Assignment for Module 6

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Problem formulation

Let x_{ij} be the number of AEDs produced at Plant i ($i = 1, 2, 3$ for Plant A, B, and C), and shipped to Warehouse j ($j = 1, 2, 3$).

The objective function is then to total cost, which contains the shipping cost and production cost, and should be minimized:

$$\begin{aligned} Z = {} & (20+400)x_{11} + (14+400)x_{12} + (25+400)x_{13} + \\ & (12+300)x_{21} + (15+300)x_{22} + (14+300)x_{23} + \\ & (10+500)x_{31} + (12+500)x_{32} + (15+500)x_{33} \\ = {} & 420x_{11} + 414x_{12} + 425x_{13} + 312x_{21} + 315x_{22} + 314x_{23} + 510x_{31} + 512x_{32} + 515x_{33} \end{aligned}$$

Now, the constraints of the problem include the following points:

1. The number of products shipped to each warehouse should meet the monthly demand:

$$x_{11} + x_{21} + x_{31} \geq 80 x_{12} + x_{22} + x_{32} \geq 90 x_{13} + x_{23} + x_{33} \geq 70$$

2. The number of products produced at each plant should not exceed its monthly production capacity:

$$x_{11} + x_{12} + x_{13} \leq 100 x_{21} + x_{22} + x_{23} \leq 125 x_{31} + x_{32} + x_{33} \leq 150$$

3. The number of products produced at each plant must not be negative:

$$x_{ij} \geq 0, \quad i = 1, 2, 3, \quad j = 1, 2, 3$$

Therefore, the entire problem can be formulated as:

$$\begin{aligned} \min\ Z = {} & 420x_{11} + 414x_{12} + 425x_{13} + 312x_{21} + 315x_{22} + 314x_{23} + 510x_{31} + 512x_{32} + 515x_{33} \\ & \text{s. t.} \\ & x_{11} + x_{21} + x_{31} \geq 80 \\ & x_{12} + x_{22} + x_{32} \geq 90 \\ & x_{13} + x_{23} + x_{33} \geq 70 \\ & -x_{11} - x_{12} - x_{13} \geq -100 \\ & -x_{21} - x_{22} - x_{23} \geq -125 \\ & -x_{31} - x_{32} - x_{33} \geq -150 \\ & x_{ij} \geq 0, \quad i = 1, 2, 3, \quad j = 1, 2, 3 \end{aligned}$$

Solving the problem

The problem can be folumated in R as:

```
library(lpSolve)

# Assume the parameters are arranged as:
# x11, x12, x13, x21, x22, x23, x31, x32, x33

# Objective function
obj.fun <- c(420, 414, 425, 312, 315, 314, 510, 512, 515)

# Constraints
constr <- matrix(c(1, 0, 0, 1, 0, 0, 1, 0, 0,
                   0, 1, 0, 0, 1, 0, 0, 1, 0,
                   0, 0, 1, 0, 0, 1, 0, 0, 1,
                   -1, -1, -1, 0, 0, 0, 0, 0, 0,
                   0, 0, 0, -1, -1, -1, 0, 0, 0,
                   0, 0, 0, 0, 0, 0, -1, -1, -1),
                 ncol = 9, byrow = TRUE)
constr.dir <- c('>=', '>=', '>=', '>=', '>=', '>=')
constr.rhs <- c(80, 90, 70, -100, -125, -150)

# Solve LP
prod.sol <- lp('min', obj.fun, constr, constr.dir, constr.rhs,
               compute.sens = TRUE)
```

Since `lpSolve` assumes that every variable is greater than or equal to zero, the third constraint in the previous section does not have to be specified here.

The solution of the LP is:

```
# LP solution
prod.sol$objval
```

```
## [1] 88250
```

```
prod.sol$solution
```

```
## [1] 10 90  0 55  0 70 15  0  0
```

Therefore, the optimal distribution of the production is:

Plant	Warehouse 1	Warehouse 2	Warehouse 3
1	10	90	0
2	55	0	70
3	15	0	0

and the minimal total cost is \$88,250.

Dual problem formulation

The dual problem can be formulated by using the number of constraints in the promal problem as the number of variables, use the RHS of the primal constraints as the coefficients in the objective function, use the coefficients in the primal objective function as the RHS of dual constraints, and reverse the sign in the constraints, as:

$$
\begin{aligned}
\max Z = 80y_1 + 90y_2 + 70y_3 - 100y_4 - 125y_5 - 150y_6 \\
\text{s. t.} \\
y_1 - y_4 \leq 420 \\
y_2 - y_4 \leq 414 \\
y_3 - y_4 \leq 425 \\
y_1 - y_5 \leq 312 \\
y_2 - y_5 \leq 315 \\
y_3 - y_5 \leq 314 \\
y_1 - y_6 \leq 510 \\
y_2 - y_6 \leq 512 \\
y_3 - y_6 \leq 515
\end{aligned}
$$

Economic interpretation of the dual

In this scenario, the primal problem solves for the optimal distribution of production allocation among different plants, and the shipping arrangements towards different warehouses, so it has 9 variables, corresponding to the number of products to produce at each plant and to deliver to each warehouse.

For the dual problem, there're 6 decision variables, corresponding to the unit price for each resource (constraint in primal problem): the production capacity at each plant, and the monthly demands from each warehouse. Therefore, the dual problem is trying to maximize the total price of the limited resources.

In detail, in the dual constraints, the variables y_1, y_2, y_3 refer to the cost per unit for products from different warehouses, while the variables y_4, y_5, y_6 combined with the RHS of the constraints stand for the revenue for products from 3 plants. Therefore, the constraints mean that the cost must be lower than or equal to the cost. At the optimal condition, at least one of the relationships should be equal on both sides.