7

Additional Topics

INTRODUCTION

In this final chapter, we'll briefly review several other areas that may be covered on the GRE Math Subject Test. We'll look at some fundamental ideas of logic, set theory, combinatorics, graph theory, algorithms, probability and statistics, point-set topology, real analysis, complex variables, and numerical analysis.

LOGIC

In mathematics, logic refers to sentential calculus (also known as propositional calculus), which is a logical and well-defined system based on sentences. A **sentence** is a well-formed expression within a language and a syntax that can be evaluated as true or false using algebraic tools. The sentence "The ball is red," can be evaluated as either true or false, but the sentence "Red is the best color," cannot be.

The language of logic includes a set of variables and a set of **connectives** from which any sentence can be built according to the rules of syntax. On the GRE, capital letters, such as X or Y, are used to represent sentences. A list of connectives follows:

iff or $\leftrightarrow$	"if and only if"	
$\neg X$	the negation of X	
$X \wedge Y$	X and Y	
$X \vee Y$	X or Y	
$X \to Y$	If X, then Y	This conditional sentence is false only if X is true and Y is false.
$X \leftrightarrow Y$	X iff Y	This biconditional sentence says X is true iff Y is true. The two sentences, X and Y, are *logically equivalent*.

For sentences X, Y, and Z, the following theorems hold:

1. Double Negation: $\neg\neg X \leftrightarrow X$
2. Commutativity: $X \wedge Y \leftrightarrow Y \wedge X$ and $X \vee Y \leftrightarrow Y \vee X$
3. Associativity: $X \wedge (Y \wedge Z) \leftrightarrow (X \wedge Y) \wedge Z$ and $X \vee (Y \vee Z) \leftrightarrow (X \vee Y) \vee Z$
4. Distribution: $X \vee (Y \wedge Z) \leftrightarrow (X \vee Y) \wedge (X \vee Z)$ and $X \wedge (Y \vee Z) \leftrightarrow (X \wedge Y) \vee (X \wedge Z)$
5. de Morgan's Laws: $\neg(X \wedge Y) \leftrightarrow \neg X \vee \neg Y$ and $\neg(X \vee Y) \leftrightarrow \neg X \wedge \neg Y$
6. Contradiction: $X \to Y \leftrightarrow \neg Y \to \neg X$

Example 7.1 Prove the statement $X \to Y \leftrightarrow \neg Y \to \neg X$ using truth tables.

Solution: To prove this theorem, show that the truth table for $X \to Y$ is equivalent to that of $\neg Y \to \neg X$

X	Y	$X \to Y$	$\neg X$	$\neg Y$	$\neg Y \to \neg X$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Since all values of the truth table are the same for $X \to Y$ and $\neg Y \to \neg X$, they are logically equivalent.