.18, 3.20.

Chapter Four: Newton’s Laws

- In this chapter we will consider Newton’s three laws of motion.
- There is one consistent word in these three laws and that is ”body” (newtonian body).
- We will define *force* through the motion it cause on mass.

Newton's Laws (1)

- First Law: Every body of matter continue in a state of rest or moves with constant velocity in a straight line unless compelled by a force to change state.
- Second Law: When net unbalanced forces act on a body, they will produce a change in the *momentum* (mv) of that body proportional to the vector sum of the force. The direction of the change in momentum is that of the line of action of the resultant force.
- Third Law: Forces, arising from the interaction of particles, act in such a way that the force exerted by one particle on the second is equal and opposite to the force exerted by the second on the first and both are directed along the line joining the two particles. (Or, action and reaction are equal and opposite).

Newton's Laws (2)

- The *average force* is defined as

$$\overline{\boldsymbol{F}}\Delta t = \Delta m\boldsymbol{v} \text{ and } \overline{\boldsymbol{F}} = m\frac{\Delta \boldsymbol{v}}{\Delta t}$$

- Let $\boldsymbol{a} = d\boldsymbol{v}/dt$, the *force* is define as

$$\boxed{\boldsymbol{F} = m\boldsymbol{a}}$$

- The forms of Newton's law that we will use are

$$\sum F_x = ma_x,\ \sum F_y = ma_y, \sum F_z = ma_z$$

Mass

- Let m_0 be the standard kilogram. If we exert a force on the mass with no other forces to interfere, we can measure an acceleration $\boldsymbol{a}_0$. If we apply this same force to a different mass m_1, we measure a different acceleration $\boldsymbol{a}_1$. Then

$$\boldsymbol{F} = m_0\boldsymbol{a}_0 = m_1\boldsymbol{a}_1$$

  and

$$m_1 = \frac{\boldsymbol{a}_0}{\boldsymbol{a}_1} m_0$$

- Force has units of mass $\times$ length/time^2 or kilogram-meter per second^2 (newton, N).
- A force of 1 N is that force which causes a mass of 1 kg to be accelerated at a rate of 1 m/sec^2.

Weight

- The rate of free fall of all objects in a vacuum at a given point on earth is the same.
- The downward acceleration at sea level is approximately the same at all locations, or g= 9.8 m/sec^2.
- Weight $= mg$.

Applications of Newton's Laws– Example 4-2

A child pulls a toy boat through the water at constant velocity by a string parallel to the surface of the water on which he exerts a force of 1 N. What is the force of resistance of the water to the motion of the boat? See Fig. 4-2.

Sol: Because constant velocity means zero acceleration,

$$\sum F_x = 0$$

$$F - f = 0$$

$$f = F = 1 \text{ N}$$

Applications of Newton's Laws– Example 4-3

Two ropes attached to a ceiling at the angles shown in Fig. 4-3 support a block of weight 50 N. What are the tensions T_1 and T_2 in the ropes?

Sol: If we examine the newtonian body, we see that it is not accelerating in either the x or y directions. We have

$$\sum F_x = 0, \sum F_y = 0$$

$$\begin{aligned} \sum F_x &= 0 \\ T_1 \cos 37^\circ - T_2 \cos 53^\circ &= 0 \\ 0.8T_1 - 0.6T_2 &= 0 \\ \sum F_y &= 0 \\ T_1 \sin 37^\circ + T_2 \sin 53^\circ - 50 \text{ N} &= 0 \end{aligned}$$

$$0.6T_1 + 0.8T_2 - 50\ \mathrm{N} \quad = \quad 0$$

Thus,

$$T_1 = \frac{0.6T_2}{0.8} = \frac{3}{4}T_2$$

Substituting into second equation

$$0.6\left(\frac{3}{4}T_2\right) + 0.8T_2 - 50\ \mathrm{N} = 0$$

$$1.25T_2 = 50\ \mathrm{N}$$

$$T_2 = 40\ \mathrm{N} \text{ and } T_1 = \frac{3}{4}T_2 = 30\ \mathrm{N}$$

Applications of Newton's Laws– Example 4-5

A block of mass 8 kg is released from rest on a frictionless incline that is at an angle of 37° with the horizontal (Fig. 4-6a). What is its acceleration down the incline?

Sol: See Figure 4-6b.

$$\sin 37^\circ = \frac{F_x}{mg}$$

$$F_x = mg\sin 37^\circ$$

From Newton's second law, we have

$$\begin{aligned} F_x &= ma_x \\ a_x &= \frac{F_x}{m} \\ &= \frac{mg\sin 37^\circ}{m} \\ &= g\sin 37^\circ = 9.8 \text{ m/sec}^2 \times 0.6 \end{aligned}$$

$$= \quad 5.9 \text{ m/sec}^2$$

Two important points:

- Because the acceleration is independent of the mass, all masses starting from rest at the same height on the same plane will have the same acceleration and, therefore, reach the bottom at the same time.
- The acceleration is less than the acceleration of gravity because only a component of the force of gravity on the body is directed down the plane.

Applications of Newton's Laws– Example 4-6

Masses of 2 kg and 4 kg connected by a cord are suspended over a frictionless pulley (Fig. 4-7a). What is their acceleration when released?

Sol: Three important facts:

1. Because the pulley is frictionless, the tension in the rope is the same on both sides.
2. The tensions are not the same as in a static situation.
3. There are two newtonian bodies and while m_1 moves upward with a positive acceleration, m_2 moves with an acceleration having the same magnitude but directed downward.

See Figure 4-7b.

For body m_1 we have

$$\begin{aligned}\sum F_y &= m_1 a\\ T - m_1 g &= m_1 a\\ T &= m_1(g+a)\end{aligned}$$

For body m_2

$$\begin{aligned}\sum F_y &= m_2(-a)\\ T - m_2 g &= m_2(-a)\\ T &= m_2(g-a)\end{aligned}$$

$$m_1(g+a) = m_2(g-a)$$

$$a(m_1+m_2) = g(m_2-m_1)$$

$$a = g\frac{m_2-m_1}{m_1+m_2}$$

$$= \quad 9.8 \text{ m/sec}^2 \times \frac{4 \text{ kg} - 2 \text{ kg}}{2 \text{ kg} + 4 \text{ kg}} = 3.3 \text{ m/sec}^2$$

Friction

- There is a force equal and opposite to the force that we exert that resists the motion of the object. This resistive force is called the *force of friction*.
- There are two types of friction, *static* and *kinetic*.
- The starting friction is called *static*. The friction of motion is called *kinetic*.
- Static friction is larger than kinetic friction. We will only consider kinetic friction.
- The force of friction is proportional to the normal force ($mg = N$). See Figure 4-8.
- The force of friction is $f = \mu N$, where μ is called the *coefficient of friction*.

Example 4-7

A force of 10 N is required to keep a box of mass 20 kg moving at a constant velocity across a level floor (Fig. 4-9). What is the coefficient of friction?

Sol: Since $a_x = 0$ and $a_y = 0$, we have

$$\begin{aligned} \sum F_x &= 0 \\ F - f &= 0 \\ f &= 10 \text{ N} \end{aligned}$$

and

$$\begin{aligned} \sum F_y &= 0 \\ N - mg &= 0 \\ N &= mg \end{aligned}$$

But

$$\begin{aligned} f &= \mu N \\ f &= \mu m g \\ \mu &= \frac{f}{mg} \\ &= \frac{10 \text{ N}}{20 \text{ kg} \times 9.8 \text{ m/sec}^2} \\ \mu &= 0.05 \end{aligned}$$

Example 4-8

A block is places on a plane inclines to the horizontal at 37°. The coefficient of friction between the plane and the block is $\mu = 0.4$. When the block is released what is its acceleration down the plane? (See Fig. 4-10)

Sol: The forces along the plane are the force of friction f upward and the component of the force of gravity F_D downward. Choose the downward direction as positive and we have

$$\begin{aligned} \sum F_{plane} &= ma_{plane} \\ F_D - f &= ma_{plane} \end{aligned}$$

Since

$$F_D = mg\sin\theta \text{ and } f = \mu mg\cos\theta$$

we have

$$mg\sin\theta - \mu mg\cos\theta = ma_{plane}$$

$$\begin{aligned} a_{plane} &= \frac{mg\sin\theta - \mu mg\cos\theta}{m} \\ &= g\sin\theta - \mu g\cos\theta \\ a_{plane} &= 9.8 \text{ m/sec}^2 \times 0.6 \\ &\quad -0.4 \times 9.8 \text{ m/sec}^2 \times 0.8 \\ &= 2.74 \text{ m/sec}^2 \end{aligned}$$

Homework: 4.2, 4.4, 4.7, 4.9, 4.11, 4.14, 4.16, 4.19, 4.21, 4.24.

Chapter Five: Work, Energy, and Power

- Definitions in physics do not always match the usage of the words.
- We consider *mechanical* work, energy, and power, for it is the treatment of these terms from First Principle that will be applied directly to electrical circuits.

Work

- Work ΔW down by a constant force $\boldsymbol{F}$ acting on a body is

$$\Delta W = F_s \Delta s$$

  where F_s represents the component of force in the direction Δs.

- We define a new unit 1 N-m = 1 joule with symbol J.
- When force and motion are not in the same direction (see Fig. 5-2), we have

$$\begin{aligned} \Delta W &= F_x \Delta x \\ &= F \cos\theta \Delta x \\ \Delta W &= F \Delta x \cos\theta \\ \Delta W &= \boldsymbol{F} \cdot \Delta \boldsymbol{x} \end{aligned}$$

Example 5-1

A box is pushed 3 m at constant velocity across a floor by a force $\boldsymbol{F}$ of 5 N parallel to the floor. (a) How much work was down on the box by the force $\boldsymbol{F}$, which clearly opposes friction (see Fig. 5-1). (b) How much work is down on the box by the force of friction?

Sol: (a) $W = 5\ \mathrm{N} \times 3\ \mathrm{m} = 15\ \mathrm{J}$

(b) Because $a = 0$

$$\begin{aligned} \sum F_x &= 0 \\ F - f &= 0 \\ f &= 5\ \mathrm{N} \\ W &= 15\ \mathrm{J} \end{aligned}$$

Potential Energy

- Work down against the gravitational force is independent of the choice of path between any two fixed endpoints. See Fig. 5-3.
- The *potential energy* E_p is defined as

$$E_p = mgy$$

where y is the height in a gravitational field.

- For potential energy a reference level must always be specified. See Fig. 5-4, 5-5.

$$\Delta E_p(\text{with the table as the reference level})$$

$$= mgy_2 - mgy_1 = mg(y_2 - y_1)$$

$$\Delta E_p(\text{with the floor as the reference level})$$

$$= \quad mg(y_2 + y_3) - mg(y_1 + y_3)$$

$$= \quad mg(y_2 - y_1)$$

- Only the difference in heights needs to be specified to give the relative difference in potential energy.

Work Done by a Variable Force

- See Fig. 5-6.

$$W = \sum_{i=1}^{N} F_{xi} \Delta x_i$$

When $\Delta x_i \to 0$, we have

$$W = \int_a^b F_x dx$$

- In more general case where $\boldsymbol{F}$ and the general displacement $\Delta \boldsymbol{s}$'s are not in the same direction, the expression for the work becomes

$$dW = \boldsymbol{F} \cdot d\boldsymbol{s}$$

or

$$W = \int_a^b \boldsymbol{F} \cdot d\boldsymbol{s}$$

Kinetic Energy

1. The force is constant:
   - The initial position is $x = 0$, we have

     $$W = F_x x$$

     By Newton's second law

     $$W = m a_x x = m a x$$

     Since

     $$v^2 - v_0^2 = 2ax$$

     where v_0 is the velocity at $x = 0$ and v is the velocity at x. Thus

     $$W = m\left\{\frac{v^2 - v_0^2}{2}\right\}$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

- The work done on a body that changes its velocity actually changes the quantity $\frac{1}{2}mv^2$, which is called the *kinetic energy* E_k.

$$E_k = \frac{1}{2}mv^2$$

2. The applied force is not constant:

-

$$\begin{aligned} W &= \int_{x_0}^{x} F\,dx \\ &= m\int_{x_0}^{x} a\,dx \end{aligned}$$

- Since $a = v\frac{dv}{dx}$ we have

$$
\begin{aligned}
W &= m\int_{x_0}^{x} v\frac{dv}{dx}dx \\
&= m\int_{v_0}^{v} v\ dv \\
&= m\ \frac{v^2}{2}\bigg|_{v_0}^{v} \\
&= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2
\end{aligned}
$$

- The *work-energy theorem* is stated as the work done by the resultant force acting on a particle is equal to the change in kinetic energy of the particle.

Energy Conservation

1. For a mechanically conservative system (one in which no energy enters or leaves the system):
   - $(E_k + E_p)_{\text{initial}} = (E_k + E_p)_{\text{final}}$
   - Let us launch an object of mass m from a point y_1 above the floor with an initial velocity v_1. Sometime later, the velocity of the object will be v_2 and its position y_2. See Fig. 5-7. We have

$$v_2^2 - v_1^2 = 2(-g)(y_2 - y_1)$$

$$v_2^2 + 2gy_2 = v_1^2 + 2gy_1$$

$$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + mgy_1$$

$$(E_k + E_p)_{\text{initial}} = (E_k + E_p)_{\text{final}}$$

   - Let us assume an idealized pendulum that swings in a vacuum so that there is no energy lost to air friction and that is no frictional loss at the pivot (see Fig. 5-8). We start the pendulum by pulling it to one side and releasing it with no initial velocity.

$$\begin{aligned} E_{p0} + E_{k0} &= E_{p2} + E_{k2} \\ mgh_0 + 0 &= mgh_2 + \frac{1}{2}mv_2^2 \end{aligned}$$

- the string does no work on the pendulum because of

$$dW = \boldsymbol{F} \cdot d\boldsymbol{s} = F \cos\theta \; ds$$

where θ is the angle between the string direction and ds, and $\theta = 90^\circ$.

2. For an accountability of energy system we have

$$E_{ki} + E_{pi} + E_{\text{in}} = E_{kf} + E_{pf} + E_{\text{out}}$$

Example 5-2

Suppose a ball is dropped from a height $h = 10$ m. What is its velocity just before it strikes the ground?

Sol:

$$\begin{aligned} E_{ki} + E_{pi} &= E_{kf} + E_{pf} \\ 0 + mgh &= \frac{1}{2}mv^2 + 0 \\ v &= \pm\sqrt{2gh} \\ &= \pm\sqrt{2 \times 9.8 \text{ m/sec}^2 \times 100 \text{ m}} \\ &= -14 \text{ m/sec} \end{aligned}$$

Example 5-3

A skier is on a 37° slope of length $s = 100$ m (see Fig. 5-9). The coefficient of friction between his skis and the snow is 0.2. If he start from rest, what is his velocity at the bottom of the slope?

Sol:

$$E_{ki} + E_{pi} + E_{\text{in}} = E_{kf} + E_{pf} + E_{\text{out}}$$

No energy is put in, but $E_{\text{out}} = |W_{\text{friction}}| = |\boldsymbol{f} \cdot \boldsymbol{s}|$.

Let us tilt our coordinate axis so that the slope becomes the x axis and the normal becomes the y axis.

$$\begin{aligned} \sum F_y &= 0 \\ N &= mg\cos 37^\circ \\ f &= \mu N = \mu mg \cos 37^\circ \\ E_{\text{out}} &= |\boldsymbol{f} \cdot \boldsymbol{s}| = \mu mg \cos 37^\circ (s) \end{aligned}$$

$$0 + mgh + 0 = \frac{1}{2}mv_f^2 + 0 + \mu mg\cos 37^\circ(s)$$

$$\begin{aligned} v_f &= [2(gh - \mu gs\cos 37^\circ)]^{1/2} \\ &= [2(9.8 \text{ m/sec}^2 \times 100 \text{ m} \times \sin 37^\circ \\ &\quad -0.2 \times 9.8 \text{ m/sec}^2 \times 100 \text{ m} \times \cos 37^\circ)]^{1/2} \\ &= 29.4 \text{ m/sec} \end{aligned}$$

Power

- Power is defined as

$$\begin{aligned} \text{Power} &= \frac{\text{work done}}{\text{time taken}} \\ P &= \frac{W}{t} \end{aligned}$$

- The unit of power is joules per second (J/sec).
- New unit: 1 J/sec = 1 watt(W).
- A 100-W light bulb uses 100 J of electrical energy each second.
- A kilowatt-hour is the energy dissipated by a device that uses 10^3 W for a period of 1 h, that is,

$$1 \text{ kwh} = 10^3 \text{ J/sec} \times 3600 \text{ sec} = 3.6 \times 10^6 \text{ J}$$

- $$\begin{aligned} P &= \boldsymbol{F} \cdot \frac{d\boldsymbol{x}}{dt} \\ P &= \boldsymbol{F} \cdot \boldsymbol{v} \end{aligned}$$
- 1 horsepower (hp) = 746 W.

Example 5-4

A tractor can exert a force of 3×10^4 N while moving at constant speed of 5 m/sec. What is its horsepower?

Sol:

$$\begin{aligned} P &= F \times v \\ &= 3 \times 10^4 \text{ N} \times 5 \text{ m/sec} \\ &= 1.5 \times 10^5 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = 200 \text{ hp} \end{aligned}$$

Homework: 5.2, 5.4, 5.8, 5.12, 5.13, 5.15, 5.16, 5.17, 5.18, 5.20.

Chapter Six: Momentum and Collisions

- Momentum is the product of the mass of a body and its velocity.
- A body may be an assembly of particles. Such an assembly can be mathematically represented by a point mass, called the *center of mass*.
- The motion of the center of mass is that predicted by Newton's second law for a particle whose mass is the sum of the masses of the individual particles and is acted on by the resultant of the forces acting on the body.

Center of Mass

- The weight of the stick supplies the downward force, and it appears to be located at the balance point , although we know that every segment of the stick has weight. We call this point the *center of gravity* of the stick.
- The center of gravity of the stick behaves as a point mass in Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$, and that the center of gravity may also be considered as the *center of mass*.
- See Fig. 6-1. The center of mass is the point such that

$$m_1 a = m_2 b$$

- From Fig. 6-1(b), we have

$$a = x_{cm} - x_1 \text{ and } b = x_2 - x_{cm}$$

$$m_1(x_{cm} - x_1) = m_2(x_2 - x_{cm})$$

$$(m_1 + m_2)x_{cm} = m_1 x_1 + m_2 x_2$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

- This relation holds true regardless of the number of masses placed on the balance, so we have

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

where M is the total mass.

- We define the center of mass as the point whose Cartesian coordinates are

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \ y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i,$$

and

$$z_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

where $x_i,\ y_i,\ z_i$ are the coordinates of the ith particle, all measured from the same arbitrary origin.

Example 6-1

Find the center of mass of the configuration in Fig. 6-2 when $m_1 = 1$ kg, $m_2 = 2$ kg, and $m_3 = 3$ kg.

Sol: If we take the position of m_1 as the origin, we have

$$\begin{aligned} x_{cm} &= \frac{1}{1 \text{ kg} + 2 \text{ kg} + 3 \text{ kg}}(1 \text{ kg} \times 0 \text{ m} \\ &\quad +2 \text{ kg} \times 0.5 \text{ m} + 3 \text{ kg} \times 1.3 \text{ m}) \\ &= 0.82 \text{ m} \end{aligned}$$

Motion of the Center of Mass

- $$\begin{aligned} Mx_{cm} &= m_1x_1 + m_2x_2 + \cdots m_nx_n \\ M\frac{dx_{cm}}{dt} &= m_1\frac{dx_1}{dt} + m_2\frac{dx_2}{dt} + \cdots m_n\frac{dx_n}{dt} \\ Mv_{xcm} &= m_1v_{x1} + m_2v_{x2} + \cdots m_nv_{xn} \end{aligned}$$

  Similar expressions can be obtained for Mv_{ycm} and Mv_{zcm}.

- Similarly, we have

  $$Ma_{xcm} = m_1a_{x1} + m_2a_{x2} + \cdots m_na_{xn}$$

- If we apply Newton's second law to each individual particle, then we have

  $$Ma_{xcm} = F_{x1} + F_{x2} + \cdots F_{xn}$$

  where F_{xi} is the x component of the resultant of the forces acting on the ith particle.

- We only need to consider all the external forces acting on any particle.
- $\boldsymbol{F} = M\boldsymbol{a}_{cm}$ where $\boldsymbol{F}$ is the resultant of the external forces acting on all the particles.

Example 6-3

Suppose a grenade is thrown that has the trajectory shown in Fig. 6-4. If it explodes in midair, only internal forces have acted on the fragments and the acceleration of the center of mass of the fragments, regardless of their subsequence dispersal, is unchanged by the explosion, and thus follows the original trajectory.

Momentum and Its Conservation

- An impulse applied to a body will change its state of momentum.

$$\begin{aligned} \boldsymbol{F}\Delta t &= \Delta m\boldsymbol{v} \\ \boldsymbol{F}\Delta t &= m\boldsymbol{v}_f - m\boldsymbol{v}_0 \end{aligned}$$

  where $\boldsymbol{v}_0$ is the velocity of the body before the force begins to act on it and $\boldsymbol{v}_f$ is the velocity when the force stops acting on the body.

- Momentum is often represented by the letter $\boldsymbol{p}$,

$$\boldsymbol{F}\Delta t = \boldsymbol{p}_f - \boldsymbol{p}_0$$

- If there is no external force exerted on a mass we have

$$\boldsymbol{p}_0 = \boldsymbol{p}_f$$

  which is the *law of conservation of momentum.*

- If the resultant of the external forces acting on all the particles is zero, the total momentum of all the particles will not change, or

$$\left(\sum_{i=1}^{n} \boldsymbol{p}_i\right)_{\text{before}} = \left(\sum_{i=1}^{n} \boldsymbol{p}_i\right)_{\text{after}}$$

- Momentum at very high speeds–*Einstein's theory of special relativity.* For the particles moving with speeds that are near the speed light, the formula $\boldsymbol{F} = d\boldsymbol{p}/dt$ is correct, provided we define the momentum of a particle not as $m\boldsymbol{v}$ but as

$$\boldsymbol{p} = \frac{m\boldsymbol{v}}{\sqrt{1 - (\frac{v}{c})^2}}$$

where c is the speed of light.

Example 6-4

A cannon of mass 1000 kg fires a 100-kg projectile with a muzzle velocity of 400 m/sec (see Fig. 6-5). With what speed and in what direction does the cannon move?

Sol: Let M be the mass of the cannon, $\boldsymbol{V}_0$ its initial velocity, and $\boldsymbol{V}_f$ its final velocity. Let m be the mass of the projectile and $\boldsymbol{v}_0$ and $\boldsymbol{v}_f$ its initial and final velocities, respectively. If we consider the cannon and projectile as our system of particles, no external force is involved in the firing of the projectile and we have

$$\boldsymbol{p}_0 = \boldsymbol{p}_f$$

$$m\boldsymbol{v}_0 + M\boldsymbol{V}_0 = m\boldsymbol{v}_f + M\boldsymbol{V}_f$$

If we choose the direction of motion of the projectile as the positive direction, we get

$$0 + 0 = m\boldsymbol{v}_f + M\boldsymbol{V}_f$$

or

$$\begin{aligned} \boldsymbol{V}_f &= -\frac{m\boldsymbol{v}_f}{M} = -\frac{100 \text{ kg} \times 400 \text{ m/sec}}{1000 \text{ kg}} \\ \boldsymbol{V}_f &= -40 \text{ m/sec} \end{aligned}$$

Example 6-5

A 10,000-kg truck travelling east at 20 m/sec collides with a 2,000-kg car travelling north at 30 m/sec. After the collision, they are locked together. With what velocity and at what angle do the locked vehicles move immediately after the collision? (See Fig. 6-6)

Sol: Because no external force is involved in the collision, momentum is conserved. In the x direction

$$\begin{aligned} p_{x0} &= p_{xf} \\ m_T v_T &= (m_T + m_c) V \cos\theta \\ V\cos\theta &= \frac{m_T v_T}{m_T + m_c} = \frac{10,000 \text{ kg} \times 20 \text{ m/sec}}{10,000 \text{ kg} + 2000 \text{ kg}} \\ &= 16.7 \text{ m/sec} \end{aligned}$$

In the y direction,

$$\begin{aligned}
p_{y0} &= p_{yf} \\
m_c v_c &= (m_T + m_c) V \sin\theta \\
V \sin\theta &= \frac{m_c v_c}{m_T + m_c} = \frac{2000 \text{ kg} \times 30 \text{ m/sec}}{10,000 \text{ kg} + 2000 \text{ kg}} \\
&= 5.0 \text{ m/sec}
\end{aligned}$$

Find the angle by dividing the two velocity components

$$\begin{aligned}
\frac{V \sin\theta}{V \cos\theta} &= \tan\theta = \frac{5.0 \text{ /sec}}{16.7 \text{ m/sec}} = 0.30 \\
\theta &= \arctan 0.30 = 16.7^\circ \\
V \sin 16.7^\circ &= 5 \text{ m/sec} \\
V &= 17.4 \text{ m/sec}
\end{aligned}$$

Collisions

- There are two types of collisions: elastic and inelastic.
- In an elastic collision kinetic energy is conserved (i.e., no energy is lost from the system).
- An inelastic collision is one in which kinetic energy is not conserved (e.g., some energy is lost to friction, crumpled fenders, or such).

Example 6-6 Elastic collisions

A neutron with a mass of $m = 1\ u$ (atomic mass unit) strikes a larger atom at rest and rebounds elastically along its original path with 0.9 of its initial forward velocity. What is the mass M, in atomic mass units, of the atom it struck?

Sol: Let v_0 be the initial velocity of the neutron and $v_f = -0.9\ v_0$ its final velocity. Let M be the mass of the atom, V_0 its initial velocity, and V_f its velocity after collision. Both momentum and kinetic energy are conserved. From the conservation of momentum

$$mv_0 + MV_0 = mv_f + MV_f$$

Since $V_0 = 0$

$$V_f = \frac{m(v_0 - v_f)}{M}$$

$$= \frac{1\ u(v_0 + 0.9v_0)}{M} = \frac{(1\ u)(1.9v_0)}{M}$$

From the conservation of kinetic energy

$$\frac{1}{2}mv_0^2 + \frac{1}{2}MV_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}MV_f^2$$

Since $V_0 = 0$

$$\begin{aligned} M &= \frac{m(v_0^2 - v_f^2)}{V_f^2} = \frac{(1\ u)(0.19v_0^2)}{V_f^2} \\ M &= \frac{(1\ u)(0.19v_0^2)M^2}{(1\ u^2)(3.61v_0^2)} \\ &= \frac{(1\ u^2)(3.61)}{(1\ u)(0.19)} \\ &= 19\ u \end{aligned}$$

Example 6-7 Elastic collisions

An important type of elastic collision at the atomic level, whose results we will use later, is the collision between a very small mass particle, such as an electron, with another particle of comparatively large mass, such as an atom. The mass of a copper atom, for example, is about 10^5 times that of an electron. In this type of collision with the copper atom one is often interested in finding the velocity of the electron after the collision with the copper atom. To solve this type of collision, we follow the usual procedure of conserving momentum and kinetic energy. We will assume a one-dimensional collision.

Sol: Let m, v_0, and v_f be the mass and the initial and the final velocity of the electron and M, V_0, and V_f those of the atom. From the conservation of momentum law

$$\begin{aligned} mv_0 + MV_0 &= mv_f + MV_f \\ M(V_0 - V_f) &= m(v_f - v_0) \end{aligned}$$

Conserving kinetic energy yields

$$\begin{aligned} \frac{1}{2}mv_0^2 + \frac{1}{2}MV_0^2 &= \frac{1}{2}mv_f^2 + \frac{1}{2}MV_f^2 \\ M(V_0^2 - V_f^2) &= m(v_f^2 - v_0^2) \\ M(V_0 + V_f)(V_0 - V_f) &= m(v_f + v_0)(v_f - v_0) \\ V_0 + V_f &= v_0 + v_f \end{aligned}$$

Let us have the atom initially at rest, $V_0 = 0$, we have

$$\begin{aligned} -M(v_0 + v_f) &= m(v_f - v_0) \\ -Mv_0 - Mv_f &= mv_f - mv_0 \end{aligned}$$

$$\begin{aligned} -v_f(M+m) &= v_0(M-m) \\ v_f &= -v_0\frac{(M-m)}{(M+m)} \end{aligned}$$

For $m \ll M$, we have

$$v_f \approx -v_0$$

Example 6-8 Inelastic collisions

A ballistic pendulum is used to measure the velocity of a bullet. The bullet is shot into a wooden block suspended by strings. It lodges in the block, losing energy in its penetration, and the increase in the height of the swinging block and bullet is measured (see Fig. 6-7). If the bullet has a mass of 0.01 kg, the block has a mass of 0.5 kg, and the swing rises 0.1 m, what was the velocity of the incident bullet and what fraction of its energy was lost during penetration?

Sol: We first conserve momentum between situation (a) and (b) in Fig. 6-7.

$$\begin{aligned} \boldsymbol{p}_0 &= \boldsymbol{p}_f \\ mv + 0 &= (m+M)V \\ 0.01 \text{ kg } v &= 0.51 \text{ kg } V \end{aligned}$$

Recall that the string does no work on the block in the pendulum.

$$\begin{aligned}(E_k)_b &= (E_p)_c \\ \frac{1}{2}(m+M)V^2 &= (m+M)gh \\ V &= \sqrt{2gh} = 1.4 \text{ m/sec}\end{aligned}$$

$$v = \frac{0.51 \text{ kg}}{0.01 \text{ kg}} \times 1.4 \text{ m/sec} = 71.4 \text{ m/sec}$$

We find the fraction of the bullet's initial energy lost in penetration by calculating the energy of the system before and after the collision.

$$\begin{aligned}(E_k)_a &= \frac{1}{2}mv^2 = \frac{1}{2}(0.01 \text{ kg})(71.4 \text{ m/sec})^2 \\ &= 25.5 \text{ J} \\ (E_k)_b &= \frac{1}{2}(m+M)V^2 = \frac{1}{2}(0.51 \text{ kg})(1.4 \text{ m/sec})^2 \\ &= 0.5 \text{ J}\end{aligned}$$

$$\text{Fraction remaining} = \frac{(E_k)_b}{(E_k)_a} = \frac{0.5\ \text{J}}{25.5\ \text{J}} = 0.02$$

$$\text{Fraction lost} = 1 - \text{fraction remaining} = 0.98$$

Homework: 6.5, 6.7, 6.9, 6.12, 6.14, 6.18, 6.19, 6.22, 6.23, 6.24.

Chapter Seven: Rotational Motion

- A roulette wheel simply rotates, whereas a car wheel both rotates and translates.
- Newton's laws and momentum and energy conservation still apply, but the formulation is somewhat different.

Measurement of Rotation

- The most common measurement of rotation is a count of the number of revolutions about an axis of rotation.
- We also use degrees as a measure, where 360° corresponds to one revolution.
- In physics we mostly use radians since the formulation affords a quick and easy bridge between linear and rotational motion.
- A measure of an angle in radians is the length of the circular arc subtended by the angle divided by the radius of the circle (see Fig. 7-1).

$$\theta(\text{in radians}) = \frac{s}{r}$$

where θ is dimensionless.

- $$\frac{s \text{ of 1 revolution}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi \text{ radians (rad)}$$
- $$\begin{aligned} 2\pi \text{ (radians)} &= 360 \text{ (degree)} \\ &= 1 \text{ revolution (rev)} \end{aligned}$$

Rotational Motion

- Suppose we have a reference marker a on the x axis of a coordinate system and a wheel whose center coincides with the origin (see Fig. 7-2).

$$\begin{aligned} \text{speed}_{a\to b} &= \frac{\Delta s}{\Delta t} \\ \boldsymbol{v} &= \frac{ds}{dt} \text{ when } \Delta t \to 0 \end{aligned}$$

- The direction of the instantaneous velocity of the marker on the rotating wheel is the tangent to the circle of motion and is called the *tangential velocity* ($\boldsymbol{v}_T$).
- The velocity $\boldsymbol{v}_T$ is constantly changing. Here we have a vector whose magnitude may remain constant while its direction always changes.

- The average rate of change of the angle θ with time is called the *average angular* or *rotational velocity.*

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

As $\Delta t \to 0$, the average angular velocity $\bar{\omega}$ becomes the instantaneous angular velocity ω

$$\omega = \frac{d\theta}{dt}$$

- Since

$$\frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt}$$

we have

$$v_T = r\omega$$

- The average tangential acceleration is defined as

$$\begin{aligned}\overline{\boldsymbol{a}}_T &= \frac{\Delta \boldsymbol{v}_T}{\Delta t} \\ &= \frac{\boldsymbol{v}_{Tf} - \boldsymbol{v}_{T0}}{\Delta t}\end{aligned}$$

  In the limit as $\Delta t \to 0$, $\overline{\boldsymbol{a}}_T$ becomes the instantaneous acceleration $\boldsymbol{a}_T$,

$$\boldsymbol{a}_T = \frac{d\boldsymbol{v}_T}{dt}$$

- We call the rate of change of ω the average *angular* or *rotational* acceleration, so that

$$\overline{\alpha} = \frac{\Delta \omega}{\Delta t}$$

  To find the instantaneous value we let $\Delta t \to 0$, and

$$\alpha = \frac{d\omega}{dt}$$

Since $v_T = r\omega$,

$$\begin{aligned} \frac{dv_T}{dt} &= r\frac{d\omega}{dt} \\ a_T &= r\alpha \end{aligned}$$

Example 7-1

A car is traveling at a constant velocity of 24 m/sec. The radius of its wheels is $r = 0.30$ m. (a) How many revolutions have the wheels turned after the car has gone 120 m? (b) How many revolutions have the wheels turned after 60 sec?

Sol:

- (a) If there is no slipping between the wheels of the car and the road, the arc length moved by a marker on the outermost radius of the wheel is equal to the distance traveled by the car.

$$\begin{aligned}\theta &= \frac{s}{r} = \frac{120\ \text{m}}{0.30\ \text{m}} \\ \theta &= 400\ \text{rad} = 400\ \text{rad}\left(\frac{1\text{rev}}{2\pi\ \text{rad}}\right) \\ &= 63.7\ \text{rev}\end{aligned}$$

- (b) Because the car travels at constant velocity,

$$x = (24 \text{ m/sec})(60 \text{ sec}) = 1440 \text{ m}$$

$$\begin{aligned} \theta &= \frac{s}{r} = \frac{1440 \text{ m}}{0.30 \text{ m}} = 4800 \text{ rad} \\ &= 4800 \text{ rad} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 764 \text{ rev} \end{aligned}$$

Example 7-2

A driver of a car traveling at 24 m/sec applies the brakes, decelerates uniformly, and comes to a stop in 100 m. If the wheels have a radius of 0.30 m, what is the angular deceleration of the wheels in rev/sec^2?

Sol:

$$v_0 = 24 \text{ m/sec},\ v_f = 0,\ x = 100 \text{ m},\ a =?$$

$$v_f^2 - v_0^2 = 2ax$$

$$\begin{aligned} a &= \frac{v_f^2 - v_0^2}{2x} \\ &= \frac{0 - (24 \text{ m/sec})^2}{(2)(100 \text{ m})} = -2.88 \text{ m/sec}^2 \end{aligned}$$

Because a is the acceleration of the car, it is also the tangential acceleration of every point on the rim of its wheels.

$$\begin{aligned}
a_T &= r\alpha \\
\alpha &= \frac{a_T}{r} = \frac{-2.88\ \mathrm{m/sec^2}}{0.30\ \mathrm{m}} \\
&= -9.6\frac{\mathrm{rad}}{\mathrm{sec}^2}\frac{1\ \mathrm{rev}}{2\pi\ \mathrm{rad}} = -1.5\ \mathrm{rev/sec^2}
\end{aligned}$$

Equations of Rotational Motion

- We will limit our discussion on the case of constant angular acceleration.
-

$$\begin{aligned}\theta &= \bar{\omega}t\\ \bar{\omega} &= \frac{\omega_0+\omega_f}{2}\end{aligned}$$

-

$$\begin{aligned}\alpha &= \frac{d\omega}{dt}\\ \int_{\omega_0}^{\omega} d\omega &= \alpha\int_0^t dt\\ \omega-\omega_0 &= \alpha t\end{aligned}$$

$$\boxed{\omega=\omega_0+\alpha t}$$

- $$\begin{aligned} \omega &= \frac{d\theta}{dt} \\ \int_{\theta_0}^{\theta} d\theta &= \int_0^t \omega dt \\ \int_{\theta_0}^{\theta} d\theta &= \int_0^t (\omega_0 + \alpha t) dt = \omega_0 \int_0^t dt + \alpha \int_0^t t dt \end{aligned}$$

$$\boxed{\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2}$$

- $$\begin{aligned} \alpha &= \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \omega \frac{d\omega}{d\theta} \\ \alpha d\theta &= \omega d\omega \\ \alpha \int_{\theta_0}^{\theta} d\theta &= \int_{\omega_0}^{\omega} \omega d\omega \end{aligned}$$

$$\alpha(\theta - \theta_0) = \frac{1}{2}(\omega^2 - \omega_0^2)$$

$$\boxed{\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)}$$

Example 7-3

A roulette wheel is given an initial rotational velocity of 2 rev/sec. It is observed to be rotating at 1.5 rev/sec 5 sec after it was set in motion. (a) What is the angular deceleration (assumed constant) of the wheel? (b) How long will it take to stop? (c) How many revolutions will it make from start to finish?

Sol:

- (a)

$$\begin{aligned}
\omega_0 &= 2.0 \text{ rev/sec } \omega = 1.5 \text{ rev/sec } t = 5 \text{ sec } \alpha = ? \\
\omega &= \omega_0 + \alpha t \\
\alpha &= \frac{\omega - \omega_0}{t} = \frac{1.5 \text{ rev/sec} - 2.0 \text{ rev/sec}}{5 \text{ sec}} \\
&= -0.1 \text{ rev/sec}^2
\end{aligned}$$

- (b)

$$
\begin{aligned}
\omega_0 &= 2 \text{ rev/sec } \omega_f = 0 \\
\alpha &= -0.1 \text{ rev/sec}^2 \; t_f =? \\
\omega_f &= \omega_0 + \alpha t_f \\
t_f &= \frac{\omega_f - \omega_0}{\alpha} = \frac{0 - 2 \text{ rev/sec}}{-0.1 \text{ rev/sec}^2} \\
&= 20 \text{ sec}
\end{aligned}
$$

- (c)

$$
\begin{aligned}
\omega_0 &= 2 \text{ rev/sec } \omega_f = 0 \; t_f = 20 \text{ sec } \theta =? \\
\theta &= \frac{\omega_0 + \omega_f}{2} t_f = \frac{2 \text{ rev/sec} + 0}{2} \times 20 \text{ sec} \\
\theta &= 20 \text{ rev}
\end{aligned}
$$

Radial Acceleration

- See Fig. 7-3.
- Because a velocity vector of a point on a circle is always tangent to the circle, it is perpendicular to the radius. For infinitesimal changes Δt and thus $\Delta \boldsymbol{v}_\perp$, there is an acceleration $\boldsymbol{a}_R$ inward along the radius called a *radial acceleration* $\boldsymbol{a}_R$ given by

$$\boldsymbol{a}_R = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t}$$

- We now examine this radial acceleration analytically (see Fig. 7-4).

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}$$

Let the marker rotate about the circle at a constant rotational velocity ω so that $\bar{\omega} = \omega$.

$$\begin{aligned} x &= r\cos\omega t \\ y &= r\sin\omega t \\ v_x &= r\frac{d}{dt}(\cos\omega t) = -r\omega\sin\omega t \\ v_y &= r\frac{d}{dt}(\sin\omega t) = r\omega\cos\omega t \end{aligned}$$

$$\begin{aligned} a_x &= \frac{dv_x}{dt} = -r\omega\frac{d}{dt}(\sin\omega t) = -r\omega^2\cos\omega t \\ a_y &= \frac{dv_y}{dt} = r\omega\frac{d}{dt}(\cos\omega t) = -r\omega^2\sin\omega t \\ a_R^2 &= a_x^2 + a_y^2 \\ a_R^2 &= r^2\omega^4\cos^2\omega t + r^2\omega^4\sin^2\omega t \\ &= r^2\omega^4(\cos^2\omega t + \sin^2\omega t) \end{aligned}$$

Since

$$\sin^2\theta + \cos^2\theta = 1$$

we have

$$\begin{aligned} a_R^2 &= r^2\omega^4 \\ a_R &= \pm r\omega^2 \end{aligned}$$

- The direction of $\boldsymbol{a}_R$ is along the radius toward the center, that is, opposite to the vector direction of the radius (see Fig 7-5).

Centripetal Force

- If there is an acceleration there must be a net force.
- The particle cannot undergo circular motion unless there is a force along the radius directed inward toward the center. This force is called the *centripetal* force.
- We indicate radial force by F_R.

$$\begin{aligned}\sum F_R &= ma_R\\ \sum F_R &= mr\omega^2\\ \sum F_R &= m\frac{v_T^2}{r}\end{aligned}$$

- In the solution of problems involving radial acceleration, two rules must be observed based on the derivations:
  1. a_R has dimensions of m/sec^2 and therefore ω must have

dimensions of rad/sec^2;

2. Radial forces directed toward the center of rotation are positive, whereas those directed away from the center are negative.

Example 7-4

A person whose weight is 600 N is riding a roller coaster. This person sits on a scale as the roller coaster passes over the top of a rise of radius 80 m. (a) What is the minimum speed of the car if the scale reads zero (the sensation of weightlessness is experienced)? (b) If the car increases its speed to 40 m/sec in descending to a dip with a radius of 80 m, what will the scale read? See Fig. 7-6.

Sol: Let us consider the forces on the rider at the rise. The rider's weight, mg, is directed toward the center. The scale exerts a normal force N upward. N is the reading of the scale.

$$\begin{aligned} \sum F_R &= m\frac{v_T^2}{r} \\ mg - N &= m\frac{v_T^2}{r} \end{aligned}$$

Since $N = 0$ we have

$$\begin{aligned} mg &= m\frac{v_T^2}{r} \\ v_T &= \sqrt{gr} \\ &= \sqrt{9.8 \text{ m/sec}^2 \times 80 \text{ m}} = 28 \text{ m/sec} \end{aligned}$$

In the dip $v'_T = 40$ m/sec, mg is downward directed away from the center whereas N' now is upward toward the center.

$$-mg + N' = m\frac{{v'_T}^2}{r}$$

$$\begin{aligned} N' &= mg + m\frac{{v'_T}^2}{r} \\ &= 600 \text{ N} + \left(\frac{600 \text{ N}}{9.8 \text{ m/sec}^2}\right)\frac{(40 \text{ m/sec}^2)^2}{(80 \text{ m})} \end{aligned}$$

$$= \quad 600 \text{ N} + 1224 \text{ N} = 1824 \text{ N}$$

Orbital Motion and Gravitation

- *Newton's law of gravitation*: any two bodies are gravitationally attracted to each other by a force proportional to the product of their masses ($m_1 m_2$) and inversely proportional to the square of the distance between them, r^2.
- If we call the proportionality constant G, the *universal gravitational constant*,

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

- The value of G is $6.67 \times 10^{-11} \mathrm{Nm}^2/\mathrm{kg}^2$.
- The way to calculate the acceleration of gravity at the earth's surface, g.

  1.

$$m_e = \text{the mass of the earth}$$

$$
\begin{aligned}
m_o &= \text{the mass of an object near the} \\
&\quad\ \text{surface of the earth} \\
m_m &= \text{the mass of the moon} \\
r_e &= \text{the radius of the earth} \\
r_{em} &= \text{the distance from the center of} \\
&\quad\ \text{mass of the earth to the center} \\
&\quad\ \text{of mass of the moon} \\
m_0 g &= \text{the force on an object at} \\
&\quad\ \text{the surface of earth}
\end{aligned}
$$

2.

$$
m_0 g = G\frac{m_0 m_e}{r_e^2}
$$

$$G = \frac{gr_e^2}{m_e}$$

3. The moon is also attracted to the earth by the gravitational force

$$F = G\frac{m_m m_e}{r_{em}^2}$$

4. This force is the centripetal force

$$F_R = m_m \frac{v_T^2}{r_{em}}$$

5.

$$\begin{aligned} G\frac{m_m m_e}{r_{em}^2} &= m_m \frac{v_T^2}{r_{em}} \\ G &= \frac{v_T^2 r_{em}}{m_e} \end{aligned}$$

$$\begin{aligned} g\frac{r_e^2}{m_e} &= \frac{v_T^2 r_{em}}{m_e} \\ g &= \frac{v_T^2 r_{em}}{r_e^2} \end{aligned}$$

6. We have $r_{em} = 3.8 \times 10^8$ m and $r_e = 6.3 \times 10^6$ m. v_T is the speed of the moon and

$$v_T = \frac{2\pi \times 3.8 \times 10^8 \text{ m}}{2.36 \times 10^6 \text{ sec}} = 1.01 \times 10^3 \text{ m/sec}$$

7.

$$\begin{aligned} g &= \frac{(1.01 \times 10^3 \text{ m/sec})^2 (3.8 \times 10^8 \text{ m})}{(6.3 \times 10^6 \text{ m})^2} \\ &= 9.8 \text{ m/sec}^2 \end{aligned}$$

Example 7-5

The radius of the earth is $r_e = 6.3 \times 19^6$ m and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. Find the mass of the earth.

Sol: There are two ways to calculate the mass of the earth.

1.

$$\begin{aligned} m_o g &= G\frac{m_o m_e}{r_e^2} \\ m_e &= \frac{r_e^2 g}{G} \\ &= \frac{(6.3 \times 10^6 \text{ m})^2 (9.8 \text{ m/sec}^2)}{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2} \\ &= 5.8 \times 10^{24} \text{ kg} \end{aligned}$$

2. Since $F_R = mr\omega^2$ by applying this to the motion of the moon

around the earth we have

$$G\frac{m_m m_e}{r_{em}^2} = m_m r_{em}\omega^2$$

Since $\omega = 2\pi/27.3 \text{ day} = 2\pi/2.36 \times 10^6 \text{ sec} = 2.66 \times 10^{-6} \text{ rad/sec}$ we have

$$\begin{aligned} m_e &= \frac{r_{em}^3\omega^2}{G} \\ &= \frac{(3.8 \times 10^8 \text{ m})(2.66 \times 10^6 \text{ rad/sec})^2}{6.67 \times 10^{-11}} \text{ Nm}^2/\text{kg}^2 \\ &= 5.8 \times 10^{24} \text{ kg} \end{aligned}$$

Homework: 7.3, 7.7, 7.11, 7.12, 7.13, 7.17, 7.18, 7.19, 7.20, 7.21.

Shell theorem on Gravitation

1. Newton solved the apple-Earth problem by solving an important theorem called the *shell theorem*:
   - A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.
   - A uniform shell of matter exerts no net gravitational force on a particle located inside it.
2. If the density of Earth were uniform, the gravitational force acting on a particle would be a maximum at Earth's surface.

Example s-1

Suppose a tunnel runs through Earth from pole to pole, as in Fig. s-1. Assume that Earth is a nonrotating, uniform sphere. Find the gravitational force on a particle of mass m dropped onto the tunnel when it reaches a distance r from Earth's center.

Sol: The force that acts on the particle is associated only with the mass M' of Earth that lies within a sphere of radius r. The portion of Earth that lies outside this sphere does not exert any net force on the particle. Mass M' is given by

$$M' = \rho V' = \rho \frac{4\pi r^3}{3},$$

in which V' is the volume occupied by M', and ρ is the assumed uniform density of Earth.

The force acting on the particle is then

$$\begin{aligned} F &= -\frac{GmM'}{r^2} = -\frac{Gm\rho 4\pi r^3}{3r^2} \\ &= -\left(\frac{4\pi m G\rho}{3}\right) r \end{aligned}$$

We have inserted a minus sign to indicate that force $\boldsymbol{F}$ and the displacement $\boldsymbol{r}$ are in opposite direction, the former begin toward the center of Earth and the latter away from that point.

Example s-2

Consider a pulsar, a collapsed star of extremely high density, with a mass M equal to that of the Sun (1.98×10^{30} kg), a radius R of only 12 km, and a rotational period T of 0.041 sec. At its equator, by what percentage does the free-fall acceleration g differ from the gravitational acceleration a_g?

Sol: To find a_g on the surface of the pulsar, we have

$$\begin{aligned} a_g &= \frac{GM}{R^2} \\ &= \frac{(6.67 \times 10^{-11} \text{ m}^3/\text{ kg sec}^2)(1.98 \times 10^{30} \text{ kg})}{(12,000 \text{ m})^2} \\ &= 9.2 \times 10^{11} \text{ m/sec}^2 \end{aligned}$$

$$\begin{aligned}
\frac{a_g - g}{a_g} &= \left(\frac{2\pi}{T}\right)^2 \frac{R}{a_g} \\
&= \left(\frac{2\pi}{0.041 \text{ sec}}\right)^2 \frac{12,000 \text{ m}}{9.2 \times 10^{11} \text{ m/sec}^2} \\
&= 3.1 \times 10^{-4} = 0.0031\%
\end{aligned}$$

Example s-3

An astronaut whose height h is 1.70 m float feet "down" in an orbiting space shuttle at a distance $r = 6.77 \times 10^6$ m from a black hole of mass $M_h = 1.99 \times 10^{31}$ kg (which is 10 times our sun's mass), what is the difference in the gravitational acceleration at her feet and head? The black hole has a surface (called the horizon of the black hole) of radius $R_h = 2.95 \times 10^4$ m. Nothing, not even light, can escape from the surface or any where inside it. Note that the astronaut is well outside the surface (at $r = 229R_h$).

Sol: The gravitational acceleration at any distance from the center of the black hole is

$$a_g = \frac{GM_h}{r^2}.$$

Then

$$\begin{aligned} da_g &= -2\frac{GM_h}{r^3}dr \\ &= -2\frac{(6.67 \times 10^{11}\ \mathrm{m}^3/\ \mathrm{kg\ sec}^2)(1.99 \times 10^{31}\ \mathrm{kg})}{(6.77 \times 10^6\ \mathrm{m})^3}(1.70\ \mathrm{m}) \\ &= -14.5\ \mathrm{m/sec}^2 \end{aligned}$$

Gravitational Potential Energy

- As before, the gravitational potential energy decreases when the separation decreases.
- We assume that the gravitational potential energy E_p is zero for $r = \infty$, where r is the separation distance.
- The potential energy is negative for any finite separation and becomes progressively more negative as the particles move closer together.
- We take the *gravitational potential energy* of the two-particle system to be

$$E_p = -\frac{GMm}{r}.$$

Gravitational Potential Energy–Proof

- Let a baseball, starting from rest at a great (infinite) distance from Earth, fall toward point P, as in Fig. s-2. The potential energy of the baseball-Earth system is initially zero.
- When the baseball reaches P, the potential energy is the negative of the work W done by the gravitational force as the baseball moves to P from its distant position.
- Thus

$$
\begin{aligned}
E_p &= -W = -\int_{\infty}^{R} \boldsymbol{F} \cdot d\boldsymbol{x} \\
&= -\int_{\infty}^{R} -F dx \\
&= -\int_{\infty}^{R} -\frac{GMm}{x^2} dx
\end{aligned}
$$

$$
\begin{aligned}
&= \int_{\infty}^{R} \left(\frac{GMm}{x^2} \right) dx \\
&= -\left. \frac{GMm}{x} \right|_{\infty}^{R} = -\frac{GMm}{R}
\end{aligned}
$$

Escape Speed

- There is a certain minimum initial speed that will cause a projectile to move upward forever, theoretically coming to rest only at infinity. This initial speed is called the *escape speed*.
- Consider a projectile of mass m, leaving the surface of a planet with escape speed v. When the projectile reaches infinity, it stops and thus has no kinetic energy. It also has no potential energy because this is our zero-potential energy configuration.
- From the principle of conservation of energy, we have

$$E_k + E_p = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0,$$

where M is the mass of the planet and R is its radius. Thus,

$$v = \sqrt{\frac{2GM}{R}}.$$

Example s-4

An asteroid headed directly toward earth, has a speed of 12000 m/sec relative to the planet when it is at a distance of 10 Earth radii from Earth's center. Ignoring the effects of the terrestrial atmosphere on the asteroid, find the asteroid's speed when it reaches Earth's surface.

Sol: Because the mass of an asteroid is much less than that of Earth, we can assign the gravitational potential energy of the asteroid-Earth system to the asteroid alone, and we can neglect any change in the speed of Earth relative to the asteroid during the asteroid's fall. Thus,

$$E_{kf} + E_{pf} = E_{ki} + E_{pi}$$

Let m represent the mass of the asteroid, M the mass of Earth $(= 5.98 \times 10^{24}$ kg), and R the radius of Earth $(= 6.37 \times 10^{6}$ m).

Thus,

$$\frac{1}{2}mv_f^2 - \frac{GMm}{R} = \frac{1}{2}mv_i^2 - \frac{GMm}{10R}.$$

$$\begin{aligned} v_f^2 &= v_i^2 + \frac{2GM}{R}\left(1 - \frac{1}{10}\right) \\ &= (12 \times 10^3 \text{ m/sec})^2 \\ &\quad + \frac{2(6.67 \times 10^{-11} \text{ m}^3/ \text{ kg sec}^2)(5.98 \times 10^{24} \text{ kg})}{6.37 \times 10^6 \text{ m}} 0.9 \\ &= 2.567 \times 10^8 \text{ m}^2/ \text{ sec}^2, \end{aligned}$$

and

$$v_f = 1.60 \times 10^4 \text{ m/sec}.$$

Kepler's Laws

- The law of orbits: all planets move in elliptical orbits, with the Sun at one focus.
- The law of areas: a line that connects a planet to the Sun sweeps out equal areas in equal times.
- The law of periods: the square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

The law of orbits

- The orbit in Fig. s-3 is described by given its *semimajor axis* a and its *eccentricity* e, the latter defined so that ea is the distance from the center of the ellipse to either focus F or F'.
- The sum of the perihelion (nearest the Sun) distance R_p and the aphelion (farthest from the Sun) distance R_a is $2a$.
- The sum of the distance from any position in the orbit to two foci is $2a$.
- The equation of any position (x, y) in the orbit is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1.$$

- An eccentricity of zero corresponds to a circle, in which the two foci merge to a single central point.

- The eccentricities of the planetary orbits are not large, so the orbits look circular.
- The eccentricity of Earth's orbit is only 0.0167.

The Law of Areas

- The planet will move most slowly when it is farthest from the Sun and most rapidly when it is nearest to the Sun.
- The law of areas is a direct consequence of the idea that all of the forces are directed exactly toward the sun.

The Law of Periods

- Consider a circular orbit with radius r. See Fig. s-4.
- Applying Newton's second law, $F = ma$, to the orbiting planet in Fig. s-4 yield

$$\frac{GMm}{r^2} = (m)(\omega^2 r).$$

If we replace ω with $2\pi/T$, where T is the period of the motion, we have

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3.$$

- the law holds also for elliptical orbits, provided we replace r with a, the semimajor axis of the ellipse.

Example s-5

A satellite in circular orbit at an altitude h of 230 km above Earth's surface has a period T of 89 min. What mass of Earth follows from these data?

Sol: From Kepler's law of periods we have

$$M = \frac{4\pi^2 r^3}{GT^2}.$$

The radius r of the satellite orbit is

$$\begin{aligned} r &= R + h = 6.37 \times 10^6 \text{ m} + 230 \times 10^3 \text{ m} \\ &= 6.60 \times 10^6 \text{ m}, \end{aligned}$$

in which R is the radius of Earth.

$$M = \frac{(4\pi^2)(6.60 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ m}^3/\text{ kg sec}^2)(89 \times 60 \text{ sec})^2}$$

$$= \quad 6.0 \times 10^{24} \text{ kg.}$$

Example s-6

Comet Halley orbits about the Sun with a period of 76 years and, in 1986, had a distance of closest approach to the Sun, its perihelion distance R_p, of 8.9×10^{10} m. (a) What is the comet's farthest distance from the Sun, its aphelion distance R_a? (b) What is the eccentricity of the orbit of comet Halley?

Sol:(a) From Kepler's law of period we have

$$\begin{aligned} a &= \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} \\ &= \left(\frac{(6.67 \times 10^{-11}\ \mathrm{m}^3/\ \mathrm{kg\,sec}^2)(1.99 \times 10^{30})(2.4 \times 10^9\ \mathrm{sec})^2}{4\pi^2}\right)^{1/3} \\ &= 2.7 \times 10^{12}\ \mathrm{m}. \end{aligned}$$

$$R_a = 2a - R_p = 5.3 \times 10^{12}$$

(b)Since

$$ea = a - R_p$$

we have

$$\begin{aligned} e &= \frac{a - R_p}{a} = 1 - \frac{R_p}{a} \\ &= 1 - \frac{8.9 \times 10^{10} \text{ m}}{2.7 \times 10^{12} \text{ m}} = 0.97. \end{aligned}$$

Satellites: Orbits and Energy

- The mechanical energy $E_k + E_p$ of the satellite remains constant.
- We first assume that the orbit of the satellite is circular.
- The potential energy is

$$E_p = -\frac{GMm}{r},$$

where r is the radius of the orbit.

- By Newton's second law,

$$\frac{GMm}{r^2} = m\frac{v_T^2}{r},$$

where v_T^2/r is the centripetal acceleration of the satellite.

- The kinetic energy of a satellite is

$$E_k = \frac{1}{2}mv_T^2 = \frac{GMm}{2r}.$$

- The total mechanical energy is

$$E_k + E_p = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}.$$

- For a satellite in an elliptical orbit of semimajor axis a, we have

$$E_k + E_p = -\frac{GMm}{2a}.$$

Chapter Eight: Rotational Dynamics

- Previously, we developed the first principles of linear dynamics; now we adapt the principles of linear dynamics to rotating bodies.
- Newton's laws, momentum, energy, and power all have equations equivalent to their linear counterparts.

Moment of Inertia and Torque

- In Newton's second law, mass is the proportionality constant between force and acceleration. Newton called it the *inertial mass*.
- This resistance to having the state of rotational motion changed is called the *moment of inertia*, with symbol I.
- See Fig. 8-1. By Newton's second law,

$$F_T = ma_T$$

Since

$$a_T = r\alpha$$

we have

$$\begin{aligned} F\sin\phi &= mr\alpha \\ rF\sin\phi &= mr^2\alpha \end{aligned}$$

$$Fh = mr^2\alpha$$

where $h = r\sin\phi$.

- The quantity $Fh = Fr\sin\phi$ is called the *torque* produced by $\boldsymbol{F}$, which is usually represented by τ.
- The quantity mr^2 is called the moment of inertia, I, of a point mass.
- Newton's second law for rotation:

$$\tau = I\alpha$$

- It is conventional to define τ as the cross product of the position vector $\boldsymbol{r}$ and the force vector $\boldsymbol{F}$, namely,

$$\tau = \boldsymbol{r} \times \boldsymbol{F}$$

- If there are a variety of masses at different distances from the pivot point, the moment of inertia of the assembly is the sum of their individual ones or

$$I = \sum_{i=1}^{n} m_i r_i^2$$

- Unlike the translational inertia (the mass), the rotational inertia (moment of inertia) of an object depends on the location of the mass relative to the axis of rotation and in general is different for different axes of rotation.

Example 8-1

A balance scale consisting of a weightless rod has a mass of 0.1 kg on the right side 0.2 m from the pivot point. See Fig. 8-2. (a) How far from the pivot point on the left must 0.4 kg be placed so that a balance is achieved? (b) If the 0.4-kg mass is suddenly removed, what is the instantaneous rotational acceleration of the rod? (c) What is the instantaneous tangential acceleration of the 0.1-kg mass when the 0.4-kg mass is removed?

Sol:

- (a) Since $\alpha = 0$, we have

$$\sum \tau = 0$$

Thus,

$$(m_2 g)(x)\sin 90^\circ - (m_1 g)(0.2 \text{ m})\sin 90^\circ = 0$$

$$\begin{aligned} x &= \frac{(m_1 g)(0.2 \text{ m}) \sin 90^\circ}{(m_2 g) \sin 90^\circ} \\ &= \frac{(0.1 \text{ kg})(9.8 \text{ m/sec}^2)(0.2 \text{ m})}{(0.4 \text{ kg})(9.8 \text{ m/sec}^2)} \\ &= 0.05 \text{ m} \end{aligned}$$

- (b)

$$\begin{aligned} \alpha &= \frac{\tau}{I} = \frac{(m_1 g)(0.2 \text{ m}) \sin 90^\circ}{(m_1)(0.2 \text{ m})^2} \\ &= \frac{(0.1 \text{ kg})(9.8 \text{ m/sec}^2)(0.2 \text{ m}) \sin 90^\circ}{(0.1 \text{ kg})(0.2 \text{ m})^2} \\ &= 49 \text{ rad/sec}^2 \text{ clockwise} \end{aligned}$$

-

$$\alpha_T = r\alpha$$

$$\begin{aligned} &= (0.2 \text{ m})(49 \text{ rad/sec}^2) \\ &= 9.8 \text{ m/sec}^2 \end{aligned}$$

Rotational Kinetic Energy

- The work down by $\boldsymbol{F}_T$ in this infinitesimal distance is dW, and

$$dW = \boldsymbol{F}_T \cdot d\boldsymbol{s}$$

Since $\boldsymbol{F}_T$ and $d\boldsymbol{s}$ are in the same direction, we have

$$dW = F_T ds$$

- Since

$$d\theta = \frac{ds}{r}$$

we have

$$\begin{aligned} dW_\theta &= F \sin\phi r d\theta \\ &= \tau d\theta \\ W_\theta &= \int_{\theta_0}^{\theta_f} \tau \, d\theta \end{aligned}$$

where θ_0 and θ_f are the initial and final angles, respectively.

- Since $\tau = I\alpha$ we have

$$\begin{aligned} W_\theta &= \int_{\theta_0}^{\theta_f} I\alpha d\theta \\ &= \int_{\omega_0}^{\omega_f} I\frac{d\omega}{dt}\omega dt \\ &= \int_{\omega_0}^{\omega_f} I\omega d\omega \end{aligned}$$

- If the distance of the particle to the point of rotation does not change. then

$$\begin{aligned} W_\theta &= I\int_{\omega_0}^{\omega_f} \omega \; d\omega \\ &= \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_0^2 \end{aligned}$$

$$(\Delta E_k)_{\text{rot}} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_0^2$$

where the quantity $\frac{1}{2}I\omega^2$ is called the *rotational kinetic energy*, $(E_k)_{\text{rot}}$.

- A point on a rotating system has an instantaneous tangential velocity V_T. Its kinetic energy is

$$\begin{aligned} E_k &= \frac{1}{2}mv_T^2 \\ &= \frac{1}{2}mr^2\omega^2 \\ &= \frac{1}{2}I\omega^2 \end{aligned}$$

since $I = mr^2$.

- The rotation of a rigid body made up of discrete masses m_i. The rotational kinetic energy is

$$\begin{aligned}(E_k)_{\text{rot}} &= \frac{1}{2}\sum_{i=1}^{n} m_i r_i^2 \omega_i^2 \\ &= \frac{1}{2}\omega^2 \sum_{i=1}^{n} m_i r_i^2 \\ &= \frac{1}{2} I \omega^2\end{aligned}$$

since the body is rigid, all point masses rotate with the same angular velocity regardless of their distance from the axis, i.e., $\omega_i^2 = \omega^2$.

- A body can be rotating as it translates through space; its total kinetic energy is therefore the sum of translational and rotational and rotational kinetic energies

$$(E_k)_{\text{total}} = (E_k)_{\text{trans}} + (E_k)_{\text{rot}}$$

$$= \quad \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2$$

where v_{CM} is the translational velocity of the center of mass.

Example 8-2

A large wheel of radius 0.4 m and moment of inertia 1.2 kg-m^2, pivoted at the center, is free to rotate without friction. A rope is wound around it and a 2-kg weight is attached to the rope (see Fig. 8-4). when the weight has descended 1.5 m from its starting position (a) what is its downward velocity? (b) what is the rotational velocity of the wheel?

Sol: (a) We may solve this problem by the conservation of energy, equating the initial potential energy of the weight to its conversion to kinetic energy of the weight and of the wheel.

$$
\begin{aligned}
mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\
\omega &= \frac{v_T}{r} = \frac{v}{r}
\end{aligned}
$$

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} \\ v &= \left[\frac{mgh}{\frac{1}{2}m + \frac{I}{2r^2}}\right]^{1/2} \\ &= \left[\frac{(2\text{ kg})(9.8\text{ m/sec}^2)(1.5\text{ m})}{(\frac{1}{2})(2\text{ kg}) + \frac{(1.2\text{ kg-m}^2)}{(2)(0.4\text{ m})^2}}\right]^{1/2} \\ v &= 2.5\text{ m/sec} \end{aligned}$$

(b) The answer to part (a) shows that any point on the rim of the wheel has a tangential velocity of $v_T = 2.5$ m/sec.

$$\omega = \frac{v_T}{r} = \frac{2.5\text{ m/sec}}{0.4\text{ m}} = 6.2\text{ rad/sec}$$

Power

- The definition of *power* is work done per unit time.
- The incremental amount of work done in moving the mass in Fig. 8-3 a distance $ds = rd\theta$ is

$$dW_\theta = \tau d\theta$$

Thus,

$$\begin{aligned} \text{power} &= \frac{dW}{dt} \\ &= \frac{\tau d\theta}{dt} \\ &= \tau\omega \end{aligned}$$

Example 8-3

A machine shop has a lathe wheel of 40-cm diameter driven by a belt that goes around the rim. If the linear speed of the belt is 2 m/sec and the wheel requires a tangential force of 50 N to turn it, how much power is required to operate the lathe?

Sol: The rotational velocity is

$$\begin{aligned}\omega &= \frac{v_T}{r} \\ &= \frac{2\ \text{m/sec}}{0.2\ \text{m}} \\ &= 10\ \text{rad/sec}\end{aligned}$$

The torque is

$$\begin{aligned}\tau &= rF\sin\phi \\ &= (0.2\ \text{m})(50\ \text{N})\sin 90^\circ\end{aligned}$$

$$= \quad 10 \text{ Nm}$$

then

$$\begin{aligned} \text{Power} &= 10 \text{ Nm} \times 10 \text{ rad/sec} \\ &= 100 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = 0.13 \text{ hp} \end{aligned}$$

Angular Momentum

- Consider, as shown in Fig. 8-5, a particle of mass m with momentum $\boldsymbol{p} = m\boldsymbol{v}$ in the x-y plane. The position vector of m is $\boldsymbol{r}$, which is not required to be a constant.

$$\begin{aligned}\boldsymbol{F} &= \frac{d}{dt}(m\boldsymbol{v}) \\ \boldsymbol{r}\times\boldsymbol{F} &= \boldsymbol{r}\times\frac{d}{dt}(m\boldsymbol{v}) \\ \tau &= \boldsymbol{r}\times\frac{d}{dt}(m\boldsymbol{v})\end{aligned}$$

Note that

$$\begin{aligned}\frac{d}{dt}(\boldsymbol{r}\times m\boldsymbol{v}) &= \frac{d\boldsymbol{r}}{dt}\times m\boldsymbol{v} + \boldsymbol{r}\times\frac{d}{dt}(m\boldsymbol{v}) \\ &= \boldsymbol{v}\times m\boldsymbol{v} + \boldsymbol{r}\times\frac{d}{dt}(m\boldsymbol{v})\end{aligned}$$

Since the cross product of two vectors in the same direction is zero,

$$\boldsymbol{v} \times m\boldsymbol{v} = m(\boldsymbol{v} \times \boldsymbol{v}) = 0$$

Therefore,

$$\frac{d}{dt}(\boldsymbol{r} \times m\boldsymbol{v}) = \boldsymbol{r} \times \frac{d}{dt}(m\boldsymbol{v})$$

Thus,

$$\tau = \frac{d}{dt}(\boldsymbol{r} \times m\boldsymbol{v})$$

- We call

$$\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v}$$

the *angular momentum* of the particle.

-

$$\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v} = rmv \sin \gamma$$

where γ is the angle between the radius vector $\boldsymbol{r}$ and the linear momentum $m\boldsymbol{v}$ (see Fig. 8-5).

But $mv \sin\gamma = mv_T$, and

$$L = rmv_T$$

Since $v_T = r\omega$ and therefore

$$\begin{aligned} L &= mr^2\omega \\ &= I\omega \\ \tau &= \frac{dL}{dt} = \frac{d(I\omega)}{dt} \end{aligned}$$

Conservation of Angular Momentum

- $\tau = \frac{dL}{dt} = \frac{d(I\omega)}{dt}$
- If we have a situation in which there is no net externally applied torque, then $\tau = 0$. Thus

$$\frac{dL}{dt} = 0$$

  and $L =$ constant. Hence, $I\omega =$ constant.

- With no net external torque

$$(I\omega)_0 = (I\omega)_f$$

  This is known as the law of *conservation of angular momentum*.

Example 8-4

Suppose the body of an ice skater has a moment of inertia $I = 4$ kg-m^2 and her arms have a mass of 5 kg each with the center of mass at 0.4 m from her body. She starts to turn at 0.5 rev/sec on the point of her skate with her arms outstretched. She then pulls her arms inward so that their center of mass is at the axis of her body, $r = 0$. What will be her speed of rotation?

Sol:

$$\begin{aligned} I_0\omega_0 &= I_f\omega_f \\ (I_{\text{body}} + I_{\text{arms}})\omega_0 &= I_{\text{body}}\omega_f \\ (I_{\text{body}} + 2mr^2)\omega_0 &= I_{\text{body}}\omega_f \end{aligned}$$

Solving for ω_f

$$\begin{aligned}\omega_f &= \frac{(I_{\text{body}} + 2mr^2)\omega_0}{I_{\text{body}}} \\ &= \frac{[4\text{kg-m}^2 + 2 \times 5\text{ kg} \times (0.4\text{ m})^2](0.5\text{ rev/sec})}{4\text{kg-m}^2} \\ &= 0.7\text{ rev/sec}\end{aligned}$$

Homework: 8.2, 8.5, 8.6, 8.7, 8.10, 8.11, 8.13, 8.14, 8.18, 8.19.

Chapter Nine: Kinetic Theory of Gases and the Concept of Temperature

- Heat is a form of energy.
- The first law of thermodynamics is a broadened statement of the law of conservation of energy.

Molecular Weight

- In this book we will consider only systems composed of identical atoms, or molecules such as oxygen (O_2) or nitrogen molecules (N_2).
- A *mole* of a substance is that quantity which contains the same number of particles as there are atoms in 12 g of carbon-12.
- Carbon-12 is said to have a mass of exactly 12 u per atom, where u is called an *atomic mass unit* and has the value

$$1\ u = 1.66057 \times 10^{-27}\ \text{kg}.$$

- The mass of an atom (or molecule) in atomic mass units is called *atomic weight* (or *molecular weight*).
- The mass, in grams, of a mole of a substance is numerically equal to the atomic weight (or molecular weight) of the atoms of that

substance, and it is referred to as the *gram atomic weight* (or *gram molecular wight*).

- The mass of 1 mole of carbon-12 is 12 g/mole. The number of atoms in 1 mole of a substance is called *Avogadro's number* and has the value

$$\begin{aligned} N_A &= 6.022 \times 10^{23} \text{ atoms/mol} \\ &= 6.022 \times 10^{26} \text{ atoms/kmol.} \end{aligned}$$

- We will use the symbol n (called *mole fraction*) to represent the number of moles present.
- If we denote M as the gram molecular weight of a substance and m as the mass of the amount present, then $n = m/M$, and, because M has N_A atoms or molecules, the number of atoms or molecules present is nN_A.

Thermometers

- Fahrenheit scale: the melting point of pure ice is 32° and the boiling point of pure water at sea level is 212°.
- Celsius scale: the melting point of pure ice is 0° and the boiling point of pure water at sea level is 100°. See Fig. 9-1.

$$\begin{aligned}\frac{{}^\circ C}{{}^\circ F - 32} &= \frac{100}{180}\\ {}^\circ C &= \frac{5}{9}({}^\circ F - 32)\\ {}^\circ F &= \frac{9}{5}{}^\circ C + 32\end{aligned}$$

Ideal Gas Law and Absolute Temperature

- The *pressure*, P, is the force perpendicular to a surface per unit surface area, or $P = F/A$.
- The dimension of pressure is newton/meter^2 (N/m^2) or pascal (Pa), where 1 Pa=1 N/m^2.
- Atmospheric pressure at sea level is approximately 1.01×10^5 N/m^2.
- An idea gas is one that has no tendency to condense. This means that the atoms are infinitesimal in size and that there is no attractive force between them.
- Idea gases do not exist.
- If the quantity of gas remained constant,

$$PV = R'T,$$

where V is the volume of gas.

- When equal volumes of the same gas are taken at the same temperature and pressure, R' remains constant.
- With the introduction of this term n the idea gas law is written as

$$PV = nRT$$

where n is the number of moles and R is now the same for all gases.

- See Fig. 9-2. When $P = 0$, the temperature is $-273.16°C$. This is called *absolute zero*, and is the lowest possible temperature.
- $0°C = +273.16K$, where K is the symbol for the new scale, called the Kelvin or *absolute* scale.
- $K = 273.16° +° C$.

- $R = 8314$ J/kmol-K.

Example 9-1

What is the temperature of absolute zero on the Fahrenheit scale?

Sol:

$$\begin{aligned} {}^\circ F &= \frac{9^\circ}{5} C + 32 \\ &= \frac{9}{5} \times (-273.16) + 32 = -459.7^\circ F \end{aligned}$$

Example 9-2

In a typical experiment to determine the value of the gas constant R, 0.152 g of neon gas (atomic weight 20.2 g/mole) is introduced into a 100-cm^3 flask that is closed and attached to a pressure gauge. It is found that when the flask is placed in a constant temperature bath at $50°C$ the pressure of the gas is 2 atmospheres (atm). What value of R is obtained?

Sol: The mole fraction n is the ratio of the number of grams present to the atomic weight in grams.

$$\begin{aligned} n &= \frac{0.152\text{g}}{20.2 \text{ g/mole}} \\ &= 7.52 \times 10^{-3} \text{ mol} \\ &= 7.52 \times 10^{-6}\text{kmol} \end{aligned}$$

The volume is

$$V = 10^2 \text{ cm}^3 \left(\frac{1 \text{ m}}{10^2 \text{ cm}}\right)^3 = 10^{-4} \text{ m}^3$$

The pressure is

$$\begin{aligned} P &= 2\text{atm}\left(\frac{1.01 \times 10^5 \text{N}/\text{m}^2}{1\text{atm}}\right) \\ &= 2.02 \times 10^5 \text{N}/\text{m}^2 \end{aligned}$$

The temperature in K is $T = 273^\circ + 50^\circ = 323$ K. The idea gas law is written as

$$\begin{aligned} R &= \frac{PV}{nT} \\ &= \frac{2.02 \times 10^5 \text{ N}/\text{m}^2 \times 10^{-4} \text{ m}^3}{7.52 \times 10^{-6}\text{kmol} \times 323\text{K}} \\ &= 8316\text{J}/\text{kmol-K} \end{aligned}$$

Example 9-3

In a diesel engine, no spark plug is required because the temperature is raised to the ignition point of the air-fuel mixture by compression. In a typical diesel engine the air intake is at $27°C$ and at a pressure of 1 atm, and it is compressed to 1/15 of its original volume with its pressure becoming about 50 atm. What is the temperature of the air-fuel mixture in the cylinder in $°C$.

Sol: We note that

$$\frac{PV}{T} = nR$$

If the quantity of gas is kept constant, then nR is a constant. Thus,

$$\frac{P_0V_0}{T_0} = \frac{P_fV_f}{T_f}$$

In this Problem

$$
\begin{aligned}
T_f &= \frac{P_f}{P_0}\frac{V_f}{V_0}T_0 \\
&= \frac{50\text{atm}}{1\text{atm}}\frac{V_0/15\ \text{m}^3}{V_0\ \text{m}^3} \times 300\ \text{K} \\
&= 1000\text{K} = 727^\circ C
\end{aligned}
$$

Kinetic Theory of Gas Pressure

- We will show how the concept of momentum conservation and the definition of pressure can be used to calculate the statistical behavior of a large number of atoms or molecules in a gas.
- One of the assumptions in this calculation is that all collisions between atoms or molecules are perfectly elastic.
- In an elastic collision of an atom with the container wall, the velocity component normal to the wall is reversed on collision with its magnitude unchanged, and the velocity component in the direction parallel to the surface of the wall is unchanged. See Fig. 9-3.
- $\bar{F}_x \Delta t = \Delta(m v_x)$, where $\bar{F}_x$ is the average force exerted by the wall of the container on the atom during the time interval Δt, and m is the mass of the atom.

- $$\begin{aligned}\bar{F}_x &= \frac{mv_{x\text{final}} - mv_{x\text{initial}}}{\Delta t} \\ &= \pm\frac{2mv_x}{\Delta t}\end{aligned}$$

- See Fig. 9-4. The area of a face is $A = l^2$ and a volume $V = l^3$. Because the atom's velocity in the x direction remains constant in magnitude, the time for a round trip between opposite walls is

$$\Delta t = \frac{2l}{v_x}.$$

- Then

$$\bar{F}_x = \frac{2mv_x}{\frac{2l}{v_x}} = \frac{mv_x^2}{l}.$$

- The total average force on a wall due to the x motion of N atoms in the box is

$$\begin{aligned}\bar{F}_x &= \frac{m}{l}v_{x1}^2 + \frac{m}{l}v_{x2}^2 + \cdots \frac{m}{l}v_{xN}^2 \\ &= \frac{m}{l}(v_{x1}^2 + v_{x2}^2 + \cdots v_{xN}^2)\end{aligned}$$

- Let the average of the squared individual x velocities be

$$\overline{v_x^2} = \frac{v_{x1}^2 + v_{x2}^2 + \cdots v_{xN}^2}{N}$$

Then

$$\bar{F}_x = \frac{mN}{l}\overline{v_x^2}$$

- By the three-dimensional pythagorean theorem

$$v^2 = v_x^2 + v_y^2 + v_z^2,$$

or

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}.$$

But in a gas in equilibrium there is no preferred direction of motion; hence

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

and

$$\overline{v^2} = 3\overline{v_x^2}.$$

- Thus

$$\bar{F} = \frac{mN}{3l}\overline{v^2}$$

$$\begin{aligned} P &= \frac{mN}{3Al}\overline{v^2} \\ &= \frac{mN}{3V}\overline{v^2} \\ &= \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}m\overline{v^2}\right). \end{aligned}$$

Kinetic Theory of Temperature

- Since $PV = nRT$ we have

$$\frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) = nRT,$$

  where N is the number of molecules present in the box and n is the number of fraction of moles present.

-

$$n(\text{number of moles}) = \frac{N(\text{number of molecules})}{N_A(\text{Avogadro's number})}$$

- Substituting $n = N/N_A$ in the above equation gives

$$\frac{2}{3}\left(\frac{1}{2}m\overline{v^2}\right) = k_B T,$$

  where $k_B = R/N_A = 1.38 \times 10^{-23}$ J/K per molecule is the

Boltzmann's constant.

- We have shown that temperature is simply proportional to the average kinetic energy E_k of the molecules, that is

$$T = \frac{2}{3}\frac{\overline{E_k}}{k_B}.$$

- $\sqrt{\overline{v^2}}$ is called the *root mean square* (RMS) velocity (v_{RMS}), and

$$v_{\mathrm{RMS}} = \sqrt{\frac{3k_BT}{m}}.$$

Measurement of Heat

- One *calorie* is the quantity of heat required to raise the temperature of 1 g of water by $1°C$.
- See Fig. 9-5. The weight is allowed to fall at constant speed so that no changes in kinetic energy have to be considered, only changes on potential energy.
- The specific heat c is defined as the amount of heat needed to raise the temperature of 1 g of a substance $1°C$. For water $c = 1$ cal/g$°C$.
- The quantity of heat ΔQ required to raise the temperature by ΔT of a mass m of a substance whose specific heat is c, is written as

$$\Delta Q = mc\Delta T$$

- We can equate this to the loss of potential energy, $\Delta E_p = m'gh$, of the falling weight $m'g$, that is

$$m'gh = mc\Delta T.$$

- By experiment,

$$4.184 \text{ J} = 1 \text{ cal},$$

which is called the *mechanical equivalent of heat*.

Specific Heats of Gases

- If we hold a quantity of gas at a constant volume so that it is can not do work by expanding, then all the heat ΔQ goes into increasing the kinetic energy of the molecules, or

$$\Delta Q = \Delta E_k.$$

- Since gases are compressible, we define a term C_v as the molar specific heat at constant volume; that is

$$C_v = c_v M$$

where M is the mass of a mole of gas and c_v is the specific heat per gram at constant volume.

- We have $m = Mn$, where m is the mass of gas. Thus

$$\Delta Q = (\text{specific heat per mole}) \times (\text{number of moles}) \times (\Delta T)$$

$$= \quad C_v n \Delta T$$

- ΔE_k per molecule$= 3/2 k_B \Delta T$. Thus,

$$\Delta E_k = (nN_A)\left(\frac{3}{2}k_B \Delta T\right)$$

$$C_v n \Delta T = \frac{3}{2} n N_A k_B \Delta T$$

$$\begin{aligned} C_v &= \frac{3}{2} N_A k_B \\ &= \frac{3}{2} R \end{aligned}$$

-

$$\begin{aligned} R &= 8314 \text{ J/kmol-K} \\ &= 8.314 \text{ J/mol-K} \left(\frac{1 \text{ cal}}{4.184 \text{ J}}\right) \end{aligned}$$

$$= \quad 1.987 \frac{\text{cal}}{\text{mol K}} \approx 2 \text{ cal/mol-K}$$

- $$C_v \approx \frac{3}{2} \times 2 \text{ cal/mol-K} \approx 3 \frac{\text{cal}}{\text{mol-K}}$$

  This value is expected to hold for all the rare gases that are monatomic, such as helium, neon, and argon.

Work Done by a Gas

- See Fig. 9-6.

$$\begin{aligned} dW &= F\,dx \\ &= PA\,dx \\ &= P\,dV \end{aligned}$$

- By the definition of work, if the gas does work by expanding, the work done is positive, whereas if the gas is compressed by the force on the position, the work done by the gas is negative.

First Law of Thermodynamics

- We have three factors relating energy and behavior of a gas:
  1. Work done on or by a gas $\Delta W = P\Delta V$.
  2. The quantity of heat ΔQ that may be added or extracted from the gas.
  3. The change in average kinetic energy of the molecules ΔE_k, which we usually call the change in internal energy ΔE.
- *First Law of Thermodynamics*:

$$\Delta Q = \Delta W + \Delta E$$

- ΔQ is taken as positive if heat enters the system and as negative if it leaves the system. The work, ΔW, is positive if it is done by the system, and negative if done on the system.

Example 9-5

Six thousand calories of heat are added to 2 moles of neon gas at $27°C$ while it does 4100 J of work. (a) How much does the internal energy of the system increase? (b) What is the final temperature of the gas?

Sol:(a)

$$\Delta Q = 6000 \text{ cal} \left(\frac{4.184 \text{ J}}{1 \text{ cal}}\right) = 2.51 \times 10^4 \text{ J}$$

$$\begin{aligned} \Delta E &= \Delta Q - \Delta W \\ &= 2.51 \times 10^4 \text{ J} - 0.41 \times 10^4 \text{ J} = 2.10 \times 10^4 \text{ J} \end{aligned}$$

(b)

$$\Delta E = (nN_A)\left(\frac{3}{2}k_B \Delta T\right)$$

$$= \frac{3}{2}nR\Delta T$$

Therefore

$$\begin{aligned}
\Delta T &= \frac{2}{3}\frac{\Delta E}{nR} \\
&= \frac{2 \times 2.10 \times 10^4 \text{ J}}{3 \times 2 \text{ mol} \times 8.314 \text{ J/mol-K}} \\
&= 842 \text{ K or } ^\circ C
\end{aligned}$$

$$T_{\text{final}} = 27^\circ C + 842^\circ C = 869^\circ C.$$

Homework: 9.4, 9.6, 9.7, 9.12, 9.14, 9.16, 9.17, 9.18, 9.19, 9.20.

Chapter Ten: Oscillatory Motion

- When a block attached to a spring is set into motion, its position is a periodic function of time.
- When we considered the motion of a particle in a circle, the components of a position vector $\boldsymbol{r}$ making an angle θ with the x axis were

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}$$

Characterization of Springs

- The *Hook's law* states that the extension or compression x of an elastic body is proportional to the applied force F. That is

$$F = kx$$

  where k is called the *force constant* or *spring constant*.

- In the case of a spring, the value of the constant k characterizes the strength of the spring.
- The extension of the spring is within its *elastic limit* if the spring returns to its original length when the weight attached to it is removed.

Frequency and Period

- Suppose we have a periodic event, that is, one that occurs regularly with time. Its *frequency* ν is one event per time. The time between periodic events is the *period*, denoted as T. Thus,

$$\nu = \frac{1}{T}.$$

- One event per second is called one hertz, abbreviated Hz.
- See Fig. 10-1.

$$\begin{aligned} x &= r\cos\omega t \\ y &= r\sin\omega t \end{aligned}$$

In every rotation θ changes by 2π rad. If the particle performs ν rotations in 1 sec, then θ will change by $2\pi\nu$ rad every second.

$$\omega = 2\pi\nu$$

$$
\begin{aligned}
x &= r\cos 2\pi\nu t \\
y &= r\sin 2\pi\nu t
\end{aligned}
$$

Amplitude and Phase Angle

- Fig. 10-2 shows a plot of $\sin\theta$ versus θ.
- The maximum value of the magnitude of this oscillation is called the *amplitude*.
- See Fig. 10-3. When $\theta = 0$ the function has the value of $\sin\pi/4$ and thereafter attains all values of $\sin\theta$ at an angle $\pi/4$ earlier.
- The general form for a function to describe a body undergoing sinusoidal oscillations is

$$A\sin(\omega t + \phi)$$

where ϕ is called the *phase angle* and its sign may be positive or negative.

Oscillation of a Spring

- See Fig. 10-4. By Newton's third law of action and reaction, if you pull on a spring with force $\boldsymbol{F}$ it pulls in the opposite direction with force $-\boldsymbol{F}$. Thus, the force that the spring exerts on the body is $-kx$ according to the Hooke's law.
- $$\begin{aligned} -F &= m\boldsymbol{a} \\ -kx &= m\frac{d^2x}{dt^2} \end{aligned}$$
- The above is a second-order differential equation. Our guess at a solution will be
$$x = A\sin(\omega t + \phi).$$
$$\frac{dx}{dt} = A\frac{d}{dt}\sin(\omega t + \phi) = A\omega\cos(\omega t + \phi)$$

$$\frac{d^2x}{dt} = A\omega\frac{d}{dt}\cos(\omega t+\phi) = -A\omega^2\sin(\omega t+\phi)$$

$$\begin{aligned} -kA\sin(\omega t+\phi) &= -m\omega^2 A\sin(\omega t+\phi) \\ k &= m\omega^2 \\ \omega &= \sqrt{\frac{k}{m}} \end{aligned}$$

- Since $\omega = 2\pi\nu$, we have

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

and the period is

$$T = \frac{1}{\nu} = 2\pi\sqrt{\frac{m}{k}}.$$

- To complete the solution of the problem, we must determine the value of the amplitude A and of the phase angle ϕ. This is done by specifying the *boundary conditions*, that is, the behavior of the body at some time such as, $t = 0$.

- At $t = 0$,

$$\begin{aligned} x &= x_0 \\ v_x &= 0 \end{aligned}$$

- Let $x = x_0$ and $t = 0$, we have

$$x_0 = A \sin \phi$$

.

-

$$\begin{aligned} v_x &= \frac{dx}{dt} = \frac{d}{dt} A \sin(\omega t + \phi) \\ &= A\omega \cos(\omega t + \phi) \end{aligned}$$

Since $v_x = 0$ and $t = 0$, we have

$$0 = A\omega \cos\phi.$$

-

$$\begin{aligned} \frac{0}{x_0} &= \frac{A\omega\cos\phi}{A\sin\phi} \\ \cot\phi &= 0 \\ \phi &= \frac{\pi}{2} \\ x_0 &= A\sin\frac{\pi}{2} = A \end{aligned}$$

- Since $\sin(\theta + \pi/2) = \cos\theta$ and $\cos(\theta + \pi/2) = -\sin\theta$, we have

$$\begin{aligned} x &= A\cos\omega t \\ v_x &= -A\omega\sin\omega t \end{aligned}$$

- $$v_{\mathrm{max}} = \pm A\omega = \pm A\sqrt{\frac{k}{m}}$$
- The maximum value of the velocity occurs when $x = 0$ or at the midpoint of oscillation.
- The amplitude of the displacement A and the maximum value of the velocity $A\omega$ are not the same because ω may be equal to or greater or smaller than unity. See Fig. 10-5.
- $$\begin{aligned} a_x &= \frac{dv_x}{dt} = -A\omega\frac{d}{dt}\sin\omega t \\ &= -A\omega^2\cos\omega t \end{aligned}$$

  See Fig. 10-6.
- When the displacement is maximum in the positive direction, the

acceleration is maximum in the negative direction.

- When the displacement is zero, so is the acceleration. See Fig. 10-7.
- Since $F = ma$ and $F = -kx$, $-kx = ma$ and a is maximum when x is maximum and x and a have opposite signs.

Example 10-1

show that $x = A\cos(\omega t + \phi)$ is also a solution of $kx = m\frac{d^2x}{dt^2}$.

Sol:

$$\begin{aligned} \frac{dx}{dt} &= A\frac{d}{dt}\cos(\omega t + \phi) = -A\omega\sin(\omega t + \phi) \\ \frac{d^2x}{dt^2} &= -A\omega\frac{d}{dt}\sin(\omega t + \phi) = -A\omega^2\cos(\omega t + \phi) \end{aligned}$$

Thus,

$$-kA\cos(\omega t + \phi) = -mA\omega^2\cos(\omega t + \phi)$$

and

$$\omega = \sqrt{\frac{k}{m}}.$$

Therefore, $A\cos(\omega t + \phi)$ is a solution of the equation for this value of ω.

Example 10-2

A given spring stretches 0.1 m when a force of 20 N pulls on it. A 2-kg block attached to it on a frictionless surface as in Fig. 10-4 is pulled to the right 0.2 m and released. (a) What is the frequency of oscillation of the block? (b) What is its velocity at the midpoint? (c) What is its acceleration at either end? (d) What are the velocity and acceleration when $x = 0.12$ m, on the block's first passing this point?

Sol:

$$k = \frac{F}{x} = \frac{20 \text{ N}}{0.1 \text{ m}} = 200 \text{ N/m}$$

(a)

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200 \text{ N/m}}{2 \text{ kg}}} = 10 \text{ rad/sec}$$

Because $\omega = 2\pi\nu$

$$\nu = \frac{\omega}{2\pi} = \frac{10 \text{ rad/sec}}{2\pi} = 1.6 \text{ Hz}$$

(b) The velocity is a maximum when $x = 0$, that is, at the midpoint. Thus,

$$\begin{aligned} v &= v_{\text{max}} = \pm A\omega \\ &= \pm(0.2 \text{ m})(10 \text{ rad/sec}) = \pm 2 \text{ m/sec} \end{aligned}$$

(c) The acceleration is a maximum at the two extremes of the motion. Therefore,

$$\begin{aligned} a_{\text{max}} &= \pm A\omega^2 \\ &= \pm(0.2 \text{ m})(10 \text{ rad/sec})^2 = \pm 20 \text{ m/sec}^2 \end{aligned}$$

(d) To determine the block's velocity and acceleration at some arbitrary value of x, we need to know the angle ωt at that position.

$$
\begin{aligned}
x &= A\cos\omega t \\
\omega t &= \arccos\frac{x}{A} = \arccos\frac{0.12\ \text{m}}{0.2\ \text{m}} \\
&= 53^\circ \\
v &= -A\omega\sin\omega t \\
&= -(2.0\ \text{m})(10\ \text{rad/sec})\sin 53^\circ = -1.6\ \text{m/sec} \\
a &= -A\omega^2\cos\omega t \\
&= -(2.0\ \text{m})(10\ \text{rad/sec})^2\cos 53^\circ \\
&= -12\ \text{m/sec}^2
\end{aligned}
$$

Energy of Oscillation

- When a body attached to a spring is displaced from its equilibrium position ($x = 0$), the spring is potentially capable, on the release of the body, to do work on the body. We can therefore associate with the spring-body system a potential energy E_p.
- This potential energy will be the work done in stretching or compressing the spring.
- When the force $\boldsymbol{F}$ and the displacement $d\boldsymbol{x}$ are in the same direction,

$$\begin{aligned} dW &= F\,dx \\ W &= \int_0^x F\,dx \end{aligned}$$

- From Hooke's law,

$$\begin{aligned} W &= k\int_0^x x\,dx \\ &= \frac{1}{2}kx^2 \end{aligned}$$

- The potential energy of the spring-body system, when the body is displaced a distance x from its equilibrium position is

$$E_p(\text{spring}) = \frac{1}{2}kx^2$$

- If the spring is initially in a position x_1 and is compressed or stretched to position x_2, the work done is as before

$$\begin{aligned} W &= k\int_{x_1}^{x_2} x\,dx \\ &= \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 \end{aligned}$$

- Because the displacement is squared the potential energy of a spring is the same whether it is stretched or compressed an equal distance x from its related position.
- By the work-energy theorem, the work done by the spring, as the body moves between two arbitrary displacements x_1 and x_2, is equal to the change in the kinetic energy of the body; that is,

$$\int_{x_1}^{x_2} F_{\text{spring}}\ dx = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

where v_1 and v_2 are the velocities of the body at x_1 and x_2, respectively. Since $F_{\text{spring}} = -kx$,

$$\begin{aligned}\int_{x_1}^{x_2} -kx\ dx &= -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right)\\ -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) &= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\end{aligned}$$

$$\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 \quad = \quad \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2$$

- $$E_{\text{total}} = E_p(\frac{1}{2}kx^2) + (\frac{1}{2}mv^2)$$

  remains constant as the body oscillates.

Example 10-3

The block of Example 10-2 is released from a position of $x_1 = A = 0.2$ m as before. (a) What is its velocity at $x_2 = 0.1$ m? (b) What is its acceleration at this position?

Sol:

(a) The velocity at x_2 can be found with the conservation of energy equation,

$$\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2.$$

Since $v_1 = 0$, we have

$$\begin{aligned} v_2 &= \left[\frac{k(x_1^2 - x_2^2)}{m}\right]^{1/2} \\ &= \left[\frac{200 \text{ N/m}[(0.2 \text{ m})^2 - (0.1 \text{ m})^2]}{2 \text{ kg}}\right]^{1/2} \end{aligned}$$

$$= \quad 1.73 \text{ m/sec.}$$

(b) We may find the acceleration at this position by using Newton's second law

$$\begin{aligned} F &= ma \\ -kx &= ma \\ a &= -\frac{kx}{m} = -\frac{(200\text{N/m})(0.1 \text{ m})}{2 \text{ kg}} \\ &= -10 \text{ m/sec}^2 \end{aligned}$$

Homework: 10.4, 10.9, 10.10, 10.11, 10.12, 10.13, 10.14, 10.16, 10.17, 10.18.

Chapter Eleven: Wave Motion

- Light can be considered wavelike by experimental analogies to the behavior of water waves.
- Experiments with fundamental particles, such as electrons, demonstrate that they also have wave characteristics.
- Travelling waves can transmit energy along a medium without any net translation of the particles in the medium through which the wave travels.
- Because of the nature of the light wave, no medium is necessary for its propagation.

Wavelength, Velocity, Frequency, and Amplitude

- The time between successive risings on the waves is called the *period*, with symbol T.
- The number of risings that we experience per unit time is called the *frequency*, with symbol ν.
- $$\nu = \frac{1}{T}$$
- See Fig. 11-1. The distance between two successive risings on the waves is called the *wavelength*, with symbol λ.
- The speed of the wave is the wavelength divided by the period, that is,
  $$v = \frac{\lambda}{T} = \lambda\nu.$$
- The *amplitude* of a wave is the maximum value of the

displacement it produces.

Travelling Waves in a String

- See Fig. 10-2. We may say the wave pulse, y, is a function of x and the time t; that is, $y = f(x, t)$.
- One of the most important and most commonly found types of travelling waves is the sinusoidal travelling wave, a wave consisting of a series of consecutive sinusoidal pulses.
- Considering the system shown in Fig. 11-3. See Fig. 11-4.

$$y(x = 0, t) = A \sin(\omega t + \phi)$$

- If the velocity of the wave in the string is v, then the time it takes to travel a distance x along the string is x/v. Let $t_0 = x/v$. We obtain

$$y(x, t) \quad = \quad A \sin[\omega(t - t_0) + \phi]$$

$$= \quad A\sin\left(\omega t - \frac{\omega}{v}x + \phi\right)$$

- If we limit ourselves to waves such that $y = 0$ when both $x = 0$ and $t = 0$, then $\phi = 0$ or π. Thus,

$$y(x,t) = A\sin(\omega t - kx) \text{ when } \phi = 0$$
$$y(x,t) = A\sin(kx - \omega t) \text{ when } \phi = \pi$$

  where

$$k = \frac{\omega}{v}$$

  and is called the *propagation constant.*

- For a wave travelling toward the left,

$$y(x,t) = A\sin(kx + \omega t)$$

- To pick a particular, fixed value of y, the argument of the sine function the above equation must be kept constant, that is,

$$kx + \omega t = \text{constant}$$

Thus, as t increases, x must decrease.

- Set t equal to an instantaneous value t_1 we have

$$y(x, t_1) = A \sin(kx - \theta_1)$$

where $\theta_1 = \omega t_1$ is a phase shift at t_1. See Fig. 11-5.

- From Fig. 11-5,

$$\lambda = x_2 - x_1$$

We identify x_1 and x_2 as two successive values of x for which the sine function equals $+1$; that is,

$$kx_1 - \theta_1 \quad = \quad \frac{\pi}{2} \text{ rad}$$

$$kx_2 - \theta_1 = \left(\frac{\pi}{2} + 2\pi\right) \text{ rad}$$

$$\begin{aligned} kx_2 - kx_1 &= 2\pi \\ x_2 - x_1 &= \frac{2\pi}{k} \\ \lambda &= \frac{2\pi}{k} \end{aligned}$$

- Set x equal to a constant value x_1, we have

$$y(x_1, t) = A\sin(\theta_1' - \omega t)$$

  where $\theta_1' = kx_1$ is a constant phase shift that depends on the point chosen in contrast to the previous analysis, which showed that the phase shift depended on the time of the snapshot.

- The above equation is similar to the one described the oscillatory motion of the body attached to a spring.

- With the position x fixed, y will vary with $\sin \omega t$ and

$$\nu = \frac{\omega}{2\pi}$$

- An alterative demonstration of the relation between frequency and wavelength:

$$\lambda\nu = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k}$$

By definition $k = \frac{\omega}{v}$, therefore

$$\lambda\nu = \frac{\omega}{k} = \frac{\omega}{\frac{\omega}{v}} = v$$

Example 11-1

A mass of 0.2 kg suspended from a spring of force constant 1000 N/m is attached to a long string as shown in Fig. 11-3. The mass is set into vertical oscillation, and the distance between successive crests of the waves in the string is measured to be 12 cm. What is the velocity of waves in the string?

Sol: The frequency of oscillation is

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m}} = \sqrt{1000 \text{ N/m} 0.2 \text{ kg}} = 70.71 \text{ rad/sec} \\ \nu &= \frac{\omega}{2\pi} = \frac{70.71 \text{ rad/sec}}{6.28} = 11.26 \text{ Hz} \end{aligned}$$

The velocity of waves in a medium is

$$\begin{aligned} v &= \lambda\nu \\ &= 12 \times 10^{-2} \text{ m} \times 11.26 \text{ sec}^{-1} = 1.35 \text{ m/sec} \end{aligned}$$

Energy Transfer of a Wave

- The wave has given energy to the particle because the wave carries energy with it.
- Wave motion is one of the two general mechanisms available to transport energy from one point to another. The other occurs when one or more particles move from one point to another and in so doing bring their kinetic energy with them.
- There are two differences between these two mechanisms:
  1. Waves transfer energy without transfer of matter, unlike the motion of particles.
  2. The energy of a beam of particles is localized. In a wave the energy is distributed over the entire space occupied at a given instant by the wave.
- We can obtain the power P of a wave by calculating the energy

crossing a given point in a string in 1 sec. See Fig. 11-6.

- $$P = (\text{energy per wavelength}) \times \nu$$
- Each particle in the string is oscillating with the same amplitude A. Because the total energy of an oscillating particle is proportional to the square of the amplitude oscillation, we conclude that all the particles in the vibrating string have the same energy.
- At any given time, the energy of a particular particle may be all kinetic or all potential or a mixture.
- To obtain the kinetic energy of the particles in the string we need an expression for the transverse velocity v_y.

$$v_y \quad = \quad \frac{\partial y}{\partial t} = \frac{\partial}{\partial t} \sin(kx - \omega t)$$

$$= \quad -\omega A \cos(kx - \omega t)$$

- We now calculate the energy of particle C of mass Δm.

$$E(\text{for particle C}) = \frac{1}{2}\Delta m \omega^2 A^2$$

- If the amplitude of the wave is small compared with the wavelength, then the mass in one wavelength is $\mu\lambda$, where μ is the mass per unit length of the string. Therefore,

$$\text{Energy per wavelength} = \frac{1}{2}\mu\lambda\omega^2 A^2$$

- Since $\lambda\nu = v$ and $\omega = 2\pi\nu$

$$P = \frac{1}{2}\mu\lambda\nu\omega^2 A^2 = 2\pi^2\mu v \nu^2 A^2$$

- The power transported by a wave is proportional to the square of the amplitude and to the velocity of propagation of the wave.

- When considering waves that propagate in three dimensions, such as sound waves or light waves, it is convenient to talk about the energy flowing through a given area of the medium traversed by the wave.
- The *intensity*, with symbol I, is defined as the power transmitted per unit area perpendicular to the direction of propagation of the wave.
- The intensity of the wave is also proportional to the square of the amplitude.

Homework: 11.4, 11.7, 11.9, 11.13, 11.14, 11.15, 11.16, 11.17, 11.20, 11.21.

Chapter Twelve: Interference of Wave

- In this chapter we will start with the behavior of two waves when they come together and the effect produced by their relative phase.
- We will first discuss this phenomenon with waves in water and then extend it to light waves.

The Superposition Principle

- The *superposition principle* is that waves can move through the same region of space independently and, as a result, when they meet the resultant wave is simply the algebraic sum of the individual waves. See Fig. 12-1.
- The superposition principle leads to a wave phenomenon known as *interference.*
- Suppose two waves with the same wavelength, velocity, and amplitude, but from difference sources, travel together in the same direction. See Fig. 12-2.
  1. If they are in phase, the total amplitude at any point will be the simple sum of the two, which is called *constructive interference.*
  2. If they are out of phase by one-half wavelength, the resulting

amplitude will be zero, which is called *destructive interference.*

- If either wave is shifted to the right or to the left by a whole wavelength, the situation is unchanged. However, if the shift is by only a half wavelength the situation is reversed.

Interference from Two Sources

- The phenomenon shown in Fig. 12-3 is called a *ripple tank*.
- Fig. 12-4 is a schematic representation of the photograph in Fig. 12-3 in which the wave crests, represented by the circular lines, proceed outward from the two sources S_1 and S_2.
- When two waves travel in the same medium, the difference in the distances travelled by them from their respective sources to a common points is called the path difference.
- When waves from two sources are emitted in phase, constructive interference occurs when the path difference is zero, or one wavelength, or an integral multiple of wavelengths $n\lambda$.
- Useful trigonometric relation:

$$\sin a + \sin b = 2 \sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$$

- Let us consider a point P whose distance from S_1 and S_2 are x_1 and x_2, respectively. The wave y_1 from S_1 and the wave y_2 from S_2 at P are

$$\begin{aligned} y_1 &= A\sin(kx_1 - \omega t) \\ y_2 &= A\sin(kx_2 - \omega t) \end{aligned}$$

From the superposition principle,

$$\begin{aligned} y &= y_1 + y_2 \\ &= A\left[\sin(kx_1 - \omega t) + \sin(kx_2 - \omega t)\right] \end{aligned}$$

Let

$$x_2 - x_1 = n\lambda, \text{ where } n = 0, \pm 1, \pm 2, \ldots$$

and

$$x_2 = x_1 + n\lambda$$

Since $\lambda = 2\pi/k$, we have

$$\begin{aligned} y &= A\left[\sin(kx_1 - \omega t) + \sin(kx_1 + 2\pi n - \omega t)\right] \\ &= 2A \sin \frac{1}{2}(kx_1 - \omega t + kx_1 - \omega t + 2\pi n) \cos \frac{1}{2}(2\pi n) \\ &= 2A \sin(kx_1 - \omega t) \end{aligned}$$

- For those points on the path where destructive interference occurs, their distances from S_1 and S_2 differ by $1/2\lambda$, or $x_2 - x_1 = \lambda/2 = \pi/k$. The resulting wave in this case is

$$\begin{aligned} y &= y_1 + y_2 \\ &= A\left[\sin(kx_1 - \omega t) + \sin(kx_1 + \pi - \omega t)\right] \\ &= 2A \sin \frac{1}{2}(kx_1 - \omega t + kx_1 - \omega t + \pi) \cos \frac{1}{2}\pi \end{aligned}$$

  But $\cos \pi/2 = 0$, and therefore $y = y_1 + y_2 = 0$.

- The same result is obtained if $x_2 - x_1 = \frac{3}{2}\lambda$ or $\frac{5}{2}\lambda$ and so on.

Double Slit Interference of Light

- If a series of either plane waves or large radius spherical waves strike a barrier with a small opening, circular waves are propagated beyond the opening as if the opening were a point source. The enlarging circumference of these waves is called a wave front. See Fig. 12-6. This phenomenon is called Huygens phenomenon.
- It is more generally stated that every point on a wave front can be considered as a source of secondary wavelets that spread out in all directions with a speed and wavelength equal to those of the propagating wave. See Fig. 12-7.
- Visible light has a wavelength that ranges from about 4×10^{-7} m to 7×10^{-7} m.
- A unit of length used in specifying the wavelength of light is the

Ångstrom, with symbol Å; $1\mathring{A} = 10^{-8}$ cm $= 10^{-10}$ m.

- Fig. 12-8 illustrates a schematic arrangement to determine the phenomenon of interference with light.
- Fig. 12-9 shows a geometric construct of lines drawn from each of the two slits to a point P of constructive interference.
- It will be seen later that the separation between the slits, d, can not be much greater than the wavelength.
- Since $D \gg d$, the two lines S_1P and S_2P are almost parallel.

$$\delta = d \sin\theta$$

- We have shown that constructive interference at point P can occur only if this path difference is an integral multiple of wavelength $n\lambda$. Thus, the condition for *constructive interference* becomes

$$d \sin\theta = n\lambda, \text{ where } n = 0, 1, 2, 3, \ldots$$

- Similarly, destructive interference will occur when the path difference is $1/2\lambda, 3/2\lambda, 5/2\lambda$, or in general $[(2n+1)/2]\lambda$, where $n = 0, 1, 2, 3 \ldots$. Thus, the condition for *destructive interference* becomes

$$d \sin\theta = \frac{2n+1}{2}\lambda, \text{ where } n = 0, 1, 2, 3, \ldots$$

- See Fig. 12-10. For a constant λ, the angular separation between successive interference maxima decreases with increasing spacing between the slits, d.
- If $d \gg \lambda$, a large number of interference maxima will occur within a small angle and, as a result, the pattern will be difficult to observe.

- Let $d = 1000\lambda$ and consider a small angle θ, for example $\theta = 1^\circ$. Since

$$1000\lambda \sin 1^\circ = n\lambda$$

we have

$$n = 1000 \sin 1^\circ = 1000 \times 0.017 = 17$$

- An opaque piece of glass with multiple slits is called a *diffraction grating*.

Single Slit Diffraction

- If the opening is greater than the wavelength but of a size comparable to a few wavelengths, then light waves passing through difference portions of a single slit will interfere with each other giving rise to a phenomenon known as *single slit diffraction.*
- Fig. 12-11 illustrates schematically the passage of individual wavelets through a single slit of width d.
- Assume that the size of the slit is much smaller than the distance from the slit to the screen. Therefore, the lines of propagation of the waves emanating from different points in the slit are approximately parallel.
- In Fig. 12-11b, wave A has a path difference of $\lambda/2$ from wave B and interferes destructively. This destructive interference effect occurs across the entire slit because any wave slightly below A

will interfere destructively with the wave at the same distance below B and so on. Therefore, a dark band will appear on the screen for angle θ_1, where

$$\sin\theta_1 = \frac{\lambda}{d}$$

- We can generalize this result by stating that the diffraction minima occur for angles satisfying the relation

$$\sin\theta = n\frac{\lambda}{d}, \text{ where } n = 1, 2, 3, \ldots$$

- If $d = \lambda$, then $\sin\theta_1 = 1$ and $\theta_1 = 90^\circ$, which means that the central maximum spreads over the entire screen. The same will be true if $d < \lambda$.
- An opening can be considered a point source when its size is equal to or smaller than the wavelength.

- If the separation between the slits, d, was much greater than the wavelength, the interference pattern would be difficult to observe. The same conclusion holds here if the size of the slit is much greater than λ. See Fig. 12-13.

Resolving Power

- Because of diffraction effects the accuracy of the determination of the precise location of a particle depends on the wavelength used to "look" at it; the smaller the wavelength employed, the greater the accuracy.
- The smaller the wavelength of the light used, the greater the momentum transferred to the particle being examined and, correspondingly, the greater the disturbance of its position in space. Thus, because of these conflicting effects there is a limit to which we may simultaneously know the location and momentum of a particle. This limit is known as the *Uncertainty Principle*.
- The determination of location depends on the wavelength, which is called the *resolving power*.
- See Fig. 12-14. If we know the distance of the sources from the

slit and the distance of the screen from the slit, we can calculate the angle $\Delta\theta$ and the distance Δx between the two sources from separation of the two central maxima.

- A rather arbitrary, although practical, criterion used to decide when the two sources S_1 and S_2 are just resolvable is the coincidence of the central maximum of one of them with the first minimum of the other (see Fig. 12-15). This is known as the *Rayleigh criterion* or the *limit of resolution*.
- The first minimum occurs when $\sin\theta_1 = \lambda/d$.
- For small angles, $\sin\theta_1 \approx \theta_1$ (in radians) and

$$\Delta\theta = \theta_1(\text{in radians}) = \text{limiting angle of resolution} \approx \frac{\lambda}{d}$$

X-ray Diffraction by Crystals: Bragg Scattering

- The smallest wavelengths that we can conveniently produce are those of X rays, whose wavelengths are about the sizes of atoms.
- See Fig. 12-16. It is clear from the figure that the path difference between the X rays scattered by atom A and atom B is $2l$ because the ray scattered from atom B must travel that extra distance to rejoin the ray scattered from atom A.
- From geometric considerations, $l = d \sin\theta$, and the path difference is $2l = 2d \sin\theta$.
- The radiation reflected by two adjacent layers of atoms will add constructively if

$$2d \sin\theta = n\lambda \text{ where } n = 1, 2, 3, \ldots$$

This is known as the *Bragg condition*.

- It is possible to have an angle θ', not necessary equal to the angle of incidence θ, for which the scattered waves recombine constructively. See Fig. 12-17.

Example 12-1

The crystal structure of silver bromide (AgBr) is represented in Fig. 12-18. The molecular weight and the density of AgBr are 187.80 g/mole and 6.47 g/cm^3, respectively. (a) Calculate the interatomic separation, d, of the atoms in AgBr. (b) If X rays of wavelength $\lambda = 1.50$ Åare incident on a AgBr crystal, at what angle will the first order ($n = 1$) diffraction maxima occur? Assume that the separation between the atomic planes producing the scattering is the interatomic spacing found in part (a).

Sol:

(a)

$$\text{Number of atoms in 1 cm}^3 = \frac{1\ \text{cm}^3}{d^3}$$

where d is expressed in cm.

The actual number of atoms can be found as follows:

$$\begin{aligned} N \text{ of atoms/cm}^3 &= N \text{ of moles/cm}^3 \times N \text{ of atoms/mole} \\ &= \frac{6.47 \text{ g/cm}^3}{187.80 \text{ g/mole}} \times 2 \times N_A \end{aligned}$$

Thus,

$$\begin{aligned} \frac{1}{d^3} &= \frac{6.47 \text{ g/cm}^3}{187.80 \text{ g/mole}} \times 2 \times 6.02 \times 10^{23} \text{molecules/mole} \\ &= 4.15 \times 10^{22} \text{ cm}^{-3} \end{aligned}$$

Therefore

$$d = 2.89 \times 10^{-8} \text{ cm} = 2.89 \mathring{A}$$

(b)

$$2d \sin\theta = \lambda$$

$$\begin{aligned}
\sin\theta &= \frac{\lambda}{2d} \\
&= \frac{1.50\mathring{A}}{2\times 2.89\mathring{A}} \\
&= 0.26 \\
\theta &= \sin^{-1} 0.26 = 15^\circ
\end{aligned}$$

Standing Waves

- Consider that the string is of finite length and the other end is clamped to a rigid support. When the wave disturbances reach fixed end, they will propagate in the opposite direction. See Fig 12-s1.
- The reflected waves will add to the incident waves according to the superposition principle and, under certain conditions, a standing waves pattern will be formed.
- If we assume that the incident waves y_1 travel toward the right in the positive x direction we have

$$y_1 = A \sin(kx - \omega t)$$

  The reflected waves y_2 will be travelling in the negative x

direction and we have

$$y_2 = A\sin(kx + \omega t)$$

The resulting waves pattern will be

$$\begin{aligned} y &= y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx + \omega t)] \\ &= 2A\sin kx \cos \omega t \end{aligned}$$

- Unlike the case of the travelling wave the amplitude of oscillation is not the same for all the points in the string.
- The points, called the *nodes*, are those for which $\sin kx = 0$.
- If the length of the string is l, then $y(x = l) = 0$. Thus,

$$0 = 2A\sin kl \cos \omega t$$

Because the above equation must be satisfied for all times t, we conclude that

$$\sin kl = 0$$

or

$$kl = \pi, 2\pi, 3\pi, \ldots, n\pi$$

where n is an integer.

- Since $k = 2\pi/\lambda$, we have

$$\frac{2\pi l}{\lambda} = n\pi$$

or

$$\lambda = \frac{2}{n} l$$

See Fig. 12-19.

Homework: 12.2, 12.4, 12.7, 12.8, 12.9, 12.11, 12.13, 12.15, 12.17, 12.18.