TRIGONOMETRY

Although trigonometry began as the study of triangles, the usefulness of the trigonometric functions now extends far beyond this simple geometric form. We'll begin our study by reviewing the definitions of the trig functions, first with acute angles in right triangles and then with arbitrary angles and real numbers.

Trig Functions of Acute Angles

The classical definitions of the six trig functions—sine, cosine, tangent, cosecant, secant, and cotangent— use the lengths of the sides of a right triangle. In the figure below, triangle ABC is a right triangle with its right angle at C. The lengths of the legs BC and AC are denoted a and b, respectively, and the length of the hypotenuse AB is denoted c.

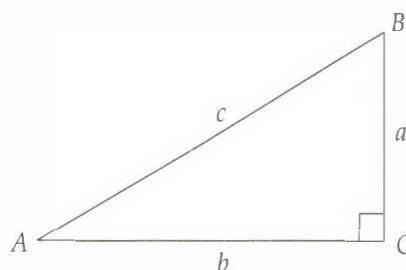$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c} \qquad \csc A = \frac{\text{hyp}}{\text{opp}} = \frac{c}{a}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c} \qquad \sec A = \frac{\text{hyp}}{\text{adj}} = \frac{c}{b}$$
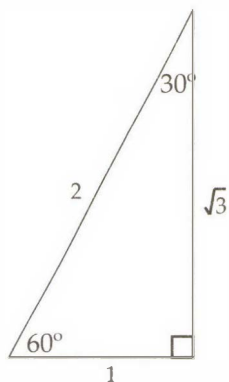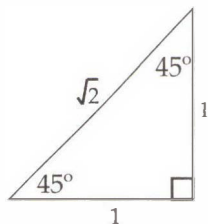
$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{a}{b} \qquad \cot A = \frac{\text{adj}}{\text{opp}} = \frac{b}{a}$$

Angles A and B are complementary (that is, the sum of their measures is 90°), and notice that $\sin A = \cos B$. That is, the sine of B's complement (namely, A) is $\cos B$, which is where the name "cosine" comes from (complement's sine). The same is true for the other pairs of cofunctions—tangent and cotangent, secant and cosecant. You'll also notice certain reciprocal relationships among the functions. Cosecant is the reciprocal of sine, secant is the reciprocal of cosine, and cotangent is the reciprocal of tangent. Finally, all of the trig functions can be written in terms of sine and cosine: The reciprocal relationships take care of secant and cosecant, and it's easy to see that tangent is the ratio of sine to cosine, and cotangent is the ratio of cosine to sine.

Two special right triangles allow us to determine the numerical values of the trig functions of the angles 30°, 45°, and 60°.

$$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}} \qquad \sin 30° = \cos 60° = \frac{1}{2} \qquad \sin 60° = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\csc 45° = \sec 45° = \sqrt{2} \qquad \csc 30° = \sec 60° = 2 \qquad \csc 60° = \sec 30° = \frac{2}{\sqrt{3}}$$

$$\tan 45° = \cot 45° = 1 \qquad \tan 30° = \cot 60° = \frac{1}{\sqrt{3}} \qquad \tan 60° = \cot 30° = \sqrt{3}$$