Gravity

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On Earth, the gravity at the planet's surface has a constant value of 10 meters per second squared in the downward direction. It can be considered to be a constant field over the surface of Earth with minor fluctuations due to elevation.

The actual value of Earth's surface gravitational field is the result of the mass of Earth and the distance of the surface from the center of the planet. Although Earth has varying surface elevations, these are actually relatively minor. So, it is acceptable to use the constant value of 10 meters per second squared for all calculations.

Gravity is an important concept and links together ideas from linear motion as well as circular motion. This chapter will review the following concepts:

- Understand Newton's law of universal gravitation and its inverse-square-law relationship.
- Visualize the gravity field surrounding masses, such as planets, and calculate its value at specific points in space.
- Calculate circular orbits governed by Newton's law of universal gravitation.
- Understand how Kepler's laws describe orbital motion.

Table 9.1 lists the variables that will be studied in this chapter.

Table 9.1 Variables Used with Gravity

New Variables	Units
F_g = Force of gravity	N (newtons)
G = universal gravitational constant	$m^3/kg \cdot s^2$ (meters cubed per kilogram seconds squared) or $\frac{N \cdot m^2}{kg^2}$ (newtons • meters squared per kilograms squared)

IF YOU SEE two planets separated by a distance

Newton's law of universal gravitation

$F_g = G\frac{mM}{r^2}$

The force acting on both planets is opposite and equal (Newton's third law).

UNIVERSAL GRAVITY

Isaac Newton determined that the **force of gravitational attraction**, F_g, between two masses was directly proportional to the product of their masses and inversely proportional to the square of the distance between them. **Newton's universal law of gravitation** can be expressed as an equation.

$$F_g = G\frac{m_1 m_2}{r^2}$$

The letter G represents the universal gravitational constant, $G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg} \bullet \text{s}^2$.

Inverse-Square Law

As the distance between two masses increases, the force of gravity between them decreases by the square of that distance. This means that a doubling of distance would result in a quartering of the gravitational force between the masses.

EXAMPLE 9.1

Calculating the Change in the Force of Gravity

Calculate the resulting force of gravitational attraction between two masses if one of the masses was to double and the distance between them were to triple.

WHAT'S THE TRICK?

The original force of attraction can be calculated using Newton's law of gravity.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Substituting $2m_1$ for m_1 and $(3r)^2$ for r^2 represents the doubling of the mass and the tripling of the distance, respectively.

$$\left(\frac{2}{3^2}\right)F_g = G\frac{(2m_1)m_2}{(3r)^2}$$

Note that the quantity $3r$ is squared.

$$\left(\frac{2}{9}\right)F_g = \left(\frac{2}{9}\right)G\frac{m_1 m_2}{r^2}$$

The force of gravity between the masses will be two-ninths of its original value.

GRAVITATIONAL FIELD

The influence, or alteration, of space surrounding a mass is known as a **gravitational field**. The gravitational field of Earth can be visually represented as several vector arrows pointing toward the surface of Earth, labeled g, as shown in Figure 9.1. The surface gravity field for Earth, g, is 10 meters per second squared. When an object of mass m is placed in Earth's gravity field, it experiences a force of gravity, F_g, in the same direction as the gravity field.

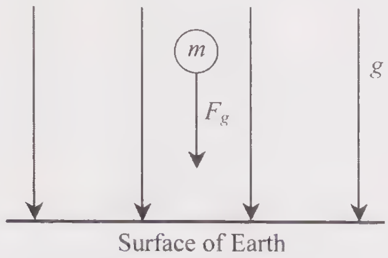

Figure 9.1. The gravitational field of Earth

The effect on mass m in the gravity field can be solved as a force problem.

$$\Sigma F = F_g$$
$$ma = mg$$
$$a = g$$

It now becomes apparent that the acceleration of a mass in a gravity field equals the magnitude of the gravity field. Although there is a conceptual difference between the gravity field and the acceleration of gravity, they both have the same magnitude and direction.

Finding Surface Gravity

The gravity field of any planet is a function of the mass of the planet and the distance of the planet's surface from its center. The exact relationship can be derived using two formulas for the force of gravity. When a mass m rests on the surface of Earth, the force of gravity can be determined using either the weight formula or Newton's law of gravitation.

$$F_g = mg \qquad \text{or} \qquad F_g = G\frac{mM_{\text{Earth}}}{r_{\text{Earth}}^2}$$

When these formulas are set equal to one another, mass m cancels. This results in a formula that describes both the strength of the gravity field and the acceleration due to gravity. Note that the mass of Earth is 5.98×10^{24} kg and that the radius is 6.37×10^6 m.

$$mg = G\frac{mM_{\text{Earth}}}{r_{\text{Earth}}^2}$$

$$g = G\frac{M_{\text{Earth}}}{r_{\text{Earth}}^2}$$

$$g = (6.67 \times 10^{-11}\ \text{N} \bullet \text{m}^2/\text{kg}^2)\ (5.98 \times 10^{24}\ \text{kg})/(6.37 \times 10^6\ \text{m})^2$$
$$g = 9.8\ \text{m/s}^2$$

IF YOU SEE a question regarding the gravity of a planet

$g = G\frac{M}{r^2}$

Remember that r is the distance from the center of the planet. To find the gravity at the surface of a planet, r is the radius of the planet.

The gravity field near the surface of Earth is considered uniform. It has the same value at all points close to the surface of Earth. The magnitude of mass m placed on the surface of Earth does not matter since it cancels. Elephants and feathers are both in the same gravitational field and experience the same acceleration of 9.8 m/s^2. For the SAT Subject Test in Physics, the value for g will be rounded to 10 m/s^2 in order to make calculations easier.

Although the formula for g was derived on the surface of Earth, it can be generalized to solve for gravity on any planet or at a point in space near any planet.

$$g = G\frac{M}{r^2}$$

In its generalized form, M is the mass of the planet and r is the distance measured from the center of the planet to the location where gravity, g, is to be calculated.

EXAMPLE 9.2

Calculating the Surface Gravity of the Moon

Calculate the surface gravity of the Moon. The Moon has a mass of 7.36×10^{22} kilograms and a radius of 1.74×10^6 meters.

WHAT'S THE TRICK?

Use the formula for finding the gravity of a planet and substitute in the values for the Moon.

$$g_{\text{Moon}} = G\frac{M_{\text{Moon}}}{r_{\text{Moon}}^2}$$

$$g_{\text{Moon}} = 1.62 \text{ m/s}^2$$

This value is roughly one-sixth of the value of the surface gravity of Earth. The SAT Subject Test in Physics does not allow you to use a calculator, and it is unlikely to present a problem requiring calculations that will be this involved. This example serves to demonstrate the universality of the equation for finding the gravitational field and the acceleration of gravity for any celestial object.

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